Rational Speculative Bubbles in the Gold Futures Market: An Application of Dynamic Factor Analysis

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The existence of speculative bubbles in financial markets has been a longstanding issue under debate. Many financial economists believe that, given the assumption of rational expectations and rational behavior of economic agents, an asset should be priced according to its "market fundamentals." Others argue that self-fulfilling rumors of market participants can influence asset prices as well. These self-fulfilling rumors are initiated by events extraneous to markets and are often called bubbles. The rationality of both expectations and behavior often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value—rational bubbles. A rational bubble can arise when the actual market price depends positively on its own ex-

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pected rate of change, as normally occurs in asset markets. Since agents forming rational expectations do not make systematic prediction errors, the positive relationship between price and its expected rate of change implies a similar relationship between price and its actual rate of change. Under such conditions, the arbitrary, self-fulfilling expectation of price changes may drive actual price changes independently of market fundamentals; we refer to such a situation as a rational *price bubble*.[1]

This article investigates gold futures markets for evidence of rational speculative bubbles. Given the relationship between futures prices and expected spot prices, if futures market bubbles exist, their source must be the spot market. Fundamental pricing relationships in the spot market are most likely to break down when the commodity at hand has a high cost of storage or transportation, is subject to wide swings in supply, or is traded only intermittently. Gold can be stored indefinitely. It does not deteriorate from climatic changes or over time. In fact, most of the gold that has ever been mined still exists today in some form. The vast majority of privately owned gold in the United States is held in vaults beneath Wall Street. Changes in ownership of this gold do not bring about physical withdrawal of the gold. Rather, when gold changes hands, an accounting entry documenting this shift in ownership is made. Clearly, storage and transportation costs associated with the ownership of gold are minimal. Gold is not subject to wide swings in supply because gold hoards are much larger than annual gold production. The stock of gold is roughly equal to 50 times the average annual production. In addition, the production of gold is unaffected by seasonality. Gold is traded 24 hours a day around the globe, helping to ensure that any divergence of the gold price from

[1] Numerous studies have been undertaken to examine the possible existence of speculative bubbles. Flood and Garber (1980) conduct econometric tests for the existence of price-level bubbles in the postwar German hyperinflation. They find no evidence of bubbles and conclude that the data are consistent with Cagan's (1956) monetary model. Although their test procedure is appealing, there are two main shortcomings. First, they test only for deterministic bubbles. The second weakness of the test is that since the regressor for the bubble parameter is an exponential function of time, as the number of observations becomes large, this regressor explodes and the asymptotic distribution of the test statistic degenerates. West (1987a) applies Hausman's (1978) specification test to look for bubbles in stock markets and finds significant evidence of bubbles. Using this same specification test, Meese (1986) reports evidence of bubbles for the dollar/Deutschemark and the dollar/British pound exchange rates. One problem of the West/Meese methodology is that the procedure tests a joint null hypothesis of no bubble and correct model specification, whereas the alternative includes misspecification of the structural model, misspecification of the market fundamental process, rational speculative bubbles, and other irrational behaviors. Any of these factors may lead to the rejection of the joint null hypothesis. Thus, one may mistakenly interpret rejection of the null as evidence of bubbles even when there is in fact no bubble. Additional studies on exchange-rate bubbles have yielded mixed results.

its fundamental value could be quickly traded away. Given these characteristics, the gold market should yield little opportunity for speculative bubbles. However, over the past 25 years, the gold spot market has experienced dramatic price changes that could well be documentation of deviations from fundamental pricing relationships. For instance, significant changes in gold spot and futures prices occurred during 1980 and 1990–1993. Although these episodes of price runs are associated with the silver crisis and the Iranian–Iraqi war; can these price changes be econometrically explained by "market fundamentals" or must an intervening variable, a price bubble, be brought into the analysis to explain the behavior of the left-hand side variable?

This article employs dynamic factor analysis to document an otherwise unobservable price bubble and tests whether the bubble is significantly different from zero over time. In addition, the impact of the bubble's representation on the equilibrium gold futures price is estimated. This article is organized as follows: after the market participants are introduced, equilibrium prices for the gold spot and gold futures markets are determined, followed by a brief discussion of dynamic factor analysis. The estimation results are then detailed.

MARKET PARTICIPANTS

A rational bubble can make no sense in the absence of a precise model detailing a market's operation. Without such a model, it is impossible to isolate the time-series trajectory characteristic of the bubble or to define the market fundamentals. We model the determination of gold spot and futures prices in a conventional four-sector two-market configuration for storable commodities. Consumers, producers, dealers, and speculators simultaneously maximize their respective utility functions and jointly determine market clearing prices.

Consumer

Gold is a relatively unique commodity, given that the cumulative flow of services generated by the stock of gold, aggregated over an infinite time horizon, is not finite. The rationale is that, even if gold is refined into bullion or manufactured into jewelry, it continues to generate an unending flow of services to the consumer. Gold has an aesthetic use and a precautionary use against major political socioeconomic catastrophes. Clearly, the market fundamentals for spot market gold prices include the factors affecting the aesthetic and the precautionary demand for gold.

These are difficult to evaluate but must be proxied. Otherwise, as noted by Flood and Garber [1980], an omitted variable problem can bias bubble tests toward rejection of the no-bubble hypothesis.

Formally, a consumer is an agent who makes a demand decision at the time he faces actual spot market prices without being exposed to price uncertainty. This price-taking consumer maximizes the utility of commodity consumption subject to a budget constraint and finds that his demand for the commodity at time t, C_t^i, is given by

$$C_t^i = a_o^i - a_1^i s_t + (a_2^i)' x_t + \varepsilon_t^i \tag{1}$$

where i is an individual consumer, a_o^i and a_1^i are scalars, while a_2^i is a $r \times 1$ vector; s_t is the time t spot price, x_t is a column vector of independent variables where the $r \times 1$ dimension will be empirically determined, and ε_t^i is the disturbance term representing idiosyncratic consumer characteristics.

The aesthetic demand for gold is inversely related to its price and to the opportunity cost of holding gold. If returns on investment opportunities decline, consumers could very well demand more gold. Obvious alternatives to gold include investments in the stock market, investment in foreign currency, or investment in fixed income securities. The real return on Standard & Poor's 500 Composite Stock Index (SAP_t) will proxy the attractiveness of stocks as an alternative to gold. Changes in the exchange rate of dollars into Swiss francs ($SWISS_t$) will represent the return on foreign currency investment. The return to fixed income securities will be represented by the 1-month U.S. Treasury Bill rate ($TBILL_t$). These three variables will be included in x_t and tested for their statistical significance.

With regard to the precautionary demand for gold, consumers potentially demand more gold as the purchasing power of currency decreases due to inflationary pressure. Consequently, price indices from several industrialized nations will be included in x_t, and, subsequently, tested as possible independent variables in the determination of the demand for gold.

Any major international political or economic disturbance, such as a default by a debtor nation may cause an increase in the precautionary demand for gold. The Money Center Bank Stock Price Index (MCB_t) is an index of stock prices of banks with large international loan portfolios. A default by a debtor nation will be captured by a drop in the value of this index. Therefore, international upheaval evidenced by price drops in the Money Center Bank Stock Price Index could cause an increase in the demand for gold. If MCB_t impacts the demand for gold, it is a RHS variable that must be included in x_t. A segment of consumer demand for gold is central bank demand. Central bank demand for gold is rather unpredictable. In the past, the Soviet Union and, more recently, Russia and other former Soviet republics have traded gold or oil for wheat. The choice of commodities with which to render payment has depended on their relative prices. As the price of oil decreases relative to gold, Soviets would trade some of their vast hoard of gold for wheat, making gold a function of the price of oil. In addition, in the face of oil price reductions, OPEC nations will have fewer dollars to exchange for gold impacting the demand for gold. While these two forces represent positive and negative demand decisions they both underscore the notion that the price of oil must be included in x_t.

Commodity Producer

Consider a producer who makes a decision in period t to produce gold for period $t + 1$.[2] At time t of the production decision, the period $t + 1$ sales price is unknown but, once the production decision is made, the quantity produced Q^p_{t+1} is certain.[3] Although the output is storable, the producer holds no inventories.[4] In this case, the price-taking risk-averse producer will maximize his expected utility of profit:

$$E_t[U^p(\pi^p_{t+1})]$$

where

$$\pi^p_{t+1} = s_{t+1}Q^p_{t+1} - \rho G(Q^p_{t+1}).$$

and where superscript p refers to an individual producer. E_t is the math-

[2]The existing stock of gold is estimated to be at least 50 times greater than the current annual production of gold. Consequently, the supply of gold schedule is clearly something much more than merely the current gold production schedule. The vast majority of supply in any one market period represents a significant number of market participants making a "negative" demand decision, i.e., reversing a previous purchase of gold. As such, their actions can be explained by the factors we have already articulated in the consumer discussion or by the factors we enumerate in our description of dealers.

[3]Output uncertainty such as weather could be incorporated in the model (e.g., McKinnon, 1976; Bray, 1981), but the derivation of commodity production and futures trading would not be a simple matter.

[4]In reality, producers may hold a stock of commodity inventories. However, we shall follow the tradition of dichotomizing traders' activities into pure production without storage and pure inventory holding (see Muth, 1961; Blanchard, 1981). Although the market analysis of inventory holding producers remains simple in the presence of a futures market, it would be extremely complicated in its absence.

ematical expectation operator conditional on information available at time t, $U^p(\cdot)$ is a strictly concave utility function, and π^p_{t+1} is the producer's profit obtained at time $t + 1$. In order to obtain a linear commodity production schedule, we assume that the cost function is quadratic

$$G(Q^p_{t+1}) = \frac{1}{2} g(Q^p_{t+1} + \varepsilon^p_t)^2$$

where Q^p_{t+1} is the quantity produced in period $t + 1$, and where ε^p_t a disturbance affecting the cost function at time t. The utility function is given as

$$U^p(\pi^p_{t+1}) = -\exp(-r^p \ \pi^p_{t+1})$$

where r^P is the Arrow-Pratt constant coefficient of absolute risk aversion.[5]

[5] $Max\ E_t[U(\Pi^p_{t+1})]$, which is tantamount to $Max\ [E_t[\Pi^p_{t+1}] - 1/2r^pV_t[\Pi^p_{t+1}]]$ (1)
where $V_t\ [\Pi^p_{t+1}] = E_t\ [\Pi^p_{t+1} - E_t\ [\Pi^p_{t+1}]]^2$
Now working on $E_t\ [\Pi^p_{t+1} - E_t\ [\Pi^p_{t+1}]]^2$

$$E_t[S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2 - E_t[S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2]]^2$$

$$E_t[S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2 - E_t[S_{t+1}]Q^p_{t+1} + \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2]^2$$

$$E_t[S_{t+1}Q^p_{t+1} - E_t[S_{t+1}]Q^p_{t+1}]^2$$

$$E_t[(S_{t+1} - E_t[S_{t+1}])Q^p_{t+1}]^2$$

$$E_t[S_{t+1} - E_t[S_{t+1}]]^2[Q^p_{t+1}]^2$$

$$V_t[S_{t+1}][Q^p_{t+1}]^2$$

Now substituting this back into (1) we obtain

$$\text{Max } E_t[\Pi^p_{t+1}] - \frac{1}{2} r^pV_t(S_{t+1})[Q^p_{t+1}]^2$$

$$\partial E_t[U(\Pi^p_{t+1})]/\partial Q^p_{t+1} = E_t[S_{t+1} - \rho g(Q^p_{t+1} + \varepsilon^p_t)] - r^pV_t[S_{t+1}]Q^p_{t+1} = 0$$

$$E_t[S_{t+1}] - \rho gQ^p_{t+1} - \rho g\varepsilon^p_t = r^pV_t[S_{t+1}]Q^p_{t+1}$$

$$E_t[S_{t+1}] - \rho g\varepsilon^p_t = [r^pV_t[S_{t+1}] + \rho g]Q^p_{t+1}$$

$$Q^{p^*}_{t+1} = \frac{E_t[S_{t+1}] - \rho g\varepsilon^p_t}{r^pV_t[S_{t+1}] + \rho g}$$

What about a futures market?

$$\Pi^p_{t+1} = S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2 + R^p_t(S_{t+1} - f_t)$$

where

Assuming that s_{t+1} is normally distributed and then maximizing the expected utility, $E_t(U^P(\pi^p_{t+1}))$, is tantamount to maximizing

$$E_t(\pi^p_{t+1}) - \frac{1}{2} r^p V_t(\pi^p_{t+1}) \tag{2}$$

where V_t is the conditional variance operator such that $V_t(\pi^p_{t+1})$ is defined to be $E_t(\pi^p_{t+1} - E_t(\pi^p_{t+1}))^2$.[6] With the possibility of futures trading, the producer at time t can enter into a forward contract to deliver, or receive

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$$V_t[\Pi^p_{t+1}] = E_t[\Pi^p_{t+1} - E_t[\Pi^p_{t+1}]]^2 = E_t[S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2$$

$$+ R^p_t(S_{t+1} - f_t) - E_t[S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2 + R^p_t(S_{t+1} - f_t)]]^2$$

$$E_t[S_{t+1}Q^p_{t+1} - \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2 + R^p_t(S_{t+1} - f_t) - E_t[(S_{t+1})Q^p_{t+1}$$

$$+ \frac{1}{2}\rho g(Q^p_{t+1} + \varepsilon^p_t)^2 - R^p_t(E_t(S_{t+1}) - f_t)]^2$$

$$E_t[S_{t+1}Q^p_{t+1} + R^p_t S_{t+1} + E_t[S_{t+1}]Q^p_{t+1} - R^p_t\, E_t[S_{t+1}]]^2$$

$$E_t[(S_{t+1} - E_t[S_{t+1}])Q^p_{t+1} - (S_{t+1} - E_t[S_{t+1}])R^p_t]^2$$

$$E_t[(S_{t+1} - E_t[S_{t+1}])(Q^p_{t+1} + R^p_t)]^2$$

$$E_t(S_{t+1} - E_t[S_{t+1}])^2(Q^p_{t+1} + R^p_t)^2\ V_t(S_{t+1})(Q^p_{t+1} + R^p_t)^2$$

Substituting the expression above into (1) yields

$$\text{Max } E_t[\Pi^p_{t+1}] - \frac{1}{2} r^p V_t(S_{t+1})(Q^p_{t+1} + R^p_t)^2$$

now the FOC for the two decision variables yields

$$\frac{\partial E_t[U(\pi^p_{t+1})]}{\partial Q^p_{t+1}} = E_t(S_{t+1}) - \rho g(Q^p_{t+1} + \varepsilon^p_t) - r^p V_t(S_{t+1})(Q^p_{t+1} + R^p_t) = 0$$

$$\frac{\partial E_t[U(\pi^p_{t+1})]}{\partial R^p_t} = E_t(S_{t+1}) - f_t - r^p V_t(S_{t+1})(Q^p_{t+1} + R^p_t) = 0.$$

Combining the FOCs yields

$$E_t(S_{t+1}) - \rho g(Q^p_{t+1}) - \rho g\varepsilon^p_t = E_t(S_{t+1}) - f_t$$

from which we obtain

$$Q^{p*}_{t+1} = \frac{f_t}{\rho g} - \varepsilon^p_t$$

Rewriting the second FOC yields

$$R^{p*}_t = \frac{E_t(S_{t+1} - f_t)}{r^p V_t(S_{t+1})} - Q^p_{t+1}.$$

[6] The sufficient conditions for an optimum in this and the following maximization problems are always satisfied owing to the strict (quasi-) concavity of the objective function with respect to the decision variable(s).

delivery of, a specified quantity of gold at time $t + 1$ at a known contract price.[7] In this case, the producer will maximize (2), where $\pi^p_{t+1} = s_{t+1}Q^p_{t+1} - \rho G(Q^p_{t+1}) + R^p_t(s_{t+1} - f_t)$ and where R^p_t is the quantity of futures contracts which can be positive (futures purchase of the commodity), zero (no futures commitment), or negative (futures sale). f_t is the period t futures contract price for delivery at time $t + 1$. ρ is the discount factor, the market rate of interest plus 1. With the quadratic cost function, the assumed utility function and the normally distributed spot prices, the optimum level of output supplied and futures contracts demanded can be expressed, respectively, as

$$Q^p_{t+1} = \frac{f_t}{\rho g} - \varepsilon^p_t \tag{3.1}$$

and

$$R^p_t = -Q^p_{t+1} + Z^p_t \tag{3.2}$$

where

$$Z^p_t = \frac{E_t(s_{t+1}) - f_t}{r^p V_t(s_{t+1})}$$

The results above show that the optimum level of output Q^p_{t+1} depends on the futures price f_t, the period t disturbance in the cost function ε^p_t, and the parameters ρ and g. The production decision is made independently of attitudes toward risk and the probability distribution of uncertain spot prices and, in this sense, it is completely separated from the futures trading decision.[8]

Futures contract demand R^p_t is divided into two parts. The first term on the right-hand side of (3.2), $-Q^p_{t+1}$, is the negative of the exact quantity that should be sold in a futures market if the producer would want to completely hedge against price risk; hence, this term represents the "hedging" component. The second term, Z^p_t, reflects the difference between the producer's expectation about the period $t + 1$ spot price and

[7]The term "futures" is used here as equivalent to a "forward" transaction. In other words, it is assumed that (1) the commodity dealt in is completely standardized; (2) a futures contract is settled by actual delivery of the commodity; and (3) a futures market reopens every period, and delivery takes place only once in each period. This set of assumptions is maintained in most of the literature on the subject (Peck 1976; Grossman, 1977; Danthine, 1978; Holthausen, 1979; Turnovsky, 1979; and Feder, Just and Schmitz, 1980). A futures contract is assumed to mature in one period, which coincides with the production (and inventory holding) period.

[8]This "separation result" can be arrived at in a general expected utility maximizing framework, as is studied similarly by Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980).

the corresponding futures price, $E_t(s_{t+1}) - f_t$, which is an anticipated gain per unit of the commodity purchased in futures. Hence, this term is called the "speculation" component. Thus, the producer enters into a futures contract not only to hedge against price risk, but also to exploit speculative opportunities as well.

Inventory Holding Dealer

For storable commodities, like gold, there are agents who carry a stock of commodities as an inventory from one period to the next. In this article, such agents are called the "dealers."[9] Typically, by acquiring and compiling commodities, they accommodate stockholding services for their customers. In the absence of commodity futures contracts, a risk-averse, price-taking dealer d would maximize

$$E_t[U^d(\pi_{t+1}^d)]$$

where

$$\pi_{t+1}^d = s_{t+1}I_t^d - \rho s_t I_t^d - H(I_t^d)$$

and where I_t^d is the stock of commodity inventories purchased in period t. $H(\cdot)$ is the inventory holding cost with the usual convexity property. The purchase of the commodity in period t is multiplied by the discount factor ρ to take into account the opportunity cost of capital outlay. It is assumed that the inventory holding cost consists of the cost of deviating from a target level of inventory $\bar{I}^d + \varepsilon_t^d$, where ε_t^d is a disturbance affecting the target stock at time t and takes a quadratic form, $H(I_t^d) = 1/2\, h(I_t^d - \bar{I}^d - \varepsilon_t^d)^2$, $h > 0$. The rationale is that the holding cost is the difference between the direct cost of carrying inventories (e.g., storage, insurance fee, depreciation), which is increasing and convex in I_t^d, and the benefit of carrying a larger inventory that reduces the probability of stockout and the loss of customers.[10]

A linear inventory demand function can be derived under the conditions of the quadratic holding cost, a constant absolute risk aversion utility function, with the risk aversion coefficient r^d, and a normally distributed random spot price:

[9] In a different context of risk neutrality, Blanchard (1981), introduced such a dealer.

[10] This inventory demand (4) is familiar in the literature on agricultural commodities with storage (e.g., Muth, 1961, Peck 1976; Turnovsky, 1978, among others).

$$I_t^d = \frac{h(\bar{I}^d + \varepsilon_t^d) + E_t(s_{t+1}) - \rho s_t}{h + r^d V_t(s_{t+1})} \tag{4}$$

This equation indicates that the level of inventory demand depends positively on the target level of inventory $\bar{I}^d + \varepsilon_t^d$ and the expected capital gain of holding a unit of the commodity (after adjusting the interest rate factor) $E_t(s_{t+1}) - \rho s_t$, and negatively on the holding cost coefficient h, the risk aversion coefficient r^d and the variance of the spot price $V_t(s_{t+1})$[11]

With futures trading available, the inventory holding dealer will now maximize the same expected utility function

$$E_t[U^d(\pi_{t+1}^d)]$$

but where $\pi_{t+1}^d = s_{t+1}I_t^d - \rho s_t I_t^d - H(I_t^d) + R_t^d(s_{t+1} - f_t)$ and where R_t^d is the quantity of futures contracts purchased (if positive) or sold (if negative). A complete "separation result" again illustrates that inventory demand is decided independent of the futures contract decision. The former depends on the current spot and futures prices, the interest rate, and the holding cost parameters (including the disturbance), while the latter is determined by the probability distribution of the random spot price and the dealer's risk aversion coefficient:

$$I_t^d = \bar{I}^d + \varepsilon_t^d + \frac{f_t - \rho s_t}{h} \tag{5.1}$$

$$R_t^d = -I_t^d + Z_t^d \tag{5.2}$$

where

$$Z_t^d = \frac{E_t(s_{t+1}) - f_t}{r^d V_t(s_{t+1})}$$

As in the producer's case, the inventory holding dealer also plays the dual role of a hedger and a speculator in a futures market.

Pure Speculator

The presence of a futures market may induce activities by pure speculators who assume open positions in futures without having to commit themselves to the production, handling or processing of the commodity. The objective of a risk-averse pure speculator s is to maximize

[11]See Kaldor (1939–1940) and Telser (1958) for the argument of convenience yield. In the case of negative inventories, the storage rule would probably have to be modified.

$$E_t[U^s(\pi_{t+1}^s)]$$

where

$$\pi_{t+1}^s = Z_t^s(s_{t+1} - f_t).$$

The term "pure" is used here since the agent in question is speculating against price movements without engaging in any real economic commitment to the market. Under the conditions of constant absolute risk aversion (with the risk aversion coefficient r^s) and normally distributed spot prices, the optimal volume of speculation is given by

$$Z_t^s = \frac{E_t(s_{t+1}) - f_t}{r^S V_t(s_{t+1})} \tag{6}$$

The expression for the pure speculator's demand is exactly the same as the speculation component of the producer's or the dealer's demand for futures trading except for the difference in risk aversion coefficients.

DETERMINATION OF EQUILIBRIUM GOLD SPOT AND GOLD FUTURES PRICES

Using both a spot and a futures market clearing condition, we will be able to solve for the equilibrium spot and futures market prices. After aggregating individual functions, we can summarize the commodity market in the presence of futures trading with the following six-equation model:

$$\text{Consumer demand: } C_t = \alpha_o - \alpha s_t + \Omega' x_t + u_t \tag{7.1}$$

$$\text{Production: } Q_t = \frac{\beta}{\rho} f_{t-1} + v_{t-1} \tag{7.2}$$

$$\text{Inventory demand: } I_t = \bar{I} + \gamma(f_t - \rho s_t) + w_t \tag{7.3}$$

$$\text{Futures speculation: } Z_t = \frac{\chi}{\theta}[E_t(s_{t+1}) - f_t] \tag{7.4}$$

$$\text{Spot market clearing: } Z_{t-1} = C_t + I_t \tag{7.5}$$

$$\text{Futures market clearing: } Q_{t+1} + I_t = Z_t \tag{7.6}$$

where the parameters are defined as

$$\alpha_o = n^i a_o,\ \alpha = n^i a_1,\ \Omega = n^i a_2,\ \beta = \frac{n^p}{g},\ \nu = \frac{n^p}{r^p},\ \gamma = \frac{n^d}{h},$$

$$\tau = \frac{n^d}{r^d},\ \omega = \frac{n^s}{r^s},\ \bar{I} = n^d \bar{I}^d\ u_t = n^i \varepsilon_t^i,\ v_t = -n^p \varepsilon_t^p,$$

$$w_t = n^d \varepsilon_t^d,\ \chi = \nu + \tau + \omega \text{ and } \theta = V_t(s_{t+1}).$$

With n^i, n^p, n^s, and n^d as given numbers of consumers, producers, speculators and dealers, respectively.[12]

The equation for futures speculation (7.4) may need a brief explanation. As we have seen in previous sections, commodity producers and inventory holding dealers participate in a futures market not only as hedgers but also as speculators. This means that the speculative demand for futures contracts should consist of the speculative components of futures demand by producers, and dealers, as well as the pure speculators. Hence, the total demand is an aggregation of Z_t^p, Z_t^d, and Z_t^s.

The spot market clearing condition (7.5) states that the amount purchased at time $t - 1$ by futures speculators appears in time t as a spot supply in the amount of Z_{t-1} and this magnitude has to match the demand by consumers and dealers.

The futures market clearing condition (7.6) indicates that the total supply of futures contracts by producers and dealers in their capacity as hedgers, $Q_{t+1} + I_t$, has to equal the quantity demanded by speculators, Z_t:

$$Q_{t+1} + I_t = Z_t$$

recall

$$Z_t = \frac{\chi}{\theta}\,[E_t(s_{t+1}) - f_t]$$

The rational expectations equilibrium solutions for the spot and futures prices can be easily found. Substitution of expressions for Q_{t+1}, I_t, and Z_t into (7.6) yields

$$f_t = \frac{1}{(\beta/\rho) + \gamma + (\chi/\theta)}\left(-\bar{I} + \frac{\chi}{\theta}E_t(s_{t+1}) + \gamma\rho s_t - v_t - w_t\right) \quad (8)$$

[12]Since u_t, v_t and w_t have unbounded support, problems of negative price and negative quantity can rise. Throughout the paper, the non-negativity constraint is ignored for simplicity. The probability of negative values can be made arbitrarily small by appropriate choice of underlying parameters and the variance of u_t, v_t, and w_t.

Lagging by one period the expression for f_t given in (8) and substituting this into a lagged version of (7.4), the spot market clearing condition (7.5) yields a difference equation for expected spot prices. The solution of the difference equation for the expected spot price of gold

$$E_{t-1}s_t = \lambda s_{t-1} + \left[\frac{\dfrac{\beta\theta}{\gamma\rho\chi}\bar{I} + \left\{\dfrac{1}{\gamma} + \left(1 + \dfrac{\beta}{\gamma\rho}\right)\dfrac{\theta}{\chi}\right\}\alpha_0}{\dfrac{\rho}{\lambda} - 1}\right]$$

$$+ \frac{\rho}{\lambda}B_{t-1} - \frac{\lambda}{\rho}\frac{1}{\gamma}v_{t-1} - \frac{1}{\gamma}\frac{\lambda}{\rho}w_{t-1}$$

$$+ \frac{\lambda}{\rho}\left[\frac{1}{\gamma} + \left(1 + \frac{\beta}{\gamma\rho}\right)\frac{\theta}{\chi}\right]E_{t-1}\left[\Omega' x_t + \frac{\lambda}{\rho}\Omega' x_{t+1} + \cdots + \cdots\right] \tag{9}$$

occasions the unexpected appearance of B_{t-1} on the RHS of equation (9), where B_{t-1} must satisfy the condition that[13]

$$E_{t-2}B_{t-1} = \frac{\rho}{\lambda}B_{t-2} \tag{10}$$

[13]Consider the following expectational equation

$$y_t = \lambda E_t y_{t+1} + x_t$$

where

$$x_t = \rho x_{t-1} + \varepsilon_t$$

we would argue that $y_t = [1/(1 - \lambda\rho)]x_t + c_t$, where $c_t = c_0\,(1/\lambda)^t$ and is given deterministically, is a solution to the expectation equation above. If we multiply the equation above by $(1 - \lambda L^{-1})$ and then use the expectation operator—we return to the original equation:

$$(1 - \lambda L^{-1})y_t = \frac{1}{1 - \lambda\rho}x_t[1 - \lambda L^{-1}] + \left(\frac{1}{\lambda}\right)^t c_0[1 - \lambda L^{-1}]$$

$$y_t - \lambda y_{t+1} = \frac{1}{1 - \lambda\rho}[x_t - \lambda x_{t+1}] + \left(\frac{1}{\lambda}\right)^t c_0 - \lambda\left[\frac{1}{\lambda}\right]^{t+1} c_0$$

Now, using the expectation operator at time t

$$y_t - \lambda E_t y_{t+1} = \frac{1}{1 - \lambda\rho}[x_t - \lambda E_t x_{t+1}] + \left(\frac{1}{\lambda}\right)^t c_0 - \left(\frac{1}{\lambda}\right)^t E_t c_0$$

$$y_t = \lambda E_t y_{t+1} + \left[\frac{1}{1 - \lambda\rho}\right][1 - \lambda\rho]x_t + 0$$

$$y_t = \lambda E_t y_{t+1} + x_t$$

In order to evaluate the expected value of the geometrically declining weighted sum of future values of the $r \times 1$ vector x_t, lets first assume that

$$x_t = \sum_{i=1}^{h} C_i x_{t-i} + \delta_t \tag{11}$$

where C_i is $r \times r$, δ_t is a $r \times 1$ vector of normally distributed error terms with zero means and finite variances. The order of the autoregression h will be given empirically. Rewriting (11) for convenience

$$X_t = AX_{t-1} + L_t$$

where $X_t = [x_t, x_{t-1}, \ldots x_{t-h}]'$ is a vector with (rh) rows and $L_t = [\delta_t, 0, \ldots]'$, with (rh) rows, with $[(r-1)h]$ rows equal to zero.

$$A = \begin{bmatrix} C_1 & C_2 & \cdot & \cdot & C_h \\ I & 0 & \cdot & \cdot & 0 \\ 0 & I & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & I & 0 \end{bmatrix}$$

where A is an (rh) by (rh) matrix and I is an $r \times r$ identity matrix. Assuming that the information set at time t contains only current and past realized values of x_t and invoking Kolmogorov's forecasting rule yields

$$E_{t-1}X_t = AX_{t-1}$$

$$E_{t-1}x_t = QAX_{t-1}$$

$$E_{t-1}x_{t+m} = QA^m X_{t-1}$$

Q is a $r \times rh$ matrix with ones in the diagonal elements of the first r columns and zeros otherwise:

$$Q = \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & \cdot & & & \\ 0 & 1 & 0 & \cdot & \cdot & \cdot & & \uparrow & \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \leftarrow & 0 & \rightarrow \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & & \downarrow & \end{bmatrix}$$

Using the formulations above, the expected spot price can eventually be written as

$$E_{t-1}s_t = \lambda s_{t-1} + \left[\frac{\frac{\beta\theta}{\gamma\rho\chi}\bar{I} + \left\{\frac{1}{\gamma} + \left(1 + \frac{\beta}{\gamma\rho}\right)\frac{\theta}{\chi}\right\}\alpha_0}{\frac{\rho}{\lambda} - 1} \right]$$
$$+ \frac{\lambda}{\rho}\left[\frac{1}{\gamma} + \left(1 + \frac{\beta}{\gamma\rho}\right)\frac{\theta}{\chi}\right]\Omega' QA \left[I - \frac{\lambda}{\rho}A\right]^{-1} X_{t-1}$$
$$- \frac{\lambda}{\rho}\frac{1}{\gamma}v_{t-1} - \frac{1}{\gamma}\frac{\lambda}{\rho}w_{t-1} + \frac{\rho}{\lambda}B_{t-1} \tag{12}$$

The final term on the RHS of (12), $\rho/\lambda\ B_{t-1}$, is the price expectation bubble. The definition of a spot price expectation bubble as a situation in which $E_{t-1}B_t \neq 0$ is attractive for two reasons. B_t is a self-contained and a self-fulfilling element of spot price expectations. A general solution for spot price expectations necessitates the existence of B_t, as such, B_t, is independent of market processes. The magnitude of other components of the RHS of (12) represent the price and quantity resolutions of other markets. It would be unrealistic to associate those market realizations with bubbles, as we would be forced to characterize them as being bubbly as well. B_t is self-fulfilling because its expected value is given by a first-order autoregressive process. In addition, if $B_t \neq 0$, the expected value of B_t changes through time changing the trajectory of spot price expectations, even if the market fundamentals do not change

$$\left[E_t B_{t+m} = \left(\frac{\rho}{\lambda}\right)^m B_t\right]$$

These changes will take place at an ever-increasing rate ($\rho/\lambda > 1$) generating the type of explosive expectations that are associated bubbles.

Plugging the solutions for $E_t(s_{t+1})$ and for $E_{t-1}(s_t)$ into equation (9) and then normalizing on s_t yields

$$s_t = G_1\bar{I} + G_2\alpha_0 + G_3 s_{t-1} + G_4 U_t + G_5 r_t + G_6 r_{t-1} + G_7 w_t$$
$$+ G_8 w_{t-1} + G_9 X_t + G_{10} X_{t-1} + G_{11} B_t + G_{12} B_{t-1}$$

where the G_is are defined in Appendix A (appendix available on request).

Substituting our solutions for s_t and $E_t(s_{t+1})$ into (8), taking the first difference of the resulting expression for f_t and, finally, rewriting the change in gold futures prices yields

$$\Delta f_t = M_1 \Delta B_t + M_2 \Delta B_{t-1} + M_3 \Delta X_t + M_4 \Delta X_{t-1} + M_5 \Delta s_{t-1}$$
$$+ M_6 \Delta u_t + M_7 \Delta v_t + M_8 \Delta v_{t-1} + M_9 \Delta w_t + M_{10} \Delta w_{t-1} \quad (13)$$

where the definition of M_i is provided in Appendix B (appendix available upon request). M_3 and M_4 are, of course, vectors that are conformable with ΔX_t and ΔX_{t-1}, respectively.

DYNAMIC FACTOR ANALYSIS

Our futures market model will be expressed in state-space form so that the Kalman filter component of the dynamic factor algorithm can be used to document a potential but unobserved stochastic price bubble.[14] Let $\underline{w}_t$ be an n x 1 vector of unobserved variables called state variables, and $\underline{g}_t$ and $\underline{z}_t$ be m x 1 and q x 1 vectors of observable variables called input and output variables, respectively. The state-space model can be written as

$$\underline{w}_{t+1} = F\underline{w}_t + G\underline{g}_t + \xi_{t+1} \quad (14.1)$$

$$\underline{z}_t = H\underline{w}_t + D\underline{g}_t + \zeta_t \quad (14.2)$$

where ζ_{t+1} and ζ_t, are, respectively, m x 1 and q $\times$ 1 vectors of disturbances with F, G, H, an D as constant real matrices of conformable dimensions. We assume that the disturbance terms, ξ_t and ζ_t, are each serially uncorrelated and are also uncorrelated with each other and that

$$E(\xi_t) = 0, \qquad E(\zeta_t) = 0$$

$$E(\xi_t, \xi_t') = V, \qquad E(\zeta_t \zeta_t') = R$$

Equation (14.1) is called the transition equation and (14.2) is called the measurement equation.

We will assume that the bubble term B_t continues to satisfy (10) according to

$$B_t = \frac{\rho}{\lambda} B_{t-1} + \eta_t \quad (15.1)$$

The complete model has the state space representation (14) if the following equations are used.

[14]The Kalman filter technique is well treated in the control literature and the interested reader is referred to Anderson and Moore (1979), Harvey (1989), or Hamilton (1991) for details.

$$\begin{bmatrix} \Delta B_{t+1} \\ \Delta B_t \end{bmatrix} = \begin{bmatrix} \frac{\rho}{\lambda} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta B_t \\ \Delta B_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta X_t \\ \Delta X_{t-1} \\ \Delta s_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{t+1} \\ 0 \end{bmatrix}$$

$$[\Delta f_t] = [M_1 M_2] \begin{bmatrix} \Delta B_t \\ \Delta B_{t-1} \end{bmatrix} + [M_3 M_4 M_5] \begin{bmatrix} \Delta X_t \\ \Delta X_{t-1} \\ \Delta s_{t-1} \end{bmatrix}$$

$$+ [M_6 M_7 M_8 M_9 M_{10}] \begin{bmatrix} \Delta u_t \\ \Delta v_t \\ \Delta v_{t-1} \\ \Delta w_t \\ \Delta w_{t-1} \end{bmatrix} \qquad (15.2)$$

For the expressions immediately above, the variance covariance matrices of the errors are given as

$$V = \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} \sigma_u^2 & & & & & \\ & \sigma_v^2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \cdot \\ & & & & & \cdot \end{bmatrix}$$

The reader should keep in mind that ΔX_t and ΔX_{t-1} continue to be $(rh) \times 1$ vectors. In this particular state space representation there are two transition equations and one measurement equation. The transition equations represent the bubble process (15.1). The measurement equation is a specification of the process that generates Δf_t. The estimation technique employed in this article—dynamic factor analysis—iterates between estimating the parameters of the state space model and generating the unobservable financial futures bubble. Details on this computation algorithm can be found in Appendix C (appendix available upon request).

ESTIMATION RESULTS[15]

In an effort to establish stationarity for the variables used in the dynamic factor analysis, augmented Dickey-Fuller (1979) tests were performed.

[15]In this paper, linked gold spot and gold futures markets were used to solve for equilibrium s_t and f_r. The reduced form solution for the gold futures price was a function of market fundamentals, speculative bubbles and a confounded error term

Test results are reported in Table I for the variables that were ultimately determined as being statistically significant in the analysis. As shown, the null hypothesis that there exists a unit root is never rejected at a conventional significance level. This suggests that there exists at least one unit root in each of the six time-series representations. We tested for a second unit root by differencing the data and applying the test procedure to the transformed data. The test statistics also reported in Table I, all rejected the null at the 1% level. Hence, there is little evidence that a second unit root exists in the time series data.[16]

$$\Delta f_t = f(\Delta B_t, \Delta B_{t-1}, \Delta X_t, \Delta X_{t-1}, \Delta s_{t-1},) + \Delta \varepsilon_t$$

$$\text{where } \Delta \varepsilon_t = f(\Delta u_t, \Delta v_t, \Delta v_{t-1}, \Delta w_t, \Delta w_{t-1},)$$

That is, the empirical specification of the gold futures price is driven by the model and nothing about the model suggests that ε_t could be determined by an ARCH or GARCH process. However, in order to validate our assumption, we tested the residual in the measurement equation for a possible ARCH or GARCH process (i.e. $\varepsilon_t = v_t \sqrt{h_t}$, where $h_t = \alpha_0 + \Sigma_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \Sigma_{i=1}^{p} \beta_i h_{t-i}$). Using Engle's multiplier test, we regressed the squared residual on an intercept and eight autoregressive terms. We then calculated a TR^2 statistic of 14.837 and compared it with a 5% level, critical value of 15.5073. Thus, we were unable to refute the hypothesis that the error term follows a white noise process. Given the evidence above, we believe no ARCH, nor GARCH (If no ARCH effects exist, then no GARCH effects can exist, see Hamilton 1989, p 664.) effects are present in the error term of the gold futures pricing equation. Consequently, we are free to perform dynamic factor analysis on our model, with no restrictions on the error term.

[16]Given the variables in the original gold futures pricing equation (when they are expressed as levels) are $I(1)$, it is possible that a cointegrating relationship exists. If this is the case, the cointegrated variables have an error correction representation of

$$\Delta x_t = \pi_0 + \pi x_{t-1} + \sum_{t=1}^{p} \pi_t x_{t-1} + \varepsilon_t \tag{1}$$

where

x_t = an (n × 1) vector containing $\{F_t, S_t, \text{MCB}_t, \text{POIL}_t, \text{PPI}_t, \text{SWISS}_t\}$
π_0 = an (n × 1) vector of intercept terms with elements π_{i0}
π_i = (n × n) coefficient matrices with elements $\pi_{jk}(i)$
π = is a matrix with elements π_{jk} such that one or more of the $\pi_{jk} \neq 0$
ε_t = an (n × 1) vector with elements ε_{it}.

With the error correction term in equation (1), each variable responds to the deviations from "long-run equilibrium". Estimating x_t as a VAR in first differences is inappropriate if x_t has an error correction representation. That is, the omission of the correction term πx_{t-1} is a specification error if x_t has the error correction representation in equation (1).

We test the gold futures equilibrium pricing equation for cointegration with the Engle-Granger method (1987). First we estimate the long-run equilibrium relationship for futures prices using 288 observations

$$F_t = \beta_0 + \beta_1 S_t + \beta_2 \text{MCB}_t + \beta_3 \text{POIL}_t + \beta_4 \text{PPI}_t + \beta_5 \text{SWISS}_t + e_t$$

Next, we preformed the augmented Dickey-Fuller test on the estimated residuals

$$\Delta \hat{e}_t = a_1 \hat{e}_{t-1} + \sum_{i=1}^{p} a_{i+1} \Delta \hat{e}_{t-1} + \varepsilon_t$$

The null hypothesis, that no cointegration exists, implies the coefficient $a_1 = 0$. The estimated value

TABLE I

Augmented Dickey-Fuller Unit Root Tests for Gold Futures Price, Gold Spot Price, Producer Price Index, Money Center Bank Index, Spot Price of Petroleum, Swiss Exchange Rate

	f	*s*	*PPI*	*MCB*	*POIL*	*SWISS*
ADF LEVEL	−0.832	−0.899	−1.125	−1.035	−1.08	−1.79
ADF DIFF	−3.873*	−4.463*	−3.11*	−3.272*	−5.010*	−4.78*

t ratios for β in augmented Dickey-Fuller tests (ADF) for the regression $\Delta x_t = \alpha + \beta x_{t-1} + \Sigma_{i=1}^{6} \phi_i \Delta x_{t-i} + \xi_t$. *Statistically significant at the 1% level.

Given the timing and relation of the independent variables in equation (13) to the gold futures price, it is possible that the market fundamentals are not exogenous, but rather depend upon the gold futures price. If this were the case, the econometric evaluation of the determinants of gold futures prices would have to be done in a multi-equation context, to avoid a possible simultaneous equation bias. Consequently, the independence of each exogenous variable from the gold futures price was statistically tested. In particular constrained and unconstrained, multivariate regressions were estimated for ΔPPI_t, ΔMCB_t, $\Delta POIL_t$ and $\Delta SWISS_t$. The RHS of each equation to be estimated included sixth-order autoregressive representations of the gold futures price and of the three variables not used on the LHS. The null hypothesis is that lagged changes in gold-futures prices have no impact upon the designated LHS variable. In a manner consistent with the null hypothesis, the constrained regressions were performed with the parameters on the lagged Δf_ts set equal to zero. To accommodate the alternative hypothesis, the unconstrained regressions had no restrictions on the estimated parameters. The sum of the squared residuals of the constrained and unconstrained cases were each computed and then were compared using a likelihood ratio test. For all four sets of the regressions, the likelihood ratio test statistic follows an $F(6, 257)$ distribution under the null hypothesis. If, for example, in the ΔPPI_t equation, the null hypothesis of all coefficients of the lagged Δf_ts being equal to zero is not rejected, the conclusion is that Δf_t does not granger cause ΔPPI_t. The test statistics and the associated critical values for rejecting the null hypothesis are reported in Table II. The null is never

of a_1 is -0.07562 with a corresponding *t*-statistic of -1.32. The critical value obtained from Engle and Yoo (1987) for five variables and 200, or more, observations at a 5% significance level is -4.48. We can therefore conclude that a_1 is not significantly different than zero. Hence, we are unable to refute the null hypothesis of no cointegration. Our analysis justifiably neglects the use of any error correction term.

TABLE II

Granger Causality Tests[a]

Null Hypothesis (H_0)	*F-Statistic*	*Critical Value for $F(6, 257)$ with $\alpha = 1\%$*	*Conclusion*
Δf does not granger cause ΔPPI	1.437	2.10	Did not reject H_0
Δf does not granger cause ΔMCB	0.973	2.10	Did not reject H_0
Δf does not granger cause ΔPOIL	1.244	2.10	Did not reject H_0
Δf does not granger cause ΔSWISS	0.599	2.10	Did not reject H_0

[a]The condition that Δf_t does not granger cause Δx_t is necessary but not sufficient for Δx_t to be exogenous with respect to Δf_t. We choose to report granger tests of granger causality, as Geweke, Meese, and Dent (1983) report strong evidence for preferring these tests to other variants proposed in the literature.

rejected at conventional significance levels. Thus, there is little evidence that Δf_t granger cause $\Delta \mathrm{PPI}_t$, $\Delta \mathrm{MCB}_t$, $\Delta \mathrm{POIL}_t$, or $\Delta \mathrm{SWISS}_t$. These results suggest that it is plausible to assume exogeneity of the market fundamentals.

Iterating between the expectation maximization step to estimate the parameters of equation (15) and the Kalman step to generate the unobservable bubble, dynamic factor analysis concluded that five variables are statistically significant in determining the futures price.[17] However, the existence of the bubble itself is very much in doubt.[18]

[17]Variables that were included in the transition equation or in the measurement equation, at any phase of the testing, are listed below with their respective sources. All the entries represent data collected at the close of the last working day of the month. For example the real return on the Standard and Poor's 500 Composite Stock Index, SAP_t, is the percentage change in SAP_t from the end of $t - 1$ to the end of t deflated by the producer's price index at the end of t. The gold and platinum futures prices are the nearest delivery month's closing price. TBILL_t is the average of the bid-ask quotes in the Wall Street Journal.

All data are for the period 1/75–6/98 monthly observations

Abbreviation	Data Item	Source
SWISS	Swiss franc/U.S. dollar	Citibase
SAP	S&P 500 Stock Index	Citibase
PPI	PPI—finished goods (not seasonally adjusted)	Citibase
s	Noon London gold price fix	Citibase
MCB	Money center bank price index	Standard & Poor's Statistical Service
CRUDE	Crude oil price-refiners cost (imported)	Citibase
f	Gold futures price	Wall Street Journal
TBILL	1 month T-Bill	Wall Street Journal
PLAT	Platinum futures price	Wall Street Journal
WHEAT	Spot price of wheat	Citibase
POIL	Price of crude oil	Citibase

[18]The EM algorithm has many attractive features. Foremost among these are its computational simplicity and its convergence properties. In our earlier Monte Carlo experiments, the method found

TABLE III

Dynamic Factor Coefficients of RHS Variables in the Measurement Equation

RHS Var	*Estimate of the Respective Reduced Form Coefficients*	*Std Error of Estimate*[a]
ΔPPI_t	0.2415	0.1055
ΔMCB_t	−0.3815	0.1573
$\Delta POIL_t$	0.0618	0.0311
$\Delta SWISS_t$	−0.2633	0.0842
ΔS_{t-1}	0.5006	0.0375
ΔB_{t-1}	0.1026	0.0616

[a]The model's initial parameter values were varied and a severe convergence criteria was employed to avoid local optimization results.

The empirical results recorded above in Table III establish the statistical importance of the producer price index in the determination of Δf_t. Given the attraction of gold as a hedge against inflation, changes in the PPI_t could conceivably produce changes in the price of gold and do increase price of gold futures contracts.[19] MCB_t was included in the analysis to proxy the level of international uncertainty. As manifested by our statistical results, increases in international uncertainty (a fall in MCB_t) increase the price of gold futures. Crude oil prices were included in the testing because we felt that if oil prices went up then the demand for gold would go up and the supply of gold would fall. These shifts in the demand and supply schedules would not only increase the price of gold but, as documented by a significant coefficient of .0618, increase Δf_t. In order to proxy the attractiveness of holding foreign exchange as an alternative to gold, $SWISS_t$ was included in x_t and used as a RHS variable in the estimation of equation (15). A significant reduced form coefficient of −0.2633 is testimony to its importance in the determination of Δf_t. Predictably, lagged gold spot prices have a statistically significant impact upon gold futures prices. A dollar increase in s_t increases subsequent futures prices by fifty cents. The role of speculative bubbles in the determination of gold futures prices is not obvious, $\Delta\hat{B}_{t-1}$ the lagged specu-

estimates in the region of the maximum reasonably quickly from even poor initial guesses. The method also has the desirable property that it constructs $\hat{R}$ and $\hat{Q}$ to be positive semidefinite matrices and therefore eliminates the need for arbitrary penalty functions to bound the parameter space.

[19]The standard errors of the maximum likelihood estimates detailed above were calculated by taking the inverse of Fisher's information matrix. All coefficients listed had asymptotic t statistics that were significant with a = 0.01.

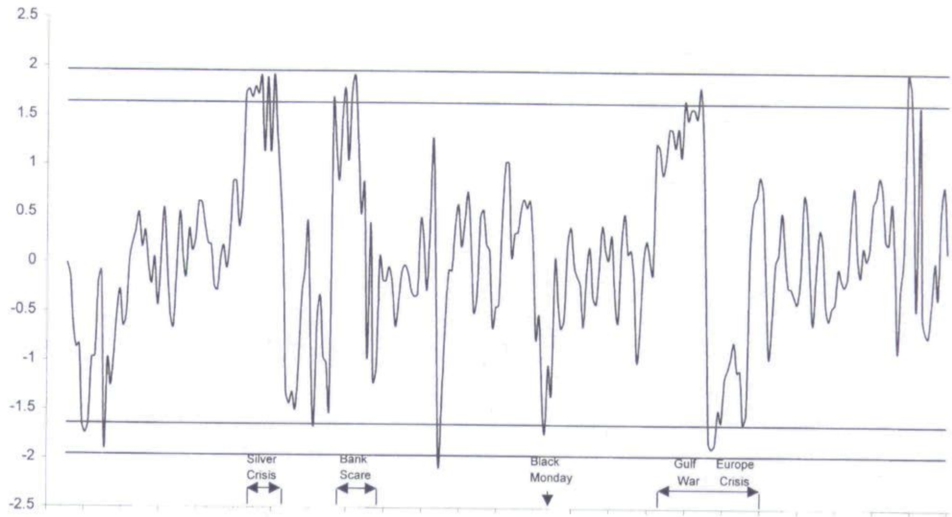

FIGURE 1
Computed t-statistics of futures market stochastic bubbles.

lative bubble has a sizeable but statistically insignificant reduced form coefficient.[20]

Dynamic Factor Coefficients of RHS Variables in the Transition Equation

RHS Var	Estimate of the Respective Reduced Form Coefficients	Std Error of Estimate[21]
$\Delta\hat{B}_{t-1}$	0.2938	0.0387

The transition equation for speculative bubbles yields a statistically significant coefficient for $\Delta\hat{B}_{t-1}$.

Estimates of the bubbles and its standard deviation for the entire sample are reported in Table IV and they are displayed in Figure 1. According to Table IV, speculative bubbles in gold futures prices were ab-

[20]The results for the measurement equation and the transition equation provide the final estimates of the parameters after convergence in the likelihood function had been obtained. Convergence took 46 iterations and less than 94 sec CPU on IBM360. Of course, many preliminary runs were performed as insignificant independent variables were dropped from the dynamic factor analysis. In addition, the state space vector was augmented with dynamic specifications of the random errors terms that appeared in the transition and measurement equations. As evidenced by the DW statistics, this generalization of the model remedied our first order serial correlation problems.

[21]The standard error of the maximum likelihood estimate detailed below was calculated by taking the inverse of Fisher's information matrix. The coefficient listed had asymptotic t-statistic that was significant at $\alpha = 0.01$.

TABLE IV

Kalman Estimates of $\hat{B}_t$ and Their Standard Deviations; Gold Futures Market Bubbles[a] (1975.1–1998.6)

Date	$\hat{B}_t$	$se(\hat{B}_t)$	*Date*	$\hat{B}_t$	$se(\hat{B}_t)$	*Date*	$\hat{B}_t$	$se(\hat{B}_t)$	*Date*	$\hat{B}_t$	$se(\hat{B}_t)$
1975.1	−0.335	31.62	1978.5	0.257	2.070	1981.9	−1.156	2.070	1985.1	−2.716	2.070
1975.2	−1.537	10.67	1978.6	0.504	2.070	1981.10	−0.682	2.070	1985.2	−1.049	2.070
1975.3	−1.617	2.576	1978.7	1.277	2.070	1981.11	−1.978	2.070	1985.3	−0.134	2.070
1975.4	−1.832	2.114	1978.8	1.268	2.070	1981.12	−2.117	2.070	1985.4	−0.141	2.070
1975.5	−1.739	2.074	1978.9	0.789	2.070	1982.1	−3.071	2.070	1985.5	0.794	2.070
1975.6	−3.401	2.070	1978.10	0.401	2.070	1982.2	3.423	2.070	1985.6	1.238	2.070
1975.7	−3.602	2.070	1978.11	0.360	2.070	1982.3	2.561	2.070	1985.7	0.362	2.070
1975.8	−3.369	2.070	1978.12	−0.490	2.070	1982.4	1.755	2.070	1985.8	0.863	2.070
1975.9	−2.016	2.070	1979.1	−0.571	2.070	1982.5	3.071	2.070	1985.9	1.515	2.070
1975.10	−1.976	2.070	1979.2	0.128	2.070	1982.6	3.678	2.070	1985.10	0.573	2.070
1975.11	−0.414	2.070	1979.3	0.359	2.070	1982.7	2.173	2.070	1985.11	−1.018	2.070
1975.12	−0.183	2.070	1979.4	−0.104	2.070	1982.8	3.630	2.070	1985.12	−0.678	2.070
1976.1	−3.897	2.070	1979.5	0.501	2.070	1982.9	3.965	2.070	1986.1	0.996	2.070
1976.2	−2.041	2.070	1979.6	1.707	2.070	1982.10	2.642	2.070	1986.2	1.135	2.070
1976.3	−2.595	2.070	1979.7	1.722	2.070	1982.11	1.037	2.070	1986.3	0.414	2.070
1976.4	−2.002	2.070	1979.8	0.770	2.070	1982.12	1.665	2.070	1986.4	0.252	2.070
1976.5	−1.053	2.070	1979.9	1.482	2.070	1983.1	−2.020	2.070	1986.5	−1.324	2.070
1976.6	−0.576	2.070	1979.10	3.600	2.070	1983.2	0.854	2.070	1986.6	−0.908	2.070
1976.7	−1.337	2.070	1979.11	3.685	2.070	1983.3	−2.490	2.070	1986.7	−0.868	2.070
1976.8	−1.106	2.070	1979.12	3.514	2.070	1983.4	−2.200	2.070	1986.8	0.966	2.070
1976.9	0.056	2.070	1980.1	3.731	2.070	1983.5	0.113	2.070	1986.9	2.137	2.070
1976.10	0.397	2.070	1980.2	3.587	2.070	1983.6	−0.342	2.070	1986.10	2.137	2.070
1976.11	0.678	2.070	1980.3	3.934	2.070	1983.7	−0.373	2.070	1986.11	0.166	2.070
1976.12	1.048	2.070	1980.4	2.361	2.070	1983.8	−0.072	2.070	1986.12	0.625	2.070
1977.1	0.328	2.070	1980.5	3.922	2.070	1983.9	−0.444	2.070	1987.1	0.677	2.070
1977.2	0.676	2.070	1980.6	2.352	2.070	1983.10	−1.333	2.070	1987.2	1.515	2.070
1977.3	−0.003	2.070	1980.7	3.982	2.070	1983.11	−0.810	2.070	1987.3	1.362	2.070
1977.4	−0.448	2.070	1980.8	2.754	2.070	1983.12	−0.178	2.070	1987.4	1.176	2.070
1977.5	0.107	2.070	1980.9	1.658	2.070	1984.1	−0.028	2.070	1987.5	1.325	2.070
1977.6	−0.894	2.070	1980.10	0.282	2.070	1984.2	−0.212	2.070	1987.6	0.290	2.070
1977.7	0.034	2.070	1980.11	−2.700	2.070	1984.3	−0.553	2.070	1987.7	−1.576	2.070
1977.8	1.153	2.070	1980.12	−2.966	2.070	1984.4	−0.697	2.070	1987.8	−1.091	2.070
1977.9	−0.031	2.070	1981.1	−2.717	2.070	1984.5	−0.650	2.070	1987.9	−2.721	2.070
1977.10	−1.091	2.070	1981.2	−3.090	2.070	1984.6	0.926	2.070	1987.10	−3.578	2.070
1977.11	−1.356	2.070	1981.3	−2.203	2.070	1984.7	0.445	2.070	1987.11	−2.147	2.070
1977.12	−0.127	2.070	1981.4	−0.697	2.070	1984.8	−0.562	2.070	1987.12	−2.744	2.070
1978.1	1.073	2.070	1981.5	−0.174	2.070	1984.9	0.947	2.070	1988.1	0.064	2.070
1978.2	0.103	2.070	1981.6	0.855	2.070	1984.10	2.624	2.070	1988.2	−0.781	2.070
1978.3	−0.293	2.070	1981.7	−1.814	2.070	1984.11	−0.901	2.070	1988.3	−1.364	2.070
1978.4	0.173	2.070	1981.8	−3.443	2.070	1984.12	−4.276	2.070	1988.4	−1.161	2.070
1988.5	0.438	2.070	1991.12	2.441	2.070	1995.7	−0.091	2.070			
1988.6	0.773	2.070	1992.1	−0.872	2.070	1995.8	−0.318	2.070			
1988.7	0.008	2.070	1992.2	−3.764	2.070	1995.9	−0.442	2.070			
1988.8	−0.263	2.070	1992.3	−3.899	2.070	1995.10	−0.223	2.070			
1988.9	−0.458	2.070	1992.4	−3.803	2.070	1995.11	0.955	2.070			
1988.10	−1.325	2.070	1992.5	−3.091	2.070	1995.12	1.625	2.070			
1988.11	−0.148	2.070	1992.6	−3.318	2.070	1996.1	0.277	2.070			
1988.12	0.344	2.070	1992.7	−2.441	2.070	1996.2	−0.250	2.070			
1989.1	−0.678	2.070	1992.8	−2.223	2.070	1996.3	0.363	2.070			

TABLE IV (Continued)

Kalman Estimates of $\hat{B}_t$ and Their Standard Deviations; Gold Futures Market Bubbles[a] (1975.1–1998.6)

Date	$\hat{B}_t$	$se(\hat{B}_t)$	*Date*	$\hat{B}_t$	$se(\hat{B}_t)$	*Date*	$\hat{B}_t$	$se(\hat{B}_t)$	*Date*	$\hat{B}_t$	$se(\hat{B}_t)$
1989.2	−0.839	2.070	1992.9	−1.957	2.070	1996.4	0.115	2.070			
1989.3	−0.136	2.070	1992.10	−1.638	2.070	1996.5	0.354	2.070			
1989.4	0.806	2.070	1992.11	−2.272	2.070	1996.6	1.263	2.070			
1989.5	0.293	2.070	1992.12	−2.251	2.070	1996.7	1.435	2.070			
1989.6	0.096	2.070	1993.1	−3.367	2.070	1996.8	1.863	2.070			
1989.7	0.584	2.070	1993.2	−3.119	2.070	1996.9	1.511	2.070			
1989.8	−0.707	2.070	1993.3	0.354	2.070	1996.10	0.541	2.070			
1989.9	−1.216	2.070	1993.4	1.263	2.070	1996.11	0.455	2.070			
1989.10	0.399	2.070	1993.5	1.435	2.070	1996.12	1.309	2.070			
1989.11	1.057	2.070	1993.6	1.862	2.070	1997.1	0.068	2.070			
1989.12	0.226	2.070	1993.7	1.509	2.070	1997.2	−1.838	2.070			
1990.1	0.294	2.070	1993.8	−0.309	2.070	1997.3	−0.477	2.070			
1990.2	−0.367	2.070	1993.9	−1.964	2.070	1997.4	0.272	2.070			
1990.3	−2.055	2.070	1993.10	−1.191	2.070	1997.5	4.011	2.070			
1990.4	−1.300	2.070	1993.11	0.031	2.070	1997.6	3.733	2.070			
1990.5	0.051	2.070	1993.12	0.223	2.070	1997.7	1.931	2.070			
1990.6	0.487	2.070	1994.1	1.099	2.070	1997.8	−0.927	2.070			
1990.7	0.126	2.070	1994.2	0.234	2.070	1997.9	3.353	2.070			
1990.8	−0.201	2.070	1994.3	−0.460	2.070	1997.10	−1.078	2.070			
1990.9	2.541	2.070	1994.4	−0.493	2.070	1997.11	−1.421	2.070			
1990.10	2.455	2.070	1994.5	−0.675	2.070	1997.12	−1.500	2.070			
1990.11	1.893	2.070	1994.6	−0.801	2.070	1998.1	−0.755	2.070			
1990.12	2.212	2.070	1994.7	−0.270	2.070	1998.2	0.078	2.070			
1991.1	2.868	2.070	1994.8	1.446	2.070	1998.3	−0.678	2.070			
1991.2	2.841	2.070	1994.9	1.069	2.070	1998.4	1.176	2.070			
1991.3	2.473	2.070	1994.10	−0.271	2.070	1998.5	1.665	2.070			
1991.4	2.881	2.070	1999.11	−1.266	2.070	1998.6	0.281	2.070			
1991.5	2.272	2.070	1995.12	−0.104	2.070						
1991.6	3.443	2.070	1995.1	0.733	2.070						
1991.7	3.061	2.070	1995.2	0.448	2.070						
1991.8	3.277	2.070	1995.3	−0.823	2.070						
1991.9	3.268	2.070	1995.4	−1.176	2.070						
1991.10	3.103	2.070	1995.5	−0.896	2.070						
1991.11	3.731	2.070	1995.6	−0.800	2.070						

[a]The residual variation documented in our paper could be attributable to regime changes in the determination of the price of gold futures contracts. If we acknowledge the possibility of structural changes, then a Markov regime switching model may well be the best way to investigate the time-series behavior of the gold futures price. A Markov-switching model allows the likelihood of a structural change to be determined by the data. Recall, we found $SPOT_{t-1}$, MCB_t, OIL_t, PPI_t and $SWISS_t$ to be statistically significant in the determination of gold futures prices and, in accordance with this analysis we model the Markov-switching regime for gold futures as

$$f_t = \alpha_1 + \beta_{11}\mathrm{Spot}_{t-1} + \beta_{12}\mathrm{MCB}_t + \beta_{13}\mathrm{Oil}_t + \beta_{14}\mathrm{PPI}_t + \beta_{15}\mathrm{SWISS}_t + \varepsilon_{1t} \qquad \text{if } s_t = 1$$

or

$$f_t = \alpha_2 + \beta_{21}\mathrm{Spot}_{t-1} + \beta_{22}\mathrm{MCB}_t + \beta_{23}\mathrm{Oil}_t + \beta_{24}\mathrm{PPI}_t + \beta_{25}\mathrm{SWISS}_t + \varepsilon_{2t} \qquad \text{if } s_t = 0$$

where s_t is an unobservable two state Markov chain with some transition probability matrix $\mathbf{P} = [\begin{smallmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{smallmatrix}]$. Note, the first column in $\mathbf{P}$ represents the probability of moving from the ith state to state one. These probabilities are unknown and endogenous, hence they are to be estimated. Consequently, given Hamilton's Markov-switching regression model ($y_t = \mathbf{z}_t'\beta_i + \varepsilon_t$), he shows the maximum likelihood estimates for the parameter coefficients and transition probabilities to be

TABLE IV (Continued)

Kalman Estimates of $\hat{B}_t$ and Their Standard Deviations; Gold Futures Market Bubbles[a] (1975.1–1998.6)

$$p_{ij} = \frac{\sum_{t=2}^{T} \mathbf{P}\{s_t = j, s_{t-1} = i|Y_T; \hat{\theta}\}}{\sum_{t=2}^{T} \mathbf{P}\{s_{t-1} = i|Y_T; \hat{\theta}\}} \tag{1}$$

$$\hat{\beta}_j = \left[\sum_{t=1}^{T} [\tilde{\mathbf{z}}_t(j)][\tilde{\mathbf{z}}_t(j)]'\right]^{-1} \left[\sum_{t=1}^{T} [\tilde{\mathbf{z}}_t(j)]\tilde{y}_t(j)\right] \tag{2}$$

$$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \sum_{j=1}^{N} (y_t - \mathbf{z}_t'\beta_j)^2 \cdot P\{s_t = j|Y_T; \theta\} \tag{3}$$

where

$$\tilde{y}_t(j) = y_t \cdot \sqrt{P\{s_t = j|Y_T; \theta\}} \tag{4}$$

$$\tilde{\mathbf{z}}_t(j) = \mathbf{z}_t \cdot \sqrt{P\{s_t = j|Y_T; \theta\}} \tag{5}$$

θ = denotes the full vector of maximum likelihood estimates.
y_t = is a left hand side endogenous variable. For our model, y_t represents futures prices $_{It}$.
$\mathbf{z}_t$ = is a matrix of right hand side exogenous variables that include for our model $SPOT_{t-1}$, MCB_t, OIL_t, PPI_t, $SWISS_t$.
$Y_T = (y_t', y_{t-1}', \ldots, y_{t-m}', \mathbf{z}_t', \mathbf{z}_{t-1}', \ldots, \mathbf{z}_{t-m}')'$ is a vector containing all observations through date t.

To start the estimation process, we obtain initial values for the parameters via ordinary least squares, while probabilities are initialized at 0.5. Starting with the initial guess for $\theta^{(0)}$, one calculates p_{ij} from (1). One then calculates the magnitudes on the right side of (2)–(5) with $\theta^{(0)}$ in place of $\hat{\theta}$. The left sides of (2)–(5) then produce new estimates $\theta^{(1)}$. These estimates $\theta^{(1)}$ are used to reevaluate (1) and recalculate the right sides of expressions (2)–(5). The left sides then produce another estimate $\theta^{(2)}$. The iteration process continues until the difference between $\theta^{(m+1)}$ and $\theta^{(m)}$ is smaller than the convergence criteria of .001.

We ran our empirical analysis in RATS (Regression Analysis of Time Series), and the estimation procedure took twenty-one iterations to converge. Given the slope estimates and the standard errors, we test for differences between the corresponding regime parameters. The t-Statistics ($t = \beta_{1i} - \beta_{2i}/\sqrt{s^2_{\beta 1i} + s^2_{\beta 2i} - 2\,\mathrm{cov}(\beta_{1i}, \beta_{2i})}$) for i = {$SPOT_{t-1}$, MCB_t, OIL_t, PPI_t, $SWISS_t$} are

$\beta_{spot1} - \beta_{spot2}$	0.9898
$\beta_{mcb1} - \beta_{mcb2}$	−0.5284
$\beta_{oil1} - \beta_{oil2}$	1.3745
$\beta_{ppi1} - \beta_{ppi2}$	−0.1028
$\beta_{swiss1} - \beta_{swiss2}$	−2.1037

According to the t-statistics above, the individual parameters for the right hand side variables $SPOT_{t-1}$, MCB_t, OIL_t, and PPI_t were not statistically distinguishable across states. The reduced form parameters for $SWISS_t$ were statistically different, however, the parameter estimate for $SWISS_t$ in the first regime was marginally significant (at only the 5% level). The second regime's estimate for the impact of $SWISS_t$ on gold futures was not significant at any reasonable significance level. In general, the parameter estimates were statistically indistinguishable across states, suggesting, gold futures follow only one regime. Thus, residual variation in gold futures could very well be attributable to a futures bubble but is certainly not an artifact of structural changes due to stochastic regime switching.

normally large negative values in 1975 and the first half of 1976. Although not statistically significant the $\hat{B}_t$s were about fifty percent larger than their standard errors. Domestically in 1975, the macro economy continued to suffer from the supply side shock of the 1973 OPEC oil embargo and endured record levels of unemployment. As the OPEC nations demonstrated their ability to cut off the supply of an indispensable good to the world, the global economy held its collective breath and the gold futures market was potentially impacted by speculative bubbles. Dynamic factor analysis suggests that speculative bubbles potentially affected gold futures prices between December 1979 and June 1980. This period corresponds to the events of the "Silver Crisis of 1980" and records $\hat{B}_t s$ that were approximately 1.8 times as large as their standard deviations. Not too surprisingly when the Hunt brothers cornered both the spot and the futures market for silver, the prices for gold futures contracts increased. Our algorithm intimates that part of the increase is attributable to the self-fulfilling prophecies of market participants.

In Table IV and Figure 1, the EM algorithm maintains that price bubbles for gold futures were uncharacteristically high from the end of 1981 to the beginning of 1983. This period is distinguished by an international banking scare. Third world debt at this time equaled $700 billion dollars—a large portion of which could not be paid on time. Many international banks were burdened with debt from developing nations, especially Mexico. Some of these banks were unable or simply refused to resume short-term lending to Mexico and other developing nations. The subsequent financial impasse generated a great deal of international economic uncertainty and created bubbles that were statistically significant at lower critical values (Fig. 1). Dynamic factor simulations produce gold futures bubbles that were negative and statistically significant immediately before Black Monday in October 1987.

Finally, our time series representation of $\hat{B}_t$ displays extraordinarily large positive values of $\hat{B}_t$ and then large negative values of $\hat{B}_t$ from Fall 1990 until the Winter 1993. The Iraqi war clearly coincides with the bubbles recorded in the early part of this time interval, whereas later the $\hat{B}_t$ s accompany stabilization problems in the European Economic Community. In 1992 German efforts to finance the rebuilding of East Germany with debt, increased interest rates, and attracted foreign capital. In the middle of recessions, other EEC members were forced with either allowing their own interest rates to rise or with devaluing their currencies relative to the Deutschmark. In the midst of this international economic uncertainty, large speculative gold futures bubbles were engendered.

A total of 288 observations are used in the estimation for gold future bubbles. At each point in time, all 288 occurrences, the Kalman filter generates an expected value for the unobserved bubble. Statistically testing for the existence of bubbles, at 5% α level, manifests very little support for the idea that bubbles are present in gold futures prices. However, relaxing the α level to 10%, we find greater incidences of statistically significant bubbles. In Figure 1, we see significant bubbles move in runs and are associated with notable socioeconomic events (i.e., Silver Crisis, Bank Scare, Black Monday, European Economic Crisis, Gulf War). Since these results hold only at lower levels of significance, we interpret them cautiously. But the association of significant bubbles with dramatic historical events, we feel adds to their credibility. The runs suggest that, although changes in the market fundamentals reflect the drama of the scenario at hand, market participant skepticism of previous structural relationships has potentially produced some trend chasing and, consequently, speculative bubbles.[22]

CONCLUSIONS

We have employed dynamic factor analysis to document price bubbles in the gold futures markets. In addition, we estimated the impact of the bubble's representation on the equilibrium futures price. First, market participants were introduced. Equilibrium prices for gold spot and futures markets were then determined, followed by a discussion of our state space model. Estimation results led to the conclusion that, at conventional levels of significance, our test statistics cannot establish the existence of rational speculative bubbles in the gold futures markets. However, if the stringency of the testing is marginally relaxed, episodes of statistically significant bubbles can be documented, and their occurrences are associated with notable historic events.

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[22]After convergence of the EM algorithm, Fisher's information matrix yielded estimates of the parameter variance-covariance matrix. Not only is this estimate useful in hypothesis testing but also in examining identifiability. Since local identifiability and asymptotic nonsingularity of the information matrix are equivalent, a nearly singular Fisher matrix indicates identification problems. In this paper, we tested for identification problems by converting the parameter variance–covariance matrix into a correlation matrix and then we examined the off diagonal elements. Interparameter correlations near unity, say ± 0.996 lead to a singular condition suggesting an overparameterized specification and a lack of complete identification. Our parameter correlations fell reassuringly short of ± 0.996 and lead us to conclude that we have no identification problems.

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