Risk Premiums on Inventory Assets: The Case of Crude Oil and Natural Gas

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This study tested for the presence of risk premiums on crude oil and natural gas. The econometric analysis followed from a stochastic model in which the equilibrium value of inventories depends on a convenience yield and an option value related to price uncertainty. The empirical findings provide rather strong support for the presence of risk premiums and also evidence for the existence of convenience yields. The risk premiums rose sharply with greater price volatility and help to explain why prices for immediate sales often exceed prices for future delivery. © 2001 John Wiley & Sons, Inc. Jrl Fut Mark 21:109–126, 2001

INTRODUCTION

Understanding the role of storage in commodity price determination has captured the interest of many researchers. Nearly all studies of storage

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build on the simple notion that the marginal benefits from stock holding must equal their marginal costs. Researchers differ, however, in how they define these costs and benefits and in how they use this equilibrium condition to specify models of prices for spot and future delivery of commodities.

The traditional theory of storage developed by Kaldor (1939), Working (1934), Brennan (1958), and Telser (1958) defined the marginal benefit of storage by expected price less the current price. Brennan argued that the net marginal cost of storage includes the financial opportunity cost of capital, a risk premium, and direct warehousing and insurance costs less the marginal benefit, or *convenience yield*, earned from holding inventories. These benefits arise from the flexibility stocks provide firms to produce when costs are low or to avoid costly changes in production. The marginal convenience yield rises sharply as inventories reach low levels. As a result, the net marginal cost of storage falls with inventories and can become negative if the convenience yield is sufficiently high, which would imply that benefits would also have to be negative. Consequently, the convenience yield plays a central role in explaining why prices for immediate sale are often higher than prices for future delivery, a situation called *commodity price backwardation*. Pindyck (1994) provided some systematic econometric evidence linking the convenience yield with the stock-to-sales ratio for several commodities. Pindyck admitted, however, that his derivation of the convenience yield was arbitrary.

Another approach, pioneered by Gustafson (1958), offers more structure with a dynamic programming framework to solve for rational storage rules. Williams and Wright (1991) developed this approach further and numerically implemented their model to examine an array of storage issues. Unlike the supply of storage literature, their equilibrium condition did not include a convenience yield. Instead, price backwardations arose from the possibility of stock-outs.

Deaton and Laroque (1996) examined whether this theory explains actual commodity prices. Assuming harvests were identically and independently distributed random variates, they found their model could not explain the high degree of autocorrelation in prices. Moreover, Deaton and Laroque illustrated that price predictions from the storage model do not detect autocorrelation when prices are high; in effect, the model completely misses price spikes. Chambers and Bailey (1996) showed that the stock-out price is constant when independent and identically distributed disturbances are assumed but varies under periodic or time-dependent disturbances. To estimate the periodic disturbance model, they arbitrarily grouped observations to correspond with various stock-out prices and found qualified support for this specification.

The results of these studies illustrate that econometric testing of commodity stock-out models is difficult because they rely on restrictive assumptions to ensure unique solutions for optimal storage rules. For example, the nonnegativity constraint on stocks is critical in generating the optimal storage rules, ensuring that storage only occurs when future prices exceed spot prices. Price backwardations occur only when stocks are zero. Williams and Wright (1991) noted that there have been no stock-outs in Chicago wheat markets during the last 120 years and conceded that storage appears to take place during periods of backwardation, in apparent violation of their price-arbitrage conditions. Similarly, in the discussion of the data sample used in this study, there appears scant evidence of stock-outs in crude oil and natural gas markets.

Litzenberger and Rabinowitz (1995) offered another explanation for commodity price backwardations. They developed an option model for crude oil production, viewing ownership of an oil field as a call option with an exercise price equivalent to extraction costs. For example, producers have two choices: to exercise the option to pump oil at marginal extraction costs or to preserve the option to produce in the future by leaving oil underground. Litzenberger and Rabinowitz argued that *weak backwardation,* when discounted futures prices are below spot prices, is necessary for current production because all producers would choose to defer production if discounted future prices were higher than spot prices. They showed that under price uncertainty, strong backwardation may be necessary for current production.

Asset valuation models developed by Brennan and Schwartz (1985) and later by Gibson and Schwartz (1990) also capture such an option value and include a marginal convenience yield. In particular, Gibson and Schwartz developed an empirical model of pricing assets that explicitly models an uncertainty premium, but they assumed an exogenously specified convenience yield. Sundaresan (1984), Pindyck (1980), and Litzenberger and Rabinowitz (1995) all considered uncertainty but not the convenience yield.

The model developed in this study contains the convenience yield and an option value under price uncertainty. Unlike the supply of storage literature, we assumed, similarly to studies on inventory behavior by Ramey (1989) and Considine (1997), that inventories are a factor of production. In this case, the shadow value of inventories is equivalent to Brennan's concept of the convenience yield. The option value is similar to the one developed by Larson (1994) to explain price backwardations for refined copper. Like Slade and Thille (1997), our model uses the capital asset pricing condition discussed by Dixit and Pindyck (1994) but does so in a short-run context with the marginal value of inventories replacing their shadow value on reserves. This formulation allows direct estimation of an inventory equilibrium condition that contains a risk premium. Brennan (1958) discussed the possibility of a risk premium on inventory holding that rises slowly with stocks but then rises sharply at high stock levels as more capital becomes exposed. In this case, the risk premium is not an integral part of price backwardations because it is small at low stocks levels. In the next section, our storage equilibrium condition includes an uncertainty premium that is a function of the variance in prices, which allows uncertainty to play a larger role in explaining price backwardations.

After discussion of the model, we discuss econometric specification, including the discrete time approximation of the continuous time model developed in the following section. We next provide a factual overview of the crude oil and natural gas sample used in the econometric analysis in the Data Sample section. We then present the empirical results. We conclude the article with a brief summary of the main findings.

A MODEL OF INVENTORIES UNDER PRICE RISK

Our model assumes that firms maximize expected profit subject to sales from current production or carryover inventories and conditional on a stochastic process for prices. Revenues are simply the state variable price, P_t, times sales, s_t, from production or inventories. Production costs are a function of the state variable inventories, X_t; input prices, w_t; production, y_t; capacity, k_t; and the state of technology, z_t, represented with a time trend in the empirical estimation below. The objective function for this problem is

$$J(X_0,P_0,w_0,y_0,k_0,z_0) = \max_{y_y,s_t} E_t \int_0^\infty [P_t s_t - C_t(X_t,w_t,y_t,k_t,z_t)]e^{-\rho t}dt \quad (1)$$

subject to

$$dX_t = (y_t - s_t)\, dt \quad (2)$$

and

$$dP_t = \mu(P_t)dt + \sigma(P_t)db \quad (3)$$

where db is a standard Weiner process, ρ is the risk-adjusted discount rate, $\mu(P_t)$ is the mean rate of price appreciation, and $\sigma(P_t)$ is the standard deviation in price.

The Bellman equation for this problem is

$$\rho J = \max_{s,y}\left\{Ps - C(X,w,y,k,z) + J_X(y-s) + \mu(P)J_P + \frac{1}{2}\sigma(P)^2 J_{PP}\right\} \quad (4)$$

Differentiating Equation 4 with respect to s and y provides the first-order conditions:

$$\rho J_s = P - J_X = 0 \Rightarrow P = J_X \quad (5)$$

$$\rho J_y = -C_y + J_X = 0 \Rightarrow P = C_y \quad (6)$$

These conditions imply that firms equate marginal revenue with the marginal value of stocks and the marginal costs of production. The marginal value of stocks follows from differentiating Equation 4 with respect to X:

$$\rho J_X = -C_X + J_{XX}(y - s) + \mu(P)J_{PX} + \frac{1}{2}\sigma(P)^2 J_{PPX} \quad (7)$$

Stochastically differentiating Equation 5 yields the following:

$$\frac{E\{dP\}}{dt} = J_{XX}(y - s) + \mu(P)J_{PX} + \frac{1}{2}\sigma(P)^2 J_{PPX} \quad (8)$$

This expression yields the expected capital gain from holding inventory, which is equivalent to the value of the option to wait and sell from inventory next period. Hence, Equation 7 states that the marginal value of inventories is equal to the shadow value of inventories, C_X, and an option value.

Substituting the first-order condition (Equation 5) and the option value (Equation 8) into Equation 7 and rearranging provide an equilibrium condition that equates the marginal costs and benefits of stock holding:

$$\rho P - \frac{E\{dP\}}{dt} = -C_X \quad (9)$$

If the expected change in price is zero, this condition collapses to a typical factor market equilibrium condition for a quasifixed input. In this case, the user cost, ρP, equals the shadow value on stocks, C_X, which is negative if stocks generate cost or production smoothing benefits. Holding inventories allows firms the flexibility to smooth production and avoid large and disruptive production changes in response to random sales shocks. Another benefit arises from the ability to produce when input costs are low and to sell from stocks when input prices are high. These production and cost smoothing benefits that arise from stocks have been the focus of many studies of inventory behavior that were surveyed by Blinder and Maccini (1991). These comments suggest that the term on the right of Equation 9, $-C_X$, is equivalent to the convenience yield from the classic theory of storage.

When expected price changes are nonzero, user costs on the left of Equation 9 become dynamic. Under price backwardation, the marginal convenience yield is positive so that variable costs decline with higher inventories, $C_X < 0$. If the expected price change is positive, the market is in *contango*. Under these conditions, expected capital gains on inventories encourage firms to build stocks beyond levels that generate cost and production smoothing benefits, which implies that the convenience yield is negative and C_X is greater than 0.

Dixit and Pindyck (1994, p. 115) showed that the risk-adjusted discount rate from the capital asset pricing model is

$$\rho = r + \phi\sigma_{X,m}\sigma \tag{10}$$

where r is the risk-free interest rate, ϕ is the market price of risk, $\sigma_{X,m}$ is the covariance between the return from holding inventory and the return on the market portfolio, and σ is the standard deviation in crude oil prices. The last term in Equation 10 is the risk premium reflecting the nondiversifiable risk of the asset. Substituting Equation 10 into Equation 9 and rearranging yield

$$\frac{[E\{dP\}/dt]}{P} = r + \frac{C_X}{P} + \phi\sigma_{X,m}\sigma \tag{11}$$

If C_X is zero, Equation 11 becomes a risk-adjusted Hotelling (1931) rule equivalent to the one developed by Slade and Thille (1997). Price backwardations occur when expected price changes are negative, which may result from a convenience yield when C_X is less than 0. If the correlation between the returns on crude oil and the market portfolio is negative[1] and if the market price of risk is positive, the risk premium will be neg-

[1]There is evidence for this. The correlations between the return from Standard and Poor's 500 Stock Price Index and the returns to storage for crude oil and natural gas calculated with the first and second nearby futures are $-.068$ and $-.120$, respectively.

ative, providing another reason why spot prices may exceed forward prices.

ECONOMETRIC SPECIFICATION

Two approaches exist for estimating the previous model. One is to specify some approximation of the value function and estimate the price function (Equation 5) and the expected price expression (Equation 8). In this case, the convenience yield follows from solving Equation 7 for $-C_X$. Another approach, adopted in the following text, involves direct estimation of the convenience yield with Equations 6 and 11. In either case, a discrete approximation of the continuous time model is required for econometric estimation. A stochastic difference equation provides a convenient approximation to Equation 11 that Slade and Thille (1997) employed in their empirical analysis and Epstein and Denny (1983) used in their estimation of a nonstochastic dynamic dual model. Employing such an approximation of Equation 11 and rearranging provide

$$E(P_{t+1}) - P_t - rP_t = C_{X,t} + \psi\sigma_t P_t \tag{12}$$

where $\psi = \phi\sigma_{X,m}$ is the market risk parameter weighted by the covariance of inventory returns with the market portfolio. This formulation avoids any arbitrary decisions that could enter the model through the definition of the market portfolio. This specification also assumes that firms myopically update or re-solve the optimization problem each period. The left side of Equation 12 is the returns to storage or the capital gain on inventory less the opportunity cost of capital. The net returns from storage equal the shadow value on stocks plus a risk premium. Marginal cost (Equation 6) and the returns to storage (Equation 12) constitute our two-equation econometric model.

This approach requires specification of a cost function, which represents the cost of producing, transporting, and delivering crude oil and natural gas. The typical specification of a short-run cost function assumes that variable costs are a function of input prices and output contingent on technology and quasifixed stocks of inventories and capital.

Several alternative functional forms are available, but they all must contain nonzero third-order derivatives because of the nature of the value function presented previously. In earlier versions of this article, we considered quadratic forms for marginal cost and the shadow value on stocks derived from a cubic cost function with results quite similar to those reported here. A translog cost function is another option. The nonlinear form for marginal cost and the convenience yield from the translog, however, requires data on variable production costs, which are unavailable. The generalized Leontief form used by Morrison (1988) avoids this problem but involves a large number of parameters typically estimated with a cost function and a set of input demand schedules. Data on input quantities, however, are unavailable. Hence, in this article we use a simple symmetric, square-root quadratic cost function:

$$C_t = \sum_{i=1}^{5} \alpha_i A_{it}^{1/2} + \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} A_{it}^{1/2} A_{jt}^{1/2} \tag{13}$$

where A_{it} includes production, y_t; inventories, X_t; input prices; production capacity; and a time trend for technological change, where γ_{ij} equals γ_{ji}. This parsimonious specification allows nonzero third-order derivatives and ensures nonnegativity for stocks and production. The marginal cost equation takes the following form:

$$P_t = \gamma_{yy} + (1/2)\alpha_y y_t^{-1/2} + \sum_{i \neq y} \gamma_{iy} (A_{it}/y_t)^{1/2} \tag{14}$$

The estimating equation for the returns-to-storage relation is as follows:

$$P_t^f - P_t - r_t P_t = \gamma_{XX} + (1/2)\alpha_X X_t^{-1/2} + \sum_{i \neq x} \gamma_{iX} (A_{it}/X_t)^{1/2} + \psi P_t \sigma_t \tag{15}$$

We assume that $E(P_{t+1}) = P_t^f$, where P_t^f is the price on the first or second nearby futures contract. Futures prices may serve as a good proxy for expectations of prices next period.[2] This relation is equivalent to the absence of the arbitrage condition widely used in the futures market literature in which the returns to storage reflect a convenience yield and a risk premium. Appending random error terms to Equations 14 and 15 to represent errors in expectations and measurement, the generalized method of moments (GMM) estimator provides consistent estimates.[3] The GMM estimator is potentially superior to conventional instrumental variable estimators, yielding consistent and efficient estimators with past or future values of the instruments correlated with the error terms.

The information set available to the crude oil and natural gas market provides a basis for selecting instrumental variables. The instruments include trend, monthly dummy variables, and lagged values of spot prices, futures prices, or both; input prices; production; total private and public stocks; capacity; and volatility. This list also includes demand shock in-

[2]An anonymous referee pointed out that Equation 15, when rearranged, provides the absence of arbitrage opportunities equation in futures markets: $P_t^f = (1 + r_t + \psi\sigma_t)P_t + C_{X,t}$.

[3]Hui (1998) found that the data used in this study were stationary.

struments, including heating and cooling degree days, and supply shock instruments, crude oil or natural gas imports, respectively. All dummy variables in the model are only included as instruments, not as explanatory variables.

Our GMM estimator allows conditional heteroscedasticity and autocorrelation in the error terms. Newey and West (1994) developed a procedure for determining the number of moving average components or bandwidths in the errors. In this study, we adapted their method to the multi-equation case by estimating Equations 14 and 15 with three-stage least squares and a series of autoregressions on the residuals, calculating bandwidths for each equation and then taking an average for the system. These calculations provide the number of moving average components in the autocorrelations for the final GMM estimation of the marginal cost and returns-to-storage relations.

DATA SAMPLE

Crude oil and natural gas provide two interesting case studies. Both commodities have similar extraction technology but different storage technologies and consumption patterns. Crude oil is less costly to transport than natural gas over long distances. In addition, natural gas consumption has much more pronounced seasonal fluctuations.

Futures trading of crude oil began with the introduction of the contract for West Texas Intermediate crude petroleum in 1983. The natural gas futures contract for delivery at the Henry Hub started trading in April 1990. This study used three different price spreads for each of these two commodities. The first was between the spot price reported by the *Wall Street Journal* and the price on the first nearby futures contract traded at the New York Mercantile Exchange. The second price spread was between the first and second nearby futures. The third spread was between the first and second nearby prices on the last trading day of the month because month-end inventories appear in the model. A daily standard deviation for prices using the previous 20 trading days, expressed at an annual rate, provided a measure of historic volatility. A monthly average of these daily values was the estimate for σ_t. Last-day-of-month values for historic volatility also were used in the analysis.

Estimating the model requires the measurement of variable input prices, production, and inventories. The selection of the arguments in the cost function, A_{it}, was very similar to the cost function estimated by Pindyck (1994). Data for production and month-ending stocks of crude oil and natural gas were from the Energy Information Administration (EIA).

The input prices included prices for labor, energy, and materials. Wages per hour equaled average weekly earnings divided by average weekly hours reported by the U.S. Bureau of Labor Statistics. Wages included benefits inferred from total compensation reported by the U.S. Bureau of Census. The energy price index is a weighted average of the fuels that producers use to extract crude oil and natural gas. This index includes prices for electricity, distillate fuel, natural gas, residual oil, and gasoline from the EIA, where the weights are cost shares reported by the U.S. Bureau of Census. Electricity comprises approximately two thirds of the energy consumed in oil and gas extraction. The material price was the price index for industrial materials less energy published by the U.S. Bureau of Labor Statistics.

An overview of the salient statistics of the samples appears in Table I. The U.S. federal government holds almost 50% of total crude oil stocks in the strategic petroleum reserve (SPR). The decision-making process of the SPR managers may be different from private inventory decisions. Variations in the SPR, however, are important, contributing to the information set available to private stockholders. Consequently, the econometric estimates reported here are with and without SPR stocks. Stocks and flows are in physical units unadjusted for seasonal variation, which avoids the measurement errors identified by Fair (1989).

Crude oil production in the United States over the sample averaged 7.5 million barrels per day (see Table I). Total private crude oil inventories are on average about 334 million barrels with a standard deviation of 16.6 million barrels. Variability in domestic production is slightly greater than the variability in domestic crude oil sales, contrary to production smoothing. The variance in crude oil imports is far higher than the variance in sales, suggesting that producers may use imports to meet swings in seasonal refining needs.

Unlike crude oil, the standard deviation in natural gas production is far less than the variability in sales. Imports also do not appear to play the smoothing role that crude oil imports serve. Natural gas storage levels vary substantially (see Table I), rising sharply during the summer and falling during the winter heating season. These market features may be one reason why historic price volatility in natural gas is more than 20% higher than in crude oil (see Table I).

Another interesting and important difference between crude oil and natural gas markets is that crude oil is often in backwardation whereas natural gas is not. The discounted forward spreads, expressed as an annual percentage in Table I, ranged from slightly over 8% to nearly 14%

TABLE I

Summary Statistics for Crude Oil and Natural Gas Markets

	M	*SD*	*Minimum*	*Maximum*
U.S. crude oil (millions of barrels)[a]				
Daily sales	13.57	0.84	11.58	15.72
Daily production	7.50	0.94	5.79	9.17
Daily imports	6.07	1.70	2.20	9.55
Total stocks	876.39	59.34	675.80	970.90
Strategic petroleum reserve	542.66	60.21	332.48	591.67
Total private stocks	333.58	16.65	283.85	384.22
U.S. natural gas (billion cubic feet)[a]				
Daily sales	56.61	12.67	39.67	82.81
Daily production	50.38	1.89	45.13	53.68
Daily imports	6.51	1.49	3.65	8.81
Total stocks	6,418.94	661.74	5,048.00	7,517.00
Spot prices from *Wall Street Journal*[b]				
West Texas Intermediate crude oil	20.73	5.07	11.17	36.09
Natural gas at the Henry Hub	2.01	0.57	1.18	4.82
Discounted forward spread(%)[c]				
West Texas Intermediate crude oil				
Spot and first nearby	8.24	18.14	−95.46	132.33
First and second nearby	14.51	25.17	−63.59	117.47
Last-day-of-the-month first and second nearby	12.93	22.62	−60.39	81.45
Natural gas at Henry Hub				
Spot and first nearby	−2.09	117.20	−305.38	813.21
First and second nearby	−7.65	74.70	−226.61	187.98
Last-day-of-the-month first and second nearby	−4.11	85.83	−294.53	246.68
Historic 20-day volatility				
West Texas Intermediate crude oil	31.00	20.26	4.69	122.31
Natural gas at the Henry Hub	50.29	23.99	11.35	170.02

[a]Energy Information Administration. [b]All prices are monthly averages unless noted otherwise. [c]These yields were computed as $\text{yield}_t = r + 12[\ln(p_{it}/p_{it+1})]$, where r is the yield on a 1-month certificate of deposit at an annual rate, p_{it} is the spot or first nearby futures price of oil, and p_{it-1} is the corresponding futures price.

for crude oil. In contrast, the same spreads for natural gas ranged from −2.1% to −7.6%.

As Figures 1 and 2 illustrate, the samples displayed considerable variation. The crude oil sample included the 1986 price collapse and the Persian Gulf War. The natural gas sample from April 1990 to December 1998 also displayed considerable price variation.

ECONOMETRIC RESULTS

The estimates for the crude oil sample appear in Table II. The objective function value, when multiplied by the number of observations (187),

FIGURE 1
Prices for West Texas Intermediate crude oil, 1983–1999.

provides a test for the overidentifying restrictions of the model, which has 34 degrees of freedom.[4] For each of the six model runs, we could not reject the maintained assumptions of the model.

Table II also presents estimates of the shadow value on stocks, $C_{X,t}/P_t$, and risk premium, $\psi\sigma_t$, evaluated with the sample means expressed in annual percentage per annum. Although an investigation of aggregation issues was not the focus of this study, the estimates of risk premiums were not sensitive to the definition of petroleum stocks. Five of the six estimates for the shadow value on stocks were negative and significant, equivalent to an annual rate ranging from slightly less than 3% to more than 8% at the mean of the data. The risk premium was negative and highly significant in all six models.[5] This finding suggests that price backwardations occur during periods of greater price uncertainty, supporting the results obtained by Litzenberger and Rabinowitz (1995), who used

[4]The degrees of freedom equal the number of instruments (25) times the number of equations (2) less the number of parameters (16).

[5]Given the estimates for $\sigma_{X,m}$ reported in Footnote 1, the implicit estimates for the market price of risk are clearly positive.

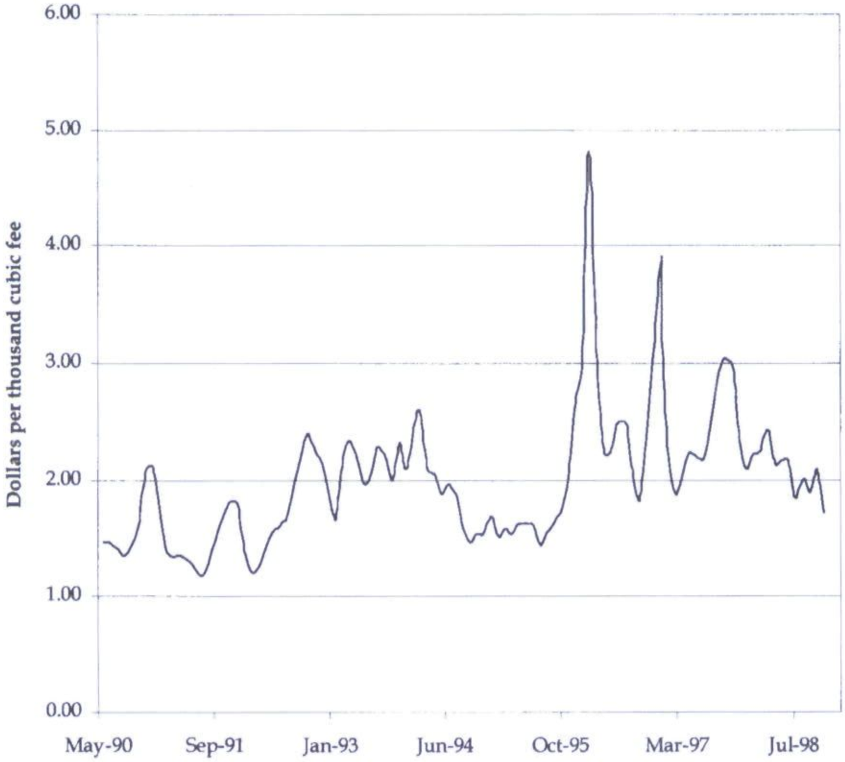

FIGURE 2
Prices for natural gas at the Henry Hub, 1990–1999.

simple regressions of various measures of price backwardations on implied volatility. Our model and findings go one step further, suggesting that the relationship between price backwardations and uncertainty is consistent with the theory of storage. The presence or absence of strategic stockpiles in our definition of inventories did not affect this conclusion.[6]

Table II also presents estimates of the elasticities of marginal cost and $C_{X,t}$ with respect to input prices, output, inventories, capital, and technological change. The elasticities of marginal cost seem intuitively plausible. Marginal costs rose with input prices. Energy and materials generally had the most sizable and statistically significant impacts. The elasticities of marginal cost with respect to output were also significant. Their inverse implied conventional supply elasticities ranging from 0.4 to 0.2, which is within the range of previous studies surveyed by Dahl and Duggan (1996). The elasticities of marginal cost with respect to capacity reflect a shift upward in the competitive supply curve as domestic pro-

[6]In previous versions of this article, we estimated the model in real terms and obtained similar results.

TABLE II

Estimates Obtained with Different Crude Oil Prices with and without Strategic Petroleum Reserve (SPR) Stocks for the Sample Period June 1983–December 1998

	Asymptotic t ratios appear in parentheses					
	Spot and First Nearby Futures		*First and Second Nearby Futures*		*Last-Day First and Second Nearby Futures*	
	With SPR	*Without SPR*	*With SPR*	*Without SPR*	*With SPR*	*Without SPR*
$\partial \ln C_Y / \partial \ln A_i$						
Wages	−0.01 (0.1)	0.75 (7.5)	0.00 (0.0)	0.70 (8.1)	0.00 (0.0)	0.60 (6.3)
Energy Prices	0.91 (10.2)	1.09 (13.1)	0.93 (9.8)	1.07 (11.9)	0.88 (7.0)	1.06 (9.6)
Materials Prices	1.29 (5.1)	1.72 (6.8)	1.25 (7.4)	1.71 (8.7)	1.24 (4.3)	1.60 (6.3)
Output	1.43 (2.3)	3.06 (5.0)	1.05 (2.4)	2.78 (6.5)	0.88 (1.2)	2.45 (4.8)
Stocks	−1.20 (6.8)	−0.36 (3.2)	−1.12 (7.8)	−0.41 (4.9)	−1.06 (5.4)	−0.32 (2.6)
Capital	−1.28 (8.5)	−0.75 (8.4)	−1.21 (8.6)	−0.75 (6.6)	−1.14 (6.0)	−0.69 (4.5)
Trend	−0.16 (2.2)	−0.48 (13.0)	−0.19 (4.6)	−0.49 (14.3)	−0.21 (3.7)	−0.46 (11.5)
$\partial C_X / \partial \ln A_i$						
Wages	0.24 (1.5)	−0.19 (1.8)	2.88 (8.2)	1.39 (7.9)	2.16 (4.4)	1.25 (7.3)
Energy prices	0.33 (2.8)	0.32 (3.2)	0.12 (0.6)	0.43 (2.7)	−0.08 (0.4)	0.10 (0.6)
Materials prices	−0.92 (4.2)	−1.13 (8.4)	−1.89 (3.4)	−1.59 (5.8)	−2.15 (3.2)	−1.75 (6.1)
Output	−0.21 (6.8)	−0.17 (3.2)	−0.20 (7.8)	−0.19 (4.9)	−0.19 (5.4)	−0.15 (2.6)
Stocks	−0.54 (0.6)	−1.81 (3.5)	1.89 (0.9)	5.86 (6.8)	−0.60 (0.2)	3.00 (3.4)
Capital	0.29 (2.1)	0.20 (2.1)	0.29 (1.2)	1.26 (5.9)	−0.12 (0.4)	0.52 (2.2)
Trend	0.01 (1.7)	0.04 (7.1)	−0.03 (2.5)	0.04 (4.7)	−0.02 (1.3)	0.03 (3.4)
$\bar{C}_X / \bar{P}$ (% year)	−3.21 (4.2)	−3.11 (5.4)	−0.70 (0.5)	−2.64 (2.7)	−7.32 (4.2)	−8.36 (7.1)
$\psi\bar{\sigma}$ (%/year)	−5.07 (7.2)	−4.43 (8.7)	−15.56 (18.2)	−11.39 (18.1)	−7.05 (5.0)	−4.86 (5.4)
Test of overid. rest. [probability value]	18.27 [0.99]	15.63 [1.00]	12.37 [1.00]	14.49 [1.00]	19.84 [0.97]	18.19 [0.99]
R^2						
Marginal cost	0.70	0.67	0.70	0.67	0.67	0.64
Returns to storage	0.16	0.18	0.47	0.60	0.34	0.47

Note: Asymptotic ± ratios appear in parentheses.

duction capacity declined. Marginal costs also declined with higher inventories. The time trend elasticities suggest that technological change reduced marginal production costs.

Because the benefits of stock holding can be negative, scaled derivatives of $C_{X,t}$ were easier to interpret. The $C_{X,t}$ function shifted up with lower production and down with higher production. The $C_{X,t}$ function also seemed to be significantly affected by input prices and capacity. The slope of $C_{X,t}$, with respect the stock levels, however, seemed sensitive to the selection of prices and the definition of stocks.

The absolute value of the risk premium for crude oil, which consti-

FIGURE 3
Estimated risk premium as a percentage of the crude oil spot price.

tutes the other component of the returns to storage, is presented in Figure 3 for the model using the monthly average first and second nearby futures prices. Notice that the premium reached over 45% in January 1991, the height of the Gulf War crisis, and fluctuated considerably. It also approached 40% during the crude oil price collapse of 1986.

The results from estimating the same model for natural gas appear in Table III. As with the crude oil sample, we could not reject the overidentifying restrictions.[7] Marginal costs for natural gas were upward sloping with a supply elasticity in the 0.9 to 0.3 range. Higher capital stocks significantly reduced marginal costs across all three specifications. Only one of the three models indicated an inverse relation between gas storage levels and marginal costs.

Like the risk premiums on crude oil, the risk premiums on natural

[7]The residuals for the crude oil and natural gas models do not contain unit roots, suggesting that our model provides a cointegrating mechanism. The Breusch–Pagan test on the residuals did not reveal significant heteroscedasticity, except for those residuals from the returns-to-storage equation for the versions using cash prices.

TABLE III

Estimates for the Natural Gas Sample: June 1990–December 1998

	Alternative Prices		
	Spot and First Nearby Futures	*First and Second Nearby Futures*	*Last-Day First and Second Nearby Futures*
$\partial \ln C_X / \partial \ln A_i$			
Wages	0.98 (2.4)	0.46 (1.3)	0.12 (0.5)
Energy prices	1.72 (9.4)	1.39 (8.3)	1.21 (8.1)
Materials prices	0.19 (0.5)	−0.67 (1.4)	−0.52 (1.5)
Output	2.66 (1.8)	3.20 (2.9)	1.08 (1.1)
Stocks	−0.17 (3.7)	0.05 (1.3)	0.09 (2.0)
Capital	−5.00 (3.1)	−2.15 (2.2)	−2.19 (2.4)
Trend	−0.26 (1.2)	0.14 (0.7)	0.30 (1.9)
$\partial C_X / \partial \ln A_i$			
Wages	0.04 (0.2)	−0.72 (−5.8)	−1.00 (−6.1)
Energy prices	−0.31 (3.3)	0.17 (2.9)	0.70 (9.0)
Materials prices	−0.78 (4.6)	−0.34 (2.8)	−0.59 (4.2)
Output	−0.003 (3.7)	0.001 (1.3)	0.001 (2.0)
Stocks	0.99 (2.1)	3.49 (6.9)	5.61 (10.9)
Capital	1.62 (2.4)	4.71 (7.9)	8.12 (12.5)
Trend	0.02 (1.9)	0.05 (7.7)	0.07 (8.7)
$\bar{C}_X / \bar{P}$ (% year)	88.38 (20.0)	59.39 (16.2)	30.60 (8.1)
$\psi\bar{\sigma}$ (% year)	−83.50 (18.2)	−43.68 (13.6)	−17.66 (4.0)
Test of overid. rest. [probability value]	13.02 [1.00]	13.15 [1.00]	13.04 [1.00]
R^2			
Marginal cost	0.36	0.39	0.37
Returns to storage	0.53	0.48	0.43

Note: Asymptotic *t* ratios appear in parentheses.

gas were negative and significant (see Figure 4). In contrast, the mean shadow value on stocks was positive, reflecting prices for future delivery persistently higher than those for immediate sales, as Table I illustrates. During most of the winter months, however, $C_{X,t}$ was negative, reflecting the benefits of gas stocks during peak consumption periods. These benefits, however, disappeared during the spring, summer, and fall seasons. The derivative of $C_{X,t}$, with respect to inventory levels, was positive and significant, consistent with the theory of storage. In other words, at low inventory levels, the cost-reducing effect of holding inventory was large, and at high stock levels, $C_{X,t}$ was positive, indicating that costs rose with additional stocks.

CONCLUSIONS

This study derived the convenience yield from the classic theory of storage from an inventory valuation model with price uncertainty. The equilib-

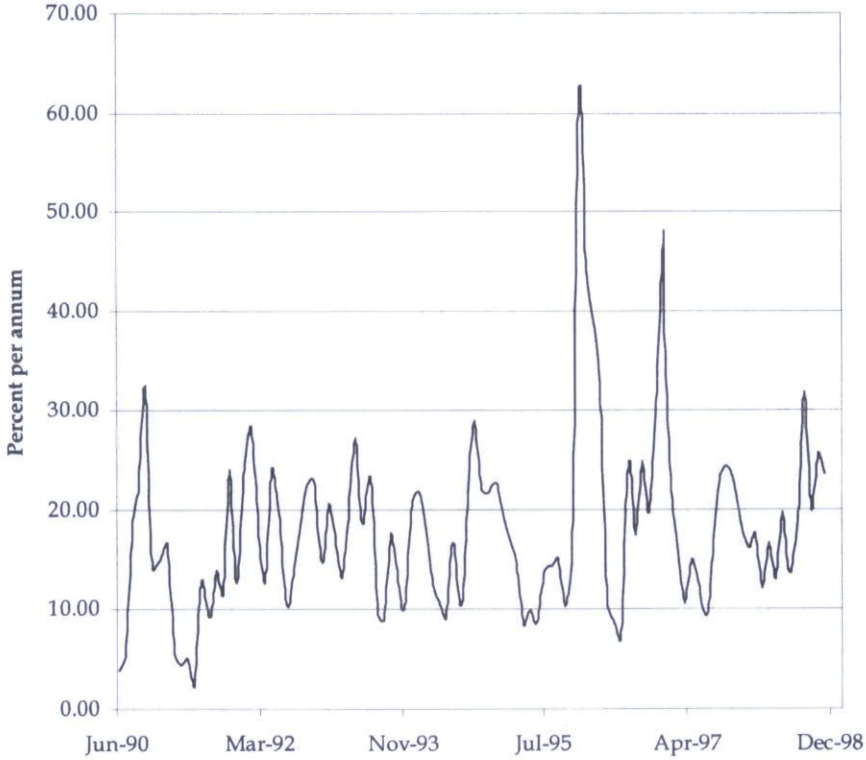

FIGURE 4
Estimated risk premium for natural gas.

rium value of inventories also contains an option value that is a function of price volatility.

The econometric findings suggest that crude oil and natural gas inventory assets contain risk premiums that rise sharply with higher price volatility. In addition, the results also indicate the existence of a convenience yield that increases as stocks fall.

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