Limits to Linear Price Behavior: Futures Prices Regulated by Limits

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This article analyzes the behavior of futures prices when the exchange is regulated by price limits. With a model analogous to exchange-rate target-zone models, we tested for the existence of a nonlinear S-shape relation between observed and theoretical futures prices. This phenomenon reflects the adjustments in traders' expectations even when limits are not actually hit. The approach is illustrated for five agricultural futures contracts traded at the Chicago Board of Trade. There is some evidence of nonlinearity in quiet periods. In cases of fundamental realignments, that is, volatile periods, this nonlinearity disappears. © 2001 John Wiley & Sons, Inc. Jrl Fut Mark 21:463–488, 2001

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INTRODUCTION

Price-Limit Regulation and Trading Behavior

Defaulting traders, market crashes, and asymmetric information problems provoke criticism of the integrity of derivatives exchanges. As a result, calls for further regulation arise with almost every large realignment of market values. Regulation thus clearly responds to the trading behavior of market participants. Whether or not trading behavior is also affected by regulation has so far largely been neglected.

When traders are confronted with market impediments, they revise their expectations accordingly. This affects their order flow and, hence, the volatility of prices. These dynamic adjustments are ignored by evaluations of the impact of market regulations on the price discovery process that consider the trading process exogenous. If trades are diverted to other outlets (e.g., nonregulated options or the cash market), this may lead to increased volatility and a reduction in order flow. Thus, the integrity of the exchange can even be further endangered because of the trading impediments invoked by regulation.

This article focuses on price-limit regulation and investigates how it affects the trading behavior of market participants. Technical (i.e., statistical) consequences of price limits have been investigated in a number of other articles. For example, Roll (1984) showed that the use of possibly stale prices implied by limit moves will inevitably affect any informational efficiency study. Two studies that carefully account for the possibility of limit moves were carried out by Kodres (1988, 1993), who analyzed the impact of price limits on tests of the unbiasedness hypothesis in foreign exchange futures markets. These studies restrict the impact of limits to actual limit move occurrences and thereby ignore any intralimit dynamics. There are also a number of studies more directly focusing on the impact of price limits, particularly on measures of intraday volatility. Most of them perform this analysis in an event-study context. Ma, Rao, and Sears (1989) and Greenwald and Stein (1991) claimed that trading halts mitigate price risk and enhance informational efficiency. Others, however, have argued that limits obstruct informational efficiency and in the process tend to inflate volatility excessively (e.g., Lee, Ready, & Seguin, 1994; McMillan, 1991; Subrahmanyam, 1994, 1995). So far, there has been no attempt to encompass the two conflicting models. This article proposes a modeling framework able to discriminate between these conflicting findings.

Trading-Behavior Models

Underlying this study's model is the notion that ex ante trading decisions incorporate the existence of price limits. For example, market makers may try to avoid or at least postpone the price hitting a boundary because that would imply interrupted trading and, therefore, a loss of earnings or profits. Kuserk and Locke (1996) investigated the impact of price limits on market-making profitability and concluded that, on average, market makers profit from trading halts. This finding contrasts with Roll's (1984) remark that limits do create informational inefficiencies but not a profit opportunity. Kuserk and Locke focused on market-maker behavior immediately preceding and following limit moves, thus ignoring potential impacts further away from those limits. Informed traders will similarly try to smooth their planned trading volume before the price hits a limit, after which the risk of a dissipation of their private information increases. Market makers protect themselves against these actions by increasing the bid–ask spread upward when the market moves toward a limit. This may exaggerate volatility and lead to the inference that price limits cause increased volatility, an argument used by gravitation proponents. Gravitationists argue that limits are self-exciting because they induce a strong drift in prices. Traders allegedly increase this drift by unwinding positions whenever the price gets close to the limits. Berkman and Steenbeek (1998) found no evidence in the Nikkei futures market to support the gravitation effect. Empirical work in this area is rather limited; some examples are McMillan (1991) and Kuserk and Locke. Some of these articles deal with circuit breakers, but their findings apply identically to price limits.

Theoretical microstructural analyses of these (opposing) dynamic trading adjustments were given by Greenwald and Stein (1991), Kodres and O'Brien (1994), and Subrahmanyam (1994, 1995). Although the first two articles tend to support trade suspension rules as (second best) optimal, the latter clearly rejects these rules on the basis of informational inefficiency. The theoretical specification has an impact on the subsequent results.

Influence of Limits on Price Expectations

This article addresses the issue of how price expectations are influenced by bounds on asset prices. In the absence of trade disruptions such as price limits and circuit breakers, observed futures prices should be equal to the unobserved fundamental futures price (as determined by the standard cost of carry pricing model) plus some random noise to reflect market microstructure effects such as bid–ask bounce. However, if there is a fixed and credible upper limit and the price is close to that limit, the probability of a further increase will be limited, whereas the probability of a decrease will be relatively larger. Thus, the probability distribution of the next price move will become increasingly skewed the closer it gets to the limit. This phenomenon implies that the existence of credible price limits will result in a nonlinear relationship between the futures price actually observed and the unobserved fundamental futures price, with the linear relationship replaced by a nonlinear S-shape functional form. The relationship will be convex for prices negatively drifting toward the lower limit and concave for prices positively drifting toward the upper limit. Thus, the effect of limits will theoretically be to stabilize prices. Shocks (demand–supply driven) in the fundamental value will have a less than proportional impact on observed prices. The statistical specification in this article also allows for nonlinearity of a different (opposite) type. Instead of a smoothing impact, price limits may have an exacerbating impact, so that shocks in the fundamental value have a more than proportional impact on the observed price, which gravitates to either the upper or lower price limit. Thus, this study nests the two competing models of stability (prices revert to the center of the price band) versus gravitation (prices gravitate to the limits of the price band).

Procedure of the Article

We investigated the existence of nonlinear relationships in futures markets that are regulated by price limits. The issue addressed here is whether the existence of price limits has an impact far away from these limits. This, in particular, distinguishes this article from previous price-limit literature. As suggested by Strongin (1995, p. 205), work on the impact of price limits away from limit moves themselves is urgently needed, and this issue is addressed.

The following hypothesis is postulated: There is a linear relationship between theoretical futures prices and observed futures prices that are regulated by limits. Rejection of this hypothesis would require analysis of the shape of the nonlinearity. If the familiar S shape in the expectation of bounded asset prices is found, it can be concluded that price stabilization occurs. If, however, an inverse S shape is observed, it can be concluded that gravitation occurs. The underlying theory and its relevance for this setting are discussed in the next section.

After developing the theoretical nonlinear model of price expectations, we apply it empirically to agricultural futures contracts traded at the Chicago Board of Trade. Daily limits apply to all of these contracts and are allegedly effective in the sense that they alter trading behavior. The sample period, which extends from January 1988 through December 1988, was chosen because of the frequency of limit moves (when the market closes up or down limit) and limit hits (when the price hits the limit but bounces back before market close) in the months of June and July, 1988. Even though the focus is on a somewhat historical sample, the issue is still very much alive today. Most U.S. commodity exchanges (and, in fact, most international commodity exchanges) still use price limits to control daily derivative price changes. The Chicago Board of Trade has expanded its futures price limits on a number of occasions since 1988, but their existence has remained effective in halting trade on a number of occasions throughout the past decade.

ABSORBING LIMITS: A TARGET-ZONE MODEL

This article considers the existence of a nonlinear relationship between fundamental (theoretical) futures prices and observed futures prices that are regulated by price limits.

We adapted a procedure recently developed in the exchange-rate target-zone literature. However, there is an important difference in that observed futures prices are regulated directly, whereas exchange rates are typically regulated indirectly by monetary authorities influencing the fundamentals (i.e., the supply and demand of foreign exchange). In addition, unlike the regulations for intramarginal adjustments prevailing with exchange-rate target zones, futures price-limit regulation operates only at the limits. However, the main distinction with the target-zone literature is the fact that futures price limits are valid for 1 day only.[1] Except when the closing price coincides exactly with the opening price on a particular trading day, the next trading day will have a different target zone. The limits are not credible on an interday basis but are perfectly credible intraday. For this reason, the focus of this study is on intraday trading behavior.

The observed intraday futures price f_{t+j} is prevented from taking values outside of the intraday price limits. The observed price change is constrained so that

[1]What we mean is that the starting point for measuring the price changes is valid for 1 day only, not the existence or size of the target zone. The size of the target zone may vary but only after successive limit-hit days (to 150% of the original point change). After an expansion of the zone, it will revert back to its original size.

$$\underline{f} - f_{t+j-1} \leq f_{t+j} - f_{t+j-1} \leq \bar{f} - f_{t+j-1} \tag{1}$$

where are $\bar{f}$ and $\underline{f}$ are the upper and lower limits, respectively, on the N intraday futures prices for any particular day symmetrically specified around f_t, the previous day's closing price, which defines the central price of the price band. The jth intraday innovation in the futures price ($f_{t+j} - f_{t+j-1}$) is conditionally distributed according to a unimodal distribution function, $\Phi(\bullet)$, with mean zero and conditional variance σ^2_{t+j}.

C. Rose (1995) generalized this conditional distribution problem in the context of doubly censored distributions. For credible reflecting barriers (e.g., futures price limits on an intraday basis), the probability of f_{t+j} exceeding the barriers can be piled up at the limit, it can be evenly distributed over the target zone, or it can even be folded inward. Thus, Φ can hardly be expected to display normality. In fact, although it is likely to be unimodal, it will typically have excessive tail probability mass. Intuitively appealing estimation methods include the Tobit model by Yang and Brorsen (1995) and the limited-dependent rational expectation approach advocated by Pesaran and Samiei (1992) and Pesaran and Ruge-Murcia (1996); these model time-varying volatility as an increasing function of the distance from the target price (the exchange-rate parity). This is the key notion exploited in this article, albeit with a different functional specification. In a commentary on McMillan (1991), Margulis (1991, p. 2) wrote the following:

> [The existence of price limits] starts affecting trading decisions at some indeterminate level [well within the price limits] and it becomes stronger and stronger as the futures price approaches the limit price.

If trading decisions are reflected in prices, a relationship will be observed between those prices and the distance from the boundaries on those prices (or alternatively, the distance from the central price within the price band). According to the target-zone literature (and following from Equation 1), this relationship is nonlinear because of the distortion in Φ. Typically, the nonlinear specifications following from the target-zone models are estimated in continuous time, but it is notoriously difficult to capture particular empirical characteristics such as fat tailedness and time-varying volatility in those models, and the assumption of normality is frequently violated. In discrete time, a simple analytical solution for this target-zone problem was suggested by Koedijk, Stork, and de Vries (1998), who permitted modification of Φ in line with the empirical characteristics.

Define the regression equation

$$f_{t+j} - f_t = E[f_{t+j} - f_t | f_{t+j-1}] + \varepsilon_{t+j} \tag{2}$$

where ε_{t+j} is orthogonal to the conditional expectation term.

If the intraday futures price is unconstrained and follows a random walk,

$$f_{t+j} = f_{t+j-1} + \varepsilon_{t+j} \tag{3}$$

However, from Koedijk et al. (1988), we know that for bounded asset prices,

$$f_{t+j} - f_{t+j-1} = g(f_{t+j-1}) + \varepsilon_{t+j} \tag{4}$$

or

$$f_{t+j} = f_{t+j-1} + g(f_{t+j-1}) + \varepsilon_{t+j} \tag{5}$$

where $g(.)$ is convex-o-concave asymmetric about f_t, the center of the price band and the first three derivatives will satisfy

$$\begin{aligned} &0 < g'(f) < 1 \\ &g''(f) \begin{cases} <0 \text{ if } f > f_t \\ =0 \text{ if } f = f_t \\ >0 \text{ if } f < f_t \end{cases} \\ &g'''(f) < 0 \end{aligned} \tag{6}$$

Also, a functional form that captures the necessary requirements for an S shape can be specified. One approach suggested in the literature is to use a Laguerre function (e.g., Miller & Weller, 1991), or one can take a Taylor series expansion around f_t (see Koedijk et al., 1998; A. K. Rose & Svensson, 1995) to obtain

$$\begin{aligned} f_{t+j} = f_t + g' \times (f_{t+j-1} - f_t) + \sum_{i=1}^{2} g'' \times I_i(.) \\ \times (f_{t+j-1} - f_t)^2 + g''' \times (f_{t+j-1} - f_t)^3 + e_{t+j} \end{aligned} \tag{7}$$

where the residual e_{t+j} consists of ε_{t+j} and any omitted higher order Taylor series expansion terms. The indicator variables $I_i(.)$ separate the positive ($i = 1$) from the negative ($i = 2$) deviations from f_t, the center of the price band. Reparameterizing Equation 7 and allowing different quadratic impacts for positive and negative deviations from the center of the price band give

$$f_{t+j} = f_t + \lambda_1 \times (f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2i} \times I_i(.) \times (f_{t+j-1} - f_t)^2 + \delta_3 \times (f_{t+j-1} - f_t)^3 + e_{t+j} \quad (8)$$

Expressing this equation in terms of the futures price changes gives

$$f_{t+j} - f_{t+j-1} = \delta_1 \times (f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2i} \times I_i(.) \times (f_{t+j-1} - f_t)^2 + \delta_3 \times (f_{t+j-1} - f_t)^3 + e_{t+j} \quad (9)$$

where δ_1 is equal to $\lambda_1 - 1$. The fact that the error term contains omitted higher order terms unfortunately implies that e_{t+j} is not necessarily orthogonal to the futures price change. This is a potential estimation problem to be addressed later.

Testing the Hypothesis

Having estimated Equation 9, we can test the hypothesis. To establish that a nonlinear relationship exists, that is, whether the limits have an impact on the observed futures prices, we test the hypothesis that all of the parameters δ_1, δ_{21}, δ_{22}, and δ_3 are zero. In the absence of market imperfections such as bid–ask spreads and infrequent trading, accepting this null implies that the random walk model for futures prices will prevail. If it is possible to reject a linear relationship, the study can proceed by investigating the shape of the nonlinearity. It can be considered that the following parameter signs imply the theoretical S shape. For a price-stabilizing model, δ_1 needs to be negative, δ_{21} (for positive deviations from f_t) needs to be negative, δ_{22} (for negative deviations from f_t) needs to be positive, and δ_3 needs to be negative. These conditions follow directly from Equation 6.

Bleaney and Mizen (1996) modeled a target zone for exchange rates by specifying two models, a linear model versus a cubic nonlinear model, both models being nested in Equation 9, and concluded that the cubic model clearly outperforms the linear model. However, by ignoring the quadratic terms, they may have a misspecified model, and because of distortion in the empirically desirable orthogonality of the error term in Equation 9, this may have a pronounced effect on the outcome. This study does not restrict the regression model but instead takes a pragmatic approach. After estimating Equation 9, we investigate the function and conclude from its shape whether or not the price-stabilizing hypothesis is to be rejected.

In general, the aforementioned signs of parameters capture the price-stabilizing behavior of the futures price and any nonlinearity involved in this behavior. The first-order term indicates whether the futures price reverts to the central price f_t (negative sign) or gravitates toward the limits (positive sign). The second-order and third-order terms indicate the speed at which this occurs, whereas the second-order terms also allow for asymmetric adjustments depending on the position (above or below f_t) of the futures price.

Next, the error term in Equation 9 needs to be addressed. Because time-varying conditional variance and fat tails most often characterize futures price changes, the innovations are modeled according to these two characteristics. Numerous authors have investigated the apparent nonnormality of futures price changes. Typically, a fat-tailed alternative has been proposed, with the evidence pointing toward the class of Student t distributions (Kofman, 1994). Allowance is made here for fatter than normal distributions under the assumption that the innovations in Equation 9 follow a t distribution:

$$e_{t+j} \sim t_v(0,\ h_{t+j}) \tag{10}$$

where e_t is assumed to follow a symmetric Student t distribution with mean zero, variance h, and degrees of freedom ν ($\nu > 2$). The ν parameter measures the extent of fat tailedness: the smaller ν is, the larger the probability mass in the tails is. For $\nu = \infty$, the t distribution converges to a normal distribution. For $\nu = 2$, the t distribution no longer has finite variance.

Note that the normal distribution based on observed excess kurtosis cannot be rejected a priori. It is possible that the fat tails are driven by time-varying conditional variance. In that case, the standardized price changes may well be normal. Thus, the conditional variance in Equation 10 is allowed to have the following well-known Generalized Auto-Regressive Conditional Heteroskedasticity [GARCH(1,1)] specification:

$$h_{t+j} = \beta_0 + \beta_1 e^2_{t+j-1} + \beta_2 h_{t+j-1} \tag{11}$$

PRICE LIMITS IN AGRICULTURAL FUTURES CONTRACTS

Having introduced the approach, we now apply the methodology to a set of agricultural futures contracts that are regulated by price limits.

The data consist of five agricultural commodities traded at the Chicago Board of Trade for 227 trading days in 1988. Contract specifications and limit regulations for these contracts are given in Appendix B. The sample period is characterized by an unusual frequency of limit moves, in particular in the month of June. The data are based on the nearby futures contracts, which roll over 2 business days prior to the delivery month. This coincides with the date the price limits are lifted on the nearby contract. A 5-min sampling interval has been chosen on the basis of a tradeoff between minimizing the number of no-trade intervals and using as much information as possible. To some extent, this choice alleviates the noise-to-signal problem endemic in using high-frequency data. If no price was recorded during this interval, the last previously recorded price was taken.

Price changes are constructed from the futures prices: $R_{t+j} = f_{t+j} - f_{t+j-1}$. The overnight price changes are omitted. The first price change of each day is computed as the difference between the last recorded price during the first 5 min of trading and the first observed price during that interval. The rollover price change is omitted, and zero price changes are imputed for empty intervals to obtain up to 9,900 observations per commodity for the time period January 5 through November 28, 1988.

The descriptive statistics for the five contracts' 5-min price changes are given in Panel A of Table I. The number of limit-move days is given in the last row. It is obvious that wheat forms an exception among these contracts because its limits have been hit on only 9 days, whereas there are between 25 and 32 limit moves for other contracts. The trading days on which the limits have been hit are eliminated because trading in limit-up (limit-down) contracts is usually interrupted for prolonged periods of time during a trading session. This implies an excessive number of non-informative zero price changes.

The information in Table I is rather homogeneous across commodities and can easily be summarized. There is clear evidence of skewness and excess kurtosis. It is, therefore, no surprise that the Jarque–Bera tests for normality are overwhelmingly rejected. The significance of the Ljung–Box tests for serial correlation in the price changes and the reported first-order serial correlation coefficient (ρ) reflect the impact of the bid–ask bounce, the discreteness of the data, or the infrequent trading effect. From a further investigation of this phenomenon, it seems that the serial correlation can be characterized as a first-order autocorrelation.

The Ljung–Box test statistic on the squared residuals and the Autoregressive Conditional Heteroskedasticity (ARCH) test for heteroskedasticity illustrate the time-varying conditionality in the variance of the price changes. The ARCH test is based on an autoregression of the squared

TABLE I

Descriptive Statistics

	Corn	*Soybeans*	*Soybean Meal*	*Soybean Oil*	*Wheat*
			Panel A: Raw Price Changes, R_{t+j}		
M	−0.00	0.02	0.01	−0.00	−0.00
SD	0.58	1.69	0.59	0.06	0.88
Skewness	−0.87*	−0.18*	0.52*	−1.17*	−0.57*
Kurtosis	31.37*	16.49*	28.91*	26.24*	24.64*
Jarque–Bera test	296890.6*	67299.8*	250881.7*	208681.6*	193741.5*
Autocorrelation ρ	−0.073*	−0.004	−0.044*	−0.053*	−0.025*
Ljung–Box (12): levels	165.63*	102.61*	80.86*	112.66*	58.44*
Ljung–Box (12): squares	3404.46*	3712.17*	1333.98*	733.26*	2182.79*
ARCH test	457.99*	729.94*	754.93*	230.48*	908.48*
Number of observations	8820	8865	8955	9180	9900
Limit move days	31	32	30	25	9
			Panel B: Adjusted Price Changes, U_{t+j}		
M	−0.00	0.03	0.03	0.01	0.01
SD	1.37	1.30	1.32	1.34	1.36
Skewness	−0.03	−0.07*	−0.00	−0.00	−0.02
Kurtosis	3.69*	3.59*	3.68*	3.84*	4.12*
Jarque–Bera test	176.3*	135.16*	172.14*	267.34*	516.2*
Autocorrelation ρ	−0.143*	−0.052*	−0.045*	−0.058*	−0.056*
Ljung–Box (12): levels	186.45*	72.42*	82.10*	94.30*	51.41*
Ljung–Box (12): squares	36.33*	52.89*	136.03*	74.08*	70.51*
ARCH test	15.97*	38.28*	77.79*	46.64*	48.70*

Note: This table reports a range of descriptive statistics for the futures price changes of five commodity futures contracts traded at the Chicago Board of Trade. The nearest delivery contract has been chosen, and rollover to the next-to-nearest contract occurs 2 trading days prior to the delivery month. The last line indicates the number of 5-min sampling intervals for 1988 for each of these contracts (45 observations per trading day). Statistics given in this table comprise the first four moments and a set of tests for normality, serial correlation, and heteroskedasticity. * indicates a significant rejection of the test at the 1% level. The Jarque–Bera test is a normality test based on skewness and kurtosis measures. Autocorrelation ρ gives the first-order autocorrelation coefficient estimate. The Ljung–Box test is a test for up to 12th order serial correlation in the levels and squares of price changes. The ARCH test is a Lagrange multiplier test for conditional heteroskedasticity in the futures price changes. Limit move days indicates the number of days on which the futures price closes at the limit (limit up or down) for each of the contracts individually.

price changes on lagged squared price changes, whereas the Ljung–Box test is a serial correlation test on the squares of the price changes.

A well-known characteristic of high-frequency prices is the diurnal pattern in intraday volatility (Andersen & Bollerslev, 1998). When intraday volatility is defined as the absolute price change over the 5-min sampling interval, a similar U-shape pattern is found in the series. This diurnal pattern implies large price changes during the opening of the trading session that taper off during the middle of the trading session before reverting to large price changes at the close of the trading session.

This pattern may bias the parameter estimates. In Appendix A, it is explained how the series is adjusted to avoid that problem. This generates a new series of adjusted price changes, U_{t+j}. Panel B of Table I indicates that the diurnal adjustment has major effects on the descriptive statistics. Skewness and kurtosis reduce to near-normal values (even though the Jarque–Bera test statistics still reject normality). The serial correlation in price changes is not affected, but serial correlation in the squares of the price changes is considerably smaller (although still significant).

The next step involves an intertemporal analysis of the descriptive statistics. Because the frequency of limit hits and moves is clustered in the middle of 1988, interest centers on the stability over time of certain key descriptive statistics. These statistics allegedly illustrate the harmful aspects of price limits.

Table II is based on a threefold sample split with reasonably comparable sample sizes: PRE from January 5 through May 1, LIMIT from May 2 through August 31, and POST from September 1 through November 28. The columns are sequentially ordered so that the first entry is PRE, the second entry is LIMIT, and the third entry is POST. The LIMIT period coincides with the months with the highest frequency of limit hits or moves during 1988. For soybeans, soybean meal, and soybean oil, a single limit hit or move is observed in the postlimit period.

A number of conclusions can be drawn from the reported measures. First, for all contracts except corn, the standard deviation is marginally larger during the LIMIT period.[2] However, this does not necessarily imply causality with limit moves. Increases in the fundamental (cash price) volatility could also lead to a higher frequency of limit hits, moves, or both. Second, skewness, kurtosis, or both are similar across the periods. The first-order autocorrelation coefficient is significantly negative in PRE and POST samples but much less so for the LIMIT sample (insignificant for soybeans and soybean meal). Bid–ask spreads are known to increase with increasing variance. Even though variance is marginally larger in the LIMIT period, first-order autocorrelation is much lower during that period. This may be due to infrequent trading that causes positive autocorrelation, thereby offsetting the bid–ask effect.

Having discussed the empirical characteristics of the data, we now estimate the nonlinear model for the adjusted price changes, U_{t+j}. For

[2]This comparison is based on the adjusted futures price changes, not the raw futures price changes. A visual inspection of Figure A-2 suggests that the raw corn futures price changes can be characterized by low volatility in the PRE sample, high volatility in the LIMIT sample, and medium volatility in the POST sample. The picture is very much the same for the other agricultural contracts. Our adjustment for diurnality with the interdaily volatility term absorbs most of this intertemporal instability.

TABLE II

Sample Split Statistics

	M	*SD*	*Skewness*	*Kurtosis*	*First-Order Autocorrelation*	*Limit Moves*
Corn						
PRE	−0.00	1.42	−0.07	3.58*	−0.206*	0
LIMIT	0.00	1.34	0.11	4.20*	−0.077*	31
POST	−0.01	1.34	−0.07	3.37*	−0.107*	0
Soybeans						
PRE	0.03	1.26	−0.13*	3.65*	−0.043*	0
LIMIT	0.06	1.34	−0.09	3.64*	−0.013	31
POST	0.01	1.32	−0.10	3.43*	−0.102*	1
Soybean Meal						
PRE	0.01	1.26	−0.07	3.49*	−0.042*	0
LIMIT	0.07	1.37	0.03	3.52*	0.001	29
POST	0.02	1.34	0.03	3.99*	−0.096*	1
Soybean Oil						
PRE	0.04	1.31	0.03	3.72*	−0.052*	0
LIMIT	0.03	1.37	−0.01	3.76*	−0.044	24
POST	−0.04	1.35	−0.04	4.03*	−0.082*	1
Wheat						
PRE	−0.01	1.31	−0.10	4.07*	−0.050*	0
LIMIT	0.01	1.39	−0.04	4.08*	−0.040	9
POST	0.04	1.37	0.10	4.16*	−0.084*	0

Note: This table reports a set of descriptive statistics for adjusted futures price changes U_{t+}, and test statistics for three subsamples. The subsample selection is based on the frequency of limit hits/moves. Limit hits/moves predominantly occur in the months of June and July, 1988. Hence, we distinguish PRE, LIMIT, and POST samples. These subsamples correspond with the first, second, and third entries, respectively. The descriptive statistics are the first, second, third, and fourth empirical moments and the first-order serial correlation coefficient. * indicates a significant rejection of the null hypothesis of normal distribution values for skewness (0) and kurtosis (3) and the null hypothesis of no autocorrelation at the 1% level. The exact number of observations differs per commodity because of the number of (excluded) limit move days. The subsample sizes are given in Appendix B.

this purpose, Equation 9 is slightly augmented by the inclusion of an intercept and a lagged dependent variable to account for possible market microstructure effects:

$$U_{t+j} = \delta_o + \delta_1(f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2i}I_i(.)(f_{t+j-1} - f_t)^2 + \delta_3(f_{t+j-1} - f_t^3) + \delta_4 U_{t+j-1} + e_{t+j} \quad (12)$$

In addition, by including appropriate interactive dummy variables D_k, we allow for different parameters in each of the three periods, PRE ($k = 1$), LIMIT ($k = 2$), and POST ($k = 3$):

TABLE III

Nonlinear Model Log Likelihoods

Error Distribution	*Model*	*Corn*	*Soybeans*	*Soybean Meal*	*Soybean Oil*	*Wheat*
Normal	Unrestricted	−14,857.3	−14,544.6	−14,801.7	−15,319.7	−16,643.3
Normal	Restricted	−14,898.2	−14,556.1	−14,811.8	−15,328.3	−16,652.6
Normal–GARCH	Unrestricted	−14,869.6	−14,528.7	−14,751.2	−15,289.7	−16,608.4
Normal–GARCH	Restricted	−14,892.7	−14,539.6	−14,759.6	−15,299.0	−16,616.3
Student *t*	Unrestricted	−14,825.3	−14,493.7	−14,740.6	−15,220.1	−16,478.3
Student *t*	Restricted	−14,847.7	−14,506.1	−14,751.2	−15,228.9	−16,488.1
Student *t*–GARCH	Unrestricted	$\underline{-14{,}821.5}$	$\underline{-14{,}480.4}$	−14,696.4	−15,190.3	−16,442.1
Student *t*–GARCH	Restricted	−14,844.1	−14,492.2	$\underline{-14{,}705.2}$	$\underline{-15{,}199.5}$	$\underline{-16{,}450.3}$

Note: This table reports the maximized log likelihood for the following unrestricted equation:

$$U_{t+j} = \sum_{k=1}^{3} D_k\left\{\delta_{0k} + \delta_{1k}(f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2ik} I_i(.)(f_{t+j-1} - f_t)^2 + \delta_{3k}(f_{t+j-1} - f_t)^3 + \delta_{4k}U_{t+j-1}\right\} + e_{t+j}$$

where $I(.)$ is an indicator function separating positive ($i = 1$) from negative ($i = 2$) deviations from the central futures price; a sample split dummy is indicated by $k = 1$ (PRE), 2 (LIMIT), and 3 (POST), and the restricted equation where the coefficients are constrained to be equal across each period (i.e., $\delta_{\cdot 1} = \delta_{\cdot 2} = \delta_{\cdot 3}$):

$$U_{t+j} = \delta_o + \delta_1(f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2i} I_i(.)(f_{t+j-1} - f_t)^2 + \delta_3(f_{t+j-1} - f_t^3) + \delta_4 U_{t+j-1} + e_{t+j}$$

We estimate this model for the unconditional normal distribution, the unconditional Student *t* distribution, and conditional versions of both models with GARCH(1,1) errors. Underlined entries indicate the best model based on a likelihood ratio test with the appropriate degrees of freedom.

$$U_{t+j} = \sum_{k=1}^{3} D_k\left\{\delta_{0k} + \delta_{1k}(f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2ik} I_i(.)(f_{t+j-1} - f_t)^2 + \delta_{3k}(f_{t+j-1} - f_t)^3 + \delta_{4k}U_{t+j-1}\right\} + e_{t+j} \tag{13}$$

The autoregressive term captures the bid–ask bounce and discreteness effects typically characterizing high-frequency intraday prices (Harris, 1990; Miller, Muthuswamy, & Whaley, 1994). For a dominating bid–ask effect, it is expected that δ_4 will be negative. Table III reports the log likelihood obtained for a number of normal and Student *t* distribution models. Likelihood ratio tests are performed to determine which model is best. The study estimates the unrestricted model, where the coefficients are defined for each subperiod, and a restricted model, where the coefficients are the same for the full sample period.

A sequence of likelihood ratio tests indicate that the Student *t* distribution outperforms the normal distribution and that allowing for time-varying conditional variance results in improved performance for the Student *t* distribution. Estimates of ν are found to be close to 11. In addition, the unrestricted model outperforms the restricted model, although mar-

TABLE IV

Tests of Restrictions

Restrictions	*Number of Coefficients*	*Corn*	*Soybeans*	*Soybean Meal*	*Soybean Oil*	*Wheat*
			Panel A: Log-Likelihood Values			
Unrestricted	22	−14,821.5	−14,480.4	−14,696.4	−15,190.3	−16,442.1
$\delta_{\cdot 1} = \delta_{\cdot 2} = \delta_{\cdot 3}$	10	−14,844.1	−14,492.2	−14,705.2	−15,199.5	−16,450.3
$\delta_{3\cdot} = 0$	19	−14,821.7	−14,482.9	−14,697.2	−15,191.3	−16,443.7
$\delta_{2\cdot} = \delta_{3\cdot} = 0$	13	−14,832.2	−14,486.9	−14,700.0	−15,196.1	−16,448.0
$\delta_{1\cdot} = \delta_{2\cdot} = \delta_{3\cdot} = 0$	10	−14,832.8	−14,490.6	−14,703.2	−15,198.6	−16,452.2
$\delta_{2\cdot} = 0$	16	−14,825.6	−14,483.0	−14,697.0	−15,193.3	−16,446.5
$\delta_{1\cdot} = 0$	19	−14,822.9	−14,482.2	−14,696.5	−15,192.8	−16,442.6

Restrictions	*Degrees of Freedom*	*Corn*	*Soybeans*	*Soybean Meal*	*Soybean Oil*	*Wheat*
			Panel B: Likelihood Ratio Test Statistics			
$\delta_{\cdot 1} = \delta_{\cdot 2} = \delta_{\cdot 3}$	12	45.2 (0.000)	23.6 (0.023)	17.6 (0.128)	18.4 (0.104)	16.4 (0.176)
$\delta_{3\cdot} = 0$	3	0.4 (0.940)	5.0 (0.172)	1.6 (0.659)	2.0 (0.572)	3.2 (0.362)
$\delta_{2\cdot} = \delta_{3\cdot} = 0$	9	21.4 (0.011)	13.0 (0.163)	7.2 (0.616)	11.6 (0.237)	11.8 (0.225)
$\delta_{1\cdot} = \delta_{2\cdot} = \delta_{3\cdot} = 0$	12	22.6 (0.031)	20.4 (0.060)	13.6 (0.327)	16.6 (0.165)	20.2 (0.063)
$\delta_{2\cdot} = 0$	6	8.2 (0.224)	5.2 (0.518)	1.2 (0.977)	6.0 (0.423)	8.8 (0.185)
$\delta_{1\cdot} = 0$	3	2.8 (0.424)	3.6 (0.308)	0.2 (0.978)	5.0 (0.172)	1.0 (0.801)
$\delta_{2\cdot} = 0 \mid \delta_{3\cdot} = 0$	6	21.0 (0.002)	8.0 (0.238)	5.6 (0.469)	9.6 (0.143)	8.6 (0.197)
$\delta_{1\cdot} = \delta_{2\cdot} = 0 \mid \delta_{3\cdot} = 0$	9	22.2 (0.008)	15.4 (0.081)	12.0 (0.213)	14.6 (0.103)	17.0 (0.049)
$\delta_{1\cdot} = 0 \mid \delta_{3\cdot} = 0$	3	14.2 (0.003)	2.4 (0.494)	3.4 (0.334)	6.2 (0.102)	1.6 (0.659)
$\delta_{1\cdot} = 0 \mid \delta_{2\cdot} = \delta_{3\cdot} = 0$	3	1.2 (0.753)	7.4 (0.060)	6.4 (0.093)	5.0 (0.172)	8.4 (0.038)

Note: Panel A gives the log-likelihood values of the unrestricted Student *t* GARCH model of Table III and for a number of restricted versions of that model. Panel B presents likelihood ratio test statistics and *p* values for the restrictions in parentheses. The conditional hypotheses are based on imposing the conditioning restrictions.

ginally so for the soybean complex and wheat, so that splitting the sample into PRE, LIMIT, and POST seems to be relevant.

The following discussion is restricted to the Student *t* cum GARCH distributional model and the unrestricted subperiod model. In the cubic functional forms for all commodities, all but one coefficient of the Taylor series approximation terms (i.e., the variables in $f_{t+j-1} - f_t$) are individually insignificant, and about half of the coefficients have the sign opposite to that suggested by the theoretical model. Collinearity[3] is clearly a problem in interpreting the model. A number of restrictions on the coefficients of this model were, therefore, considered. The log likelihoods for these restricted models and likelihood ratio test statistics for testing these restrictions are presented in Table IV. For each commodity, the null hypothesis that the coefficients on the cubic terms are zero can be accepted. In the remaining quadratic functional form for the five commod-

[3]In the nonlinear Equation 13, collinearity occurs because different parameter combinations for the squared and cubic terms can generate similar nonlinear relationships.

TABLE V

Quadratic Nonlinear Models

	Corn	*Soybeans*	*Soybean Meal*	*Soybean Oil*	*Wheat*
$\delta_{0,\text{pre}}$	−0.0022 (−0.077)	0.0362 (1.595)	0.0155 (0.649)	0.0186 (0.727)	−0.0193 (−0.754)
$\delta_{0,\text{limit}}$	−0.0057 (−0.181)	0.0725 (2.287)	0.0658 (2.116)	0.0285 (0.993)	0.0209 (0.823)
$\delta_{0,\text{post}}$	0.0239 (0.747)	0.0269 (0.928)	0.0313 (1.040)	−0.0510 (1.678)	0.0073 (0.235)
$\delta_{1,\text{pre}}$	0.1266 (2.568)	0.0067 (0.728)	0.0135 (0.466)	0.5767 (2.362)	0.0089 (0.468)
$\delta_{1,\text{limit}}$	0.0226 (1.306)	0.0087 (1.261)	0.0063 (0.344)	0.0328 (0.206)	0.0011 (0.125)
$\delta_{1,\text{post}}$	0.0626 (2.398)	0.0044 (0.524)	0.0406 (1.735)	0.1863 (0.684)	0.0274 (1.171)
$\delta_{21,\text{pre}}$	−0.0504 (−2.545)	−0.0004 (−0.571)	−0.0035 (−0.449)	−0.9915 (−1.563)	−0.0017 (−0.531)
$\delta_{21,\text{limit}}$	−0.0025 (−1.316)	−0.0008 (−2.667)	−0.0037 (−1.423)	−0.2662 (−0.996)	−0.0009 (−1.125)
$\delta_{21,\text{post}}$	−0.0180 (−2.769)	−0.0006 (−0.857)	−0.0094 (−1.709)	−0.7621 (−0.854)	−0.0040 (−1.081)
$\delta_{22,\text{pre}}$	0.0964 (3.110)	0.0013 (1.182)	0.0057 (0.445)	1.9409 (2.582)	0.0123 (1.864)
$\delta_{22,\text{limit}}$	0.0053 (1.767)	0.0008 (2.000)	0.0047 (1.270)	0.1653 (0.811)	0.0005 (0.714)
$\delta_{22,\text{post}}$	0.0104 (1.576)	0.0002 (0.333)	0.0069 (1.408)	0.8150 (1.334)	0.0085 (1.889)
$\delta_{4,\text{pre}}$	−0.2050 (−12.275)	−0.0469 (−2.635)	−0.0558 (−3.100)	−0.0565 (−3.247)	−0.0615 (−3.534)
$\delta_{4,\text{limit}}$	−0.0827 (−3.919)	−0.0155 (−0.760)	−0.0070 (−0.338)	−0.0401 (−2.046)	−0.0464 (−2.651)
$\delta_{4,\text{post}}$	−0.1160 (−5.771)	−0.1168 (−5.869)	−0.1069 (−5.189)	−0.0859 (−4.295)	−0.0953 (−4.862)
β_0	0.8682 (2.208)	0.4827 (2.887)	0.6192 (6.236)	0.6212 (4.375)	0.7498 (4.943)
β_1	0.0302 (2.517)	0.0416 (4.078)	0.0887 (7.040)	0.0732 (5.588)	0.0798 (6.045)
β_2	0.4995 (2.294)	0.6719 (6.454)	0.5536 (8.900)	0.5828 (6.824)	0.5169 (5.914)
ν	12.1996 (6.818)	11.5559 (7.126)	11.0375 (7.278)	7.9738 (8.846)	6.7075 (10.299)
Skewness	−0.049	−0.084*	−0.024	0.004	−0.047
Kurtosis	3.747*	3.676*	3.860*	4.045*	4.408*
Jarque–Bera test	187.00*	175.21*	270.80*	408.42*	802.94*
Autocorrelation ρ	−0.005	0.012	0.015	0.012	0.017
Ljung–Box (12): levels	22.48	37.33*	37.76*	65.01*	14.93
Ljung–Box (12): squares	24.48	28.74*	57.05*	55.21*	59.75*
ARCH test	3.64	4.94	22.57*	19.68*	31.32*

Note: This table reports the parameter estimates for the Student *t* cum GARCH error distribution:

$$U_{t+j} = \sum_{k=1}^{3} D_k \left\{ \delta_{0k} + \delta_{1k}(f_{t+j-1} - f_t) + \sum_{i=1}^{2} \delta_{2ik} I(.)(f_{t+j-1} - f_t)^2 + \delta_{4k} U_{t+j-1} \right\} + e_{t+j}$$

where the third-order term coefficients are restricted to zero. The coefficient estimates are given for the three subsamples PRE, LIMIT, and POST, and *t* statistics are given in parentheses. For the degrees of freedom parameter, the null hypothesis is that $\nu = 2$. For the diagnostic statistics, * indicates a rejection of the null hypothesis at the 1% level.

ities, although the majority of coefficients remain individually insignificant, it is notable that across the five commodities every quadratic coefficient has the correct sign but every linear coefficient has the incorrect sign, as predicted by the theoretical model.

The coefficient estimates of this quadratic functional form are presented in Table V. In contrast with the cubic specification, there are consistent parameter estimates across commodities and across subperiods for the explanatory variables. Except for soybeans and soybean meal in the LIMIT period, the constant, δ_0, is insignificantly different from zero. The first-order term, δ_1, is also frequently insignificantly different from zero. It is, however, significant for corn and does have a negative sign consistent with gravitation. The second-order terms, δ_{21} and δ_{22}, are occasionally significant (or close to significance), particularly in the PRE and POST periods and do have the required signs for stabilization. The nonlinear model apparently performs best (in terms of significance) for corn. Formal statistical tests indicate that the random walk model is the preferred model for the soybean complex and wheat, whereas the quadratic functional form is the preferred model for corn. The diagnostic test statistics indicate that there is still residual excess kurtosis in the model for each commodity, but this is consistent with the estimated Student *t* models. The first-order serial correlation has been adequately modeled. There appears to be some remaining dependence in the first 12 lags of the levels and the squares of the residuals; however, for none of the commodities does the absolute value of any of these autocorrelation coefficients exceed 0.06.

Concluding the parameter estimate discussion, we find that only for corn is there very limited evidence of nonlinearity in futures prices potentially driven by the existence of price limits. The estimated coefficients indicate that even within the limits, there may be a discernible impact, in particular in the PRE and POST corn samples. The inconsistency in the signs of the linear and quadratic coefficients suggests a straightforward conclusion cannot be drawn with regard to the gravitation versus price-stabilization theory.

Given that linearity is rejected by the estimates, we would expect that either the S shape or its inverse would be found. For this purpose, consider Figure 1, where the fitted model for corn from Table V is plotted according to the estimated model. The dashed lines indicate the 95% confidence interval. Panel A is related to the PRE estimates; Panel B is related to the LIMIT estimates; and Panel C is related to the POST estimates.

If the price-stabilizing model is the true model, a (increasing) negative deviation from the central futures price would be expected to generate a (increasing) positive futures price change. If the gravitation model is the true model, a (increasing) negative deviation from the central futures price would be expected to generate a (increasing) negative futures price change. Analogous conclusions can be drawn for positive deviations from the central futures price.

A similar shape is observed for the three subperiods. The PRE and POST graphs clearly display price-stabilizing nonlinear behavior. The

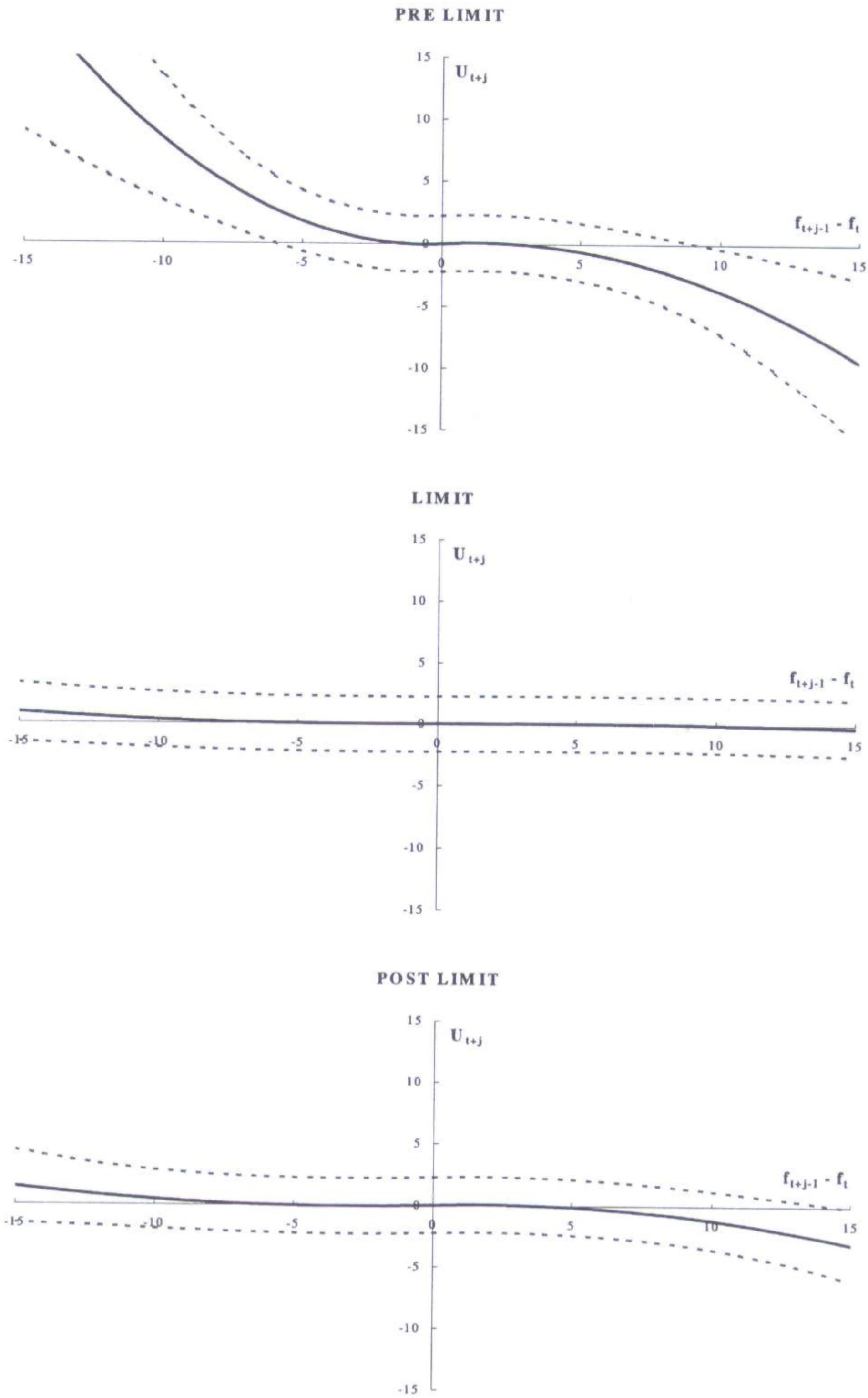

FIGURE 1
Estimated functions for corn futures.

LIMIT graph also displays this behavior, but the confidence bands indicate that it is much less impressive in nonlinear terms; there is hardly any reaction or weak evidence of stability (where prices revert to the center of the price band). The cases where we cannot reject no reaction correspond to the random walk model. The evidence for the other commodities corroborates these results. Although all graphs indicate price stabilization, in most cases it is insignificant.

Thus, when the fundamental cash price is strongly drifting upward or downward so that it seems that limit hits (and moves) are imminent, the futures price process behaves like a random walk with drift. For normal periods (PRE and POST LIMIT), price limits seem to have a stabilizing impact on prices, with prices reverting to the center of the price band. As suggested in the introduction, we conclude that price limits are not the preferred regulatory tool for fundamental market realignments. When most needed, their impact is insignificant.

CONCLUDING REMARKS

Price-limit regulation is a hotly debated issue provoked by the occasional major realignment in prices. Despite their limited occurrence, antagonists argue that their very existence may deter prospective traders from entering futures trading. Protagonists, however, argue that the occasional mayhem and chaos sufficiently underline the need for even tighter price limits.

We have made an attempt to identify the impact of futures price limits on traders' expectations and, hence, price discovery. The current literature on futures price limits can be divided between those that argue in favor of the gravitation of prices toward the limits versus those that claim the stabilizing behavior of prices bouncing back from the limits. Both hypotheses imply nonlinearities in futures price changes. This study specifies a model for intraday futures prices that allows proper testing for the existence of this nonlinearity. By applying the model to a set of agricultural futures contracts, we illustrate that even without actual limit hits (or moves) for a prolonged period of time, there may be some impact, albeit limited, from the bounds on prices.

All parameter estimates have signs consistent with the S-shape price-stabilizing hypothesis. For corn futures in particular, evidence is found for this type of nonlinearity in futures price changes. For the soybean complex and wheat, conclusive evidence for price stabilization is unable to be found, but the gravitation hypothesis can still be rejected. Hence, there is either no impact of price limits on traders' expectations, or there is some stabilization impact.

It is, therefore, tentatively concluded that price limits might not be the preferred tool for major market realignments. For regular market behavior, however, price limits seem to have a modifying impact on volatility. Given their cost of impeding a continuous trade flow in times when price discovery is most urgently needed, this seems to be insufficient evidence in favor of price limits.

APPENDIX A: ADJUSTING THE FUTURES PRICE CHANGES FOR DIURNAL INTRADAY EFFECTS

Theory

As in Andersen and Bollerslev (1998) and Kofman and Martens (1997), the flexible Fourier form (FFF) specification originally introduced by Gallant (1981) is used to account for deterministic intraday volatility. The FFF models volatility as a sum of low-order polynomial and trigonometric terms:

$$|R_{t,d}| = \sum_{k=0}^{K} \sigma_d^k \left\{ \alpha_{0k} + \alpha_{1k}\left(\frac{t}{T}\right) + a_{2k}\left(\frac{t}{T}\right)^2 + \sum_{j=1}^{J} \left[\tau_{jk} \cos\left(\frac{2jt\pi}{T}\right) + \gamma_{jk} \sin\left(\frac{2jt\pi}{T}\right) \right] \right\} + \zeta_{t,d} \quad \text{(A1)}$$

where absolute intraday futures price changes are used as the volatility measure. The FFF is partly a quadratic function of time $t = 1 \ldots T$ to capture the intraday U shape and partly a trigonometric function of time t to capture any other smooth patterns (e.g., regularly timed agricultural news releases). The σ_d^k term measures the standard deviation of futures price changes on day d. By including this interactive volatility term, we allow for the fact that daily volatility levels are commonly found to be correlated. Hence, inclusion absorbs level shifts in the volatility intraday effect during the sample period. The number of (co-)sinusoids J and interactive volatility terms K is determined by the overall goodness of fit of the regression. The futures price changes are then adjusted by being divided by their diurnal fitted value:

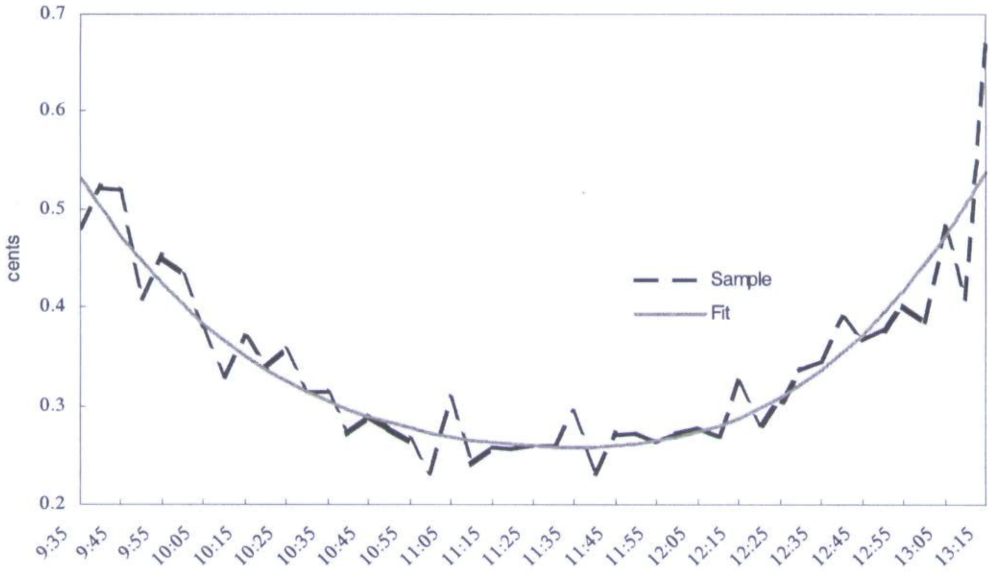

FIGURE A-1
Intraday diurnal effects, corn futures (1/5/88 to 11/28/88).

$$U_{t,d} = \frac{R_{t,d}}{|\hat{R}_{t,d}|} \tag{A2}$$

Both raw price changes and adjusted futures price changes have been used in the model estimation.

Empirical Intraday Diurnal Effect

Consider the absolute value of futures price changes as a measure of volatility. These volatility measures are averaged for each 5-min sampling interval across trading days. This generates an average sample volatility measure per 5-min interval. Figure A-1 illustrates the well known intraday U shape (the broken line) for the average sample volatility. Next, it is investigated whether the interactive (interdaily) volatility level should be included in the estimation of the diurnal effect. Daily standard deviations are computed from the (45) intradaily price changes per day. Note that the opening interval uses the difference between the last price and the first observed price within that interval. The remaining intervals use the difference between the last observed price in that interval and the last observed price in the previous interval. Figure A-2 plots this series. Although volatility is rather stable for the first 4 months, it clearly explodes in the following 3 months, after which it becomes more stable but at a

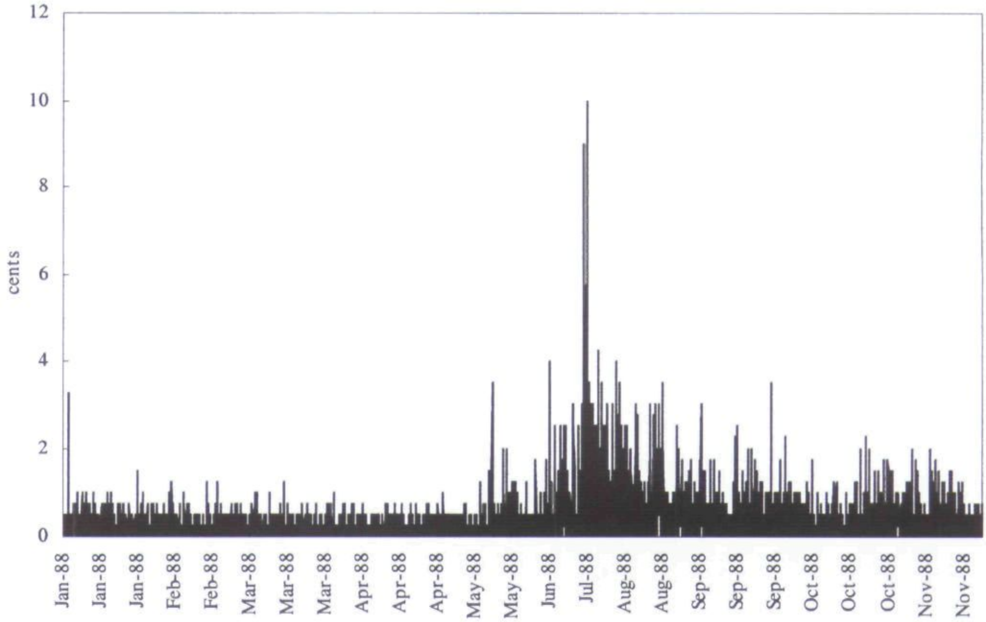

FIGURE A-2
Sample volatility of futures price changes, corn futures (1/5/88 to 11/28/88).

much higher level than before. These structural changes may have a substantial impact on intradaily volatility diurnal effects.

Table A-I, therefore, uses the interactive volatility level in the estimation of the diurnal effects. Let $J = K = 1$ in Equation A1. Higher order terms do not contribute to the shape and significance of the diurnal effects.

The smooth solid line in Figure A-1 shows the fitted average U shape for corn. Finally, Figure A-3 shows these diurnal effects across days, clearly capturing the interdaily volatility changes.

APPENDIX B. DATA AND CONTRACT SPECIFICATIONS

The data used in this study cover the time period January 5, 1988 through November 28, 1988. The intraday data (all frequencies) are derived from the Tick Data, Inc. time and sales tapes. The details of the agricultural futures contracts traded at the Chicago Board of Trade are given in Table A-II. According to Regulation 1008.01 Trading Limits, the following rules have been active for our sample period: Price limits are symmetric above and below the previous business day's settlement price. Limits are lifted 2 business days before the cash month (i.e., the delivery month). For the

TABLE A-I

Diurnal Estimation Results

Variables	*Corn*	*Soybeans*	*Soybean Meal*	*Soybean Oil*	*Wheat*
C	−0.0368 (−0.60)	0.3877 (1.85)	−0.3011 (−4.09)	−0.0179 (−2.24)	−0.5340 (−5.04)
$\sigma \cdot C$	1.3912 (13.10)	1.1458 (9.26)	2.1925 (17.48)	1.9876 (15.52)	2.5311 (21.02)
$\frac{t}{T}$	0.0014 (0.01)	−2.3332 (−2.04)	1.3897 (3.45)	0.0338 (0.78)	2.4961 (4.31)
$\sigma \cdot \frac{t}{T}$	−3.3639 (−5.80)	−2.9991 (−4.43)	−7.7483 (−11.30)	−5.9372 (−8.49)	−9.3484 (−14.21)
$\left(\frac{t}{T}\right)^2$	0.0717 (0.22)	2.4651 (2.21)	−1.1148 (−2.85)	0.0065 (0.15)	−2.0366 (−3.62)
$\sigma \cdot \left(\frac{t}{T}\right)^2$	3.1422 (5.57)	3.1650 (4.81)	7.1158 (10.68)	5.0545 (7.43)	8.4181 (13.16)
cos(.)	−0.0377 (−1.10)	−0.3028 (−2.59)	0.0791 (1.92)	−0.0040 (−0.90)	0.1364 (2.31)
$\sigma \cdot$ cos(.)	−0.0207 (−0.35)	−0.0240 (−0.35)	−0.4245 (−6.06)	−0.2058 (−2.88)	−0.4886 (−7.27)
sin(.)	0.0517 (3.40)	0.1339 (2.57)	0.0994 (5.44)	0.0127 (6.39)	0.1547 (5.89)
$\sigma \cdot$ sin(.)	−0.0986 (−3.74)	−0.0369 (−1.20)	−0.2113 (−6.79)	−0.2897 (−9.12)	−0.2734 (−9.16)

This table reports the estimation results for the diurnal effects in each of the five commodity futures price volatility series. The diurnal specification is given by

$$|R_{t,d}| = \sum_{k=0}^{1} \sigma_d^k \left\{\alpha_{0k} + \alpha_{1k}\left(\frac{t}{T}\right) + \alpha_{2k}\left(\frac{t}{T}\right)^2 + \left[\tau_k \cos\left(\frac{2t\pi}{T}\right) + \gamma_k \sin\left(\frac{2t\pi}{T}\right)\right]\right\} + \zeta_{t,d}$$

The t statistics are given in parentheses.

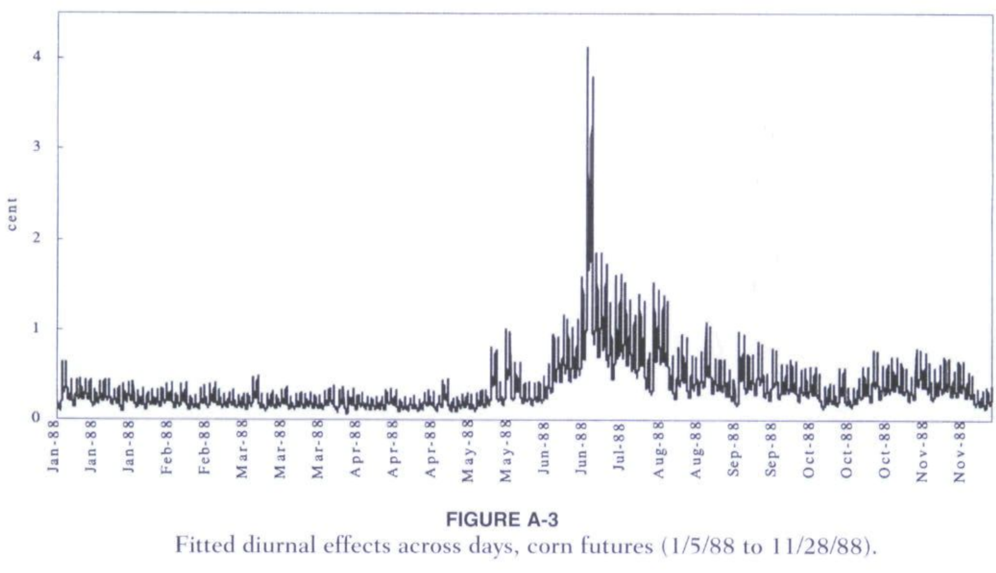

FIGURE A-3

Fitted diurnal effects across days, corn futures (1/5/88 to 11/28/88).

TABLE A-II

Contract Specifications

Contract	*Corn*	*Soybeans*	*Soybean meal*	*Soybean oil*	*Wheat*
Contract size	5,000 bushels	5,000 bushels	100 tons	60,000 lbs	5,000 bushels
Delivery months	March, May, Jul, Sep, Dec	Jan, Mar, May, Jul, Aug, Sep, Nov	Jan, Mar, May, Jul, Aug, Sep, Nov	Jan, Mar, May, Jul, Aug, Sep, Nov	Mar, May, Jul, Sep, Dec
Minimum tick size	0.25¢/bushel	0.25¢/bushel	10¢/ton	0.01¢/lb	0.25¢/bushel
Initial limit	12¢	30¢	$10	1¢	20¢
Expandable limit	18¢	45¢	$15	1.5¢	30¢
Sample periods	5 Jan to 29 Apr	5 Jan to 29 Apr	5 Jan to 29 Apr	5 Jan to 29 Apr	5 Jan to 29 Apr
	2 May to 31 Aug	2 May to 31 Aug	2 May to 31 Aug	2 May to 31 Aug	2 May to 31 Aug
	1 Sep to 28 Nov	1 Sept to 28 Nov	1 Sep to 28 Nov	1 Sep to 28 Nov	1 Sep to 28 Nov
Observations	3645	3690	3690	3690	3690
in samples	2475	2475	2565	2790	3465
	2700	2700	2700	2700	2745

first half of our sample period (until June 23, 1988), limits were expanded by 150% of the current level if, for 2 successive business days, the market closed at the limit for three or more simultaneously traded maturities for a contract year. If there were less than three maturities remaining, this rule was changed to all traded maturities. For the second half of our sample period (as of June 24, 1988), the sequential limit days requirement was reduced to a single business day. Expanded limits remained in action for 3 successive business days. Limits remained at 150% for successive 3-day periods unless at the end of such a period three or more maturities did not close at the limit. The limit expansion operated concurrently on the soybean complex futures. Limits reverted to 100% only if all futures in this complex met the conditions for reversal.

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