Clustering and Psychological Barriers: The Importance of Numbers

JASON MITCHELL

The contemporary press frequently makes mention of, and identifies significance with, specific numerical values in financial markets. This focus has been suggested to result in clustering. Furthermore, these symbolic numbers are often referred to as *psychological barriers*. These effects are identified as a widespread phenomenon permeating into the economic decision making and financial market environments. This article outlines various arguments and rationale from the cultural, economic, and behavioral literature why clustering and other effects such as psychological barriers may be expected in financial markets. Evidence across a variety of literatures suggests that financial market clustering derives from a variety of sources. There is evidence from a cultural and conventional basis to suggest that number preference exists and that there is a natural tendency to round that derives from the development of the modern decimal system. The literature also provides valid behavioral and economic reasons as to why these effects may occur. However, no evidence is found to support the notion that clustering or barriers in financial markets

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For correspondence, Jason Mitchell, Department of Accountancy, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, SAR, China; e-mail: acjason@inet.polyu.edu.hk

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■ *Jason Mitchell is an Assistant Professor in the Department of Accountancy at Hong Kong Polytechnic University in Kowloon, Hong Kong.*

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would occur as a result of a natural order or as a product of the number progression or simply from the numbers themselves. Jrl Fut Mark 21:395–428, 2001

One year, the winner of the Christmas drawing of the Spanish National Lottery, the El Gordo, was interviewed on television. He was asked: "How did you do it? How did you know what ticket to buy?" Our winner replied that he had searched for a vendor who could sell him a ticket ending in 48. "Why 48?" he was asked. "Well I dreamed of the number seven for seven nights in a row, and since seven times seven is 48 . . . "

—*Russo and Schoemaker (1989)*

INTRODUCTION

The common fascination and preoccupation with symbolic or interesting numbers in general life also pervades financial markets. For instance, although many economists regard the financial markets as a paragon of efficiency, traders often talk about "reading the mood of the market" and the importance of "identifying and trading on support and resistance levels" (Slezak, 1989; Tvede, 1990, pp. 35–36). Similarly, the fixation with rounded numbers is contradictory at a superficial level to the ideas of market efficiency and rationality (Donaldson, 1990). It also contradicts an underlying assumption of neoclassical demand theory, whereby demand functions are assumed to be homogeneous of degree zero with regard to prices. The existence of such seemingly number-induced effects may give rise to clustering, psychological barriers, or both in financial market prices. This study surveys the literature and considers to what extent observed number preferences bias decision making and give rise to psychological barriers or clustering in financial market prices.

If clustering or psychological barriers occur, the inference is that traders are imputing some form of relevant information into a particular digit of price or barrier. If this is persistent, it also implies the information content of current prices is relevant for future prices. This suggests current asset prices are influenced by or depend on past prices, and possible trading strategies exist to take advantage of this information. This is akin to the rationale originally suggested by Osborne (1965) and Niederhoffer (1965) why clustering should not be observed for efficient markets. The theoretical irrelevance of numbers is even stronger in some markets, for instance, foreign exchange markets (Mitchell, 1998). This is because (a) the fact that an exchange rate ends with or contains a zero or any other number should be irrelevant information, as an exchange rate can always be transformed by multiplication of some arbitrary number, that is, a constant, and (b) an exchange rate can always be defined in two ways, namely, a stated value and its inverse.[1]

Questions considered in this study are: Do individuals exhibit number preference? Is there a bias in decision making, recording, or measuring that suggests that clustering is possible? Do certain types of data or particular series exhibit clustering because of a natural order, or is clustering in data a property of the number system itself? The consideration of these issues provides prima facie support for the clustering of data in general and, furthermore, an a priori expectation of the degree of clustering and other related effects in financial markets.

The article proceeds as follows. A review of the literature as well as anecdotal evidence of clustering and psychological barriers is provided in the second section. In the third section, a brief overview of the concepts of clustering and psychological barriers is documented. The fourth examines the importance of the concept of numbers as it relates to decision making and financial markets. This is done to understand why clustering potentially occurs in a variety of natural and manmade measurements or data and why it may, or may not, be expected in financial markets. It covers a brief illustration of the preference and behavioral quirks related to certain numbers used in decision making. It examines the history, importance, and meaning of numbers; the incidences of number clustering displayed in measurement, recording, and economic decisions; and behavioral explanations for this clustering. In addition, this section discusses whether some data observations display clustering as a result of some inherent law of nature or as a product of the number system. The last section contains the conclusion and implications.

REVIEW OF THE LITERATURE AND ANECDOTAL EVIDENCE

Overview of the Literature on Clustering and Psychological Barriers

There has recently been a renewed focus on the clustering phenomenon in relation to financial markets. This focus has predominantly been on

[1]Mitchell (1998) commented that if the AUD/DEM (German Deutsche Mark to Australian Dollar) rate is, for example, 3.00, a round number, the corresponding DEM/AUD (Australian Dollar to German Deutsche Mark) rate is 0.33333°, an unrounded number. All exchange rates can be priced according to the designated currency of interest. All foreign exchange representations in this article are made in terms of the conventional written form; that is, the first currency is the unit item whose value is being quoted (McGrath, 1994).

the U.S. and other equity markets (Aitken, Brown, Buckland, Izan, & Walter, 1996; Buckland, 1994; Hameed & Terry, 1998; Harris, 1991; Osborne, 1962) and, to a lesser degree, on currency markets (De Grauwe & Decupere, 1992; Goodhart & Curcio, 1991). Clustering is further documented in derivatives, that is, options and futures markets (Gwilym, Clare, & Thomas, 1998a, 1998b), and other financial markets and variables.[2] The recent renewed interest in the clustering phenomenon is in part driven by the need to understand and explain why it occurs in financial markets. It is potentially of interest and concern as it contradicts at a superficial level the notion of rationality or efficient markets (Aitken et al., 1996). In addition, it is contrary to the tenet of a uniform value in prices (De Grauwe & Decupere, 1992; Niederhoffer, 1965, 1966).

In a similar vein, psychological barriers have attracted a great deal of interest over recent years (see Donaldson, 1990; Donaldson & Kim, 1991, 1993; Koedijk & Stork, 1994; and Ley & Varian, 1994, in relation to stock indexes, and De Grauwe & Decupere, 1992, for currency markets). This again stems from the symbolic number syndrome noted previously. Anecdotal evidence to suggest that psychological barriers are perceived to exist in a variety of financial markets is discussed in more detail later. The anecdotal evidence suggests that a section of the investment community forms the view that behavioral and psychological factors are important in the setting of price.

If one makes the assumption that participants or traders in the market are well-informed price setters, those activities such as economically irrational price clustering and psychological barriers should not be observed. Such erratic and nonoptimal behavior will be traded out of the market. However, clustering, behavioral factors, or both may not be traded out of financial markets under the following conditions: (a) a few traders or trades dominate the market, (b) trading impediments exist that prevent the observance of some values, (c) number preference is widespread, and (d) there is a bias in the decision-making environment.

A progressive focus has arisen on the behavioral aspects of participants making judgements and economic decisions (for summaries of this literature, see Kahneman, Slovic, & Tversky, 1982; Plous, 1993; Thaler, 1991) and also as part of consumer theory (Thaler, 1980, 1985; Thaler & Johnson, 1990). This interest is again motivated by the need to explain whether economic decisions, such as in the setting of price and of which price clustering is a part, are consistent with economic rationality. In the literature on experimental markets, Plott (1986), among others, showed

[2]Buckland (1994) documented that in addition to clustering in bid, offer, and trade prices of equity markets, the phenomenon exists in initial public offer prices, takeover bids, and rights issues.

that individuals display frequently erratic or irrational behavior. Although in an overall sense rational behavior seems to reflect market actions adequately, there are some notable exceptions. For financial markets, Wood (1989), Thaler (1991, 1992, 1993), and Heisler (1994) all stressed the importance of behavioral phenomena. A general awareness of both conscious and subconscious behavioral factors that exist in price setting has become increasingly evident. Roll (1988) in his presidential address to the American Finance Association noted that much of the variation in speculative prices is not explained by the arrival of fundamental economic information.

Implications for Trading

Some examples of how traders can take advantage of the aforementioned effects have in fact been tested. Niederhoffer (1965, 1966) concluded that profitable trading rules existed based on the clustering of transaction frequencies after allowing for the bid–ask spread. As a general overview, these trading rules are based on the supposition that a reversal in the direction of the price of the financial asset becomes likely as the price moves to some round number. An example of this is provided in Niederhoffer (1965, 1966). Niederhoffer and Osborne (1966) also documented a dependent structure in the form of direction reversals in the movement of prices.[3] This was partly a result of the nonuniform distribution of orders that produces some nonrandom effects in price movements. Again, traders cognizant of clustering and the dependent effects can take advantage of this for trading opportunities.

Curcio and Goodhart's (1992) tested predictions that most technical analysts tend to accept, namely, that particular levels of the exchange rate, called *support* and *resistance*, can provide useful buying and selling signals. They investigated this for three exchange rate series, the DEM, JPY, and GBP, priced in terms of the USD. Signals generated with these chartist support and resistance levels as inputs were found to be more frequent and profitable than other trading rules. This confirmed previous work by Brock, Lakonishok, and LeBaron (1991) and Levich and Thomas (1991) that trading range breaks do generate profitable signals relative to simple buy and hold strategies, even after the inclusion of transaction costs. These support and resistance levels may work by warning traders against

[3]Price reversals are more concentrated at integers where stable, slow moving participants offer to buy and sell (Niederhoffer & Osborne, 1966).

holding currencies subject to adverse trends.[4] The identification of support, resistance, and other psychological barriers is thus of interest in terms of generating trading rules to take advantage of this information.

Anecdotal Evidence on Number Focus and Psychological Barriers

Financial markets, from society in general, have an affinity with round, identifiable, symbolic or interesting numbers. Aitken et al. (1996) noted that rounding fractions to the nearest whole number, rather than to the alternative +0.00001, is an efficient measurement convention entrenched in and preserved by the educational system. Literature on stock market price clustering originated from illustrations by Gann (1930), who found that "the popular trading prices are 25, 40, 50, 60, 75, 100" (pp. 63–64), and Wyckoff (1963), who declared that "we think in round numbers and try to sell at round numbers" (pp. 106–107). Furthermore, there is an increasing tendency to refer to these numbers in a behavioral context. Expressions including *psychological barriers, psychological levels,* and *critical levels* or simply *barriers, support,* and *resistance levels* permeate the financial press, technical analysis, and trade investment literature.[5]

Psychological Barriers in Financial Markets

The financial press frequently reports the movement of financial market prices in terms of barriers, psychological levels, and critical levels. This pervades across all the financial markets, namely, equity, bond, gold, commodities, and futures markets. For share markets, the main concern appears to be on zero-based round number levels of the prime market index, such as the 10,000 level of the Dow Jones Industrial Index or the 2,000 level of the All Ordinaries Index in Australia.[6] Analogous emphasis occurs

[4]One of the fundamental claims of technical analysis is that investor psychology is important and that the level at which an investor has bought an asset is an important determinant of the level at which he will sell. The technical analysis literature reports several alternative ways of computing where support and resistance levels are going to occur (Levich & Thomas, 1991). However, they all seem to agree on one prediction, that once a support or resistance level has been broken, this is a signal that a trend in that direction has started and that it is likely to continue.

[5]The article "In Search of Inefficiency" in the Economist (Economic issues, 5/9/1995, p. 85) cited De Grauwe and Decupere (1992) and Curcio and Goodhart (1992) and considered the issue of inefficiency arising from psychological barriers.

[6]"Added to the market's gnawing uncertainty about where the $US rates are headed is a growing 'roundophobia', the word coined by the UK strategist David Fuller to describe the markets fear of round numbers—in Australia's case the looming 2000 point on the All Ordinaries index" (Chartists warn of super bear, Dunstan, AFR 20/4/94, p. 21).

for the Nikkei and the Dow, mainly on multiple levels of 100 and 1,000.[7] Gold, oil, and other commodity markets have their own significant numbers. The obvious example is the preoccupation of the gold market and the community at large with the USD 400 per ounce mark. Here again is a rounded number feature that, apart from tradition, has little to justify its significance. The same focus is apparent for futures[8] and currency markets.[9]

Discussion of Anecdotal Evidence

This demonstrates that behavioral and psychological effects have widespread acceptance in markets. It seems tenable that people do or are conditioned to attach unwarranted significance to numbers across a wide variety of situations. The media continually reinforces this widespread acceptance. Shiller (1988) noted the importance of the media in inciting or raising public awareness to patterns of social interactions and belief. Shiller also noted the lumpiness or irregular concern of the media. This suggests these behavioral effects may not be continually evident.

The fact that clustering occurs leads to the suggestion that some individuals may have an impediment to the selection of or preference for various digits. What is also interesting is that in many cases the focus of the market appears to be self-induced. It is only in a few cases that the psychological barrier or resistance point is coupled with a commentary regarding the basis for such a conclusion. Occasionally, it is suggested that barriers relate to fundamentals, but more often than not they appear to be derived from anchoring or *roundophobia*, the fear of round numbers syndrome, where the round number gives a focus or an anchor for decision making rather than relating to underlying economic value.

[7] "The Yen's strength against the US dollar depressed Tokyo stocks yesterday and the 225-Share Nikkei average closed lower after briefly falling bellow a psychological support level of 16,000 in the morning" (Strong yen slugs Nikkei, Briefs, AFR 21/3/95, p. 34).

[8] "If (Share Price Index) futures break through the 2100 level and close above it, it would add back psychological and technical confidence to the market." (Future traders look to sustained upward trend, Salmons, AFR 13/4/94, p. 28).

[9] Mitchell (1998) noted that the focus on psychological barriers for the AUD is almost exclusively reported relative to the USD. Round numbers such as USD 70¢ and USD 75¢ receive the predominant market focus and the majority of the press coverage. However, press coverage and market attention is not always restricted to round numbers. Numbers such as USD 69¢, USD 72¢, and even USD 68.30¢ and USD 72.2¢ are reported as having, or warranting, special attention or significance by the market.

DISTINGUISHING THE NOTIONS OF CLUSTERING AND PSYCHOLOGICAL BARRIERS

This article adopts the view that psychological barriers are different from and are not necessarily related to clustering in financial markets.

Clustering is broadly defined as the grouping together or congestion of items. *Price clustering* is a concentration of the distribution associated with a particular value or values when various values of price (digits) occur more frequently than other prices (digits). However, the notion of *barriers* suggest a dealer's reluctance to trade at or approach certain values of price, and *psychological barriers* rely on the interpretation of the findings for why the individual or market is hesitant to approach or break the barrier. In a general sense, a psychological barrier can be viewed as an impediment to an individual's mental outlook, that is, an obstacle created by the mind, barring advance or preventing access. Thus, although the items may have similar behavioral interpretations and explanations for their existence, they do not necessarily occur synonymously, nor must they be related.

The view here is that clustering does not necessarily imply that a barrier, psychological or otherwise, exists. On a conceptual level, a barrier can exist regardless of the degree of clustering observed, and clustering can occur regardless of the price or value being a barrier, or not. Clustering is thus neither a necessary nor sufficient condition for a (psychological) barrier, and the two concepts are essentially different. Clustering or the concentration of digits or prices or the reluctance to approach various values can be linked to various explanations. In some instances, the potential explanation for clustering may also be a potential reason for a barrier and, if it is behaviorally induced, a psychological barrier.[10]

It is clear that the importance of numbers, number preference, and the related explanations relate to both clustering and additional behavioral effects associated with numbers, such as psychological barriers. For convenience, the focus will be on the aspect of clustering from this point on. Accordingly, the reader is reminded that the rationale presented in the remainder of the article is not restricted solely to explaining why clustering occurs but also may provide insight and explanation as to why other related number-induced effects such as psychological barriers may be expected in financial markets.

[10]This distinction in this section is further elaborated by Mitchell (1998). In relation to the empirical evidence testing of such barriers (see De Grauwe & Decupere, 1992; Koedijk & Stork, 1994; Ley & Varian, 1994), Mitchell also noted that the correct interpretation of barriers is more likely an explanation of a positional transgressional effect, that is, movement across or passing the number of interest rather than any clustering effect.

NUMBER PREFERENCE: MYTH OR REALTY?

Illustration of Number Preference

The opening quotation depicting the selection process of the winner of the Spanish Lottery illustrates how number preference or behavioral aspects of decision making may eventually lead to number clustering or psychological barriers.

Specifically, an unwarranted significance is placed on certain numbers, in the aforementioned case, the number seven. Logic or a rational approach would suggest that any number based on an equiproportionate likelihood would be the appropriate decision.[11] This decision was, therefore, based on a behavioral reaction to numbers, regardless of whether this was considered logical. In fact, the decision maker displayed an intrinsic form of number preference. The second decision represents a violation of basic arithmetic and was obviously incorrect, although the decision maker was none the wiser. More importantly, the individual was not logical or optimal as presumably no time or effort was made to work out the answer. The individual in this instance could be called a *noncalculator.*[12] The decision can be viewed as an extreme example of how behavioral factors and quirks, that is, heuristics and biases, permeate decision making.[13] Generally, this results in inconsistent decisions, when contrasted to the decision maker's previous decisions, or incorrect, as against some obvious rule. A pattern will emerge if such decision processes are perpetuated through all lottery players or over a length of time. A clustering (concentration of observations) will arise, either from a similar selection of numbers, that is, number preference, or a consistent bias in the decisions from a fixed set of inputs. Similarly, a psychological barrier could be viewed as occurring in the situation where clustering per-

[11]Lottery gambles have an equiproportionate likelihood of a number outcome but not an equiproportionate payoff because of number preference that reduces the payoff for frequently selected numbers.

[12]This description is akin to the behavioral decision research literature's non-Bayesian (pseudoleper) status, alluding to individuals who fail to allow for prior probabilities, or the nonstatistician, that is, one who fails to take account of the power of the sample, or other similar nonoptimizers commonly encountered in this literature.

[13]Plous (1993) noted that heuristics are general rules of thumb that are used to arrive at judgements or to make decisions. They reduce the time and effort required in making decisions by giving rough approximations or estimates for the decision in question rather than adopting more formal logic processes and calculations that would give more precise estimates. Kahneman et al. (1982) demonstrated how individuals frequently employ representative, availability and anchoring and adjustment heuristics or biases in general decision making. Thaler (1986, 1987) provided further support that these and other peculiarities permeate into economic and financial decisions at the individual and aggregate level, which leads to the premise that behavioral forces are evident and important within financial markets.

petuates itself or appears irrational, and there is no alternative rational explanation for the continued observation, that is, where individuals repeatedly exhibit number preference or have a bias in their decision practices.

As a general framework, it is possible for clustering to potentially arise from a variety of sources: (a) number preference; (b) a bias in decision making or judgement practices caused by behavioral factors, economic factors, or both; (c) the natural order of things, as it is argued some data display a natural clustering; or (d) the way in which numbers progress, that is, clustering arises because of numbers themselves. From the outset, it is important to distinguish the first two categories. The notion of number preference per se or the inherent importance assigned to numbers is seen to be a distinct conscious choice. The second category considers the use of, or abstraction of, a given number in a decision-making or judgmental context. Naturally, there is some overlap because often in making any decision, reading a scale, or recording a number, a selection process is involved. However, in the second category the bias is more unconscious judgement rather than an intrinsic conscious number preference. Given that clustering may arise from one of the aforementioned scenarios, it is thus a natural progression to determine whether these conditions exist or whether there is any evidence to support the assumption that clustering, in a general sense, is possible.

Development and Importance of Numbers

The origin of number development in terms of both counting and numerals, as well as a consideration of the "What are numbers?" question, either on a metaphysical or scientific basis and regardless of interpretation, provides valuable input for understanding number preference behavior and the culture of number preference that exists today.

Development of Numbers

An important aspect that arises from the development of counting is the way in which humans view numbers. It seems clear that we like to think, count, or measure in distinct groups or patterns. This facilitates the notion of group-to-group correspondence. It allows the decomposition of large numbers into subgroups and places a decreased demand on memory and cognitive processing ability. Even at the rudimentary level of counting and in collections and measurement systems largely derived therefrom, there is a preference for decimal (10), duodecimal (12), or vigesimal (20) systems.

The history of numerals (see Schimmel, 1993, and Appendix A) highlights two important points: (a) In terms of numerology, numbers are associated with various degrees of significance (magical powers) that influences number preference among individuals and (b) the development of our present fully ciphered decimal system has arisen in preference to other forms of numeral script. The place–value system of the decimal notation is particularly relevant in reinforcing the importance of powers of 10 and the group association with them. It is evident that conditioning to groups and powers of 10 is enforced and entrenched in the decimal number system itself, virtually to the exclusion of other systems with different bases.[14]

Modern Numbers, Symbolism, and Numerology

Modern Numbers. Number counting and number representation (ciphers) of the decimal system suggest a natural tendency to think in terms of 10s or powers of 10. The ability and desire to decompose numbers into these groups are facilitated by the decimal system.[15] Grouping by the powers of 10 facilitates the mental order and the correspondence process and provides computationally convenient breaks between groups. Psychological importance is thus placed when a number moves from one collective group to another. The numerousness associated with the next group is 10 times that of the previous group. This is regardless of the fact that the actual difference is considerably less, for example, 950 relative to 1,050. This numerousness is also reflected in the word association of thousands relative to hundreds, for instance.

The place–value system enhances the focus on the leading digit, or where there are many digits, the leading digits, which convey the most necessary information.[16] A digit (d) is, for example, three decimal places

[14]"In modern times, of course, the decimal (base ten) system is in worldwide use among civilised peoples and will undoubtedly continue to be so. But lest the complacent think, Pangoss-like, that Divine Providence has given us ten fingers because we get such a fine number system from it, I remark again that base ten while adequate is not ideal. For practical purposes i.e., computation, it would have been better if we had been endowed with twelve fingers so the base twelve might have been chosen giving rise to what is called the duodecimal system. Or on the other hand for theoretical reasons, i.e., the investigations of mathematicians into the deeper secrets of numbers, a prime number such as seven or eleven would have been a better choice" (Dubisch, 1952, p. 12).

[15]Numbers such as hundreds, thousands, millions, or googols (10^{100}) would not have the same fascination or preoccupation under a duodecimal system, for example.

[16]The term *digit* is employed in a mathematical context, that is, the 10 Arabic numerals, 0 to 9, currently used in the decimal number system.

away from the first digit in a number. The digit must be at least 1, the initial digit must be 1, and the intervening digits must be at least 0. In this case, the selected digit has a value in relation to the number of $d/(1{,}000 + d)$, where d is the selected digit value. If the selected digit value (d) is the maximum of 9, the digit value in relation to the number is about 0.9%. The bulk of the value contained in the first digits suggests (a) the trailing digits are redundant for quick or approximate computational purposes and (b) emphasis should be placed on the first digits for impromptu (ad hoc) mental decisions and calculations. The second suggestion is important in light of the numerousness concept of moving to another (higher) group.

In terms of consumer behavior, the use of $999 as a ticket price rather than $1,000 relies on this concept. $999 involves one less decimal place so the numerousness association would suggest the value is not in the thousands. Also, any relative association of price, that is, comparable or substitute products, is made with the first digits. As these are in hundreds, that will be the relative benchmark. Obviously, if the calculations are performed precisely, the ideas of numerousness and rough relative comparisons would not hold. However, as reflected in the human judgement theory and behavioral decision research literature (Thaler, 1980), similar decision aids to the numerousness concept are employed and often result in mental illusions. Individuals are governed by bounded rationality (Simon, 1955) as they do not make the effort to retain all the information necessary for complete rationality. Similarly, individuals display nonoptimal search cost behavior for information and do not precisely calculate or formulate decisions even when they have the capacity to do so (Thaler, 1980).

Symbolism and Numerology. It seems logical then that increased psychological significance should occur at 10s and increasing powers of 10. Moving into the next decimal digit group alters the degree of numerousness. This presumably has the effect of imparting a certain significance, consciously or subconsciously, to the number. Number symbolism reinforces this, originating from the tetrad/decad where 10 (or powers of 10) is the perfect number because it is a count of all members of that group. Numbers below 10, that is, 9, are symbolic of less than group totality. Similarly, 11 and 12 denote movement beyond the current group representation (Schimmel, 1993, gave details of symbolism and the importance assigned to various numbers). The same holds true for all powers of 10. The similarity of number symbolism and the psychological interpretation of the decimal system are readily apparent, but it is not clear whether the two interpretations have a common source. Today, numerology is widespread. It is found in astrology, superstition, and fear of the numbers 4 and 13 (see Bell, 1933)[17] and has even graduated to become part of sharemarket folklaw (see Appendix B).[18]

Overview: The Importance of Numbers

The introduction to this section illustrated number preference and behavioral influences in the context of lottery gambles. Clear evidence that number superstition or symbolism plays a role in the process of number selection is provided by research into lottery numbers. Ziemba, Brumelle, Gautier, and Schwartz (1986) investigated the 6/49 Lotto system in the United States and found that some 15 to 20 of the lotto numbers were unpopular. These unpopular numbers were high numbers, that is, nonbirthdays, possibly because of the availability heuristic, and also numbers ending in 0, 8, or 9. The most popular number selected was 7. Of interest is the fact that some numbers remain unpopular despite their selection having higher rewards, that is, increasing the payoff from $0.50 to $2.25 for a $1 bet. Although some learning effect is noticed, it has not removed the unpopularity. This leads to the paradoxical prediction that superficially in an efficient market "no one would choose the most popular numbers" (Thaler, 1992, p. 138). Both number symbolism and the psychological framework of numerousness, existing from the development of the decimal system, suggest that number preference is a reality; numbers in themselves have importance. Decision rules preserving aspects of the mental order and numerousness concepts are propagated by the decimal place–value system.

Clustering as a Behavioral and Psychological Issue for Individuals

Clustering and Behavioral Effects

In a seminal article, Tversky and Kahneman (1974) argued that in many decision contexts individuals rely on a number of heuristic principles that reduce complex tasks such as assigning probabilities and predicting values to simpler and, in some cases, nonoptimal judgmental operations. Exten-

[17]Fear of the number 13 is called *triskaidekaphobia*.
[18]Appendix B discusses numerology in sharemarket investing through the use of the Fibonacci ratio to determine investment strategies (Armstrong, 1991; Fischer, 1993; Slezak, 1989).

sive literature has been devoted to expounding and elucidating the intricacies of these and other heuristics (behavioral impairments) on judgements and decision processes (see Kahneman et al., 1982). An implication of these behavioral idiosyncrasies (heuristics) is that clustering or overrepresentation of round and other preferred numbers is a distinct possibility in economic decision making.

Psychological experiments demonstrate generally that clustering of outcomes at round and other numbers is a fundamental attribute of human behavior. This is evident in terms of simply recording or measuring items, analyzing responses to numerical stimuli, and making decisions. A further crucial factor is that this clustering effect translates to economic decision making at both the individual and aggregate level. Two explanations for this are the number preference phenomenon already documented and the fact that individuals use decision rules for numbers. These decision rules may be dictated by convention, cultural factors, and psychological stimuli, or they may simply be attempts to simplify the information level or overcome uncertainty. These behaviors seem to be widespread, and it is both logical and tenable that they permeate financial decision making.

Documentation of Clustering on Reading Scales and Recording Primary Variates

On Reading a Scale. Yule (1927) documented evidence of clustering at round or other numbers that arose from either reading a scale or recording data measures that are termed *primary variates.*[19] Through an examination of the final digits of recorded measurements, Yule demonstrated the human tendency to select and identify with round numbers (0s), halves (5s), quarters, and even numbers.[20] There was, however, a tendency for the clustering effect to differ depending on the type of measurement observed and also the observer. For instance, there was a greater occurrence of 5s as a final digit, especially on scales where the numbers

[19]Preece (1981) noted that readings from a scale for physical apparatus measurements and counting are *primary variates. Derived variates* are those obtained by arithmetic calculation from primary variates.

[20]To illustrate, from the 1911 population census of England and Wales, the final digits for the age last birthday response for both males and females indicated an excess of frequency on the 0, 2, 5, 6, 8, and 9 (with a smaller excess for 5 and 6) and, correspondingly, a deficit on 1, 3, 4, and 7. 0 followed by 8 had the greatest excess, and 1 had the greatest deficit followed by 7 and 3. Mallet and Stevenson (1913) suggested that the misstatements of age occur from three errors: (a) round number, (b) age next birthday, and (c) even number errors. In addition to these three, there were also many potential willful misstatements of age.

were visible to the recorder or had graduating marks of various lengths, that was attributed to a rounding effect. By contrast, there were other instances where an disproportionately low number of 5s resulted. The second situation was construed as an odd-number effect.

Rosenbaum (1954) observed an excess of even digits relative to odd digits in the final digits of recorded heights and weights of army inductees. Also, Keen (1966) reported a concentration at round and even numbers for measurements of characteristics of electrical valves, and Cunliffe (1976) noted the concentration of whole numbers and round numbers in measurements of beer-cask volumes and beer-bottle contents. Stockwell (1966) further confirmed Yule's (1927) observation of patterns (clustering) in certain digits of age statistics of a national census. Yule (1927) and Preece (1981) in an equally comprehensive article documented a plethora of other similar instances of clustering. In a recent article, integer clustering was evident in the Current Population Survey employment data, as respondents typically reported the length of their current unemployment spell as an integer of 1 month (Baker, 1992).

Behavioral Explanations for Clustering. Behavioral explanations of the nonuniformity of the final digits of scale readings and recording of other primary variates have long been recognized. Yule (1927), as in many of the studies mentioned, stated that one explanation was the preference or attraction to certain digits and the avoidance of others. Myers (1906) indicated that there is an unconscious tendency to record nearby measurements as a whole number, namely, round, and that this action is a product of convention or culture. Yule argued that the different graduations on some measurement scales or the displayed numbers themselves draw the reader "with the force of personal suggestion" (p. 571).[21] Kolthoff and Sandell (1943) noted that when selecting a number in small measurements, individuals are likely to be influenced by the previous number selected. This suggests that a form of anchoring and adjustment or availability heuristic was prevalent (see Tversky & Kahneman, 1973, 1974); that is, previously known or provided values were employed in the selection of uncertain values.

Haziness, Imprecision of Measurement, and Mixed Degrees of Precision. Preece (1981) also pointed out that clustering arises from the uncertainty of the measurement, that is, the inability to decide (discriminate) between

[21] Yule (1927) commented that the bias on the 0s and 5s and so forth stems from a number stimulus. The measurer is able to identify the scale with numeration and/or graduation of various lengths. Again, this is reminiscent of an anchoring effect where the digit or numerical value selected is influenced by a value that is available. This led Yule to suggest that individuals should not see the numbers or graduations.

two adjacent values. A common psychological reaction used to counter this uncertainty was to adopt a measuring rule such as rounding or the selection of even numbers (Miller, 1950). Carr and Garner (1952) found that subjects tended to round off readings when the measuring interval was small rather than large, suggesting a lack of ability in discriminating values when the measuring scale was very finely partitioned. Clustering can thus be related to the degree of uncertainty (haziness) of the value being measured. Moreover, the subconscious assumption that the estimated figure was of little importance led to the measurer considering that the figure was immaterial. Consequently, increased clustering was observed where the figure was perceived to be immaterial. Richards (1925), for instance, demonstrated that where the importance of the observational unit was stressed, precision was high and little nonuniformity (clustering) was observed.

Both Yule (1927) and Preece (1981) suggested that the degree of departure from uniformity (clustering) would differ, or be mixed, between informed and uninformed recorders. This was as a result of various abilities in coping with and recording the items with the desired degree of precision. Corns and Corns (1972) confirmed that the precision of autoanalyzer chart readings was related to the experience of the technicians and the terminal digits of the number label presented on the actual chart, that is, further evidence of number fixation or an anchoring effect.

Expectation of a Uniform Distribution. The expectation of a uniform distribution for the digit of interest is related to the range (magnitude) of the measurements available compared to the range (magnitude) of the unit represented by the final or other digit of interest.[22] This naturally applies to any digit, not just a final digit. There is need to evaluate (a) the range (magnitude) of the data or measurement and (b) the ranges, both potential and realizable, of the digit(s) of interest and (c) to compare the ranges of (b) and (a), the *relative data range.* A further unrelated issue is to consider the number of observations in the data series. The greater the sample is, the less likely that any idiosyncrasies, that is, sample bias, will occur.

The previous quote from Yule focuses on the relative data range of the measurement (data) rather than a formal analysis of the functional

[22]According to Yule (1927), "It is obvious that the measurements available must cover a range considerable as compared with the unit represented by the final digit, or the frequency distribution of the final digits will be affected by the form of the distribution of the measurements themselves. Thus if statures are measured to a millimetre, it is safe to assume that absolutely correct measurements should give a uniform distribution of the final digits, for statures of humans have a range in most populations of the order of 50 cm. But it will not be safe to make the same assumption in the case of, say, measurements to the same degree of precision with a range of only 2 or 3 cm, as in the case of a small measurement on the head" (p. 572).

relationship between the range of the data and the digits. There are potentially a number of statistical characteristics of a distribution besides the data range that could affect the distribution of the frequency of the digits, that is, the moments of the data and the sample size. Ley and Varian (1994) considered this in assessing the assumption of the expectation of equal frequencies of digits as the appropriate testable hypothesis. They noted that the distribution of the data played a pivotal role in assessing the expectation of digit frequencies.

Numerical Stimuli and Psychological Effects

Prototypic Values. Clustering in terms of focusing on prototypic values is widespread in the collection of survey information of the self-reporting of time-based activities. Hornick, Cherian, and Zakay (1994) analyzed the influence of "prototypic values" (p. 145) and their validity for studies using time estimates. They noted that surveys using self-reports of time-based activities contained a rounding bias where prototypic decimal digits 0 and 5 occurred with relatively greater frequency. The bias was especially marked for larger values and certain data collection methods, such as telephone and retrospective reports.[23] Hornick et al. argued that this form of clustering was a product of two processes: (a) Stored values become rounded as memory becomes less exact and (b) rounded values are spontaneously generated with heuristics and simple schemata. The latter is potentially more relevant as an explanation in economic decision making or financial markets. It is difficult to envisage rounding occurring in this context because of memory loss, but rounding may function as a means to reduce information load.

Digit Differential Processing. The difference between odd and even digits is a fundamental one, not only mathematically but also psychologically. Individuals exhibit a difference in terms of both initial recognition and paired comparison with other digits depending on whether the digits are odd or even. Shepard, Kilpatric, and Cunningham (1975) initially discovered that an odd–even distinction was a major dimension in subjects' judgements concerning similarity among single digits. Judgements were based on the digits as abstract concepts. This importance of the oddness or evenness of the digits emerged regardless of whether they were given

[23]Hultsman, Hultsman, and Black (1989), studying leisure activities, demonstrated clustering, more 0s and 5s, in self-reports of the number of days and years. Tarrant and Manfredo (1993) also analyzed self-reports of recreation participation and noted that recall and nonresponse bias may be a function of unequal digit selection in the response, again an excess of 0s and 5s.

as Arabic numerals, rows of dots, or words. Krueger (1986) and Krueger and Hallford (1984) found that subjects use an odd–even rule to judge whether a visually presented equation featuring a sum or product was correct or not. Sudevan and Taylor (1987), in analyzing the processing of mental operations of recognizing odd and even digits, found that the response to odd digits was some 24 ms slower than the response to even digits. Hines (1990) confirmed this result, and the disparity of digit processing speed was even more pronounced when pairs or triplets were used. Hines found further that paired comparison, that is, matching of one digit with another, takes longer for odd–odd combinations than for even–even combinations.

Numerical Stimuli. A number of studies have examined human multidigit processing (see Takayanagi, Cliff, & Fidler, 1995, for details). There are two major competing human information-processing theories. The first is a parallel or holistic approach, where a numerical comparison takes place only after the whole digital representation of numbers is transformed into an internal magnitude code. Under this approach, digit length per se does not determine speed of processing. The second, a lexicographic or serial model, proposes that digit place and digit value are sequentially processed. Takayanagi et al. analyzed response time (RT) to various numerical stimuli involving redundant leading and following zeros, for example, 00080 and 800.00.[24] They found that (a) redundant leading zeros hindered processing (increased RT); (b) the effect of redundant following zeros on RT was conditional on how the number was presented; (c) overall, longer digits required more processing time, which in this regard was partly consistent with the lexicographic approach; (d) minimal processing time was required for numbers containing no leading or following redundant zeros; and (e) nonlexicographic processing occurred when feature identification could be used for numerical identification, that is, when the format was consistent.

Two points from Takayanagi et al. (1995) are the following: first, a consistent type of visual digital representation increased the performance of the visual detection and numerical comparison, which did not support the lexicographic model and confirmed Farrell's (1984) finding that visual appearance was important. Second, individuals have an internal array that determines the length of digits to make judgements at least in part holistically (nonlexicographic). This agrees with the findings of Tullis

[24]The digit 0 was selected as it is unique from other digits in that it becomes automatically redundant depending on its position. Moreover, previous studies using a zero showed some unique effects relative to the other digits.

(1981, 1983) that people preferred a fixed formatting so they could use concurrent processing. An inconsistent format caused increased operational and processing time as well as physical fatigue. For nonredundant digits, that is, digits other than zero, studies also find support for both the lexicographic and holistic models. The visual appearance of the digits as a whole is important for comparative tasks, and a consistent format aids processing.

The adoption of a fixed format for numbers and a fixed selection of digits may be in line with convention or a way to facilitate those who are used to dealing with a particular type and format of number. Representation using a particular decimal place or rounding, causing conventional clustering, may thus be an attempt to facilitate comparison and reduce information load and mental processing time.

Evidence and Rationale for Clustering in Economic Decision Making

Focal Points. Sugden (1995) aptly described the theory of focal points in a recent article. The thrust of the argument in relation to economic decisions was that rather than focus on the purely numerical attributes of a decision, decision makers use labels to identify strategies. The implication was that the use of labels influences the decision. In some cases, this induces a property of salience that provides "a focal point for each person's expectation of what the other expects him to be expected to do" (Schelling, 1960, p. 57).[25] Mehta, Starmer, and Sugden (1994) illustrated how this applies through a series of controlled experiments in a variety of general situations. In the Mehta et al. study, respondents were asked to name any time of day, for example, 10.22 p.m. or 7 p.m. Round identifiable values were most often selected, for example, noon, and thus represent a focal point with regard to selecting a time of day.

It is possible that rounding and clustering in other situations are further examples of focal points. People select numbers that they believe others recognize or that are readily discernible to other individuals to facilitate the decision-making process and achieve equilibrium. These focal points also draw on culture and the decimal place–value convention.

Clustering in Economic Decisions. Loomes (1988) identified a clustering or rounding phenomenon in economic decisions involving the certainty

[25] *Salience* in this context is the notion that one equilibrium stands out from others or is noteworthy. Where this is due to a property from labeling that decision makers recognize, it is termed a *focal point.*

equivalent valuations of risky gambles. When confronted with a situation where a valuation had to be made to the nearest penny, individuals displayed a marked tendency to select rounded pounds or half-pounds (£0.50). A tendency for the clustering to vary (increase) with the parameters of the risky action (variance) was also noted.

Two explanations are presented for the rounding or clustering phenomenon. First, the rounding was a function of a cost–benefit analysis that weighed the benefits of increasing the fineness of the valuation relative to the loss of value resulting from rounding, that is, an imprecise estimate. Butler and Loomes (1988) noted that this was consistent with a suggestion by Beach, Beach, Carter, and Barclay (1974) of a subjectively acceptable error in such decisions.[26] The subjectively acceptable error is proportional in size to the magnitude of the correct answer. Second, both Loomes (1988) and Butler and Loomes suggested that economic decision makers do not measure utilities exactly but operate in a *sphere of haziness*. The sphere of haziness represents the degree of difficulty in ascertaining a value with a high degree of precision. The higher the sphere of haziness is, the higher the propensity to cluster is. This again suggests that clustering is directly related to the variance (uncertainty) of the value, which is similar to the argument put forward by Preece (1981) and others discussed previously.

Clustering as a Law of Nature or a Product of the Number System

Relevance of the Literature

Two additional reasons for clustering were suggested previously. It was put forward that clustering in any data series may be the result of (a) a natural order, that is, a law of nature; or (b) the order of number progression, that is, a product of numbers themselves. These reasons arise from the mathematics literature, which has devoted some attention to both documenting and explaining the commonly observed and encountered phenomenon of clustering arising in everyday data series. These

[26]Butler and Loomes (1988) used one of the decision gambles in Loomes (1988) where subjects indicated the degree of decision difficulty for the gambles in terms of a difficulty score. Results indicated individuals found it difficult to be precise regarding certainty equivalent valuations. Grossman, Miller, Cone, Fischel, and Ross (1997) posited a similar theory for price clustering in competitive asset markets, suggesting that the degree of the coarseness of price depends on a balance of the benefits and costs of having a finer price. Finer trades allow more accurate pricing but increase the time and effort as well as the cost of negotiation.

data series are taken from statistical and scientific handbooks or street address numbers, which tend to cluster on certain numerical values (digits).[27]

Law of Anomalous Numbers

Newcomb (1881) was the first to draw attention to this phenomenon.[28] Benford (1938), unaware of Newcomb's work, further provided a model to describe the frequency of first and other higher placed digits. The formal presentation or model of this phenomena is known as *Benford's law* (Rami, 1976).

Distribution of First Digits. In brief, Benford's law (model) is that the frequency (probability) of first digits follows the following logarithmic relationship:

$$F_a = \log_{10}\left(\frac{a + 1}{a}\right)$$

where F_a is the frequency (probability) of the digit a in the first place of used numbers regardless of decimal place. a is equal to 1, 2 . . . 9 because 0 is not admitted as a possible first digit. Under Benford's law (model), the probability of $a = 1$ is 0.301, and the probability of $a = 2$ is 0.176; this decreases to a probability of 0.046 for $a = 9$.

Interpretation of Benford's Law. A number of mathematical complexities have arisen in regard to the notion of Benford's law, most of which are not relevant to this discussion. In terms of a philosophical consideration, two broad-based interpretations have arisen. One, introduced by Benford (1938) himself, was that the law is essentially a phenomenon of nature. Rami (1976) argued, however, that the fact that most geometrical sequences obey Benford's law did not make them a phenomenon of nature. There was no rationale to state that nature should be counted in geo-

[27]The observed data did not include numbers whose digits obviously cluster either because of an inability to observe a particular digit or because of an inherent natural clustering in the frequency of the digits. All data "had to be not too restrictive in its natural range or conditioned in some way too sharply" (Benford, 1938, p. 552). The data also had to be random to some degree. Benford further stated that "the logarithmic law applies particularly to those outlaw numbers that are without known relationship rather than to those that individually follow an orderly course; and therefore the logarithmic relation is essentially a Law of Anomalous Numbers" (p. 557).

[28]"That the ten digits do not occur with equal frequency must be evident to anyone making use of logarithmic tables, and noticing how much faster the first pages wear out than the last ones. The first significant digit is oftener 1 than any other digit, and the frequency diminishes up to 9" (Newcomb, 1881, p. 39).

metrical steps. Another consideration was that Benford's law was a property of numbers themselves (Rami, 1969), a property of counting, or a property of our writing of numbers (Goudsmit & Furry, 1944). Rami (1969, 1976) demonstrated that the derivation of the law, as applied to the arithmetic series of natural numbers, was subject to the summation technique employed. For the arithmetic series of numbers, the law is not unique and is in fact conditional on the averaging process adopted in the analysis. Benford's law is thus considered not to be unique and does not stem from the progression of numbers or from numbers themselves.

In all, although the law applies to most geometrical sequences of data and some arithmetical sequences, it does not hold unequivocally, as demonstrated by empirical evidence, and there is no reason to infer that it results from either a law of nature or the numbers themselves. Furthermore, it applies equally to ordered geometrical series and unordered series. The title, the Law of Anomalous Numbers, is, therefore, open to question.

Distribution of Other (Nonfirst) Place Digits. Benford's law applies to digits other (higher) than first-placed digits. An unconditional frequency of the higher placed digits can, furthermore, be computed on the basis of the sum of the conditional frequencies relative to all the preceding placed digits. The general digit formulae for Benford's law, as well as an illustration of its application, were given by Hill (1995a, 1995b).[29] Note the second and higher placed digits are 10 in number, as the 0 digit is now taken into account.

Two points arise from the general digit law: (a) the *i*th digit of interest (a_i) is dependent (conditional) on all those preceding it, and (b) "the distribution of the *i*th significant digit approaches the uniform distribution where each digit (0, 1, . . . , 9) occurs with frequency 1/10 exponentially fast as $i \rightarrow \infty$" (Hill, 1995b, p. 324). This is illustrated in Table I. Newcomb (1881) and Benford (1938) provided the frequencies for the second digit, and the calculation is further extended to the third digit. As can be seen, the probability of observing a particular third-placed digit is nearly uniform, and Newcomb noted that for the fourth digit any difference from a uniform distribution is imperceptible.

Implications of Benford's Law. One further pertinent point needs to be raised. From the analysis, if a decimal point exists or 0 occurs before the

[29]The conditional frequency of a second digit (a_2) allowing for a first place digit (a_1) is $F_{a_2|a_1} = \log_{10}$ $((a_2a_1 + 1)/a_2a_1)/((a_1 + 1)/a_1)$; the unconditional frequency of (a_2) F_{a2} is $\Sigma_{a_1=1}^{9} \log_{10}((a_2a_1 + 1)/a_2a_1))/((a_1 + 1)/a_1)$; the sum for a_2 across all the conditional frequencies of a_1. Note $a_1 \in \{1, 2 \ldots 9\}$ and $a_2 \in \{0, 1, 2 \ldots 9\}$. Higher (third) digit probabilities are computed in a similar vein with the same dependent relationship across all previous digits (see Hill, 1995a, 1995b).

TABLE I

Benford's Law: Frequency (Probability) of First-, Second-, and Third-Place Digits Occurring

Digit	*First Place*	*Second Place*	*Third Place*
0	0.000[a]	0.120	0.102
1	0.301	0.114	0.101
2	0.176	0.108	0.101
3	0.125	0.104	0.101
4	0.097	0.100	0.100
5	0.079	0.097	0.100
6	0.067	0.093	0.099
7	0.058	0.090	0.099
8	0.051	0.088	0.099
9	0.046	0.085	0.098

[a]0 is not permitted as a first place digit, so the value of 0.000 is by construction rather than observation.

first natural number (digit), it is ignored. Benford (1938) stated that "no attention is to be paid to magnitude other than that indicated by the first digit" (p. 552). Thus, Benford's law investigates and depicts a floating place digit representation that has no regard for the potential range or likelihood of the realization of a digit. This may vary in a data series for equivalent or other digits depending on their place order (e.g., 1 vs 1,000) or their relation to a decimal point or zeros (e.g., 4.0 vs 0.00007). 1, 4, and 7 would all qualify as the first digit in the aforementioned numbers.

It seems logical that the range and likelihood of realization would differ for digits depending on their place order. In fact, the numerical range would be greater and the expectation of a uniform distribution increased for digits that occur in place orders higher relative to the initial place or further to the right of any decimal place. For most data series, notably financial data, digits in higher place orders rapidly give an increasingly fine partitioning of the value of the item. This results in an increased confidence of no deterministic clustering, that is, clustering conditional on the data series, being inherent in the digits as the potential of each digit being included in the data range and theoretically realizable for each observation is increased.

CONCLUSION AND IMPLICATIONS

First, it has been demonstrated that the behavioral or culturally conventional consideration of numbers is evident through the process of number or numeral development as well as the abstract notion of number. Two points are apparent: (a) The existing decimal place–value system encourages individuals to think in groups of 10 or multiples thereof and encourages a numerousness concept, particularly through the adoption of the place–value system and notation, and (b) coupled with this symbolism, mysticism and even cultural convention may dictate some form of number preference.

Second, various behavioral explanations have been put forward to explain clustering. These include (a) individuals use simple heuristics such as anchoring to provide rough approximations to questions and decisions rather than precise estimates and (b) there is a tendency for people to simplify the information level when mentally processing numbers, which enables quicker and potentially more cost-effective decisions. The investigation of numerical stimuli of digits suggest that rounding and fixed formats speed up numerical processing and comprehension and that individuals also process even numbers faster than odd digits.

Third, rational economic explanations are relevant. The focal point literature suggests that individuals are not simply influenced by the numerical attributes of a decision but tend to use number labels to identify strategies. People may select numbers that they believe others will recognize or that are readily discernible to other parties to facilitate the decision-making process. In short, individuals use numbers with which they are familiar, depending on the circumstances (Niederhoffer, 1965). In addition, clustering is influenced by what error is acceptable in decisions based on a cost–benefit trade-off. Finally, in most cases individuals operate in a sphere of haziness (Butler & Loomes, 1988; Loomes, 1988) concerning the value of items, and clustering results from attempts to overcome this uncertainty. Many of these interpretations are not mutually exclusive and indicate similar behavioral effects. They appear concurrently in many discipline areas.[30]

Clustering in a data series does not result from a natural order, or alternatively, it is not a product of the number progression or the numbers themselves as this depends on the particular series itself. The idea of a general clustering phenomenon, propagated by Benford's law, appears especially invalid for financial data and markets. Any such relationships (a) are restricted to certain data series, (b) depend on the place digit, and (c) are influenced by the way in which the analysis is conducted and how the clustering is specified (e.g., whether it is relative to a floating or fixed digit place format). This conclusion is contrary to De Grauwe and Decupere (1992) and Ley (1994), who suggested Benford's law describes

[30]See, for instance, the attraction and price resolution hypotheses developed by Ball, Torous, and Tschoegl (1985) and embellished by Harris (1991) to explain clustering evident in financial markets.

many data series, including financial data, so that widespread clustering due to nonbehavioral factors is possible.

Overall, it seems that the documented evidence of clustering and psychological barriers in financial markets is not as strange or anomalous as it is often presented. It is clear that society has a fascination with and an affinity for numbers. This is more than adequately supported by the anecdotal reference to the focus of important and/or psychologically relevant numbers. This provides prima facie support for the clustering of data in general and, furthermore, an a priori expectation of the degree of clustering and other related effects in financial markets. This provides a rationale to support the documented evidence.

Obviously, there are potential implications for trading strategies based on the identified structural dependency of prices. The potential for various trading strategies needs to be identified and evaluated through further investigation. For instance, trading rules based on the clustering of transaction frequencies are possible. These trading rules are based on the supposition that a reversal in the direction of the price of the financial asset becomes likely as the price moves to some round number. Examples of these trading rules and dependent structures were provided by Niederhoffer (1965, 1966) and Niederhoffer and Osborne (1966). Similarly, signals generated with support and resistance levels as well as perceived psychologically important values as inputs as part of a trading rule may be profitable. Again, traders cognizant of clustering and the dependent effects can take advantage of these for trading opportunities. Market efficiency implications of the documented effects would hinge on the outcome of any observable profits from the trading strategies developed.

APPENDIX A: THE DEVELOPMENT AND IMPORTANCE OF NUMBERS[31]

The origin of number development in terms of both counting and numerals, as well as consideration of the "What are numbers?" question, either on a metaphysical or scientific basis and regardless of interpretation, provides valuable input for understanding number preference behavior and the culture of number preference that exists today.

Early Cultural Development of Numbers: Counting Systems

The origin of numbers is lost in the proverbial murky mists of antiquity. A lucid insight is that the first application of numbers was for counting.

[31]Material in Appendix A is based on Brainerd (1979) and Flegg (1989).

Primitive counting methods consisted of duo (2)-count, neo-2-count, 4-count, 5-count, 20-count, and hybrid 5/20-count systems in addition to the standard 10 or decimal-count system.[32] Primitive man used a 2- or 4-count system, and this is still evident in the native languages in some indigenous populations of Africa, that is, the Bushman, South America, Australia, and New Guinea. The hybrid 5/20-count system was the most popular in Europe and South and North America, as evidenced in the Maya and Aztec culture. It still exists in the Eskimo language, for instance. It probably existed throughout Europe before the spread of the Indo-European languages (see Flegg, 1989).

The decimal count system developed and spread through the medium of Indo-European languages (e.g., Gothic, Celtic, and Romance) via Europe, replacing the existing 5/20-count system and eventually reaching the rest of the world.[33] Remnants of the original 5/20-count system still remain in the existing decimal count system, that is, in French: *quatre-vingt* or four twenties for 80, *quatre-vingt-dix* or four twenties and ten for 90. An alternative measuring (counting) unit is the 12-system, as reflected in the word *dozen* from the French *douzaine.*[34] An important aspect that arises from consideration of the development of counting is the way in which humans view numbers. It suggests that we think, count, or measure in distinct groups or patterns. This facilitates the notion of group-to-group correspondence. It allows the decomposition of large numbers into subgroups and places a decreased demand on memory and cognitive processing ability. Even at the rudimentary level of counting and further displayed in collections and measurement systems largely derived therefrom, there is a preference for decimal (10), duodecimal (12), or vigesimal (20) systems.

[32]For example, the English word *thirty-five* comes from the expression *three tens and a five*. The decimal count system is distinguished from the decimal number system, that is, using numerals, namely, *fifty-five* relative to 55.

[33]An exception to this is the 10/60 or sexagesimal system developed by the Sumerians and Babylonians. Why 60 was chosen as the base for both counting and ciphers provides an interesting question. The answer has been postulated to arise from the mercantile system of the day where the monetary unit the *mina* was divided into 60 *shekels*. The sexagesimal system survives to this day in the seconds and minutes of time measurement as well as the degrees, minutes, and seconds of geographical, astronomical, and angle measurement. The Babylonian and Greek astronomers, who added a non-place value zero, adopted the sexagesimal system.

[34]The word *douzaine* was an alternative for the French word *sou* (shilling), which was divided into 12 *livre*. A gross is a *grosse douzaine*, or strong dozen. Similarly, a *score* today represents counting by twenties; score means to keep count. Old Saxon *sceran* means to shear or cut. Commercial transactions and measurement systems of the Romans and Anglo-Saxons made frequent use of 12, 20, and other more elaborate systems, such as 12 inches (*pollices*) to a foot (*pes*). These developed from counting and collation methods.

Abstract Notion of *Number*

A number has four different meanings, as a numeral, as a number word, as a concept, and as a property posed by every collection of such objects (Flegg, 1983, p. 3). In its most rudimentary form, a number represents a quality, property, or collection of sensible objects (Brainerd, 1979, p. 4). This property of numbers, that is, numerousness, derives from our direct experience in an analogous way to colors, flavor, or other perceptions. However, for number to progress to an abstract concept, it is necessary to assume that number is a property independent of sensible objects. This facilitates rejection of direct experience as a use for numbers. The first instance of number as an abstract concept is in the recordings of the ancient Egyptians, who were able to calculate (estimate) inventories, the volume capacity of grain silos, land allotments, and the days between the flooding of the Nile. The use of large obviously estimated numbers confirms this notion of the abstract concept, that is, beyond direct experience.

Development of Numerals (Ciphers)

There is a close link between the number system, the written word, and the numeral (cipher) that depicts the number in notational form. Various number systems, up to the medieval period, generally inscribed numbers with one or more of the following forms:

1. Iterative place–value (positional) systems, such as that of the Babylonians.
2. Fully ciphered systems, such as those of the Greeks (Ionian) and Hebrews, in which all numbers are fully represented or ciphered, usually by alphabetical notation.[35]

[35]The concept of a cipher is defined as a mark, notation, sign, or symbol that designates a number. Originally, it denoted simply the symbol zero (see Flegg, 1989, p. 127). Flegg (p. 79) defined a fully ciphered system as one where all the number values have a distinctive mark and this is not repeated; in other words, for a decimal system first 1–9 and then each of the nine integral multiples of the powers of 10, that is, tens (10–90); hundreds (100–900), and so forth, would have unique ciphers. An iterative (partial) ciphered system has, relative to a value V, symbols for each of the units from 1 to $V - 1$ and then further symbols for the steps of values from V. These symbols need not be unique (e.g., I, II, III). The iterative place–value system has symbols, not necessarily unique, for units from 1 to $B - 1$ and then places the units in the appropriate order to denote base value (e.g., B^2, B^3). The term *digits* from the Latin *digiti* means written numerals, and its interpretation is generally similar to the concept of a cipher. However, the mathematical interpretation is the 10 Arabic numerals from 0–9, which is the meaning adopted here.

3. Iterative (partial) ciphered systems, such as those of the Egyptians and Romans, where numbers are derived from a repetition of the ciphers (e.g., the Roman numeral for 1990 is MCMXC).[36]

Our present number system finally evolved from a combination of some of these systems. It includes zero as a number, which the Babylonians originally had but only as a place value, and incorporates a fully ciphered decimal system.

APPENDIX B: THE FIBONACCI SERIES, RATIO, AND NUMEROLOGY IN INVESTING

Numerology is defined as the esoteric significance of numbers. This is never more evident than in interpretations attributed to a certain arithmetic series of numbers known as the Fibonacci series. This significance of numbers arises from the belief in a mathematically ordered world, or *harmonia mundi*. Kepler's (1571–1630) belief in such an order is said to have provided inspiration for some of his scientific discoveries. He had an unshakeable conviction that there existed a harmony between human beings, the earth, and the cosmos, a harmony in number. In the Fibonacci series, each number is the sum of the previous two numbers. The starting two numbers are denoted the seed numbers and are 1 and 1. Therefore, the series is the following:

Number	1	2	3	4	5	6	7	8	9	10	11	12	...
Series	1	1	2	3	5	8	13	21	34	55	89	144	...

The Fibonacci system describes the growth process of many forms in nature, in fir (pine) cones, pineapples, and petals of flowers (phyllotaxis); the disposition of the chambers in the nautilus shell; and the spiraling shapes of galaxies. More recently, it has been assumed investors in capital markets follow this same growth spiral (Armstrong, 1991; Fischer, 1993; Slezak, 1989). The origin of the arithmetic Fibonacci series comes from the solution to the following problem: How many pairs of rabbits can be produced from a single pair in a year if every month each pair begets another pair, which in turn becomes productive from the 2nd month onward?

The Fibonacci ratio embodies an interesting property of the series. Asymptotically, the ratio of any number to its next highest number is 0.618 and to the next lowest number is 1.618. The ratio thus effectively

[36]The Chinese had a slightly different (partial) iterative ciphered system. In a fully iterative system, the value of the base symbol is repeated; for example, the Roman 400 is CCCC. In Chinese, 400 is written as two symbols, *szu* (four) and *pai* (hundred). This is called a *named place–value system*.

depicts the growth of the series. Pacioli referred to this ratio as the *golden ratio, golden section,* or *divine proportion.* It was known to the ancient Greeks and Egyptians and was used in applications of architecture, geometry, and art. The ratio is irrational and is only defined here to three decimal places.

One approach to the use of Fibonacci series as an investing tool in capital markets is based on the identification of an initial move in the market and then the use of the ratio(s) to determine the potential turning points, swings, or resistance and support levels of the market. An initial move in the market is always at a point reversing a previous trend, and this determines the direction of the trend (see Slezak, 1989, for more details). The appropriate potential resistance levels are then computed with the initial move multiplied by the ratio (1.618) and this added to the price level of the initial move. Thus, the resistance levels are a function of the size of the first upward move. Normally, there are supposed to be up to five legs of resistance levels, which are computed on the basis of the powers of the Fibonacci ratio relative to the initial move, that is, 1.6, 2.6, 4.2, and 6.8. Fischer (1993) used the same basic philosophy and the Fibonacci ratio, with a different application, to calculate the peaks and troughs of the five upward and three downward trends predicted by the Elliott wave theory of market movements.

The Fibonacci ratio is evidence of number symbolism applied to investment strategy. Two questions arise in considering this approach to investment. Why should a system that obviously describes a deterministic growth in living organisms and flowers (phyllotaxis) apply to the process of stock market movements? Why is this particular ratio, that is, growth pattern, appropriate? Is there no other series that provides ratios that would more adequately describe the market growth patterns? Vajda (1989) provided evidence that other related series do exist, for example, Lucas numbers, and also that the growth pattern depends on the original seed. The link between the breeding behavior of rabbits and the growth of capital market prices seems tenuous unless one is prepared to make gross generalizations regarding the series. The generalizations are fully admitted in most cases.

The series is also used to describe an infinite arithmetic progression or growth at a constant rate ad infinitum. There is no mention of regression, reversal, or a plateauing of the growth series, which is how the ratio is in fact used to predict movement in market prices. The arithmetic progression of the Fibonacci series is also a discrete approximation to a continuous fixed growth rate that derives from a reproduction cycle. This growth rate per period is not carried over to the market analysis because the market is allowed many or indefinite periods to achieve the next point in the series. If the market has some positive growth, it will eventually reach this level. As to whether the market, in fact, turns or plateaus is no more likely to be predicted by the Fibonacci ratio than a 50/50 (not allowing for drift) guess.

Finally, the implicit relationship of the Fibonacci series is that the increase (growth) in a period is only allowed to influence the series after two periods have lapsed. Even if such a growth were applicable and were measured by the change in price or the ratio of prices, it would appear difficult to rationalize the delay in influencing the future period's growth rate in a competitive market. It is also clear that the market will not necessarily grow at the rate suggested. Slezak's (1989, p. 36) rationale for the ratio relates to a growth in traders rather than the market itself and an allegory relating to the growth in rabbits (uninformed traders) and eventually to wolves (informed traders) pouncing upon them, thereby reversing the trend at the resistance level. Although this is amusing, it seems that the number of traders does not proportionately influence the growth in price that is the supposed purpose for which the ratio is intended. Finally, who the wolves are, or where they fit into the scenario, remains elusive in relation to the original Fibonacci series, which simply models a particular growth pattern taken from nature.

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