FUTURES HEDGING UNDER DISAPPOINTMENT AVERSION

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This article considers optimal futures hedging decisions when the hedger is disappointment-averse (Gul, 1991). When the futures contract is a perfect hedge instrument, a disappointment-averse hedger always holds a position closer to the full hedge than a nondisappointment-averse hedger. In the presence of basis risk, the optimal futures position is either a partial hedge or a full hedge. Neither Texas hedge nor overhedge could be optimal. The effects of different degrees of disappointment aversion on futures trading are also analyzed. © 2001 John Wiley & Sons, Inc. Jrl Fut Mark 21:1029–1042, 2001

INTRODUCTION

Traditional portfolio theory is established within the expected utility framework (and the restricted mean-variance version). Dissatisfied with the explanatory power of this approach, researchers began to consider the behavioral decision-making process and examine its implications for financial markets. Two behavioral factors, loss (realization) aversion and overconfidence, have recently been integrated into financial market literature.

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Notwithstanding the explanatory power, many field experiments indicated the fallacies of the expected utility hypothesis. Among them, the Allais paradox has a profound history. Gul (1991) adopted an axiomatic approach to resolve this paradox. As a result, he proposed a disappointment-averse utility function. The new utility function does not fall into the expected utility framework. However, it can be generated by the conventional utility function augmented with an additional parameter, the loss-aversion coefficient. Both disappointment aversion and loss aversion, according to the prospect theory of Kahneman and Tversky (1979), define the utility function asymmetrically over gains and losses relative to a reference point. For a loss-aversion utility function, the reference point is arbitrarily exogenously chosen; see Benartzi and Thaler (1995) and Barberis, Huang, and Santos (in press). The disappointment-averse utility function determines the reference point endogenously.

Aizenman (1998) found that the effects of buffer stock on stabilization under disappointment aversion differ sharply from those derived with conventional expected utility functions. Aizenman and Marion (1999) found empirically a significant negative correlation between volatility and private investment. They adopted an analytical model with a disappointment-averse agent to support the empirical finding. Ang, Bekaert, and Liu (2000) considered the optimal portfolio choice for a disappointment-averse agent. They found that disappointment aversion helps to explain the large equity premium observed in the data.

This article investigates the optimal hedging decision for a disappointment-averse hedger. Barberis et al. (in press), Gomes (2000), and Berkelaar and Kouwenberg (2000a, 2000b) constructed optimal portfolio choice under loss aversion. Albuquerque (1999) and Lien (in press) examined the optimal futures position for a loss-averse hedger. Ang et al. (2000) characterized optimal (dynamic) portfolios under disappointment aversion. To my knowledge, there is currently no article incorporating disappointment aversion into the futures hedging framework. This article provides the first attempt.

BASIC FRAMEWORK

Let $u(.)$ denote a strictly increasing and strictly concave von Neuman–Morganstern utility function defined over a random wealth level W. A disappointment–loss-averse utility function, denoted $\Phi_{u,\beta}(.)$, is defined as follows:

$$\Phi_{u,\beta}(W) = u(W) \quad \text{for} \quad W \leq V \tag{1}$$

$$\Phi_{u,\beta}(W) = [u(W) + \beta u(V)]/(1 + \beta) \quad \text{for} \quad W > V \tag{2}$$

where $u(V) = E[\Phi_{u,\beta}(W)]$, the expectation of $\Phi_{u,\beta}(W)$, and $\beta \geq 0$. V is the reference point. Utilities for gains and losses are treated asymmetrically, with the gains being discounted such that $\Phi_{u,\beta}(W)$ is a linear, convex combination of $u(W)$ and $u(V)$. With the relationship $E[\Phi_{u,\beta}(W)] = u(V)$, it can be shown that

$$E[\Phi_{u,\beta}(W)] = E[u(W)] - \beta \int_{-\infty}^{V} [u(V) - u(W)]g(W)\, dW \qquad (3)$$

where $g(W)$ is the probability density function of W.

When $W < V$, there is a loss of utility of $u(V) - u(W)$ with respect to the reference point. A disappointment-averse agent is agonized by this loss. His expected utility is, therefore, reduced by the disappointment of losses. The degree of disappointment is measured by β. When $\beta = 0$, there is no disappointment, and thereafter $\Phi_{u,\beta}(W)$ is the same as $u(W)$. The two utility functions differ as long as $\beta \neq 0$. A larger β indicates the individual is more disappointed by losses (i.e., more disappointment-averse). In their analysis, Ang et al. (2000) assumed $u(.)$ is a constant-relative-risk-aversion utility function. I consider a general utility function here but restrict the realization of W to be discrete.

A simple hedging problem is investigated here. At time 0, a hedger has a nontradable cash position x to be liquidated at time 1. A futures contract that expires at time 1 is available for trading. Let p_k and f_k denote cash and futures prices at time k, respectively; $k = 0$ or 1. Assume the hedger sells z amount of futures contract. Let W_0 denote the initial wealth at time 0. The hedger's wealth at time 1 is

$$W_1 = W_0 + (p_1 - p_0)x + (f_0 - f_1)z \qquad (4)$$

The terminal wealth is a random variable because both p_1 and f_1 are unknown at time 0. We assume the hedger has a disappointment-averse utility function, $\Phi_{u,\beta}(W)$. Below I adopt a simplified notation, $\Phi(W)$. Given x, the hedger chooses the optimal futures position, denoted z^*, to maximize the expected utility, $E[\Phi(W)]$. Let W^* denote the terminal wealth when the optimal futures position is undertaken. Also, let $W_{10} = W_0 + (p_1 - p_0)x$, the terminal wealth level when futures trading is not available. The welfare gain of a futures market is then $E[\Phi(W^*)] - E[\Phi(W_{10})]$, which is always nonnegative.

PERFECT FUTURES CONTRACT

A futures contract is perfect when the futures price coincides with the cash price at the expiration date (i.e., $f_1 = p_1$). There is no basis risk. Consequently,

$$W_1 = W_0 + (p_1 - p_0)x + (f_0 - p_1)z \tag{5}$$

Suppose that the cash price has only two possible realizations, high (p_H) or low (p_L), with probabilities π_H and π_L, respectively: $0 < \pi_H, \pi_L < 1$, and $\pi_H + \pi_L = 1$. A similar approach was adopted by Gomes (2000). As a result, there are two possible realizations for the terminal wealth, W_H and W_L, such that

$$W_H = W_0 + (p_H - p_0)x + (f_0 - p_H)z \tag{6}$$

$$W_L = W_0 + (p_L - p_0)x + (f_0 - p_L)z \tag{7}$$

When z is smaller than x, W_H is larger than W_L. The reference point V must lie in the interval (W_L, W_H). Moreover,

$$\pi_L u(W_L) + \pi_H \frac{u(W_H) + \beta u(V)}{1 + \beta} = u(V) \tag{8}$$

From Equation 8, we solve for the following:

$$u(V) = \frac{1}{1 + \beta - \beta\pi_H}\{(1 + \beta)\pi_L u(W_L) + \pi_H u(W_H)\} \tag{9}$$

The hedger chooses the optimal futures position z^* to maximize $E[\Phi(W)]$, or simply $u(V)$. On taking the partial derivative with respect to z, we have

$$\frac{\partial u(V)}{\partial z} = \theta\{(1 + \beta)\pi_L u'(W_L)(f_0 - p_L) + \pi_H u'(W_H)(f_0 - p_H)\} \tag{10}$$

where $u'(W)$ denotes the first derivative of $u(W)$ and $\theta = 1/(1 + \beta - \beta\pi_H)$. In an unbiased futures market, $f_0 = E(f_1)$. Therefore,

$$f_0 = \pi_H p_H + \pi_L p_L \tag{11}$$

from which we derive $\pi_H(f_0 - p_H) = \pi_L(p_L - f_0)$. On substituting this relationship into Equation 10, we find

$$\frac{\partial u(V)}{\partial z} = \theta\{u'(W_H) - (1 + \beta)u'(W_L)\}\pi_H(f_0 - p_H) > 0 \tag{12}$$

The inequality is established because $u(.)$ is strictly concave and $W_H > W_L$. Also, $f_0 < p_H$. Thus, full hedge (i.e., $z = x$) dominates any underhedge strategy (i.e., $z < x$).

Suppose, instead, an overhedge is considered: $z > x$. Here, W_H is smaller than W_L. The reference point V satisfies $W_H < V < W_L$. Also,

$$\pi_H u(W_H) + \pi_L \frac{u(W_L) + \beta u(V)}{1 + \beta} = u(V) \tag{13}$$

From Equation 13, we have

$$u(V) = \frac{1}{1 + \beta - \beta\pi_L}\{(1 + \beta)\pi_H u(W_H) + \pi_L u(W_L)\} \tag{14}$$

On taking the partial derivative with respect to z, we derive

$$\frac{\partial u(V)}{\partial z} = \gamma\{(1 + \beta)\pi_H u'(W_H)(f_0 - p_H) + \pi_L u'(W_L)(f_0 - p_L)\} \tag{15}$$

where $\gamma = 1/(1 + \beta - \beta\pi_L)$. If we assume the futures market is unbiased, Equation 15 can be reduced to

$$\frac{\partial u(V)}{\partial z} = \gamma\{(1 + \beta)u'(W_H) - u'(W_L)\}\pi_H(f_0 - p_H) \tag{16}$$

Evaluated at $z = x$, $W_H = W_L$. Consequently, $\partial u(V)/\partial z < 0$. Because $u(v)$ is a strictly concave function of v, $\partial u(V)/\partial z < 0$ whenever $z \geq x$. Thus, any overhedge strategy is outperformed by the full hedge.

Proposition 1

In an unbiased, perfect futures market, the optimal futures position for a disappointment-averse hedger is the full hedge.

In other words, disappointment aversion has no impact in an unbiased, perfect futures market. Proposition 1 applies to the most general case. In a perfect futures market,

$$\frac{\partial E[\Phi(W_1)]}{\partial z} = u'(W_0 + (f_0 - p_0)x)E[f_0 - p_1] \tag{17}$$

when evaluated at $z = x$. When the futures price is unbiased, the aforementioned value is equal to zero. Therefore, the optimal hedge is the full hedge.

BACKWARDATION AND CONTANGO

A futures market is in backwardation (or contango) when f_0 is smaller (or larger) than $E(f_1)$. The hedger has a speculative motive to buy futures under backwardation and to sell futures under contango. Thus, a nondisappointment-averse hedger will underhedge for the former case and overhedge for the latter.

Consider first the backwardation case. Herein $f_0 < \pi_H p_H + \pi_L p_L$, and so $\pi_H(f_0 - p_H) < \pi_L(p_L - f_0)$. From Equation 15, we have

$$\frac{\partial u(V)}{\partial z} < \gamma\{(1 + \beta)u'(W_H) - u'(W_L)\}\pi_L(p_L - f_0) < 0 \qquad (18)$$

whenever $z \geq x$. Consequently, the optimal futures position z^* must be smaller than x. Moreover, z^* must satisfy the following condition:

$$\frac{\partial u(V)}{\partial z} = \theta\{(1 + \beta)\pi_L u'(W_L)(f_0 - p_L) + \pi_H u'(W_H)(f_0 - p_H)\} = 0 \quad (19)$$

With comparative static analysis, it can be easily shown that $\partial z^*/\partial\beta > 0$. The result is similar to the effect of risk aversion in a mean-variance framework (e.g., Lien, in press). That is, as the hedger becomes more disappointment-averse, a larger futures position will be held. Note that $\beta = 0$ corresponds to the case of a nondisappointment-averse hedger.

Proposition 2

Suppose that the perfect futures market is in backwardation equilibrium. The optimal futures hedge strategy is to hedge less than full. However, the deficiency for a disappointment-averse hedger is smaller than that for a nondisappointment-averse hedger.

Intuitively, backwardation promotes the hedger to buy futures for speculation purposes. Disappointment aversion, however, discourages the hedger from buying too much.

There are other comparative static results available:

$$\frac{\partial z^*}{\partial \pi_H} = \frac{-\theta\{(1 + \beta)u'(W_L)(f_0 - p_L) + u'(W_H)(f_0 - p_H)\}}{-\partial^2 u(V)/\partial z^2} < 0 \quad (20)$$

$$\frac{\partial z^*}{\partial p_L} = \frac{\theta(1 + \beta)[u''(W_L)(x - z)(f_0 - p_L) - u'(W_L)]}{-\partial^2 u(V)/\partial z^2} < 0 \qquad (21)$$

The optimal futures position must equate two (adjusted expected) marginal utilities, $(1 + \beta)\pi_L(f_0 - p_L)u'(W_L)$ and $\pi_H(p_H - f_0)u'(W_H)$. When

π_H increases, the latter increases. As a result, $u'(W_L)$ must increase to restore the equality. That is, W_L must decrease or, equivalently, z^* must decrease. An increase in the probability of a high-price state occurring reduces the optimal futures position.

Similarly, when p_L increases, $u'(W_L)$ must increase to restore the equality between the two (adjusted expected) marginal utilities. Thus, z^* must decrease. An increase in the price realization in the disappointment state depresses futures hedging. However, the effect of p_H is undetermined.

Next, we turn to the contango equilibrium. Because $f_0 > \pi_H p_H + \pi_L p_L$, we have $\pi_H(f_0 - p_H) > \pi_L(p_L - f_0)$. From Equation 10, we have

$$\frac{\partial u(V)}{\partial z} > \theta\{(1 + \beta)u'(W_L) - u'(W_H)\}\pi_L(f_0 - p_L) > 0 \qquad (22)$$

whenever $z \leq x$. In other words, the optimal futures position is an overhedge. Furthermore, z^* satisfies the following condition:

$$\frac{\partial u(V)}{\partial z} = \gamma\{(1 + \beta)\pi_H u'(W_H)(f_0 - p_H) + \pi_L u'(W_L)(f_0 - p_L)\} = 0 \quad (23)$$

From which we derive $\partial z^*/\partial\beta < 0$. Again, the result is similar to the effect of risk aversion in a mean-variance framework (Lien, in press). As the hedger becomes more disappointment-averse, a smaller futures position is assumed.

Proposition 3

Assume a contango equilibrium prevails in the perfect futures market. The optimal futures hedge strategy is to hedge more than full. However, the excess for a disappointment-averse hedger is smaller than that for a nondisappointment-averse hedger.

Indeed, a contango induces the hedger to sell additional futures contracts for speculation purposes. Disappointment aversion, however, prevents the hedger from selling too much.

By comparative static analysis, we also have the following results:

$$\frac{\partial z^*}{\partial \pi_H} = \frac{\gamma u'(W_L)(f_0 - p_L)}{\pi_H(\partial^2 u(V)/\partial z^2)} < 0 \qquad (24)$$

$$\frac{\partial z^*}{\partial p_H} = \frac{\gamma(1 + \beta)\pi_H[u''(W_H)(x - z)(f_0 - p_H) - u'(W_H)]}{-\partial^2 u(V)/\partial z^2} < 0 \quad (25)$$

Interpretations of these results are similar to the backwardation case. The optimal futures position is chosen to equate two (adjusted expected) marginal utilities, $(1 + \beta)\pi_H(p_H - f_0)u'(W_H)$ and $\pi_L(f_0 - p_L)u'(W_L)$. When π_H increases, the former increases and $u'(W_L)$ must increase to restore the equality. Thus, z^* must decrease: An increase in the probability of a high-price state occurring reduces the optimal futures position. If p_H increases, $u'(W_L)$ must increase to restore the equality between the two (adjusted expected) marginal utilities, resulting in a decrease in z^*. An increase in the price realization in the disappointment state depresses futures hedging. The effect of p_L is, however, undetermined.

EFFECTS OF BASIS RISK

Because of mismatches in timing, location, or grade, a futures contract settled by physical delivery is almost always an imperfect hedging instrument. To examine the effects of basis risk, we adopt the framework of Kamara and Siegel (1987) and Lien (1988). Assume two different grades can be delivered against the futures contract at par. An example would be No. 2 red hard wheat and No. 2 red soft wheat against the wheat futures contract traded at the Chicago Board of Trade. Let the price of the alternative grade be q_1 at time 1. Because the short hedger has the choice of delivery grade, the futures price is equal to the minimum of the two cash prices: $f_1 = \min(p_1, q_1)$.

There are only two possible realizations for q_1: low (p_L) or high (p_H). Let π_{jk} denote the probability that $p_1 = p_j$ and $q_1 = p_k$ and let W_{jk} denote the corresponding terminal wealth level, where j and k are H and L, respectively. In total, there are three possible terminal wealth levels:

$$W_{HH} = W_0 + (p_H - p_0)x + (f_0 - p_H)z \tag{26}$$

$$W_{HL} = W_0 + (p_H - p_0)x + (f_0 - p_L)z \tag{27}$$

$$W_{LH} = W_{LL} = W_0 + (p_L - p_0)x + (f_0 - p_L)z \tag{28}$$

Also, let $\pi_L = \pi_{LH} + \pi_{LL}$, the probability that $p_1 = p_L$.

To analyze the optimal futures position, we first consider the possibility of a Texas hedge, $z < 0$. In this case, $W_{HH} > W_{HL} > W_{LH} = W_{LL}$. The reference point V must lie in $[W_{LH}, W_{HL}]$ or $[W_{HL}, W_{HH}]$. Suppose that $W_{LH} \le V \le W_{HL}$. Then,

$$\pi_{HH}\frac{u(W_{HH}) + \beta u(V)}{1 + \beta} + \pi_{HL}u(W_{HL}) + \pi_L u(W_{LH}) = u(V) \tag{29}$$

Solving for $u(V)$, we derive

$$u(V) = \phi_{HH}[\pi_{HH}u(W_{HH}) + (1 + \beta)\pi_{HL}u(W_{HL}) + (1 + \beta)\pi_{L}u(W_{LH})] \tag{30}$$

where $\phi_{HH} = 1/(1+ \beta - \beta\pi_{HH})$. The partial derivative of $u(V)$ with respect to z is

$$\frac{\partial u(V)}{\partial z} = \delta_{HH}\left\{\frac{\pi_{HH}\cdot}{1 + \beta}u'(W_{HH})(f_0 - p_H) + [\pi_{HL}u'(W_{HL}) + \pi_{L}u'(W_{LH})(f_0 - p_L)]\right\} \tag{31}$$

where $\delta_{HH} = \phi_{HH}(1 + \beta)$. If the futures price is unbiased, then

$$\pi_{HH}(f_0 - p_H) = (\pi_{HL} + \pi_L)(p_L - f_0) \tag{32}$$

With this relationship, Equation 30 can be rewritten as follows:

$$\frac{\partial u(V)}{\partial z} = \delta_{HH}(f_0 - p_L)\left\{\pi_{HL}\left[u'(W_{HL}) - \frac{u'(W_{HH})}{1 + \beta}\right] + \pi_L\left[u'(W_{LH}) - \frac{u'(W_{HH})}{1 + \beta}\right]\right\} \tag{33}$$

When $z = 0$, $W_{HH} = W_{HL} > W_{LH}$. As a result, $\partial u(V)/\partial z > 0$ at $z = 0$. Note that $u(V)$ is a strictly concave function of z; we have $\partial u(V)/\partial z > 0$ whenever $z \leq 0$.

Now, suppose instead that $W_{HL} \leq V \leq W_{HH}$. The reference point V is defined by the following:

$$\pi_{HH}\frac{u(W_{HH}) + \beta u(V)}{1 + \beta} + \pi_{HL}\frac{u(W_{HL}) + \beta u(V)}{1 + \beta} + \pi_L u(W_{LH}) = u(V) \tag{34}$$

From which we have

$$u(V) = \phi_L[\pi_{HH}u(W_{HH}) + \pi_{HL}u(W_{HL}) + (1 + \beta)\pi_L u(W_{LH})] \tag{35}$$

where $\phi_L = 1/(1+ \beta\pi_L)$. In an unbiased futures market,

$$\frac{\partial u(V)}{\partial z} = \phi_L(f_0 - p_L)\{\pi_{HL}[u'(W_{HL}) - u'(W_{HH})] + \pi_L[(1 + \beta)u'(W_{LH}) - u'(W_{HH})]\} \tag{36}$$

When $z = 0$, $W_{HH} = W_{HL} > W_{LH}$. Therefore, $\partial u(V)/\partial z > 0$ at $z = 0$, implying $\partial u(V)/\partial z > 0$ whenever $z \leq 0$.

Proposition 4

In an unbiased future market with basis risk, the Texas hedge is outperformed by the no-hedge strategy. That is, a hedger with a long cash position will not hold a long futures position.

In other words, the optimal futures position z^* is no smaller than zero.

POSSIBILITY OF AN OVERHEDGE

When $z \geq 0$, there are two cases: (I) $x \leq z$ and (II) $x \geq z$. In Case I, $W_{HL} \geq W_{LH} = W_{LL} \geq W_{HH}$. Thus, we have either $W_{LH} \leq V \leq W_{HL}$ or $W_{HH} \leq V \leq W_{LH}$. For the former case, following a similar approach to the previous sections, we can solve for $u(V)$ as follows:

$$u(V) = \phi_{HL}[\pi_{HL}u(W_{HL}) + (1 + \beta)\pi_{HH}u(W_{HH}) + (1 + \beta)\pi_{L}u(W_{LH})] \tag{37}$$

where $\phi_{HL} = 1/(1 + \beta - \beta\pi_{HL})$. Let $\delta_{HL} = \phi_{HL}(1 + \beta)$. Given an unbiased futures market,

$$\frac{\partial u(V)}{\partial z} = \delta_{HL}(f_0 - p_L)\left\{\pi_{HL}\left[\frac{u'(W_{HL})}{1 + \beta} - u'(W_{HL})\right] + \pi_L[u'(W_{LH}) - u'(W_{HH})]\right\} \tag{38}$$

which is negative.

If $W_{HH} \leq V \leq W_{LH}$, by the definition of $u(V)$, we derive

$$u(V) = \lambda_{HH}[\pi_{HL}u(W_{HL}) + \pi_{L}u(W_{LH}) + (1 + \beta)\pi_{HH}u(W_{HH})] \tag{39}$$

where $\lambda_{HH} = 1/(1+ \beta\pi_{HH})$. As a result,

$$\frac{\partial u(V)}{\partial z} = \psi_{HH}(f_0 - p_L)\left\{\pi_{HL}\left[\frac{u'(W_{HL})}{1 + \beta} - u'(W_{HH})\right] + \pi_L\left[\frac{u'(W_{LH})}{1 + \beta} - u'(W_{HH})\right]\right\} \tag{40}$$

in an unbiased futures market, where $\psi_{HH} = \lambda_{HH}(1 + \beta)$. Again, we find $\partial u(V)/\partial z < 0$. We conclude that $u(V)$ decreases with increasing z whenever z is larger than x.

Proposition 5

In an unbiased future market with basis risk, the hedger will not overhedge.

The aforementioned result is consistent with the conventional expected utility framework. Basis risk tends to depress the demand for futures contracts.

OPTIMAL FUTURES POSITION

We are now left with only Case II, to which the optimal futures position belongs. When $0 \leq z \leq x$, $W_{HL} \geq W_{HH} \geq W_{LH} = W_{LL}$. There are two possible scenarios: (A) V lies in the interval $[W_{HH}, W_{HL}]$ and (B) V lies in $[W_{LH}, W_{HH}]$. In Scenario A, the reference point is solved by the following equation:

$$\pi_{HL}\frac{u(W_{HL}) + \beta u(V)}{1 + \beta} + \pi_{HH}u(W_{HH}) + \pi_L u(W_{LH}) = u(V) \quad (41)$$

Recall that $\phi_{HL} = 1/(1 + \beta - \beta\pi_{HL})$. Then,

$$u(V) = \phi_{HL}[\pi_{HL}u(W_{HL}) + (1 + \beta)\pi_{HH}u(W_{HH}) + (1 + \beta)\pi_L u(W_{LH})] \quad (42)$$

which is the same as Equation 36. If we assume an unbiased futures market, the partial derivative of $u(V)$ with respect to z is, therefore, Equation 37; it is reproduced as Equation 43:

$$\frac{\partial u(V)}{\partial z} = \delta_{HL}(f_0 - p_L)\left\{\pi_{HL}\left[\frac{u'(W_{HL})}{1 + \beta} - u'(W_{HL})\right] + \pi_L[u'(W_{LH}) - u'(W_{HH})]\right\} \quad (43)$$

The first term inside the bracket is negative, whereas the second is positive.

Evaluated at $z = x$, $W_{HL} > W_{HH} = W_{LH}$. Thus, $\partial u(V)/\partial z < 0$, implying $z^* < x$. However, $W_{HL} = W_{HH} > W_{LH}$ at $z = 0$. The sign of $\partial u(V)/\partial z$ is determined by the sign of M:

$$M = \pi_L u'[(p_L - p_0)x] - \left\{\pi_L + \frac{\beta\pi_{HL}}{1 + \beta}\right\}u'[(p_H - p_0)x] \quad (44)$$

An interior solution, z^*, satisfying $\partial u(V)/\partial z = 0$ exists when $M > 0$. From Equation 44, M is more likely to be positive if π_L is large, π_{HL} is small, β is small, or $p_H - p_L$ is large. When the interior solution exists, by comparative static analysis we have $\partial z^*/\partial\beta < 0$.

Proposition 6

Assume the futures market is unbiased. When the probability of low prices for own grade is large, or the probability of high price for own grade and low price for the other is small, or the hedger is not too disappointment-averse, or the difference between the low and high prices is large, the optimal hedge strategy is a partial hedge, $0 < z^* < x$. Moreover, as the hedger becomes more disappointment-averse, he will choose a smaller futures position.

Under Scenario B, the reference point is defined as follows:

$$\pi_{HH}\frac{u(W_{HH}) + \beta u(V)}{1+\beta} + \pi_{HL}\frac{u(W_{HL}) + \beta u(V)}{1+\beta} + \pi_L u(W_{LH}) = u(V) \quad (45)$$

which is the same as Equation 34. Following the analysis there, we have

$$\begin{aligned}\frac{\partial u(V)}{\partial z} &= \phi_L(f_0 - p_L)\{\pi_{HL}[u'(W_{HL}) - u'(W_{HH})] \\ &\quad + \pi_L[(1+\beta)u'(W_{LH}) - u'(W_{HH})]\}\end{aligned} \quad (46)$$

At $z = 0$, $\partial u(V)/\partial z > 0$. Thus, $z^* > 0$. When evaluated at $z = x$, the sign of $\partial u(V)/\partial z$ is the same as that of N:

$$N = \pi_{HL}u'[(f_0 - p_0 + p_H - p_L)x] + (\beta\pi_L - \pi_{HL})u'[(f_0 - p_0)x] \quad (47)$$

An interior solution exists if $N < 0$, which in turn is more likely when β is small, π_{HL} is large, π_L is small, or $p_H - p_L$ is large. From Equation 47, $\partial z^*/\partial\beta > 0$.

Proposition 7

Assume the futures market is unbiased. When the probability of low prices for own grade is small, or the probability of high price for own grade and low price for the other is large, or the hedger is not too disappointment-averse, or the difference between the low and high prices is large, the optimal hedge strategy is a partial hedge, $0 < z^* < x$. Moreover, as the hedger becomes more disappointment-averse, he will choose a larger futures position.

Both propositions 6 and 7 indicate that a boundary solution may be possible when the hedger is very disappointment-averse and the volatility of the cash price (as measured by $p_H - p_L$) is small. Therefore, the hedger may choose no hedge (i.e., $z = 0$) or full hedge (i.e., $z = x$). By straightforward calculation, we derive

$$u(V) = u[(p_H - p_0)x] + (1 + \beta)u[(p_L - p_0)x] \tag{48}$$

when $z = 0$. Also,

$$u(V) = u[(p_H - p_0 + f_0 - p_L)x] + (1 + \beta)u[(f_0 - p_0)x] \tag{49}$$

at $z = x$. Because $f_0 > p_L$, the latter is larger than the former. Consequently, full hedge strictly dominates the no-hedge strategy. We now summarize the aforementioned results.

Proposition 8

In an unbiased futures market, full hedge is the optimal strategy when the hedger is very disappointment-averse and when the cash price is not very volatile. Otherwise, the optimal hedge strategy is a partial hedge. There are two possible solutions. The probability distribution of cash prices determines which one may prevail. When π_L is large or π_{HL} is small, the optimal hedge leads to $W_{HH} < V < W_{HL}$ and decreases with increasing disappointment aversion. However, if π_L is small or π_{HL} is large, the optimal hedge satisfies $W_{LH} < V < W_{HH}$ and increases with increasing disappointment aversion.

In other words, disappointment at possible losses encourages the hedger to assume the full hedge position unless the cash price is volatile. However, conventional expected utility analysis shows that basis risk reduces the hedging effectiveness of the futures contract and, therefore, leads to a smaller futures position.

CONCLUSIONS

Behavioral factors have become more important in the recent finance literature. Following the research line, this article incorporates disappointment aversion into the optimal futures hedging decision framework. Our main finding is that disappointment aversion discourages the hedger from deviating from the full hedge position. When the futures contract is a perfect hedge instrument, it is shown that a disappointment-averse hedger always holds a position closer to full hedge than a nondisappointment-averse hedger in cases of backwardation or contango.

If basis risk prevails, the optimal futures position is either a partial hedge or a full hedge. Neither Texas hedge nor overhedge could be optimal. A very disappointment-averse hedger will assume a full hedge when the cash market is not very volatile.

The aforementioned conclusions were derived within a two-state general utility framework. An alternative approach would rely on a specific utility function and a specific joint distribution function for cash and futures prices. I expect the conclusions to remain valid. Indeed, the prevalence of disappointment aversion leads the hedger to not divert too far away from the naive strategy. Any trading gains move up the reference point. The corresponding utility will be discounted more because of disappointment aversion. The benefits of trading gains are smaller for a disappointment-averse hedger than a nondisappointment hedger. Consequently, the former will assume an optimal futures position closer to the full hedge.

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