HEDGING MULTIPLE PRICE AND QUANTITY EXPOSURES

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We examined the general hedging problem faced by a global portfolio manager or a pure exporting multinational firm. Most hedging models assume that these economic agents hold only a single asset in the spot market and are exposed only to a single source of price–quantity uncertainty. Such models are less relevant to many financial and exporting firms that face multiple sources of risk. In this study, we developed a general hedging model that explicitly recognizes that these hedgers are faced with multiple price and quantity uncertainties. Our model takes advantage of the full correlation structure of changes in spot prices, quantities, and forward prices. We performed simulations of the hedging model for a firm with two pairs of price and quantity exposures to demonstrate potential gains in hedging efficiency and effectiveness. © 2001 John Wiley & Sons, Inc. Jrl Fut Mark 21:145–172, 2001

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INTRODUCTION

Two general characteristics of many hedging problems are (a) the hedger faces both price and quantity uncertainties and (b) the hedger is exposed to multiple sources of price and quantity uncertainties. A classic example of the general hedging problem is provided by a multinational firm whose cash flows are denominated in many currencies. Such a global firm is exposed to a portfolio of currencies for which not only the quantities of foreign cash flows but also the domestic currency value of foreign cash flows are uncertain. In a similar fashion, an international equity or bond portfolio manager faces an uncertain end-of-period market value and an uncertain exchange rate in each country. In all these cases, the manager is exposed to multiple sources of price risk (the exchange rate) and quantity risk (the foreign currency cash flow and the equity value or bond market value).

However, price and quantity uncertainties are not specific to multinational firms or international portfolio managers. Most firms, including even purely domestic firms, are exposed to both uncertainties and may wish to hedge these exposures. For example, a farmer simultaneously growing more than one crop faces multiple price and quantity uncertainties because the harvest-time crop yields and prices are unknown at the planting time. Similarly, the inventory risk faced by a domestic bond dealer comprises uncertainties in the quantity and market prices of bonds of various maturities and degrees of default risk. Thus, many types of hedgers face both price and quantity uncertainties in multiple assets.

The extant hedging models address only simplified versions of the general hedging problem. Traditional models assume that the hedger holds only a single asset in the spot market (not a portfolio of assets) and that he or she is exposed to only price uncertainty. For example, it is common to assume that the magnitude of the future foreign currency cash flow is known and that the hedger faces only the exchange rate uncertainty (price uncertainty or translation exposure). Kerkvliet and Moffett (1991; henceforth, KM) developed a model that recognizes not only price uncertainty but also quantity uncertainty (stochastic transaction exposure). A separate strand of the literature, Rolfo (1980), Benninga, Eldor, and Zilcha (1985), and Fung and Lai (1991) and others, has looked at the hedging behavior of firms facing uncertain production with and without forward currency and commodity markets.

A common assumption in this literature is that the hedger is exposed to transaction and translation exposures in only one currency (the single-asset case), which is a special case of the general hedging problem. Other studies have noted that a firm with multiple sources of risky income may be systematically over or under hedging relative to the true variance minimizing hedge because of the potential correlation between exchange rates and the sources of risky income (see Lien, 1996; Zilcha & Broll, 1992). Also, finance practitioners have recognized the need to measure risk and exposure in a portfolio context. This is evidenced by the increased popularity of value-at-risk products, such as Riskmetrics, which use a variance–covariance matrix of all positions to determine overall exposure. Effective risk management, therefore, requires a more general framework that recognizes the interactions between different currency exposures and currency-specific hedges.

In this article, we examine the general hedging problem faced by pure exporters and global portfolio managers, who are not exposed to production uncertainty. Limiting our attention to exporters and portfolio managers allowed us to focus on financial hedging issues without regard to operational hedging strategies (i.e., strategies that affect the investment decisions of a firm). We assumed that a pure exporter derives revenues denominated in several foreign currencies. Equivalently, we assumed that a global portfolio manager invested in stocks and bonds traded in many foreign countries. Furthermore, we considered hedging strategies that used forward/futures instruments but no option contracts. Our objective was to develop an optimal hedging model that addressed both price and quantity uncertainties in multiple assets. In the section entitled Hedging Models, we develop a general hedging model in which it is explicitly recognized that hedgers face multiple price and quantity uncertainties; we also review a number of special cases and show that these may be obtained from our model through the restriction of parameter values. In the section entitled Simulations of Optimal Hedge Ratios, we report the results of four simulations of the optimal hedge ratio for a multinational firm. The last section concludes the article.

HEDGING MODELS

Although price and quantity uncertainties are generic to most hedging problems, for the sake of clarity we focus on the case of managing foreign exchange risk. A manager enters into cross-border trades at the beginning of the period (time t_0), but the agent does not know the amount of cross-border cash flows it will realize until the end of the period (time t_1). Q_i is the quantity of cash flow denominated in foreign currency i ($i = 1, 2 \ldots I$), S_i is the spot exchange rate (dollars per unit of foreign currency i), F_j is the price of the jth forward contract ($j = 1, 2 \ldots J$), N_j is the number of the jth forward contract, and N_jF_j is the aggregate price of the

jth forward position. The aggregate (unhedged) end-of-period cash flow UC_{t_1} consists of the sum of the individual country flows: $UC_{t1} = Q_{1t_1}S_{1t_1} + \ldots + Q_{It_1}S_{It_1}$. To be consistent with previous research in this area and without a loss of generality, we deal with changes rather than levels.[1] Thus, ΔUC is the change in the aggregate (unhedged) cash flow between t_0 and t_1; similarly, $\Delta(Q_iS_i) = Q_{it_1}S_{it_1} - Q_{it_0}S_{it_0}$ stand for the change, denominated in the domestic currency, of the cash flow from country i. To hedge the risk inherent in both the exchange rate (S) and the quantity (Q) of foreign currency, a firm may wish to use a number of forward contracts J, with J not necessarily equal to I. Then, the change in the aggregate cash flow, including the changes in the forward contracts, ΔHC, is

$$\Delta HC = \sum_{i=1}^{I} \Delta(Q_iS_i) + \sum_{j=1}^{J} N_j\ (\Delta F_j) \tag{1}$$

where $\Delta F_j = F_{jt_1} - F_{jt_0}$ is the gain or loss (per contract) from the jth position. Mathematically, the problem is to choose, at the beginning of the period, the number of contracts N_j for each currency j to minimize the volatility of the total cash flow or, equivalently, the change in flow. The variance of ΔHC is given by

$$\begin{aligned}\mathrm{Var}(\Delta HC) &= \mathrm{Var}\left[\sum_{i=1}^{I} \Delta(Q_iS_i)\right] + \mathrm{Var}\left[\sum_{j=1}^{J} N_j(\Delta F_j)\right] \\ &\quad + 2\mathrm{Cov}\left[\sum_{i=1}^{I} \Delta(Q_iS_i),\ \sum_{j=1}^{J} N_j(\Delta F_j)\right]\end{aligned} \tag{2}$$

The minimum may be determined by direct differentiation with respect to N_j, but first one needs an explicit expression for this variance. Before developing a solution to this problem, we consider a number of special cases.

Special Case 1: The Traditional Hedging Model

Traditional hedging models assume that the exposure of the firm is limited to one foreign currency and the magnitude (quantity) of this exposure is known at the beginning of the period (Adler & Dumas, 1984; Babbel,

[1]Ederington (1979), Hill and Schneeweis (1982), and Kroner and Sultan (1993), to name just a few studies, used changes in prices. Hill and Schneeweis (1981) pointed out that the estimation of hedge ratios on the basis of price levels would likely result in serially correlated residuals; they recommended the use of price changes.

1983; Ederington, 1979; Hill & Johnson, 1960; Kwok, 1987; Schneeweis, 1982; Stein, 1961; Swanson & Caples, 1987). Setting $I = 1$ and $J = 1$ in Equation 1 and omitting the i and j subscripts for simplicity, the cash flow change is given by $Q_{t_0}\,\Delta S + S_{t_0}\,\Delta Q + \Delta Q \Delta S + N\Delta F$, where $\Delta Q = Q_{t_1} - Q_{t_0}$ denotes the change in quantity and $\Delta S = S_{t1} - S_{t0}$ represents the change in the spot exchange rate. The volatility of the hedged change is $\mathrm{Var}(\Delta HC) = (Q_{t_0} + \Delta Q)^2 \mathrm{Var}(\Delta S) + N^2 \mathrm{Var}(\Delta F) + 2N(Q_{t_0} + \Delta Q)\mathrm{Cov}(\Delta S, \Delta F)$. Minimizing this variance with respect to N yields $N^* = -Q_{t_1}H$, where H stands for the traditional hedge ratio:

$$H = \frac{\mathrm{Cov}(\Delta S, \Delta F)}{\mathrm{Var}(\Delta F)} \tag{3}$$

In this case, the optimal hedge is proportional to (minus) the quantity of the foreign currency cash flow. In the next section, we show that when both price and quantity are stochastic, the optimal forward position may be larger or smaller than the currency exposure (in absolute value).

The effectiveness of the hedge, e, is given by:

$$e = 1 - \frac{\mathrm{Var}[\Delta(QS) + N^*\Delta F]}{\mathrm{Var}[\Delta(QS)]} \tag{4}$$

It is well known that e corresponds to the squared correlation coefficient between the change in spot and forward prices. The hedging effectiveness will range from 0 to 1 depending on the degree of correlation between the spot and forward price changes. Clearly, maximum effectiveness will be achieved when there is perfect correlation between the two.

An interesting corollary of this analysis is that the variance minimizing hedge ratio, N^*, also maximizes expected utility for a general von Neumann–Morgenstern utility function. To see this point, observe that at N^*, the change in the hedged cash flow is uncorrelated with the change in the forward price: $\mathrm{Cov}(\Delta HC, \Delta F) = 0$. Now, consider a general utility function $U(\Delta HC)$: the first-order condition for maximum expected utility is $E[U'(\Delta HC)\Delta F] = 0$; applying Stein's lemma (Ingersoll, 1987, p. 13), we obtain $E[\Delta F] + EU''/EU' \,\mathrm{Cov}[\Delta HC, \Delta F] = 0$. This expression implies that if forward prices follow a martingale process and N^* is given by $-Q_{t_1}H$, then the optimal hedge ratio will minimize the variance of the hedged cash flow and at the same time maximize a quite general expected utility function.[2] Thus, the traditional hedging theory, based on minimizing the variance, is more general than typically assumed. However, it is

[2]An anonymous reviewer pointed out to us that Benninga et al. (1984) derived the same result with an assumption of linearity rather than normality (as required by Stein's lemma).

unclear at this time whether this result may be generalized to a richer setting with multiple price and quantity exposures.

Special Case 2: Hedging Both Price and Quantity Exposures

KM relaxed the assumption of known quantity and assumed that the firm did not know at the beginning of the period the magnitude of its single foreign currency exposure (see also Adler & Dumas, 1984; Eaker & Grant, 1985; Jacque, 1981; Shapiro, 1984). Under a mild distributional assumption, they showed that the variance-minimizing number of contracts is $N^* = -E(Q_{t_1})H$, where $E(Q_{t_1})$ is the expected foreign currency cash flow and H is given by

$$H = \frac{1}{\mathrm{Var}(\Delta F)}\left[\mathrm{Cov}(S_{t_1}, F_{t_1}) + \frac{E(S_{t_1})}{E(Q_{t_1})}\mathrm{Cov}(Q_{t_1}, F_{t_1})\right] \tag{5}$$

When Q_{t_1} is deterministic, as assumed in Special Case 1, $\mathrm{Cov}(Q_{t_1},F_{t_1})$ is equal to 0 and Equation 5 yields the same solution as Equation 3, even though Equation 5 is expressed in levels and Equation 3 is in changes. Furthermore, if the covariance between the quantity and forward price is positive (negative), the revised hedge ratio in Equation 5 will be higher (lower) than that in Equation 3 because the stochastic change in the quantity of cash flow does not (does) act as a natural hedge against the stochastic change in exchange rates.

A drawback of Equation 5 is that it is based on both the level and change in the spot and forward cash flows; also, the estimation of H in this form may be problematic because of potential unit roots in the forward and spot prices. Therefore, before moving to the general hedging problem, we derive the optimal hedge ratio when all variables are expressed as changes rather than levels. From Equation 2, we know that the variance of the change in the hedged cash flow is $\mathrm{Var}(\Delta HC) = \mathrm{Var}[\Delta(QS)] + N^2\mathrm{Var}(\Delta F) + 2N\mathrm{Cov}[\Delta(QS), \Delta F]$. Minimizing this variance with respect to N yields

$$\begin{aligned} N^* &= -\frac{\mathrm{Cov}[\Delta(QS), \Delta F]}{\mathrm{Var}(\Delta F)} \\ &= -\frac{Q_{t_0}\mathrm{Cov}(\Delta S, \Delta F) + S_{t_0}\mathrm{Cov}(\Delta Q, \Delta F) + \mathrm{Cov}(\Delta Q \Delta S, \Delta F)}{\mathrm{Var}(\Delta F)} \end{aligned} \tag{6a}$$

To simplify this expression further, the last term in the numerator may be shown to be

$$\text{Cov}(\Delta Q\Delta S, \Delta F) = E(\Delta S)\text{Cov}(\Delta Q, \Delta F) + E(\Delta Q)\text{Cov}(\Delta S, \Delta F) + E\{[\Delta S - E(\Delta S)][\Delta Q - E(\Delta Q)][\Delta F - E(\Delta F)]\} \tag{6b}$$

There are several reasons to expect the last term in Equation 6 to be zero. First, each component inside the square brackets is a deviation from the mean, so it may be regarded as a zero mean random error. If we assume that these errors are normally distributed, the expectation of the triple product equals zero (see Bohrnstedt & Goldberger, 1969). Second, and perhaps more compellingly, it follows from the theory of conditional expectations that the triple product may be represented as

$$E\{[\Delta S - E(\Delta S)][\Delta Q - E(\Delta Q)][\Delta F - E(\Delta F)]\} = E_{\Delta Q}\{[\Delta Q - E(\Delta Q)]\text{Cov}(\Delta S, \Delta F|\Delta Q)\} \tag{7}$$

If spot and forward prices are determined by an arbitrage process (independently of the actions of a particular firm), the covariance between the spot and forward price changes will be the same at all levels of ΔQ. This implies that in Equation 7 the covariance term may be taken out of the expectation operator, and thus obtain the result that the triple product in Equation 6b has zero expected value.[3] In other words, the covariance between the ith currency cash flow and the change in the forward price (i.e., the numerator of Equation 6a) is the expected spot exchange rate times the covariance between the change in quantity and the change in the forward price plus the expected quantity times the covariance between the change in the exchange rate and the change in the forward price:

$$\text{Cov}(\Delta Q\Delta S, \Delta F) = [E(\Delta S)]\text{Cov}(\Delta Q, \Delta F) + [E(\Delta Q)]\text{Cov}(\Delta S, \Delta F) \tag{8}$$

Next, let $\beta^S = \text{Cov}(\Delta S, \Delta F)/\text{Var}(\Delta F)$ and $\beta^Q = \text{Cov}(\Delta Q, \Delta F)/\text{Var}(\Delta F)$ denote the slope in the simple linear regression of ΔS on ΔF, and ΔQ on ΔF, respectively.[4] It will be helpful later to interpret β^Q as a measure of quantity exposure and β^S as a measure of price exposure. Now, the optimal number of forward contracts is given by

[3]A further rationale for this result may be obtained with

$$\text{Cov}(\Delta S, \Delta F) = E[\text{Cov}(\Delta S, \Delta F|\Delta Q)] + \text{Cov}[E(\Delta S|\Delta Q), E(\Delta F|\Delta Q)]$$

Market efficiency requires that futures prices, conditional on Q, form a martingale process; hence, the covariance between changes in spot and futures prices must be independent of changes in quantity. Of course, there may be exceptions to this rule as in the case of small economies with few market players.

[4]We discuss estimation issues in the subsection Description of Simulations.

$$N^* = -\{[S_{t_0} + E(\Delta S)]\beta^Q + [Q_{t_0} + E(\Delta Q)]\beta^S\} \tag{9}$$

The General Hedging Problem

In general, a pure exporter or portfolio manager faces a portfolio of foreign currency exposures as characterized in Equation 1. Our objective was to generalize the hedging model in Equations 1 and 2 to the case where the firm is exposed to price and quantity uncertainties in I currencies and uses J forward contracts to hedge against those exposures. To this end, we first develop the case of price and quantity uncertainties in two foreign currencies while limiting the number of forward contracts to one ($I = 2$ and $J = 1$). This is analogous to a case where the two-currency exposure is hedged with an index forward contract. Next, we discuss the case of $I = 2$ and $J = 2$. Finally, we present the general hedging model that covers I currencies and J forward contracts.

Case I: Hedging Two Pairs of Price and Quantity Exposures with One Forward Contract

To begin, consider the case of a firm that faces both price and quantity uncertainties in two countries and uses one forward contract to hedge these exposures. The total unhedged cash flow consists of the sum of the individual country flows denominated in dollars: $UC_{t_1} = Q_{1t_1}S_{1t_1} + Q_{2t_1}S_{2t_1}$. To hedge the risk inherent in both the exchange rate and the quantity of foreign currency, a firm may wish to use just one contract. The change in the total hedged cash flow is

$$\Delta HC = \Delta(Q_1S_1) + \Delta(Q_2S_2) + N\Delta F \tag{10}$$

and its variance is given by

$$\begin{aligned}\mathrm{Var}(\Delta HC) &= \mathrm{Var}[\Delta(Q_1S_1)] + \mathrm{Var}[\Delta(Q_2S_2)] + N^2\mathrm{Var}(\Delta F)\\ &+ 2\mathrm{Cov}[\Delta(Q_1S_1), \Delta(Q_2S_2)] + 2N\mathrm{Cov}[\Delta(Q_1S_1), \Delta F]\\ &+ 2N\mathrm{Cov}[\Delta(Q_2S_2), \Delta F]\end{aligned} \tag{11}$$

Following the methodology of the previous section, we found that the minimum variance is achieved at

$$N^* = -\sum_{i=1}^{2} \{[S_{it_0} + E(\Delta S_i)]\beta_i^Q + [Q_{it_0} + E(\Delta Q_i)]\beta_i^S\} \tag{12}$$

where the price and quantity betas (for $i = 1, 2$) are computed in a manner analogous to that of the single cash flow. Once again, we interpret β_i^Q as a measure of quantity exposure to currency i and β_i^S as a measure of price exposure to the ith currency. Equation 12 shows that the optimal number of forward contracts is a function of the expected future prices (exchange rates) and quantities (magnitude of foreign cash flows), as well as the price and quantity betas.

When the firm is indeed exposed to both price and quantity uncertainties in two foreign currencies but chooses to hedge only the single currency (first currency) exposure, Equation 12 shows that the resulting hedge ratio will in general be upward or downward biased. The sign of the bias is difficult to predict and depends on the values of the expected prices and quantities and the sign and magnitude of the beta coefficients.

Case II: Hedging Two Pairs of Price and Quantity Exposures with Two Forward Contracts

If the firm in Case I wishes to hedge both cash flows with two forward contracts, the total unhedged cash flow is the same as in the previous section. The change in hedged cash flow is $\Delta HC = \Delta(Q_1S_1) + \Delta(Q_2S_2) + N_1\Delta F_1 + N_2\Delta F_2$, and its variance is given by

$$\begin{aligned}\mathrm{Var}(\Delta HC) &= \mathrm{Var}[\Delta(Q_1S_1)] + \mathrm{Var}[\Delta(Q_2S_2)] + N_1^2\,\mathrm{Var}(\Delta F_1)\\ &+ N_2^2\,\mathrm{Var}(\Delta F_2) + 2\mathrm{Cov}[\Delta(Q_1S_1), \Delta(Q_2S_2)]\\ &+ 2N_1\mathrm{Cov}[\Delta(Q_1S_1), = \Delta F_1] + 2N_2\,\mathrm{Cov}[\Delta(Q_1S_1), \Delta F_2]\\ &+ 2N_1\,\mathrm{Cov}[\Delta(Q_2S_2), \Delta F_1] + 2N_2\,\mathrm{Cov}[\Delta(Q_2S_2), \Delta F_2]\\ &+ 2N_1N_2\,\mathrm{Cov}[\Delta F_1, \Delta F_2] \qquad (13)\end{aligned}$$

The first-order conditions for a minimum yield the following optimal number of contracts for the first ($j = 1$) and second ($j = 2$) forward contracts:

$$\begin{aligned}N_1^* &= -\frac{1}{1-\rho^2}\sum_{i=1}^{2}\{[S_{it_0} + E(\Delta S_i)]\beta_{i1}^Q + [Q_{it_0} + E(\Delta Q_i)]\beta_{i1}^S\}\\ &- \sqrt{\frac{\mathrm{Var}(\Delta F_2)}{\mathrm{Var}(\Delta F_1)}}\frac{\rho}{1-\rho^2}\sum_{i=1}^{2}\{[S_{it_0} + E(\Delta S_i)]\beta_{i2}^Q\\ &+ [Q_{it_0} + E(\Delta Q_i)]\beta_{i2}^S\} \qquad (14a)\end{aligned}$$

and

$$N_2^* = -\frac{1}{1-\rho^2}\sum_{i=1}^{2}\{[S_{it_0} + E(\Delta S_i)]\beta_{i2}^Q + [Q_{it_0} + E(\Delta Q_i)]\beta_{i2}^S\}$$
$$- \sqrt{\frac{\text{Var}(\Delta F_1)}{\text{Var}(\Delta F_2)}}\frac{\rho}{1-\rho^2}\sum_{i=1}^{2}\{[S_{it_0} + E(\Delta S_i)]\beta_{i1}^Q$$
$$+ [Q_{it_0} + E(\Delta Q_i)]\beta_{i1}^S\} \tag{14b}$$

where ρ is the correlation coefficient between the changes in the two forward prices, $\beta_{ij}^S = \text{Cov}(\Delta S_i, \Delta F_j)/\text{Var}(\Delta F_j)$ is the slope in the simple linear regression of ΔS_i on ΔF_j, and β_{ij}^S is the slope in the regression of ΔQ_i on ΔF_j.

Equations 14a and 14b constitute our primary analytical results for the case of two pairs of price and quantity risks and two forward contracts. It is instructive to contrast this model with the two predecessors, the traditional hedging model and the KM model. Consider the case of non-stochastic foreign currency cash flows and uncorrelated forward contracts ($\rho = 0$); under these restrictions, Equation 14 reduces to

$$N_j^* = -[Q_{1t_1}\beta_{1j}^S + Q_{2t_1}\beta_{2j}^S]$$

The traditional hedging model (Equation 3) is obtained by restricting the cross betas to zero: $N_1^* = -Q_{1t_1}\beta_{11}^S$ and $N_2^* = -Q_{1t_1}\beta_{22}^S$. Similarly, if the cash flows are random, but ρ and all cross betas are null, we obtain

$$N_j^* = -[ES_{jt_1}\beta_{jj}^Q + EQ_{jt_1}\beta_{jj}^S]$$

This equation is a straight forward extension of the KM model (Equation 5) to the case of two pairs of price and quantity risk and two hedging instruments.

These comparisons highlight the advantages of our model to its predecessors in Equations 3 and 5. Equation 14 recognizes the complete correlation structure of the two pairs of price and quantity exposures; in contrast, its predecessors are valid only when certain restrictions are imposed on the covariance matrix.

Case III: Hedging I Currency Exposures with J Forward Contracts

In actual practice, the most common problem is that of a firm with lines of business in many different currencies. In this section, we present the most general result: how to hedge I foreign currency cash flows with J forward contracts.

As we state at the beginning of the section entitled Hedging Models, the goal is to choose the number of contracts N_j for each currency j to minimize the volatility of the total cash flow. The minimum may be determined by direct differentiation of Equation 2 with respect to N_j and then the solving of the first-order conditions. The optimal number for each contract j is given by

$$N_j^* = -\sum_{k=1}^{I} v_{jk} \operatorname{Var}(\Delta F_k) \sum_{i=1}^{I} \{[S_{it_0} + E(\Delta S_i)]\beta_{ik}^{Q} + [Q_{it_0} + E(\Delta Q_i)]\beta_{ik}^{S}\} \qquad (15)$$

where v_{jk} is the jk element in the inverse of the variance–covariance matrix of forward price changes and $\operatorname{Var}(\Delta F_k)$ is the variance of the kth forward price changes.

SIMULATIONS OF OPTIMAL HEDGE RATIOS

In this section, we present the results of four simulations of the optimal hedge ratios for a multinational pure exporter that uses U.S. dollars as its base currency. For simplicity, the most general case we consider is that of two foreign currency cash flows hedged with the corresponding two forward contracts. We compute the optimal hedge ratios with our model (i.e., Equation 14) and compare the results to those obtained from the traditional hedge (nonstochastic cash flows) and to those computed with a naive extension of KM to the two-exposure two-forward contracts case.

Description of Simulations

Given the two pairs of price and quantity exposures and two hedging instruments, a multinational firm may consider the following three hedging strategies: (a) Apply the traditional hedging model (Equation 3) separately to each currency exposure, (b) apply the KM model (Equation 5) separately to each currency exposure, or (c) apply our model (Equation 14) simultaneously to both pairs of price and quantity exposures. The simulations that follow demonstrate the advantages of our portfolio approach to hedging over the first two alternatives: namely, the portfolio model takes advantage of the full correlation structure of changes in spot prices, quantities, and forward prices.

The empirical evaluation of these hedging strategies requires two types of data: firm-specific and market-wide. Foreign currency cash flows,

Q_i, vary from firm to firm, but data on foreign exchange spot and forward rates (S_i and F_j) are market-wide factors. Because it was difficult to obtain the data on firm-specific factors (unlike the widely accessible data on exchange rates), we resorted to simulations that relied on hypothetical data on firm-specific foreign currency cash flows.

The three parameters that captured the cross-correlations considered in our study were the following:

1. The correlation between the changes in prices of two foreign currency forward contracts, ρ.
2. The spot exchange rate-forward contract cross-correlations between the changes in the spot price of currency i (S_i) and the forward price for foreign currency j (F_j), for i not equal to j: $\rho_{ij}^S \equiv \beta_{ij}^S \sqrt{\mathrm{Var}(\Delta F_j)/\mathrm{Var}(\Delta S_i)}$, and
3. The quantity-forward cross-correlations between the changes in the size of the foreign currency cash flow i and the forward price for currency j, for i not equal to j: j: $\rho_{ij}^Q \equiv \beta_{ij}^Q \sqrt{\mathrm{Var}(\Delta F_j)/\mathrm{Var}(\Delta Q_i)}$,

Failure to properly account for these three variables inevitably will lead a manager to systematically overhedge or underhedge relative to the optimal hedge.

We conducted four simulations to determine the impact of the cross-correlations, described in the previous list, on the optimal hedge ratios and hedging effectiveness measures.

Simulation I. In this simulation, we restricted cross-correlations 1–3 to zero. In this case, our model reduced to a naive extension of the KM model to the case of two pairs of exposures, each hedged only with its corresponding forward contract. The optimal hedge ratios for the two foreign currency exposures were computed in isolation without regard to quantity-forward, spot-forward, or forward cross-correlations. This simulation provided a benchmark for comparing our model to that of a naive extension of KM to the case of two pairs of exposures.

Simulation II. In this case, only the change in the foreign currency cash flow was uncorrelated with ΔF_j for $i \neq j$ ($\rho_{ij}^Q = 0$); all other parameters were unrestricted. This simulation isolated the effect on the optimal hedge ratio of a positive or negative correlation between the ith exchange rate and the jth forward contract (ρ_{ij}^S), as well as the forward contract cross-correlation (ρ).

Simulation III. In this simulation, both ρ_{ij}^S and ρ were restricted to be zero; however, the size of each cash flow was correlated with the other forward contract: $\rho_{ij}^Q \neq 0$. If we assumed that the two forward contracts were independent ($\rho = 0$), we could safely assume that the other restriction ($\rho_{ij}^S = 0$) was also satisfied because the changes in the spot rate and forward price for a given currency were nearly perfectly correlated.[5] Although this situation may not be very common in practice, this case isolated the importance of firm-specific quantity-forward cross-correlations.

Simulation IV. No restrictions were imposed in this simulation. We simultaneously considered the effect of the forward contract, price-forward, and quantity-forward cross-correlations on the optimal hedge ratio. This simulation provided an unrestricted application of our model, as presented in Equation 14, to the two-exposure two-forward contract case. This case illustrated the major contribution of our study. When hedging multiple exposures with multiple hedging instruments, it is important to recognize firm-specific price–quantity cross-correlations and correlations among forward price changes.

To illustrate the results of these four simulations, we considered a United States-based firm that wished to hedge the monthly end-of-period cash flows from two foreign subsidiaries: one was a Canadian operation whose cash flow was denominated in Canadian dollars and the other was a French operation whose cash flow was denominated in French francs. We assumed that the management wished to minimize the total risk of its end-of-month cash flow.

The firm used both Canadian dollar and French franc forward contracts to hedge currency risk; the realizations of the end-of-month foreign currency cash flows were unknown to the firm at the beginning of the month. The expectation of the end-of-month foreign currency cash flow was assumed to be equal to the foreign currency cash flow of the previous month. The foreign currency cash flows for the next month were assumed to be independent, with expectations equal to 1,000,000 Canadian dollars and 4,000,000 French francs.[6,7] These foreign currency cash flows were firm-specific, and their variances and covariances with the forward prices were varied in our simulations.

The remaining information needed to implement our hedging model was driven by market-wide forces and was obtained from historical data in the foreign exchange markets. We assumed that the exporter entered into 30-day forward contracts at time t_0 to implement the optimal hedge.

[5]Because S_i and F_i are perfectly correlated, $\lambda S_i = F_i$ with λ equal to some nonzero constant. If F_i and F_j are independent random variables, $\mathrm{Cov}(F_i, F_j) = 0 = \mathrm{Cov}(\lambda S_i, F_j)$, so $\rho_{ij}^S = 0$.

[6]It was not necessary to designate the exposure as long (i.e., a cash inflow) in the foreign currency as we have. Our model can accommodate short exposures (i.e., a contractual cash outflow) and portfolios of long and short exposures.

[7]The choice of 4 million French francs and 1 million Canadian dollars provided for approximately equal foreign currency exposure when denominated in U.S. dollars.

TABLE I

Sample Estimation Parameters for French Francs (FFr) and Canadian Dollars (CDN $)

January 1994 to November 1999 (Monthly)[a]	*M (U.S. $/FCU)*[b]	*SD*	*Maximum*	*Minimum*
	FFr			
Spot rate (S_t)	0.1806	0.0152	0.2089	0.1538
Forward rate (F_t)	0.1807	0.0151	0.2090	0.1542
Δ spot rate ($S_{t_1} - S_{t_0}$)	−0.0002	0.0047	0.0134	−0.0119
Δ 30-day forward rate ($S_{t_1} - F_{t_0}$)	−0.0003	0.0047	0.0135	−0.0123
Spot rate (November 30, 1999)	0.1538			
30-day forward rate (November 30, 1999)	0.1542			
	CDN $			
Spot rate (S_t)	0.7105	0.0295	0.7515	0.6376
30 day forward rate (F_t)	0.7107	0.0295	0.7511	0.6375
Δ spot rate ($S_{t_1} - S_{t_0}$)	−0.0010	0.0094	0.0238	−0.0237
Δ 30-day forward rate ($S_{t_1} - F_{t_0}$)	−0.0012	0.0096	0.0240	−0.0241
Spot rate (November 30, 1999)	0.6786			
30 day forward rate (November 30, 1999)	0.6786			

Correlation Matrix	*Δ FFr Spot*	*Δ FFr Forward*	*Δ CDN $ Spot*	*Δ CDN $ Forward*
Δ FFr spot	1			
Δ FFr forward	0.9974	1		
Δ CDN $ spot	−0.1099	−0.1085	1	
Δ CDN $ forward	−0.0934	−0.0888	0.9960	1

[a]The source of the data was the *Wall Street Journal*, January 1994 to November 1999. Spot and 1 month forward rate prices were taken as the close on the last trading day of each month. [b]All currency exchange rates are expressed as U.S. dollars per foreign currency unit (U.S. $/FCU).

Furthermore, we assume that the exporter held the contract until its expiration at time t_1 and realized the gain/loss from the position. The expiration of the forward contract at time t_1 was assumed to be coincident with the end of the hedging horizon. Thus, we defined the gain/loss on the forward contract as $N^*(\Delta F)$ where N^* is the number of contracts prescribed in the optimal hedge and ΔF is defined as $S_{t_1} - F_{t_0}$ (given the convergence of the forward price/rate at delivery, F_{t_1}, with the prevailing spot price, S_{t_1}). The end-of-month expected spot exchange rate was approximated by the 30-day forward rate at the beginning of the month. The necessary data on the first two moments of the foreign exchange market data are summarized in Table I. The data were taken from the end-of-month closing prices as reported in the *Wall Street Journal* for the period January 1994 to November 1999.

In the simulations that follow, we used unconditional estimates of variances and covariances to determine optimal hedge ratios. This approach suffered from two criticisms. First, it assumed that the joint distribution of spot and futures price changes was stationary. Second, because ΔS and ΔF were likely to be cointegrated, the traditional ordinary least squares might have yielded biased estimates; a GARCH error correction methodology should have been used (see Kroner & Sultan, 1993). However, the purpose of the simulations was to quantify the potential risk reduction from employing the full covariance structure. Although the issues of how to best estimate the second moments and the effect of long-run relationships on the computation of hedge ratios are interesting in their own right, to introduce these complications into our simulations would detract from the focus of the article. Thus, we assumed for the purposes of our simulation that the relevant joint distributions were stationary.

Economic Rationale for Price–Quantity Correlations

The optimal hedge ratios were estimated for selected combinations of three firm-specific factors that determined the foreign exchange exposure of the firm (along with the market-wide factors shown in Table I). The first firm-specific factor was the volatility of the change in the future foreign cash flows, which was denoted by the standard deviation, σ_{Qi}, divided by the expected value of foreign currency cash flows. We varied this ratio from zero (representing the traditional hedging model of non-stochastic foreign cash flows) to 100% (see the columns in Tables II–VI). The second firm-specific factor was the level of correlation between the changes in the quantity of foreign currency cash flows and the corresponding changes in the forward prices, ρ_{ii}^{Q}. These correlations varied from -0.6 to $+0.6$ (see the rows in Tables II–VI) in our simulations. The third specific factor was the level of cross-correlation between the changes in the quantity of foreign currency cash flows from country i and the changes in the forward prices for currency j for i not equal to j, ρ_{ij}^{Q}. These cross-correlations varied from -0.3 to $+0.3$ in our simulations.

Shapiro (1975) examined the effect of a change in exchange rates on the noncontractual cash flows of a multinational firm with uncertain production. He showed that the effect is a function of the level of competition between imports and exports, the existing cost technologies, the degree of substitutability between domestic and foreign inputs, and the classical elasticities of supply and demand. As Shapiro (1975) made clear, the exposure of a multinational firm to changes in the foreign currency exchange rate is a complex matter. This reinforces the notion that the magnitude and sign of the correlation between the changes in the quantity of foreign currency cash flows and the corresponding changes in the forward prices, ρ_{ii}^{Q}, is a firm-specific factor. These arguments apply to a pure exporter as well as a global portfolio manager. Therefore, we varied ρ_{ii}^{Q} from -0.6 to 0.6 in our simulations to capture a wide range of plausible values. To the extent that the foreign markets for the exports of the firm, the available substitutes for these exports, and the relevant foreign currencies were not completely segmented, we expected some level of interaction between them. The reality of increasingly integrated financial and goods markets throughout the world underscores the importance of the cross-correlations. As for the quantity-forward correlations discussed earlier, the sign and magnitude of the quantity-forward cross-correlations were firm-specific and dependent on the complex interaction of numerous factors. However, it was intuitive that the magnitude of the cross-correlations would be smaller than the correlations. As a result, we varied ρ_{ij}^{Q} from -0.3 to $+0.3$ in our simulations to capture a wide range of plausible values.

Simulation Results and Analysis

Table II reports the results of Simulation I. The optimal hedge ratios (H) are reported as a fraction of the expected foreign currency exposure for both the Canadian dollar and French franc. The results in Table II reveal several interesting patterns. The first concerns the effect of ρ_{ii}^{Q} on the optimal hedge ratio H. When the foreign cash flows of the firm were positively correlated with forward exchange rates ($\rho_{ii}^{Q} > 0$; see the first row), its overall currency exposure was magnified (relative to the base case of nonstochastic cash flows). The firm needed a larger short position to hedge this added exposure. This is reflected by the larger optimal hedge ratios reported in the first two rows of both Panels A and B. For example, in Panel A the optimal hedge ratio increases from 1.97 to 2.96 as ρ_{ii}^{Q} increases from 0.3 to 0.6, and the volatility of cash flows is 10%. In contrast, when ρ_{ii}^{Q} was less than 0, the market-wide price exposure of the firm (i.e., fluctuations in spot currency rates) was in part offset by the firm-specific quantity exposure. That is, the firm enjoyed the benefits of a natural hedge between its price and quantity exposures, a natural price–quantity hedge. This is reflected by the lower optimal hedge ratios reported in the last two rows of both Panels A and B. A negative estimate of the optimal hedge ratio implies that because of the natural hedging between price and quantity exposures, even a firm with a long cash market exposure needs to buy forward contracts to minimize portfolio variance.

TABLE II

Optimal Hedge Ratios (H)[a] under Foreign Cash Flow Uncertainty for Simulation I,[b] where Forward, Spot-Forward, and Quantity-Forward Cross-Correlations Are All Assumed To Be Zero

Panel A: The U.S. Dollar/French Franc (FFr) Case

The expected next period cash flow, $E(Q_{1,t})$, is 4,000,000 FFr.

Quantity-Forward Correlations (ρ_{ii}^{Q})	*SD of Foreign Currency Cash Flow Changes* (σ_{Q_1}) *Expressed as a Percentage of* $E(Q_{1,t})$				
	0.00%	*10.00%*	*33.00%*	*50.00%*	*100.00%*
0.6	0.99	2.96	7.48	10.83	20.66
0.3	0.99	1.97	4.24	5.91	10.83
0.0	0.99	0.99	0.99	0.99	0.99
−0.3	0.99	0.01	−2.26	−3.93	−8.85
−0.6	0.99	−0.98	−5.50	−8.85	−18.69

Panel B: The U.S. Dollar/Canadian Dollar (CDN \$) Case

The expected next period cash flow, $E(Q_{1,t})$, is 1,000,000 CDN \$.

Quantity-Forward Correlations (ρ_{ii}^{Q})	*SD of Foreign Currency Cash Flow Changes* (σ_{Q_2}) *Expressed as a Percentage of* $E(Q_{1,t})$				
	0.00%	*10.00%*	*33.00%*	*50.00%*	*100.00%*
0.6	0.97	5.22	14.99	22.21	43.44
0.3	0.97	3.10	7.98	11.59	22.21
0.0	0.97	0.97	0.97	0.97	0.97
−0.3	0.97	−1.15	−6.04	−9.65	−20.26
−0.6	0.97	−3.28	−13.04	−20.26	−41.50

[a] $H = -N^{*}/E(Q_{it})$ where N^{*} is the value of the optimal forward position and $E(Q_{it})$ is the expectation of the next period's foreign currency cash flow. [b] Simulation I assumed all (forward, spot-forward, and quantity-forward) cross-correlations were zero. Under these assumptions, optimal hedge ratios computed with our model were the same as obtained with Equation 5, the model of Kerkvliet and Moffett (1994), extended to two exposures.

A second pattern of interest is the effect of the volatility of foreign cash flows on the optimal hedge ratio. When the volatility increased from zero to 100%, the absolute value of H increased; this result was consistent with KM. Another interesting finding to note in Simulation I is that, *ceteris paribus,* with any foreign currency cash flow uncertainty, the optimal hedge ratio for the Canadian dollar exposure was consistently larger (in absolute value) than the ratio for the French franc exposure. The reason for this pattern is readily seen with an inspection of Equation 5. The first term in Equation 5 represents the traditional hedge ratio, and it is approximately unity. The difference in optimal hedge ratios arises because of the second term, which represents the effect of the quantity-forward correlations. The ratio of the expected spot rate to the standard deviation of the change in the forward contract prices is the only currency-specific factor in the simulation and thus provides the source of the differences in the optimal hedge ratios. This ratio is 71.5 for the Canadian dollar and 32.8 for the French franc. This ratio for the Canadian dollar is 2.18 times that of the French franc; this is the approximate magnitude of the differences in optimal hedge ratios in Table II.

Table III reports the results for Simulation II. The values of the optimal hedge ratios are computed with Equation 14 with the following restrictions imposed: the quantity-forward cross-correlations (ρ_{ij}^{Q}) are set at zero while the forward correlations (ρ) and forward-spot exchange rate cross-correlations (ρ_{ij}^{S}) are accounted for. The traditional hedge ratio ($H \sim 1.00$) was not obtained for nonstochastic foreign currency cash flows (i.e., $\sigma_{Q1} = \sigma_{Q2} = 0$) as for KM. Our optimal hedge ratios were 0.91 and 0.63 for the French franc and Canadian dollar exposures, respectively. Why did our model suggest hedging less than that of KM and the traditional hedging formulation even when hedging nonstochastic cash flows? The answer lies in the explicit consideration of the forward correlation and spot-forward cross-correlations in our model. In this simulation, $\rho = -0.09$ and both spot-forward cross-correlations were also negative (see the bottom panel of Table I).

If forward price changes in the two currencies are negatively correlated, spot price changes in those currencies will also be negatively correlated. Notice that, in Table I (bottom), the correlation between the two spot exchange rate changes is -0.11. The resulting natural hedge reduces the overall exposure, $\mathrm{Var}(\Delta HC)$, in Equation 13. Furthermore, positive price changes in the French franc will in part be offset by negative price changes in the Canadian dollar forward contract. This leaves less residual exposure in French francs to be hedged by the corresponding forward contract. Consequently, the optimal hedge ratio in that forward contract is reduced (in absolute value) for both nonstochastic and stochastic foreign cash flows. Similar arguments explain why the absolute value of all estimates of the Canadian dollar optimal hedge ratios reported in Panel B of Table III are lower compared than those in Panel B of Table II. The opposite result would hold when the forward and spot-forward correlations are positive. Thus, when forward correlations or forward-spot cross-correlations are nonzero, naively extending the KM model to hedge multiple exchange rate exposures leads to overhedging (underhedging) when ρ is negative (positive).

The use of two forward contracts to hedge the two exposures, as in Equation 14 or 15, is not appropriate if the squared correlation between

TABLE III

Optimal Hedge Ratios $(H)^{a,b}$ under Foreign Cash Flow Uncertainty for Simulation II,[c] where Both Quantity-Forward Cross-Correlations Are Zero and Table I Values for Exchange Rate Correlations and Cross-Correlations Are Used

Panel A: Optimal Hedge Ratios (H) for the French Franc (FFr) Exposure

The expected next period cash flow, $E(Q_{1t})$, is 4,000,000 FFr.

Quantity-Forward Correlations (ρ_{ii}^{Q})	*SD of Foreign Currency Cash Flow Changes* (σ_{Q_1}) *Expressed as a Percentage of* $E(Q_{1,t})$				
	0.00%	*10.00%*	*33.00%*	*50.00%*	*100.00%*
0.6	0.91	2.70	6.81	9.85	18.80
0.3	0.91	1.80	3.86	5.38	9.85
0.0	0.91	0.91	0.91	0.91	0.91
−0.3	0.91	0.01	−2.05	−3.57	−8.04
−0.6	0.91	−0.88	−5.00	−8.04	−16.99

Panel B: Optimal Hedge Ratios (H) for the Canadian Dollar (CDN \$) Exposure

The expected next period cash flow, $E(Q_{t,2})$, is 1,000,000 CDN \$.

Quantity-Forward Correlations (ρ_{ii}^{Q})	*SD of Foreign Currency Cash Flow Changes* (σ_{Q_2}) *Expressed as a Percentage of* $E(Q_{1,t})$				
	0.00%	*10.00%*	*33.00%*	*50.00%*	*100.00%*
0.6	0.63	4.57	13.62	20.31	39.99
0.3	0.63	2.60	7.13	10.47	20.31
0.0	0.63	0.63	0.63	0.63	0.63
−0.3	0.63	−1.34	−5.86	−9.21	−19.05
−0.6	0.63	−3.30	−12.36	−19.05	−38.73

[a]$H = N^*/E(Q_{i,t})$, where N^* is the value of the optimal forward position and $E(Q_{i,t})$ is the expectation of the next period's foreign currency cash flow. [b]Optimal hedge ratios for Simulation II were computed with Equation 14. [c]Simulation II assumed that the price-quantity cross-correlations with both zero. Table I values were used for the spot and forward market correlations and cross-correlations.

the changes in the two forward contract prices (ρ^2) approaches unity. In this case, the use of a single hedging instrument to hedge both exposures is warranted (i.e., see Equation 12). Mathematically, when the two hedging instruments are highly correlated, the variance–covariance matrix of the changes in the forward contract prices is singular and thus cannot be inverted. Intuitively, if one hedging instrument is a substitute for another, use only one, not both. In practice, if the liquidity and hedging effectiveness of the two forward contracts are comparable, the forward contract with the higher variance of price changes should be used. This reduces the size of the optimal hedge position and thus minimizes transaction costs.

TABLE IV

Optimal Hedge Ratios $(H)^{a,b}$ under Foreign Cash Flow Uncertainty for Simulation III,[c] where Both Quantity-Forward Cross-Correlations (ρ_{ij}^{Q}) Are Equal to -0.3 and Forward and Spot-Forward Cross-Correlations (ρ_{ij}^{S}) Are Assumed To Be Zero

Panel A: Optimal Hedge Ratios (H) for the French Franc (FFr) Exposure

The expected next period cash flow, $E(Q_{1t})$, is 4,000,000 FFr.

Quantity-Forward Correlations (ρ_{ii}^{Q})	SD of Foreign Currency Cash Flow Changes (σ_{Q_1}) Expressed as a Percentage of $E(Q_{1,t})$				
	0.00%	10.00%	33.00%	50.00%	100.00%
0.6	0.99	1.87	3.90	5.41	9.82
0.3	0.99	0.89	0.66	0.49	−0.01
0.0	0.99	−0.09	−2.59	−4.43	−9.85
−0.3	0.99	−1.08	−5.83	−9.35	−19.69
−0.6	0.99	−2.06	−9.08	−14.27	−29.52

Panel B: Optimal Hedge Ratios (H) for the Canadian Dollar (CDN \$) Exposure

The expected next period cash flow, $E(Q_{1,t})$, is 1,000,000 CDN \$.

Quantity-Forward Correlations (ρ_{ii}^{Q})	SD of Foreign Currency Cash Flow Changes (σ_{Q_2}) Expressed as a Percentage of $E(Q_{2,t})$				
	0.00%	10.00%	33.00%	50.00%	100.00%
0.6	0.97	3.29	8.63	12.57	24.17
0.3	0.97	1.17	1.62	1.95	2.93
0.0	0.97	−0.96	−5.39	−8.66	−18.30
−0.3	0.97	−3.08	−12.40	−19.28	−39.54
−0.6	0.97	−5.20	−19.40	−29.90	−60.77

[a]$H = -N^*/E(Q_{it})$, where N^* is the value of the optimal forward position and $E(Q_{it})$ is the expectation of the next period's foreign currency cash flow. [b]Optimal hedge ratios for Simulation III were computed with Equation 14. [c]Simulation III assumed nonzero quantity-forward cross-correlations. However, the forward and spot-forward cross-correlations were assumed to be zero to isolate the effect of considering nonzero quantity-forward cross-correlations.

Table IV reports the results for Simulation III. The values of the optimal hedge ratios were computed with Equation 14 with restrictions 1 and 2 imposed. Simulation III assumes the forward and spot-forward cross-correlations are zero (i.e., $\rho = \rho_{ij}^{S} = 0$); however, the quantity-forward cross-correlations (ρ_{ij}^{Q}) are not constrained to zero and are assumed to equal -0.3. In general, positive correlations and cross-correlations between ΔQ and ΔF lead to a larger foreign exchange exposure and require more forward contracts to minimize risk. However, if the own correlations (ρ_{ii}^{Q}) are positive but the cross-correlations (ρ_{ij}^{Q}) are negative, the overall exposure may be reduced, resulting in a lower optimal hedge ratio. If both the own and cross-correlations are negative, even a firm with a long expected exposure may need to buy, rather than sell, forward contracts to minimize the portfolio variance.

A comparison of Table IV to Table II reveals that with foreign currency cash flow uncertainty, if the quantity-forward correlations (ρ_{ii}^{Q}) and cross-correlations (ρ_{ij}^{Q}) are all negative, optimal hedging now requires a greater long position (i.e., increased foreign currency exposure) compared to a naive extension of KM. However, if the quantity-forward correlations and cross-correlations are all positive, optimal hedging will require a greater short forward position compared to a naive extension of KM (not reported in Table IV).[8] Also, when the signs of the quantity-forward correlations and cross-correlations differ, optimal hedging in our model is accomplished with a smaller forward position compared with the optimal hedge positions of the KM model naively extended to the two-exposure case (e.g., compare the estimates in the first two rows of Panel A of Table II to those of Panel A in Table IV).

Also, *ceteris paribus*, when the quantity-forward correlations are positive (i.e., $\rho_{ii}^{Q} > 0$), as the quantity-forward cross-correlations (ρ_{ij}^{Q}) increase, optimal hedging in our model requires a larger forward short position. As an example, consider the optimal hedge ratio (H) for the French franc forward contract position for the case when the standard deviation of both future cash flow changes is 10% and the quantity-forward correlations (ρ_{ii}^{Q}) are both $+0.3$. Now, as the quantity-forward cross-correlations increase from -0.3 (Table IV) to 0.0 (Table II) to $+0.3$ (this table is not presented), the value of H increases from 0.89 to 1.97 to 4.02. Obviously, the optimal hedge ratio is sensitive to changes in the quantity-forward cross-correlations.

The patterns of the optimal hedge ratios in Table IV are also similar to those noted in KM. The optimal H is unbounded, surpassing unity for positive values of quantity-forward correlations (ρ_{ii}^{Q}) and increasing cash flow uncertainty. The acquisition of increased foreign currency exposure (i.e., $H < 0.0$) occurs in cases of sufficiently high cash flow uncertainty with nonpositive quantity-forward correlations.

Table V reports the results for Simulation IV. The values of the optimal hedge ratios were computed with Equation 14 without imposing any restrictions. Simulation IV extends Simulation III by considering the effect of negative forward and spot-forward cross-correlations in addition to the quantity-forward cross-correlations. Specifically, the bottom panel in Table I shows that the correlation between the French franc and Ca-

[8] A table detailing the optimal hedge ratios when the price–quantity cross-correlations are positive is not presented to conserve space. For price–quantity cross-correlations equal to $+0.3$, the results are very similar to Tables IV and V, except that the signs of the price–quantity correlations are reversed.

TABLE V

Optimal Hedge Ratios $(H)^{a,b}$ under Foreign Cash Flow Uncertainty for Simulation IV,[c] where Both Quantity-Forward Cross-Correlations (ρ_{ij}^{Q}) Are Set Equal to -0.3 and Table I Values for Exchange Rate Correlations and Cross-Correlations Are Used

Panel A: Optimal Hedge Ratios (H) for the French Franc (FFr) Exposure

The expected next period cash flow, $E(Q_{1t})$, is 4,000,000 FFr.

Quantity-Forward Correlations (ρ_{ii}^{Q})	SD of Foreign Currency Cash Flow Changes (σ_{Q_1}) Expressed as a Percentage of $E(Q_{1,t})$				
	0.00%	10.00%	33.00%	50.00%	100.00%
0.6	0.91	1.69	3.50	4.83	8.75
0.3	0.91	0.80	0.54	0.36	−0.19
0.0	0.91	−0.10	−2.41	−4.12	−9.14
−0.3	0.91	−0.99	−5.36	−8.59	−18.08
−0.6	0.91	−1.89	−8.31	−13.06	−27.03

Panel B: Optimal Hedge Ratios (H) for the Canadian Dollar (CDN \$) Exposure

The expected next period cash flow, $E(Q_{1,t})$, is 1,000,000 CDN \$.

Quantity-Forward Correlations (ρ_{ii}^{Q})	SD of Foreign Currency Cash Flow Changes (σ_{Q_2}) Expressed as a Percentage of $E(Q_{2,t})$				
	0.00%	10.00%	33.00%	50.00%	100.00%
0.6	0.63	2.82	7.84	11.55	22.47
0.3	0.63	0.85	1.34	1.71	2.79
0.0	0.63	−1.12	−5.15	−8.13	−16.89
−0.3	0.63	−3.09	−11.65	−17.97	−36.57
−0.6	0.63	−5.06	−18.14	−27.81	−56.25

[a]$H = N^*/E(Q_{i,t})$, where N^* is the value of the optimal forward position and $E(Q_{i,t})$ is the expectation of the next period's foreign currency cash flow. [b]Optimal hedge ratios for Simulation IV were computed with Equation 14. [c]Simulation IV considered all futures, futures-spot, and quantity-forward cross-correlations.

nadian dollar forward price changes, ρ, was approximately −0.09, and the spot-forward cross-correlations (ρ_{ii}^{S}) were approximately −0.10. These negative cross-correlations were associated with a negative correlation of 0.11 between the spot exchange rate changes of the two currencies. The overall unhedged exposure of the firm declined because of this negative correlation in the spot exchange rate changes. In addition, if the quantities of foreign cash flows were negatively cross-correlated with the spot and forward exchange rate changes as assumed in Simulation III $(\rho_{ii}^{Q} = -0.3)$, the overall exposure of the firm was further reduced, and it required fewer forward contracts to hedge this reduced exposure. As expected, the optimal hedge ratios computed for this case were less in absolute value than the ratios for Simulation III (see Table IV).

Hedging Effectiveness

We have demonstrated that when a firm faces multiple price and quantity exposures, our model gives optimal hedge ratios different from those obtained by the naive extension of KM. In this section, we examine the *ex post* hedging effectiveness of our model. We employ the traditional measure of hedging effectiveness (e) defined as one minus the ratio of the hedged cash flow variance to the unhedged cash flow variance. The variance of the hedged portfolio is obtained by the substitution of N_j^* for N_j in Equation 13. The variance of the change in the unhedged cash flow is defined as

$$\begin{aligned}\mathrm{Var}(\Delta UC) &= \mathrm{Var}[\Delta(Q_1S_1)] + \mathrm{Var}[\Delta(Q_2S_2)] \\ &\quad + 2\mathrm{Cov}[\Delta(Q_1S_1), \Delta(Q_2S_2)] \qquad (16)\end{aligned}$$

Panel A of Table VI reports the hedging effectiveness (e) achieved when the two foreign currency exposures were hedged with the optimal hedge ratios we computed in Simulation IV (see Table V). Panel B of Table VI reports the hedging effectiveness (e) when the optimal hedge ratios were used that were computed in Simulation I, the two-exposure naive extension of KM (see Table II). Recall that in Simulation IV, no restrictions were imposed on the forward, spot-forward, or quantity-forward cross-correlations. The hedging effectiveness estimates reported in Panel A were invariably larger than the corresponding values in Panel B. The differences between positive estimates ranged from 5 to 19% and represented the potential increase in hedging effectiveness from using our model versus a naive extension of KM. Table VI suggests that the ability of the multinational firm to hedge a portfolio of foreign cash flows was enhanced by the consideration of the cross-correlations.[9] The increased effectiveness of our model in reducing portfolio variance in comparison with a naive extension of KM was an increasing function of the absolute magnitude of the quantity-forward cross-correlations. For example, assuming the quantity-forward cross-correlations (ρ_{ij}^{Q}) were both -0.1, consider the case when the quantity-forward correlations (ρ_{ii}^{Q}) were both -0.6 and the standard deviations of the foreign cash flows were 10%:

[9]Hedging effectiveness is increased for all nonzero choices for magnitude and/or the sign of the quantity-forward cross-correlations (ρ_{ij}^{Q}). Only one case is presented here.

TABLE VI

Comparison of the *ex post* Hedging Effectiveness (e) for Simulation IV, where Both Quantity-Forward Cross-Correlations Are −0.3 and Table I Values for Exchange Rate Correlations and Cross-Correlations Are Used

Quantity-Forward Correlations (ρ_{ii}^{Q})	*Panel A: Hedging Effectiveness (e)*[b,c] *SD of Foreign Currency Cash Flow Changes Expressed as a Percentage of $E(Q_{1,t})$*				
	0.00%	*10.00%*	*33.00%*	*50.00%*	*100.00%*
0.6	*[a]	0.22	0.13	0.12	0.11
0.3	*	0.04	0.00	0.00	0.00
0.0	*	0.02	0.06	0.07	0.08
−0.3	*	0.23	0.34	0.35	0.37
−0.6	*	0.81	0.85	0.85	0.86
Panel B: Hedging Effectiveness (e) for Naive Extension of Kerkvliet and Moffett (1991)					
0.6	*	0.17	0.07	0.06	0.05
0.3	*	−0.04	−0.08	−0.08	−0.08
0.0	*	−0.09	−0.04	−0.02	−0.01
−0.3	*	0.08	0.22	0.24	0.26
−0.6	*	0.62	0.72	0.73	0.74

[a]*indicates an infeasible combination of simulation parameters. [b]Measure of hedging effectiveness (e) = 1 − (variance hedged/variance unhedged), where the variance hedged is defined by Equation 13; the variance unhedged is defined by Equation 16. [c]Hedged variances for Panel A were computed with optimal hedge ratios from Table V. Hedged variances for Panel B were computed with optimal hedge ratios from Table II for the two-exposure naive extension of Kerkvliet and Moffett (1991).

the improvement in hedging effectiveness with our model was only 2.5%: $e = 0.38$ for our model and $e = 0.35$ for the naive extension of KM.[10] However, in Table VI, which assumes the quantity-forward cross-correlations were −0.3, the improvement in hedging effectiveness when our model was used increased to 19%, $e = 0.81$ as opposed to $e = 0.62$.

Consistent with our general hedging model, the estimates reported in Table VI show that the ability to hedge the variance of a portfolio was a function of the magnitude and sign of the quantity-forward correlations and cross-correlations as well as the foreign cash flow uncertainty. When the signs of the quantity-forward correlations and cross-correlations (i.e., ρ_{ii}^{Q} and ρ_{ij}^{Q}) were the same, hedging effectiveness remained high relative to the case when their signs differed. Note the wide range of hedging effectiveness in Table VI for stochastic cash flows; hedging effectiveness remained high only in the case of strong quantity-forward correlations

[10]The optimal hedge ratios and hedging effectiveness measures for $\rho_{ij}^{Q} = -1.0$ are not presented in table form to preserve space. The information is available from the authors on request.

(ρ_{ii}^{Q}) if the correlations had the same sign as the quantity-forward cross-correlations (ρ_{ij}^{Q}). Given quantity-forward correlations and cross-correlations of the same sign, *ceteris paribus,* a decrease in the standard deviation of cash flow changes, an increase in the magnitude of the quantity-forward correlations, or an increase in the absolute magnitude of the price–quantity-forward cross-correlations all led to increased hedging effectiveness.

In the cases for which the correlations and cross-correlations were approximately equal in magnitude with opposite signs, hedging effectiveness was essentially zero. The findings of very low or even negative hedging effectiveness for various simulation parameter combinations may be a partial explanation for the finding that many multinational firms choose not to hedge at all.

The negative values of hedging effectiveness (e) in Panel B of Table VI at first seem improbable. How can optimal hedging increase the variance of the portfolio? Recall that the variance of the unhedged and hedged portfolios are misspecified in a naive two-exposure extension of KM; the naive KM extension fails to recognize the forward, spot-forward, and quantity-forward cross-correlations in forming the optimal hedge ratio. This results in overhedging relative to the true optimal hedge. As a result, the variance of the hedged portfolio can exceed that of the unhedged portfolio.

The results reported in Table VI used the full sample to estimate hedge ratios and hedging effectiveness. The *ex post* nature of these measures implies that the reported hedging effectiveness is indicative of the maximum potential gain from hedging. A more realistic measure of hedging effectiveness is given by an *ex ante* measure that evaluates the performance of hedging strategies using out-of-sample data.

CONCLUSIONS

We have examined the risk-management problem of pure exporters and global portfolio managers who face multiple quantity and price risks and wish to hedge these exposures by using forward/futures contracts. In hedging these multiple exposures, we suggest that the firm should not consider the individual price and quantity risks in isolation as would be the case if it decentralized risk management to the regional or country level. A firm that proceeds in this manner is implicitly ignoring the covariance structure of its multiple exposures and hedging positions. However, if the firm does centralize the management of its foreign currency exposures, the extant hedging literature does not provide a general framework relevant to the case of multiple price and quantity exposures. Consequently, a multinational exporting firm is left with little guidance but to proceed with the existing single-exposure hedging models (e.g., Ederington, 1979; KM, 1991), which often lead to systematic underhedging or overhedging.

Our contribution in this article is the development and application of a more general hedging framework to accommodate hedging multiple price and quantity exposures. In developing our model, we explicitly considered the cross-correlations between the changes in the prices of hedging instruments themselves and the correlations between changes in the prices of each of the hedging instruments and the changes in the price and quantity of each exposure. Our more general framework includes the hedging of a single uncertain cash flow and the traditional hedging model as special cases. We have shown by simulation that the ability of a multinational firm to hedge multiple foreign currency exposures is enhanced by the use of our model vis à vis the traditional hedge model or a naive multiple exposure extension of KM. The improvements in hedging effectiveness that we obtained by applying our model were robust to a wide range of firm-specific model parameters. This gain in hedging effectiveness was an increasing function of the absolute magnitude of the cross-correlations among hedging instruments, spot and forward price changes, and quantity and forward price changes. Indeed, in the unlikely event when all of these cross-correlations were zero, our model collapsed to the extant single-exposure models. To the extent that the relevant cross-correlations were nonzero, our model provided management with the necessary framework to more effectively manage multiple price and quantity exposures.

Our analysis suffered from two limitations. Limiting the scope of this study to the case of pure exporters and global portfolio managers allowed us to focus on financial hedging strategies; we have not addressed the more challenging hedging problems confronting multinational firms with uncertain production. Optimal hedging strategies of such producers typically consist of not only financial hedges but also operational hedges, such as global plant locations (e.g., Chowdhry & Howe, 1999). Financial strategies are relevant to multinational producers for hedging only short-term exposures, but operational hedges are more suitable for managing long-term exposures. Moreover, we have focused on linear hedging strategies based on forward/futures contracts but ignored nonlinear strategies involving option contracts. Finally, we have provided an *ex post* analysis of the effectiveness of multiple price and quantity exposures so that we can compare the performance of our model with that of the previous studies in the area, such as KM. It would be interesting to examine the *ex ante* effectiveness of our hedging model with out-of-sample data; these extensions represent topics for future research.

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