HETEROGENEOUS EXPECTATIONS OF TRADERS IN SPECULATIVE FUTURES MARKETS

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The representative agent hypothesis is disputable on theoretical grounds because it is inconsistent with observed trading behavior and the existence of speculative markets. In such markets, the representative agent hypothesis implies agents hold homogeneous expectations. If this were true, speculative markets would fail as only one side of the market would be represented, either demand or supply. Nonetheless, the homogeneity assumption has been maintained in the past to ensure tractability because of the difficulty of explicit aggregation across heterogeneous expectations. In this article, we present and apply an approach for analyzing heterogeneity in specific market settings. To do so, our approach specifies an underlying distribution of expectations that is consistent with heterogeneity across expectations. To demonstrate the utility of the approach, we present results from its application to a time series of commodity

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futures prices. Results are consistent with the conclusion that significant heterogeneity in expectations exists in speculative futures markets. Jrl Fut Mark 21:429–446, 2001

INTRODUCTION

Marshall's (1961) notion of a representative firm has permeated modern economic thought. Despite criticisms and concerns (e.g., Hartley, 1996; Kirman, 1992; Levy & Levy, 1996; Robbins, 1928; Sraffa, 1926; Young, 1928), the representative agent hypothesis has continued to be maintained. It renders a measure of elegance and simplicity to otherwise complicated analytical models and establishes tractability in empirical applications. Although these advantages may be justifiable at some levels of enquiry, the frequent maintenance of the hypothesis must not obscure the importance of evidence concerning its appropriateness.

A key implication of the representative agent hypothesis is homogeneous expectations of uncertain future prices. The alternative hypothesis of heterogeneous expectations has historically been recognized as valid. For example, the importance of heterogeneous expectations for the existence of speculative markets has often been noted (e.g., Grossman, 1977; Keynes, 1930). In this context, there is no incentive for trade if all agents are identical with respect to information, endowments, and trading strategies. The persistence and implications of heterogeneous expectations has also received some attention. For example, Black (1986) explored persistence, noting the role of differential capital reserves. Brock and Hommes (1997), Cartapanis (1996), and Dufey and Kazemi (1991) showed that persistence of heterogeneity can result in boom and bust behavior under incomplete information.

Perhaps the most widely recognized incentive for trade is heterogeneous prior beliefs. Speculative traders disagree about the nature and extent of market moves, taking bullish or bearish positions in opposition to their counterparts. The source of disagreement is the variety of interpretations possible for the same available information. Different traders filter information differently and, consequently, form different expectations. Harrison and Kreps (1978), Varian (1985), De Long, Shleifer, Summers, and Waldman (1990), Harris and Raviv (1993), and Wang (1998) showed that this sort of heterogeneity can lead to market behavior much like that observed empirically.

At an empirical level, considerable evidence suggests that speculative traders do not hold a homogeneous rational expectation (e.g., Beach, Fernandez, & Uri, 1995; Frankel & Froot, 1988, 1990; Ito, 1990; Levine, 1993; L'Her & Suret, 1995; MacDonald & Macmillan, 1994; MacDonald & Marsh, 1996; Van-Hoek, 1992). Furthermore, evidence of heterogeneity across agents follows from the persistence of different trading rules. For example, the persistence of commodity pools trading in futures markets using simple trading rules alongside traditional speculators and hedgers in commodity markets has been noted (Irwin, Krukemeyer, & Zulauf, 1993; Irwin & Yoshimaru, 1999; Murphy, 1986). Events such as occurred in the October 1987 equity market reminds us that when expectations converge toward homogeneity, a one-sided market emerges. Prices may adjust precipitously if trade involves only atomistic agents that respond to arbitrage incentives for only a buy or a sell, but not for both.

In response to these concerns, recent articles have considered specifications incorporating heterogeneous expectations. Chavas (1999) divided agents into three categories based on how they form expectations in the pork market: naive (7%), quasi-rational (73%), and rational (20%). Frechette (1999) divided agents into two groups in the copper market (quasi-rational, 16%, and rational, 84%), the corn market (quasi-rational, 3%, and rational, 97%), and the wheat market (quasi-rational, 1%, and rational, 99%). Chavas assumed agents were risk neutral and concluded that homogeneous expectations are not appropriate for modeling the pork market. Frechette assumed agents were risk averse and concluded that homogeneous expectations are appropriate for the corn and wheat markets but not for the copper market. Both authors indicated that further research is needed on aggregation and the nature of price expectations. Weaver and Zhang (1999) allowed for a continuum of heterogeneity in expectations and explored the implications of the extent of heterogeneity for price level and volatility in speculative markets.

This article develops a framework for evaluating empirical evidence of heterogeneous expectations in futures markets and applies it to estimate the evolution of the distribution of heterogeneous agent-level expectations. The article differs from those of Chavas (1999) and Frechette (1999) by classifying agents by the direction of the bias in their expectations, their bullish or bearish sentiment, rather than by how they form expectations. A sample of U.S. soybean futures prices is used to illustrate the utility of this framework for examining evidence of heterogeneity and implementing a test of the representative agent hypothesis. The homogeneous expectation implication of the representative agent hypothesis is nested within the heterogeneous expectations model and is tested empirically. Additional tests help characterize the nature and extent of heterogeneity in the market and how it affects the behavior of prices.

THEORETICAL FRAMEWORK

To motivate a means of empirically characterizing heterogeneity in expectations, consider the following competitive market model. For homogeneous expectations,

$$\text{Current supply } S_t = S(p^e_{t-1,\,t}, Z^s_t, \beta) + v_t;\ v_t \sim V(\mu^v_t, \sigma^v_t) \quad (1)$$

$$\text{Current demand } D_t = D(p_t, Z^d_t, \alpha) + w_t;\ w_t \sim W(\mu^w_t, \sigma^w_t) \quad (2)$$

$$\text{Current balance } S_t = D_t \quad (3)$$

where p_t is the current price, $p^e_{t,t-1}$ is last period's expectation of the current price, Z^s_t and Z^d_t are exogenous shifters that can include lagged endogenous variables, α and β are parameter vectors, and v_t and w_t are stochastic shocks. S_t is net of storage. Given linear forms, Equations 1 to 3 imply a stochastic excess demand that implicitly defines the equilibrium price:

$$p_t = P(p^e_{t-1,t}, Z^d_t, \alpha, Z^s_t, \beta) + u_t = 0;\ u_t \sim U(\mu_t, \sigma^u) \quad (4)$$

where $u_t = w_t - v_t$. This price function immediately suggests an empirical approach for identifying the underlying homogeneous price expectation. Given a time series of prices $\{p_t\}$ and supposing stationarity such that p_t are independent and identically distributed with mean μ_t and variance Σ we have T observations available to estimate K parameters in (α,β) and σ^u and $p^e_{t-1,t}$.

A similar approach is motivated under rational expectations. Here, in addition to homogeneity, the hypothesis may be written as follows:

$$p^{re}_{t,t-1} = E_{t-1}p_t \quad (5)$$

Again, where the market structure is linear, the expectation implicit in Equation 4 defines the representative agent's rational expectation, and by substitution into Equation 4, the equilibrium price is again related to the underlying expectation. An examination of either homogeneity or rationality of the expectations based on this framework may be pursued, although inferences in both cases remain conditional on the specification of Equation 4. A weak test of rationality would focus on unbiasedness by examining consistency with Equation 5.

Consider next an approach that specifies a clearly stated alternative hypothesis that admits heterogeneous expectations. We note the kth agent's expectation $p^e_{k,t,t-1}$. To establish a relationship between this expectation and the cash equilibrium price, we rely on the equilibrium condition for each agent's arbitrage. That is, Equations 1 to 3 no longer provide a complete system for determination of p_t and its relationship with price expectations. Under heterogeneous expectations, the arbitrage equilibrium conditions (e.g., Equation 4) no longer link current price to expected prices. The implications of this specification can now be assessed.

The relationship between an agent's expectation and the current price can be established by consideration of the storage arbitrage entry equilibrium condition. Suppose the agent is risk neutral with expected profits determined as follows:

$$\pi^e_{k,t-1} = (p^e_{k,t,t-1} - p_{t-1})I_{k,t} - C(I_{k,t}) \tag{6}$$

where $C(I_{k,t})$ represents the cost of holding inventories $I_{k,t}$. Given that agent expectations are conditional on the agent's expectation of market-wide storage, I^{k*}_t, the traditional zero profits entry equilibrium condition would be achieved, establishing an equilibrium relationship across heterogeneous expectations and the market price. For example,

$$p^e_{k,t,t-1}|I^{k*}_t - p_{t-1} - AC(I_{k,t}) = 0 \text{ for all agents } k = 1 \ldots K \tag{7}$$

Averaging over K agents and denoting market averages in bold, we find that entry occurs until

$$\mathbf{p}^e_{t,t-1} - p_{t-1} - \mathbf{AC}_t = 0 \tag{8}$$

We define the mean of agent expectations as the market-wide expectation.

Thus, although agent-specific decisions establish heterogeneous arbitrage equilibrium conditions based on Equation 6, competitive entry also establishes a relationship between what we label as the market-wide expectation, $\mathbf{p}^e_{t,t-1}$, and the current price, p_{t-1}. Furthermore, Equations 7 and 8 define a useful equilibrium relationship between agent and market-wide expectations:

$$p^e_{k,t,t-1} = \mathbf{p}^e_{t,t-1} + AC(I_{k,t}) - \mathbf{AC}_t \tag{9}$$

Defining the difference $AC(I_{k,t}) - \mathbf{AC}_t$ as the idiosyncratic component of individual average costs, $e_{k,t}$, we establish an important link between agent and market-wide expectations:

$$p^e_{k,t,t-1} = \mathbf{p}^e_{t,t-1} + e_{k,t} \tag{10}$$

Now, to proceed toward an empirical examination of the distribution of the heterogeneous expectation structure, recall that $\mathbf{p}^e_{t,t-1}$ was defined as the definite integral over the distribution of agent expectations, for example, defining that distribution as $f_t(p,\Gamma)$, with Γ representing an appropriate vector of parameters. Then,

$$p^e_{t,t-1} = \int_{p_-}^{p^+} pf_t(p,\Gamma)dp$$

implying

$$p_{t+1} = \int_{p_-}^{p^+} pf_t(p,\Gamma)dp + e_t,\ e_t \sim \phi(0,\gamma^2) \tag{11}$$

The advantage of Equation 11 is its empirical tractability. Given a functional specification for the density functions $\phi(.)$, the time series distribution of the mean idiosyncratic component of average costs, and $f_t(.)$, the density function of agent-level expectations over agents, we illustrate in the next section a fruitful approach that allows estimation of the underlying parameters of these distributions.

EMPIRICAL APPLICATION

We have specified the existence of a continuum of traders with heterogeneous expectations. Trader k at time t expects next period's futures price to be $p^e_{k,t,t+1}$. The density of expectations across agents at time t is denoted $f_t(.)$. The area to the left of $f_t(p)$ corresponds to the portion of agents who expect the price to be less than p at time $t + 1$. For a particular population of traders, finite limits exist for the level of those expectations; for example, (p_-,p^+), that define the support of $f_t(.)$. $f_t(p)$ is bounded between zero and one for all p and t. Denoting $\int_{p_-}^{p^+} f_t(p,\Gamma)dp$ as F_t, the number of agents who expect price to be p at time $t + 1$ is represented by the density function $f_t(p)/F_t$. In our empirical application, we suppose time is measured in discrete units (days).

Specifically, let $f_t(p,\Gamma)$ be a mixture of two normal distributions:

$$\begin{aligned} f_t(p) = \alpha \exp\left[-\frac{1}{2}\left(\frac{p-\mu_{1t}}{\sigma_{1t}}\right)^2\right]\Big/\sigma_{1t}\sqrt{2\pi} \, + \\ (1-\alpha)\exp\left[-\frac{1}{2}\left(\frac{p-\mu_{2t}}{\sigma_{2t}}\right)^2\right]\Big/\sigma_{2t}\sqrt{2\pi} \end{aligned} \tag{12}$$

where μ_{it} and σ_{it} are means and standard deviations of the individual densities, respectively, and α is a weighting parameter between zero and one. This distributional assumption allows for, but does not require, a bimodal shape, corresponding to concentrated density reflecting the presence of both bulls and bears in the market. The mixture function $f_t(.)$ is not a density function itself unless α is 0 or 1. This approach is similar to one used by Cukierman and Wachtel (1982) to study inflation expectations. Any sort of density function could be chosen for $f(.)$, depending on the modeling assumptions. Other distributions were considered, including nonsymmetric distributions, but none seemed markedly superior to Equation 12.

We would like to describe empirically the distribution of agent price expectations for each time t. To do so, we adopt a parametric approach and specify the following time series structure for the parameters of $f(.)$ in Equation 12. For example,

$$\mu_{it} = a_{i0} + a_{i1}p_{t-1} + a_{i2}t \tag{13}$$

$$\sigma_{it} = b_{i0} + b_{i1}t;\ i = 1,2. \tag{14}$$

These parameters could be specified in many different ways. Equations 13 and 14 were chosen to include some richness by allowing μ_{it} and σ_{it} to vary but to keep computation time reasonable. Additional lags were considered but were found to have little additional explanatory power over the data. Conceivably, any variables from Z^s_{t-1} and Z^d_{t-1} could be included in μ_{it} and σ_{it}. On the basis of Equation 11, the reduced form for price comes from embedding these time series specifications into the distribution and integrating them numerically. The time series parameters can then be estimated with a nonlinear estimator, such as maximum likelihood. The specification and approach allow the distributions to be described for each time t and the convergence properties to be tested.

There are many ways to interpret this approach. If heterogeneity exists, this methodology exposes and quantifies it, but it is possible that something else entirely is driving the results. The density function may be misspecified or may be describing an underlying price distribution generated in another way. Regardless, this approach is the first attempt to distill a pattern of heterogeneous price expectations from nothing but price dynamics. The cost of performing market surveys (that may be unreliable anyway) makes this approach all the more valuable as a practical alternative for quantifying market sentiment.

Soybean futures prices provide a natural and important case for empirical study. Oilseeds are important in both domestic and global food systems. They also trade in cash markets that are relatively undisturbed by national government interventions. We analyzed daily closing futures prices (nominal) for the September 1997 futures contract for yellow #1 soybeans traded on the Chicago Board of Trade from August 27, 1996 through September 19, 1997 (270 observations). The data were purchased from Prophet Information Services, Inc. but are available to the public in the *Wall Street Journal*.

On the basis of our specification, 12 parameters were estimated: six a_{ij}'s, four b_{ij}'s, α, and γ. To proceed, we developed estimates through maximum likelihood. Estimates were found to be robust to starting values. This application involves iteration over two steps: (a) integrating over the specified distribution of expectations conditional on a vector of parameter values for a, b, and α and (b) estimating the parameter γ that determines the dispersion of the error in market-wide expectation. The density functions could be specified any number of ways, depending on the modeling assumptions. Exhaustive robustness diagnostics and pretesting were not performed for $\phi(.|\gamma)$. With the distribution function $\phi(.|\gamma)$ of the error in expectation specified as $N(0, \gamma^2)$, the log likelihood of our sample of prices is

$$L = \sum_{t=1}^{269} \log\left\{\phi\left[p_{t+1} - \int_{p_-}^{p^+} pf_t(p|a_{ij},b_{ij},\alpha)dp|\gamma\right]\right\}, \tag{15}$$

where (p_-,p^+) is the support of $f_t(.)$.

Estimation proceeds by the empirical choice of the support interval (p_-,p^+). The choice of support is important for econometric identification of the model. A wide interval such as $(0,+\infty)$ would mean that $\mathbf{p}^e_{t,t-1}$ equals $\alpha\mu_{1t} + (1 - \alpha)\mu_{2t}$, leaving the model unidentified in the parameter space. The lack of identification under infinite support follows intuitively from the definition of the market-wide expectation. Under this specification, the gradient of the likelihood function with respect to α is constant, and any value for $\mathbf{p}^e_{t,t-1}$ can be reproduced by the choice of α, regardless of the remaining parameter values. A finite support interval implies the market-wide expectation $\mathbf{p}^e_{t,t-1}$ is approximated by a definite integral, where the limits of integration are estimated empirically to be consistent with observed data. Furthermore, the use of the finite support renders α identifiable.

A side benefit is that estimation of the finite limits provides estimates of the bounds on the range of variation in agent expectations that are consistent with observed price data. In themselves, these estimated bounds are important evidence of heterogeneity in expectations. Where expectations are homogeneous, the estimates of upper and lower limits

TABLE I

Parameter Estimates for Equations (12 to 15)

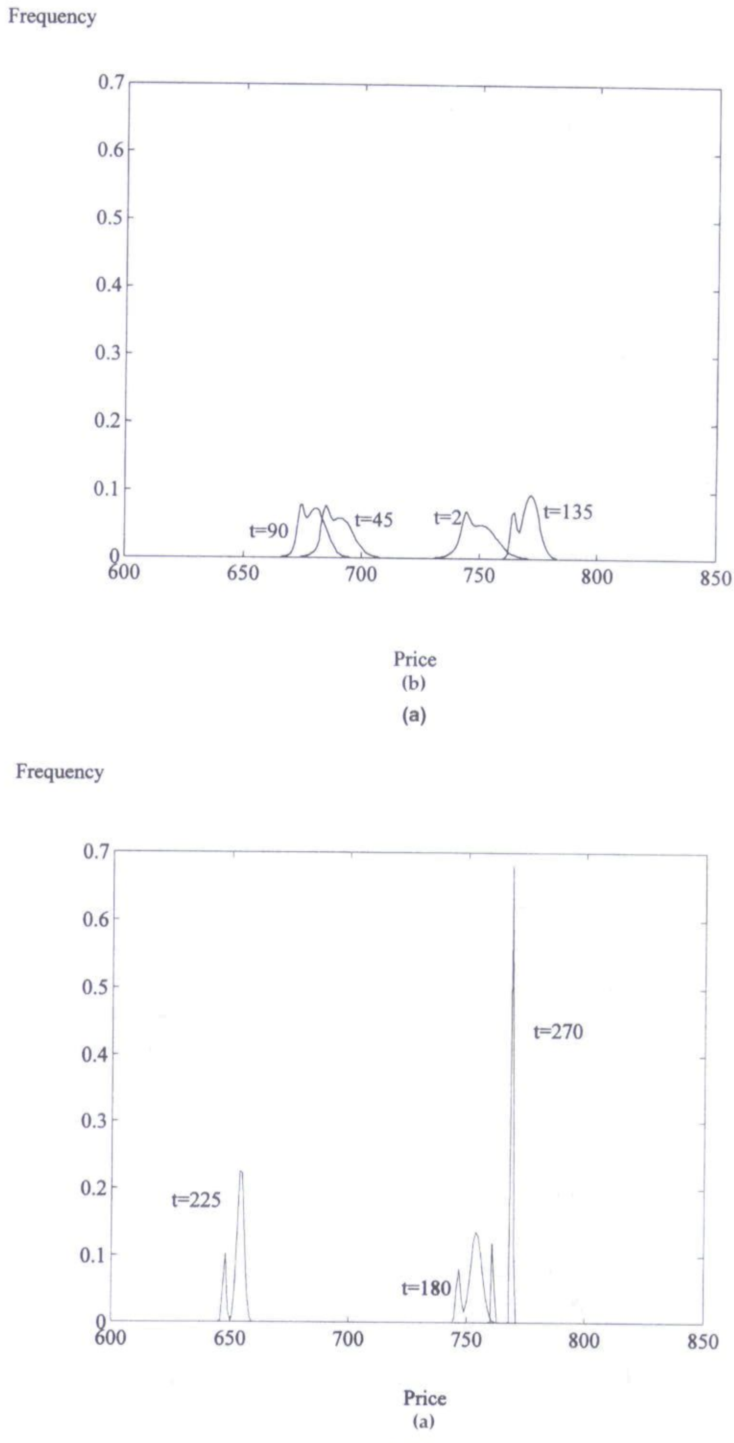
Parameter	*Estimate*	*Asymptotic SE*	t *Ratio*
a_{10}	0.08961	1.028	0.087
a_{20}	−0.1458	0.4352	−0.335
a_{11}	0.9922	0.00058	1,702**
a_{21}	1.0014	6.005×10^{-5}	16,675**
a_{12}	−0.001062	0.002321	−0.458
a_{22}	0.003254	0.001941	1.676*
b_{10}	1.724	0.251	6.859**
b_{20}	6.537	1.072	6.097**
b_{11}	−0.004838	0.000890	−5.434**
b_{21}	−0.02291	0.00385	−5.955**
α	0.1736	0.1024	1.696*
p^{+}	18.57	3.10	6.110**
p_{-}	18.93	2.34	7.924**
γ	8.143		

*95% statistical significance. **99% statistical significance.

would be equal. The support interval is estimated parametrically by maximum likelihood simultaneously with the other 12 parameters.

In an efficient futures market, the expectation of tomorrow's price is today's price. For example, the market is efficient if $a_{i0} = 0$, $a_{i1} = 1$, and $a_{i2} = 0$. As shown in Table I, the estimates for a_{i0} and a_{i2} are close to zero, and the estimates for a_{i1} differ only slightly from one. If every participant's expectations were identical, $b_{ij} = 0$ for all i and j. However, the estimates for b_{i0} are approximately 1.7 and 6.5 cents per bushel. Correspondingly, the estimates for b_{i1} also differ; they are −0.0048 and −0.023. They are negative, implying that agents' price expectations become less dispersed within each mode as expiration approaches, but expectations do not appear to converge fully. The estimate of α is 0.17, indicating that one side of the market is weighted five times more heavily than the other.

The density function changes shape and position every day, based on the previous day's data. Thus, a series of daily frames is the best way to describe the evolution of the expectation distribution over time. Figure 1 displays seven daily frames equally spaced between Day 2 (the 2nd day of trading) and Day 270 (the last day of trading). The distribution initially is closer to being symmetric and unimodal, but as time passes it becomes highly skewed and peaked. Late in the sample, the bullish and bearish groups become more homogeneous, but a gap opens between them. These groups may be likened to speculators and hedgers or to bulls and bears. The distinction between the groups is most evident at expiration.



(b)

FIGURE 1

Estimated distributions of heterogeneous expectations of the closing price after *t* days of trading.

HYPOTHESIS TESTS

Inference is not necessarily straightforward for this model. The standard methods to estimate the covariance matrix of the parameter estimates cannot always be implemented because the domain of definition of the parameters depends on the data, specifically causing a problem if σ_{it} approaches zero for any value of t. If it does, the information matrix cannot be estimated, and maximum likelihood estimation fails. Implementation of numerical maximization algorithms must account for this possibility.

If maximum likelihood estimation succeeds, hypothesis testing can proceed with Wald statistics and t statistics. A Wald statistic was calculated for each hypothesis test and compared with the 10, 5, and 1% significance critical values for the appropriate chi-squared distribution. The results are summarized in Table II.

There are four hypothesis tests to perform: a representative agent test, a unimodal test, a martingale test, and a normal martingale test. The tests indicate whether heterogeneity is present in the sample. The first of these tests is the representative agent test.

The representative agent test determines whether the data could have been generated by a single representative agent. Formally, the representative agent test is

$$H_0\text{: } \alpha = 0 \text{ and } \sigma_{2t} = 0 \text{ versus } H_1\text{: } \alpha > 0 \text{ or } \sigma_{2t} > 0$$

There are only three free parameters under these restrictions, so the number of effective restrictions is 10. The Wald statistic for this test is enormous (2.09×10^9) compared with the 1% critical value for $\chi^2(10)$, 23.21. Therefore, the representative agent hypothesis is rejected at the 99% level of confidence. This result provides statistical evidence that market participants did not always share a common expectation of price.

The unimodal test determines whether the distribution of expectations has distinct modes or humps. Formally, the unimodal hypothesis is

$$H_0\text{: } \mu_{1t} = \mu_{2t} \text{ versus } H_1\text{: } \mu_{1t} \neq \mu_{2t}$$

There are 10 free parameters under this restriction, so the number of effective restrictions is three. The Wald statistic for this test is 3.63×10^5, and the 1% critical value for $\chi^2(3)$ is 11.34. Therefore, the unimodal hypothesis can be rejected at the 99% level of confidence. This result lends strong support to the hypothesis that there were distinct modes corresponding to distinct groups of traders, such as hedgers and speculators.

Another way to test the unimodal hypothesis is to examine the estimates of μ_{it} and σ_{it} without reference to their statistical properties. The

TABLE II

Hypothesis Test Results

Hypothesis	*Wald Statistic*	*Result*
Representative agent	2.09×10^9	Rejected at 99% level
Unimodal	3.63×10^5	Rejected at 99% level
Martingale	2.82	Not rejected
Normal martingale	9.04×10^5	Rejected at 99% level

distinctness of the modes, in an economic sense, depends on the difference between the means of the modes and the spread of the modes. This distinctness property can be quantified by calculation of the overlap area of the two subdistributions. The overlap is the area under both subdistributions and has a maximum value equal to $\max(\alpha, 1 - \alpha)$. In this case, α is 0.17, so the maximum overlap is 0.17. On Day 1, the overlap was 0.153. Midsample, on Day 135, the overlap was 0.050. At the end of the sample, the overlap was effectively zero. The overlap is plotted in Figure 2 as a function of time. Figure 2 indicates that the modes diverged slowly at first and then quickly as expiry approached.

The martingale test determines whether the mean price expectation is the current price:

$$H_0\colon \alpha\mu_{1t} + (1 - \alpha)\mu_{2t} = p_t \text{ versus } H_1\colon \alpha\mu_{1t} + (1 - \alpha)\mu_{2t} \neq p_t$$

There are 10 free parameters under this restriction, so the number of effective restrictions is three. The Wald statistic for this test is 2.82, the 5% critical value for $\chi^2(3)$ is 7.82, and the 10% critical value is 6.25. Therefore, the martingale hypothesis cannot be rejected at either the 95% level of confidence or the 90% level. This result is evidence that prices followed a martingale during the sample period.

A more restrictive test is the normal martingale test, which determines whether price expectations are distributed normally around the current price with a fixed variance ($\sigma_{2t} = \sigma_2$). The martingale hypothesis was not rejected, so this test indicates whether the distribution of expectations changed over time or was nonnormal. Formally, the normal martingale test is

$$H_0\colon \mu_{2t} = p_t,\ \sigma_{2t} = \sigma_2, \text{ and } \alpha = 0 \text{ versus}$$

$$H_1\colon \mu_{it} \neq p_t,\ \sigma_{2t} \neq \sigma_2, \text{ or } \alpha \neq 0$$

There are only three free parameters (σ_2, p^+, and p_-) under this restriction, so the number of effective restrictions is 10. The Wald statistic is

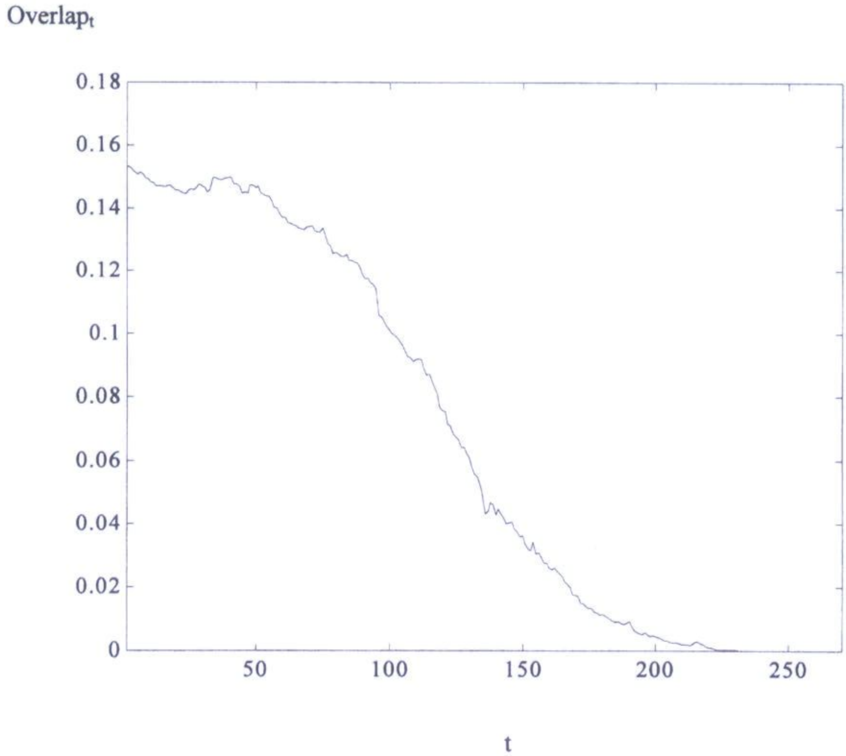


FIGURE 2
Overlap values for subdistributions.

9.04×10^6, and the 1% critical value for $\chi^2(10)$ is 23.21. Therefore, the normal martingale test is rejected at the 99% level of confidence. This result is evidence that the distribution of price expectations around the current price was not normal, verifying the pattern of changes indicated by Figure 1.

Another issue to consider is whether expectations converge over time. The distribution of expectations ought to narrow considerably as information unfolds over the life of the futures contract. Figure 1 displays the convergence properties of the estimated distributions of expectations, but the qualitative results therein are ambiguous on this issue. Quantitative measures of dispersion are therefore preferred, such as mean absolute dispersion (MAD) and mean squared dispersion (MSD):

$$\text{MAD}_t = \int_{p_-}^{p^+} \frac{|p - \bar{p}_t|}{\bar{p}_t} f(p)dp$$

$$\text{MSD}_t = \int_{p_-}^{p^+} \left(\frac{p - \bar{p}_t}{\bar{p}_t}\right)^2 f(p)dp$$

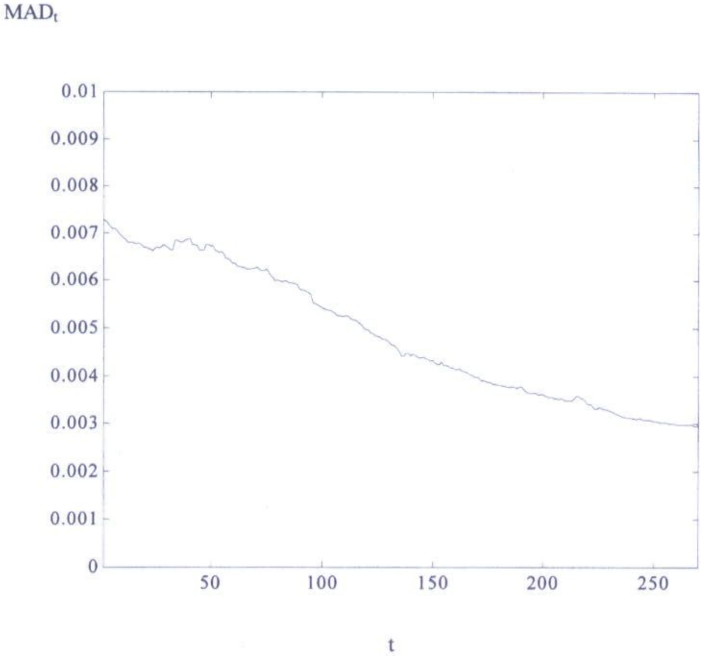
Figure 3 displays the pattern of dispersion under these two metrics. Both are downward sloping, indicating that opinions gradually became less dispersed as information entered the market. Figure 1 indicates that opinions did not converge totally, but it is not clear whether they converged at all. Figure 3 indicates that two common measures of dispersion both declined almost monotonically as the contract neared expiration. Expectations remained heterogeneous throughout, yet somewhat less dispersion was evident.

Overall, the results demonstrate that heterogeneous expectations play a role in futures price dynamics. The coincident existence of hedgers, speculators, and commodity pools makes the market less homogeneous. Prices may follow a martingale, but they do not act in accordance with the representative agent hypothesis. Significant heterogeneity is evident in the market. The representative agent hypothesis is disputable on theoretical grounds because it is not consistent with market trading and the existence of the market place. It has been favored in the past because of intractable aggregation problems associated with heterogeneity. Now, because of improved computing technology, those aggregation problems are more tractable, and heterogeneous expectations may be considered a more serious alternative for modeling commodity market dynamics. The lack of agent-level data on traders' expectation formation processes remains an obstacle to empirical work.

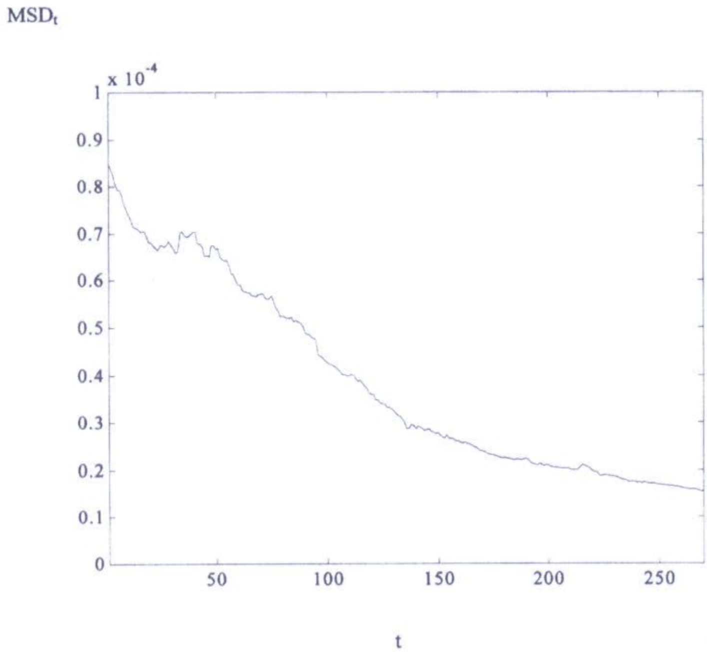
CONCLUSION

Homogeneous expectations and the representative agent hypothesis are incompatible with what we know to be true about markets: that traders have different visions of the future, take positions on opposite sides of the market, and initiate speculative trade that determines observable prices in contingent markets. Nonetheless, hypotheses of homogeneity and representative agents have been maintained in the past for the sake of convenience or because aggregating over heterogeneous agents is difficult. Overall, empirical validation of these hypotheses has been mixed, and authors have recommended that better ways to incorporate heterogeneity to economic models should be developed.

The major result of this article is that the distribution of heterogeneous price expectations is estimable and may be forecast over time. This achievement was attained by the imposition of a parametric structure on the distribution of expectations. The structure we imposed was a mixture of two normal distributions, representing bulls and bears in the market. Structural flexibility could be improved by an increase in the number of



(a)



(b)

FIGURE 3
Convergence of expectations over time: (a) mean absolute deviation and (b) mean squared deviation.

distributions in the mixture, with distributions other than the normal, or an increase in the number of explanatory variables and parameters in the model.

There were several limitations to the methodological approach. For example, the expectations density function, $f_t(.)$, can be interpreted as evidence of heterogeneity or in another way. Transaction costs, nonlinear dynamics, distributional assumptions, or some other model misspecification could be affecting the empirical results. If so, $f_t(.)$ is just another general specification for prices, and its parameters are not imbued with any structural meaning. If, however, misspecification has not tainted the results, the parameters of the model have clear structural interpretations consistent with heterogeneous expectations.

The model was applied to the U.S. soybean futures market as an illustration. Several hypotheses were tested, including the representative agent hypothesis and the hypothesis that bulls and bears are indistinguishable. The most important empirical result was that the representative agent hypothesis was rejected at the 95% level of confidence. Further analysis confirmed the existence of distinct groups of bulls and bears that differed sufficiently in their expectations to suggest substantial agent-level incentives for trade. In addition to this economic significance, statistical significance of heterogeneity was also evident. This result indicates that trading behavior is not characterized by a unique homogeneous expectation.

Indeed, these results were consistent with our priors; we would have been surprised to find the opposite. Speculative agricultural markets have existed for centuries, and anecdotal evidence abounds that traders disagree. If they did not, markets could not function and would cease to exist. The contribution of this article has been to verify and quantify this age-old phenomenon.

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