THE COST OF CARRY MODEL AND REGIME SHIFTS IN STOCK INDEX FUTURES MARKETS: AN EMPIRICAL INVESTIGATION

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This article examines empirically the dynamic relationship between spot and futures prices in stock index futures markets employing a class of nonlinear, regime-switching-vector-equilibrium-correction models, which is novel in this context. Using data for the S&P 500 and the FTSE 100 over the post-1987 crash period, it is shown that a long-run relationship between spot and futures prices exists, which implies mean reversion of the basis. After providing strong evidence against the hypothesis of linear dynamics in the relationship under investigation, regime-switching-vector-equilibrium-correction models for spot and futures price movements are developed and shown to capture well the time-series properties of our data, consistent with

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INTRODUCTION

Over the last two decades or so, a large body of both theoretical and empirical research focusing on the dynamic relationship between spot and futures prices in stock index futures markets has developed, initially stimulated largely by the early influential studies of, *inter alios*, Modest and Sundaresan (1983) and Figlewski (1984). In particular, a number of empirical studies have focused on the persistence of deviations from the Cost of Carry and have investigated the relationship between spot and futures prices in the context of vector autoregressions. These studies often were motivated based on the effects of arbitrage in futures markets along the lines of models of the type developed by Garbade and Silber (1983), where a continuum of traders induces movements in spot and futures prices such that the basis returns to a constant equilibrium level (e.g., see Chan, 1992; Kawaller, Koch, & Koch, 1987; Stoll & Whaley, 1990). However, the assumption of a continuum of traders acting in futures markets often has been debated; notably, in an attempt to shed light on the time-series properties of the basis, Miller, Muthuswamy, and Whaley (1994) provided convincing evidence that the S&P-500 basis change is mean reverting and argued that much of the mean reversion may be explained by infrequent trading in the cash market.

Nevertheless, from an empirical standpoint, the Cost of Carry model and variants of it predict that spot and futures prices cointegrate, and their long-run relationship is characterized by a long-run equilibrium that is essentially defined by the basis, hence implying both mean reversion of the basis and the existence of an equilibrium-correction model characterizing the dynamic relationship between spot and futures prices movements (Davidson, Hendry, Srba, & Yeo, 1978; Engle and Granger, 1987; Granger, 1986).[1] Also, an interesting strand of the literature recently has developed that allows for nonlinear mean reversion in the basis and suggests that the dynamic relationship between spot and futures prices may be characterized by a *nonlinear* equilibrium-correction model due to factors such as non-zero transactions costs, infrequent trading, or simply the existence of regime shifts in the dynamic adjustment of spot

[1]Hendry recently has started using the term "equilibrium correction" instead of the traditional "error correction" as the latter term now seems to have a different meaning in his theory of economic forecasting (e.g., see Clements & Hendry, 1998, p. 18). Since the term "equilibrium correction" conveys the idea of the adjustment considered in the present context quite well, this article has adhered to the new term below.

and futures price changes towards the long-run equilibrium level defined by the basis (e.g., see Akgiray, 1989; Dwyer, Locke, & Yu, 1996; Gao & Wang, 1999; Taylor, 1985; Yadav, Pope, & Paudyal, 1994; Yang & Brorsen, 1993, 1994; and the references therein).[2]

The present article contributes to this literature in that it reexamines the dynamic relationship between spot and futures prices using a very general nonlinear-equilibrium-correction approach that essentially is based on the extension of Markovian regime shifts to a nonstationary framework, whose underlying econometric theory recently has been developed significantly (for an overview, see Krolzig, 1997 and the references therein). In particular, using post-1987 crash data for the S&P 500 and FTSE 100, first, it was shown that, while a long-run equilibrium relationship between spot and futures prices consistent with mean reversion of the basis easily can be established, conventional linear-vector-equilibrium-correction models produced residuals that contained strongly significant nonlinearity and were easily rejected when tested against nonlinear, regime-switching-vector-equilibrium-correction models. Then, employing a Markov-switching vector equilibrium correction approach, the authors were able to characterize the dynamic relationship between spot and futures prices and easily outperform the alternative of linear equilibrium correction, ultimately obtaining models that are both statistically satisfactory and of natural interpretation.

MARKOV-SWITCHING REGIMES IN COINTEGRATED VECTOR AUTOREGRESSIONS AND VECTOR EQUILIBRIUM CORRECTION MODELS

In this section, the authors outline the econometric procedure employed in order to model regime shifts in the dynamic relationship between spot and futures prices in stock index futures markets. The procedure is essentially the result of extending the concept of Markovian regime shifts in time-series analysis proposed by Hamilton (1988, 1989) to nonstationary systems and applying it to cointegrated vector autoregressions and vector-equilibrium-correction models (for a rigorous account of the techniques discussed in this section, see Krolzig, 1997, 1999).

Consider the following M-regime p-th order Markov-switching vector autoregression (MS(M)-VAR(p)) that allows for regime shifts in the intercept term:[3]

[2]Other related studies of interest are due to, *inter alios*, Blank (1991), Fujihara and Mougoue (1997a,b), Parhizgari and de Boyrie (1997).

[3]Although this article focuses on eq. (1) since it makes sense in the present context, clearly a more general formulation of eq. (1) may be considered that allows for all parameters of the model to be conditioned on the state s_t.

$$y_t = \nu(s_t) + \sum_{i=1}^{p} \Gamma_i y_{t-i} + \varepsilon_t \quad (1)$$

where the K-dimensional observed time series vector $y_t = [y_{1t}, y_{2t}, \ldots, y_{Kt}]'$; the intercept terms vector $\nu(s_t) = [\nu_1(s_t), \nu_2(s_t), \ldots, \nu_K(s_t)]'$; the Γ_i's are $K \times K$ matrices of parameters; $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Kt}]'$ is a K-dimensional vector of Gaussian-white-noise processes with covariance matrix Σ, $\varepsilon_t \sim NID(0, \Sigma)$; $y_0, \ldots, y_{1-p}$ are fixed; the regime-generating process is assumed to be an ergodic Markov chain with a finite number of states $s_t \in \{1, \ldots, M\}$ governed by the transition probabilities $p_{ij} = \Pr(s_{t+1} = j \mid s_t = i)$, and $\Sigma_{j=1}^{M} p_{ij} = 1 \ \forall i, j \in \{1, \ldots, M\}$.[4]

A standard case in economics and finance is that y_t is nonstationary, but first-difference stationary, i.e., $y_t \sim I(1)$. Then, given $y_t \sim I(1)$, there may be up to $K - 1$ linearly independent cointegrating relationships that represent the long-run equilibrium of the system, and the equilibrium error (the deviation from the long-run equilibrium) is measured by the stationary stochastic process $u_t = \alpha' y_t - \beta$ where $\beta = E(\alpha' y_t)$, with E denoting the mathematical expectation operator (Engle & Granger, 1987; Granger, 1986). The cointegrated MS-VAR (1) implies a Markov-switching-vector-equilibrium-correction model (MS-VECM) of the form:

$$\Delta y_t = \nu(s_t) + \sum_{i=1}^{p-1} \Lambda_i \Delta y_{t-i} + \Pi \, y_{t-p} + \varepsilon_t, \quad (2)$$

where Δ denotes the first-difference operator, $\Lambda_i = (-\mathrm{I}_K - \Sigma_{j=1}^{i} \Gamma_i)$ for $i = 1, \ldots, p-1$ are matrices of parameters, and $\Pi = \mathbf{I}_K - \Sigma_{j=1}^{p} \Gamma_j = \Gamma(1)$ is the long-run impact matrix whose rank r determines the number of cointegrating vectors. The matrix Π in eq. (2) may be written as AB, where A and B' are $K \times r$ matrices of rank r and typically are termed cointegration matrix and loading matrix, respectively (e.g., Johansen, 1995; Krolzig, 1999).[5]

It is instructive to note that the intercept term in a VECM such as eq. (2) may be written as $\nu = -A\beta + \Lambda(1)\mu$ where $\mu = E(\Delta y_t)$ with β

[4]Precisely, s_t is assumed to follow an ergodic M-state Markov process with transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix},$$

where $p_{iM} = 1 - p_{i1} - \ldots - p_{i,M-1}$ for $i \in \{1, \ldots, M\}$.

[5]In this section, it is assumed that $0 < r < K$, implying that y_t is neither purely difference-stationary (i.e., $r = 0$) nor a stationary vector (i.e., $r = K$).

$= E(By_t)$ representing the long-run equilibrium of the model, given the cointegrating relation $u_t = \alpha' y_t - \beta$. Given that cointegration implies that the expected first difference of the system satisfies the restriction that $E(B\Delta y_t) = B\mu = 0$, μ reflects the common deterministic trends of the system. Then, one may consider various alternative specifications of the MS-VECM model eq. (2) depending on the case at hand, hence allowing, for example, for (i) unrestricted shifts of the intercept $\nu(s_t)$, (ii) shifts in the drift $\mu(s_t)$, (iii) shifts in the long-run equilibrium $\beta(s_t)$, or (iv) contemporaneous shifts in the drift $\mu(s_t)$ and in the long-run equilibrium $\beta(s_t)$.[6] Essentially each specification from (i) to (iv) is a multiple equilibria model, in the sense that the attractor of the system is defined by the equilibrium value of the cointegrating vector and the drift.

The present application focuses on a bivariate model comprising the futures price f and the spot price p (hence $y_t = [f_t, p_t]'$), for which conventional finance theory in the spirit of the Cost of Carry model implies that a unique cointegrating relationship exists with a cointegrating vector consistent with mean reversion (stationarity) of the basis. As discussed later on, after considerable experimentation, the authors selected a specification of the MS-VECM that captures the behavior implied by the Cost of Carry model, while allowing for regime shifts in the drift term $\mu(s_t)$, as well as the in the variance–covariance matrix Σ. The resulting model essentially may be seen as reflecting regime shifts in factors underlying the Cost of Carry model, such as interest rates or dividends, through changes in the intercept term. This model, termed MSMH-VECM, may be written as follows:

$$\Lambda(L)(\Delta y_t - \mu(s_t)) = A(By_{t-p} - \beta) + \varepsilon_t$$
$$\varepsilon_t \sim NID(0, \Sigma(s_t)) \; s_t = 1, \ldots, M. \tag{3}$$

An MS-VECM can be estimated using a two-stage-maximum-likelihood procedure. The first stage of this procedure essentially consists of the implementation of the Johansen (1988, 1991) maximum-likelihood cointegration procedure in order to test for the number of cointegrating relationships in the system and to estimate the cointegration matrix. In fact, in the first-stage, use of the conventional Johansen procedure is legitimate without explicitly modeling the Markovian regime shifts (see the theoretical results of Saikkonen, 1992; Saikkonen & Luukkonen, 1995). The second stage then consists of the implementation of an expectation-

[6]Note that (i) differs from (iv) in that (i) essentially captures complex dynamic adjustment after the transition from one state into another, while (iv) allows for a one-time jump of both the drift and the mean after a change in regime has occurred.

maximization algorithm for maximum-likelihood estimation, which yields estimates of the remaining parameters of the model (see Dempster, Laird, & Rubin, 1977; Hamilton, 1990; Krolzig, 1999).

DATA

The data set comprises weekly time series on futures written on the S&P 500 index and the FTSE 100 index, as well as price levels of the corresponding underlying cash indices. The data were obtained from *Datastream.* The time series of interest are the log of the futures price, f_t, and the log of the spot price, p_t, and the sample period examined spans from January 1, 1988 to December 26, 1997. Given the focus of the present article on investigating the importance of allowing for regime shifts in modeling the dynamic relationship between spot and futures prices in stock-index futures markets, the authors deliberately chose to use data after the 1987 crash in order to reduce the risk that the regime shifts identified in the empirical analysis could be determined by a unique and, perhaps, exceptional event occurring over the sample. Nevertheless, the data set under examination covers a ten-year period, which may be sufficiently long to capture some of the main features of the unknown stochastic process characterizing the dynamic relationship between spot and futures prices, while also providing a sufficiently large number of observations, $T = 522$, to be fairly confident of the estimation results.[7]

In Figure 1, the authors plot the graphs of each of the four time series under investigation, providing clear visual evidence in favor of the existence of trending behavior for each of the series. In fact, in addition to the existence of a stochastic trend (which will be formally detected below through unit root tests), each of the series also appears to contain an upward sloping deterministic trend.

EMPIRICAL RESULTS

Preliminary Data Analysis

Panel A of Table I provides some summary statistics of the basic time series used in the empirical analysis, namely f_t and p_t. As one would expect for both the S&P 500 and the FTSE 100, the first moment of the futures price is larger than the first moment of the spot price (although it is not the case that $f_t > p_t$ at each point in time), while the second moment of

[7]Obviously, the futures contract is paired up with the spot price comparing the spot price to the contract nearest to maturity.

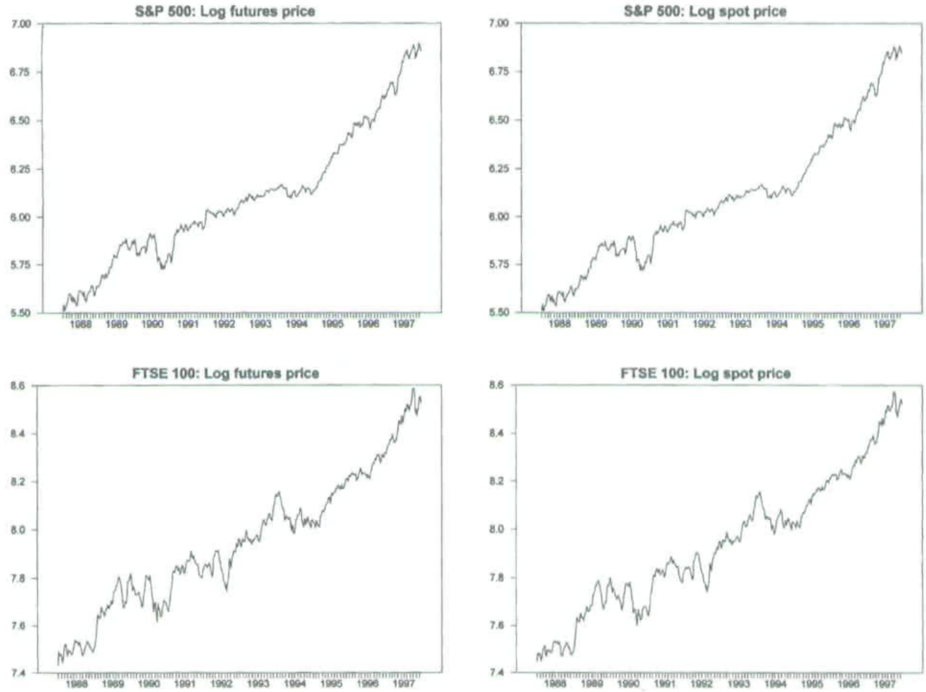

FIGURE 1
Data set.

the spot price is larger than the second moment of the futures price, suggesting that the futures price is larger on average and less volatile than the spot price. The partial autocorrelation functions, reported in Table I up to order 12, suggest that all four time series examined display very strong first-order serial correlation, while they do not appear to be significantly serially correlated at higher lags.

Considering that some of the papers in the relevant literature—such as, notably, Miller et al. (1994)—were engendered by considerations of a microstructural nature and focused on price and basis changes, the authors also reported the autocorrelation function (up to ten lags) of spot price changes, futures price changes, and basis ($b_t \equiv f_t - p_t$) changes for both indices used (Table I, Panel B). The results are not surprising in that changes in both spot and futures prices for both indices display significant autocorrelation, fairly strong, especially at lag 1.[8] Further, for both indices, changes in the basis display the well-documented negative autocor-

[8]Miller et al. (1994) report the same finding using intra-day data and attribute the significant autocorrelation to infrequent trading of some index stocks. In particular, they stress the importance of the fact that the lagged adjustment of prices of portfolio stocks to new market information induces positive autocorrelation in observed index level changes.

TABLE I

Preliminary Data Statistics

Panel A: Summary Statistics for f_t and p_t

	S&P 500: f_t	*S&P 500: p_t*	*FTSE 100: f_t*	*FTSE 100: p_t*
Minimum	5.5084	5.5081	7.4325	7.4458
Maximum	6.8977	6.8835	8.5864	8.5725
Mean	6.0969	6.0921	7.9467	7.9387
Variance	0.1171	0.1172	0.0764	0.0777
PACF:				
lag 1	0.9915	0.9917	0.9904	0.9912
lag 2	−0.0134	−0.0146	−0.0227	−0.0381
lag 3	−0.0388	−0.0368	−0.0182	−0.0202
lag 4	−0.0105	−0.0117	0.0158	0.0159
lag 5	0.0373	0.0309	0.0226	0.0183
lag 6	0.0070	0.0079	−0.0156	−0.0116
lag 7	0.0183	0.0145	0.0085	0.0066
lag 8	−0.0176	−0.0172	−0.0069	−0.0091
lag 9	0.0323	0.0314	0.0285	0.0207
lag 10	−0.0466	−0.0443	−0.0306	−0.0335
lag 11	−0.0067	−0.0110	−0.0147	−0.0193
lag 12	−0.0117	−0.0092	0.0173	0.0148

Panel B: Autocorrelation Functions for Δf_t, Δp_t, and Δb_t

	S&P 500			*FTSE 100*		
	Δf_t	Δp_t	Δb_t	Δf_t	Δp_t	Δb_t
lag 1	0.0980	0.0980	−0.0798	0.1538	0.2010	−0.0516
lag 2	0.0393	0.0425	−0.0540	4.73E-5	0.0452	−0.0762
lag 3	−0.0501	−0.0377	−0.0599	−0.0246	−0.0111	−0.1277
lag 4	−0.0383	−0.0537	−0.0647	−0.0019	−0.0002	−0.0603
lag 5	−0.1280	−0.1247	−0.1051	−0.0156	−0.0218	−0.0539
lag 6	0.0051	0.0239	−0.1087	−0.0599	−0.0587	−0.0621
lag 7	0.0336	0.0433	−0.1103	−0.0488	−0.0499	−0.0586
lag 8	−0.0458	−0.0404	−0.1140	−0.0524	−0.0564	−0.0714
lag 9	0.0688	0.0542	−0.0622	−0.0263	−0.0427	−0.0448
lag 10	−0.0188	−0.0046	−0.0813	−0.0692	−0.0440	−0.1008

Notes: f_t and p_t denote the log level of the futures price and the log level of the spot price, respectively; $b_t \equiv f_t - p_t$ denotes the basis; Δ is the first difference operator. PACF is the partial autocorrelation function, and its standard deviation can be approximated by the square root of the reciprocal of the number of observations, $T = 522$, hence being about 0.0438.

relation. As suggested by Miller et al. (1994), the negative autocorrelation of the basis has important implications for microstructural finance, and—at least at high frequency—presumably may be generated by the fact that index stocks do not trade continuously. Clearly, the nature of mean reversion is complex and depends upon a number of factors; however, mean reversion not only is expected, but also becomes more severe as trading frequency increases—see Miller et al. (1994).

TABLE II

Unit Root Tests

Futures Prices	f_t	$f_t^{(\tau)}$	Δf_t	$\Delta^2 f_t$
S&P 500	−0.5239	−1.5353	−14.6855	−25.2663
FTSE 100	−0.3332	−2.2220	−15.0592	−25.5788
Spot Prices	p_t	$p_t^{(\tau)}$	Δp_t	$\Delta^2 p_t$
S&P 500	−0.5577	−1.5204	−14.6468	−25.4636
FTSE 100	−0.2590	−2.2065	−14.2267	−24.8368

Notes: Statistics are augmented Dickey–Fuller test statistics for the null hypothesis of a unit root process; f_t and p_t denote the log level of the futures price and the log level of the spot price, respectively; the (τ) superscript indicates that a linear trend also was included in the augmented Dickey–Fuller regression; Δ is the first-difference operator. The critical value at the 5% level of significance is −2.86 to two decimal places if no linear trend is included in the regression, and −3.41 if a linear trend also is included in the regression (Fuller, 1976; MacKinnon, 1991).

As a preliminary exercise, the authors then tested for unit root behavior of each of the (log) futures price and spot-price time series by calculating standard augmented Dickey–Fuller test statistics (Table II). In each case, the number of lags was chosen such that no residual autocorrelation was evident in the auxiliary regressions.[9] In keeping with the very large number of studies of unit root behavior for these time series and conventional finance theory, the authors, in each case, were unable to reject the unit root-null hypothesis at conventional nominal levels of significance. On the other hand, differencing the series did appear to induce stationarity in each case. Overall, the unit root tests clearly indicate that both f_t and p_t are realizations from stochastic processes integrated of order one, which suggests that testing for cointegration between f_t and p_t is the logical next step.[10]

Cointegration Results

The implementation of the Johansen procedure is essentially the first stage of the two-stage procedure designed to estimate a bivariate MS-

[9]Moreover, using nonaugmented Dickey–Fuller tests or augmented Dickey–Fuller tests with any number of lags in the range between 1 and 12 yielded results qualitatively identical to those reported in Table II. Also, note that a deterministic trend was found to be statistically significantly different from zero at conventional nominal levels of significance in the auxiliary regressions, confirming the suspect in favor of the existence of a deterministic trend generated by Figure 1.

[10]In addition to the ADF tests, the authors also executed unit root tests of the type proposed by Phillips and Perron (1988), as well as Johansen likelihood ratio tests (Johansen, 1988, 1991) in a vector autoregression with one series. The results were perfectly consistent with the ADF tests results, indicating that both f_t and p_t are I(1) (results not reported but available from the authors upon request).

VECM for Δf_t and Δp_t, as has already been discussed in brief. The authors employed the well-known Johansen (1988, 1991) maximum-likelihood procedure in a VAR, allowing for a lag length of five and an unrestricted constant term, hence testing for cointegration in the long-run model:[11]

$$p_t = \omega + \phi f_t + e_t. \tag{4}$$

Both Johansen likelihood-ratio (LR) test statistics (based on the maximum eigenvalue and on the trace of the stochastic matrix, respectively) clearly suggested that a cointegrating relationship existed. Also, the restriction suggested by conventional finance theory in the spirit of the Cost of Carry model that the cointegrating parameter associated to f_t equals unity, $\phi = 1$ could not be rejected at conventional nominal levels of significance for each of the two VARs. In fact, estimation of the VAR with ϕ left unrestricted produced estimated values of ϕ equal to 1.0008 for the S&P 500 and equal to 1.0012 for the FTSE 100. Then, estimation of the VAR with the imposition of the restriction $\phi = 1$ produced the results given in Table III, suggesting that a unique cointegrating relationship exists between f_t and p_t for both the S&P 500 (Panel A) and the FTSE 100 (Panel B).[12] Given the restriction $\phi = 1$, the cointegrating residuals retrieved from the cointegrated VAR may be seen as the estimated deviation of the basis from its mean, and $\hat{\omega}$ may be interpreted as the long-run equilibrium of the relationship between f_t and p_t over the sample period as implied by the Cost of Carry model inspiring eq. (4).[13,14]

[11]The authors were very careful in selecting the number of lags in the VAR, being aware of the sensitivity of vector autoregression analysis to the lag length in this context. Both the Akaike Information Criterion (AIC) and the Schwartz Information Criterion (SIC) suggested a lag length of five for both systems. This adds confidence to the results, especially in the light of the Monte Carlo study by Cheung and Lai (1993), who showed that, for autoregressive processes with no moving average dependencies, the AIC and the SIC indicate the correct lag order of a VAR used for testing for cointegration in 99.86 and 99.96% of cases, respectively. Furthermore, in the present application, the results were found to be very robust to the choice of the lag length. Also, note that the VAR considered is essentially model $H_1^*(r)$ in Johansen (1995, p. 83) notation, where a linear deterministic trend is implicitly allowed for but this can be eliminated by the cointegrating relations, and the process contains no trend stationary components; hence, the model allows for a linear trend in each variable but not in the cointegrating relations.

[12]LR tests of the hypothesis $\phi = 1$ could not be rejected with p-values equal to 0.616 and 0.603 for the S&P 500 and the FTSE 100, respectively.

[13]Obviously, testing for cointegration in eq. (4) under $\phi = 1$ is tantamount to testing for nonstationarity of the basis.

[14]Note that the finding of cointegration between the spot and futures price may be expected, but it is not a trivial result. Several authors have not been able to detect cointegration between the spot and the futures price or mean reversion in the basis. Notably, Miller et al. (1994) found that the S&P 500 index basis is nonstationary, implying that the spot and the futures price do not cointegrate. Indeed, their study focused on mean reversion in basis *changes* and was motivated largely by considerations of microstructural nature, while the present study and its associated econometric techniques are more concerned with basis *levels*. Nevertheless, it is clear that microstructure effects and Cost of Carry effects can co-exist in a legitimate fashion.

TABLE III

Johansen Maximum-Likelihood Cointegration Procedure

Panel A: S&P 500			
LR Tests Based on the Maximum Eigenvalue of the Stochastic Matrix			
H_0	H_1	*LR*	*5% Critical Value*
$r = 0$	$r = 1$	56.60	14.06
$r \leq 1$	$r = 2$	0.27	3.84
LR Tests Based on the Trace of the Stochastic Matrix			
H_0	H_1	*LR*	*5% critical value*
$r = 0$	$r \geq 1$	56.87	15.41
$r \leq 1$	$r = 2$	0.27	3.84
Panel B: FTSE 100			
LR Tests Based on the Maximum Eigenvalue of the Stochastic Matrix			
H_0	H_1	*LR*	*5% critical value*
$r = 0$	$r = 1$	60.47	14.06
$r \leq 1$	$r = 2$	0.02	3.84
LR Tests Based on the Trace of the Stochastic Matrix			
H_0	H_1	*LR*	*5% Critical Value*
$r = 0$	$r \geq 1$	60.49	15.41
$r \leq 1$	$r = 2$	0.02	3.84

Notes: The model being tested for cointegration is eq. (4). H_0 and H_1 denote the null hypothesis and the alternative hypothesis, respectively; r denotes the number of cointegrating vectors, and the 5% critical values reported in the last column are taken from Osterwald-Lenum (1992).

Linearity Tests and MS-VECM Estimation Results

Preliminary to estimating a MS-VECM, the authors estimated a standard linear bivariate VECM for Δf_t and Δp_t, which is implied by the finding of cointegration between f_t and p_t reported in the previous sub-section (Engle & Granger, 1987; Granger, 1986). Thus, using full-information maximum-likelihood (FIML) methods, the authors estimated a bivariate VECM of the form

$$\Delta y_t = v + \sum_{i=1}^{p-1} \Lambda_i \Delta y_{t-i} + \Pi y_{t-p} + u_t \tag{5}$$

where $y_t = \{f_t, p_t\}'$, assuming a lag length of five as suggested by both the AIC and the SIC. Employing the conventional general-to-specific procedure, the authors obtained fairly parsimonious models for each of Δf_t and Δp_t and for both the S&P 500 and the FTSE 100, which displayed high coefficients of determination (about 0.9 in each case) and no residual serial correlation.[15] As a check of adequateness of the models, as well as an additional motivation for the need of employing a nonlinear process in modeling the dynamic relationship between spot price and futures prices, however, the authors employed two fairly general tests for linearity of the residuals from the VECM eq. (5). First, the authors employed a RESET (Ramsey, 1969) test of the null hypothesis of linearity of the residuals from the linear VECM eq. (5) against the alternative hypothesis of general model misspecification. Under the RESET test statistic, the alternative model involves a higher-order polynomial to represent a different functional form; under the null hypothesis, the statistic is distributed as $\chi^2(q)$ with q equal to the number of higher-order terms in the alternative model. Second, the authors employed the test statistic developed by Brock, Dechert, and Sheinkman (BDS; 1991) for testing the null hypothesis that the residuals from eq. (5) are independent and identically distributed (iid) against an unspecified alternative. The BDS test for a series u_t is calculated in the following way. Let $\mathbf{u}_{t,v}$ be a set of consecutive terms from u_t: $\mathbf{u}_{t,v}$ $\{u_t, u_{t+1}, \ldots, u_{t+v-1}\}$. The pair of vectors $\mathbf{u}_{t,v}$ and $\mathbf{u}_{s,v}$ are said to be no more than ζ apart if $|u_{t+j} - u_{s+j}| \leq \zeta$ for $j = 0, 1, \ldots, v - 1$. Thus, the correlation integral $C_v(\zeta)$ is defined as the product of the limit of T^{-2} (T being the number of observations) times the number of ζ-close pairs (s, t), essentially measuring the probability that the pairs of points (s, t) are within ζ of each other. The BDS statistic then is constructed as follows

$$S(v, \zeta) = \hat{C}_v(\zeta) - [\hat{C}_1(\zeta)]^v \tag{6}$$

for some v and ζ. Under the null hypothesis that u_t is iid, $\sqrt{T}[S(v, \zeta)] \sim N(0, \xi)$, where the variance ξ is a function of v and ζ. Rejection of the null implies that some form of nonlinearity is present in u_t, although the type of nonlinearity cannot be determined exactly under the BDS test. Brock et al. (1991) suggested that the choice of v, and particularly of ζ, are crucial for the power of the test, which is reasonably powerful only in large samples. Brock et al. (1991) also suggested values of ζ between 0.5 and 1.5 times the standard deviation of u_t, whereas the value of v should preferably be such that $(T/v) > 200$.[16]

Panel A of Table IV reports the results from executing RESET test statistics, where, in the alternative model, we included a quadratic and a

[15]Full details on these estimation results are available from the authors upon request, but are not reported so as to conserve space, and because this paper mainly is interested in linearity tests on the VECM residuals.

[16]Note that it is implicitly assumed that u_t is asymptotically normal.

TABLE IV

Linearity Tests on the Residuals from the VECM eq. (5)

Panel A: RESET Tests: p-Values

	S&P 500	FTSE 100
Futures equation	2.48E-08	4.31E-08
Spot equation	3.36E-08	5.72E-08

Panel B: BDS Tests: p-Values

	$v = 2$			$v = 3$		
	$\varsigma = 0.5$	$\varsigma = 1.0$	$\varsigma = 1.5$	$\varsigma = 0.5$	$\varsigma = 1.0$	$\varsigma = 1.5$
S&P 500						
futures equation	0.0001	0.0036	0.0124	0.0004	0.0059	0.0132
spot equation	0.0008	0.0079	0.0150	0.0011	0.0081	0.0196
FTSE 100						
futures equation	0.0005	0.0057	0.0293	0.0007	0.0064	0.0349
spot equation	0.0012	0.0212	0.0373	0.0040	0.0023	0.0391

Notes: Panel A: RESET test statistics were computed considering an alternative model with a quadratic and a cubic term; in this context, therefore, the RESET test statistics were distributed as χ^2 (2) under the null hypothesis of linearity. *Panel B:* The BDS test statistic tests the null hypothesis that a series is iid against the alternative of a realization from an unspecified nonlinear process, as described in the text. The critical values, from the normal distribution, are 1.960 and 2.576 at the 5 and 1% nominal levels of significance, respectively. For both RESET tests and BDS tests, only *p*-values are reported.

cubic term; the null hypothesis is rejected very strongly for all countries considered with *p*-values of virtually zero. The authors then also executed BDS tests on the estimated u_t for values of $v = 2$, 3 and $\zeta = 0.5$, 1.0, 1.5; the results, reported in Panel B of Table IV, provide additional strong empirical evidence that the residuals from the linear VECM eq. (5) contain nonlinearities. Clearly, RESET and BDS tests are general linearity tests. Nevertheless, other linearity tests proposed by Granger and Teräsvirta (1993, Chapter 6) also might be applied, and rejection of the hypothesis of linearity may be taken as evidence in favor of Markov-switching models, although these tests have power against several nonlinear alternatives. To the best of the authors' knowledge, there is no test of linearity that has a Markov-switching model as an alternative that does not require specification and estimation of the alternative model. Also, there seems to be no descriptive tool to detect reliably Markovian shifts (see Tjostem, 1990). However, the evidence of nonlinearity from the residuals of the linear VECM, coupled with the likelihood ratio tests of linear VECM against MS-VECM reported in Table VI and the satisfactory goodness of fit shown by the MS-VECM proposed below, suggest adequateness of the MS-VECM in this context.[17]

[17]One additional test executed is the 'third moment' test discussed in *Brock, Hsieh and LeBaron*

TABLE V

Likelihood Ratio Tests for No-Regime Dependence

	$LR1 \sim \chi^2(12)$	$LR2 \sim X^2(6)$
S&P 500	102.3 {0.0000}	72.68 {0.0000}
FTSE 100	83.82 {0.0000}	16.82 {0.0106}

Notes: $LR1$ and $LR2$ are test statistics of the null hypothesis of no-regime-dependent residual-variance–covariance matrix and no-regime-dependent mean, respectively. They are constructed as $2(lnL^* - lnL)$, where L^* and L denote the unconstrained likelihood and the constrained likelihood, respectively, and have an asymptotic distribution of $\chi^2(R)$, where $R = (M - 1)K(K + 1)/2$, with M being the number of regimes and K the number of equations. Figures in braces denote p-values.

TABLE VI

MSMH(3)-VECM(1) Estimation Results

Panel A: S&P 500

$$\tilde{\Lambda}_1 = \begin{bmatrix} 0.9137 & 0.8607 \\ (0.2320) & (0.1739) \\ 1.1070 & 1.0447 \\ (0.2837) & (0.2245) \end{bmatrix}$$

$$\tilde{\mu}_f = \begin{bmatrix} 5.4020E\text{-}04 \\ (1.3286E\text{-}04) \\ 1.8058E\text{-}03 \\ (0.6390E\text{-}03) \\ 4.8616E\text{-}03 \\ (1.3057E\text{-}03) \end{bmatrix}; \tilde{\mu}_p = \begin{bmatrix} 2.9398E\text{-}04 \\ (0.8322E\text{-}04) \\ 2.0246E\text{-}03 \\ (0.8361E\text{-}03) \\ 2.8651E\text{-}03 \\ (1.2035E\text{-}03) \end{bmatrix}; \tilde{A} = \begin{bmatrix} 0.2841 \\ (0.0830) \\ -0.2236 \\ (0.0569) \end{bmatrix}$$

$$\tilde{\Sigma}(s_1) = \begin{bmatrix} 6.2765E\text{-}05 & 6.1708E\text{-}05 \\ 6.1708E\text{-}05 & 6.0938E\text{-}05 \end{bmatrix};$$

$$\tilde{\Sigma}(s_2) = \begin{bmatrix} 1.4586E\text{-}05 & 1.4553E\text{-}05 \\ 1.4553E\text{-}05 & 1.4624E\text{-}05 \end{bmatrix};$$

$$\tilde{\Sigma}(s_3) = \begin{bmatrix} 3.8204E\text{-}05 & 3.1955E\text{-}05 \\ 3.1955E\text{-}05 & 2.8432E\text{-}05 \end{bmatrix}$$

$$\tilde{P} = \begin{bmatrix} 0.7870 & 0.0765 & 0.4507 \\ 0.1279 & 0.8393 & 0.1709 \\ 0.0852 & 0.0842 & 0.3784 \end{bmatrix}; \tilde{\Xi} = \begin{bmatrix} 0.4191 \\ 0.4609 \\ 0.1199 \end{bmatrix}$$

LR linearity test: 496.06 {0.0000}

	$\bar{R}^2$	$ARCH(1)$	$ARCH(4)$	$RESET$
Futures equation	0.9995	{0.2483}	{0.5810}	{0.2912}
Spot equation	0.9995	{0.2742}	{0.3841}	{0.6532}

TABLE VI (Continued)

MSMH(3)-VECM(1) Estimation Results

Panel B: FTSE 100

$$\tilde{\Lambda}_1 = \begin{bmatrix} 0.0179 & 0.1384 \\ (0.0052) & (0.0439) \\ 1.1080 & 0.0651 \\ (0.0432) & (0.0129) \end{bmatrix}$$

$$\tilde{\mu}_f = \begin{bmatrix} 4.5718E\text{-}04 \\ (1.3006E\text{-}04) \\ 5.2547E\text{-}04 \\ (1.2930E\text{-}04) \\ 4.0194E\text{-}03 \\ (1.2253E\text{-}03) \end{bmatrix}; \tilde{\mu}_p = \begin{bmatrix} 5.9265E\text{-}04 \\ (1.3827E\text{-}04) \\ 9.8900E\text{-}04 \\ (3.3271E\text{-}04) \\ 1.2411E\text{-}03 \\ (0.4052E\text{-}03) \end{bmatrix}; \tilde{A} = \begin{bmatrix} 0.1646 \\ (0.0458) \\ -0.1055 \\ (0.0305) \end{bmatrix}$$

$$\tilde{\Sigma}(s_1) = \begin{bmatrix} 6.7722E\text{-}05 & 6.2083E\text{-}05 \\ 6.2083E\text{-}05 & 5.8329E\text{-}05 \end{bmatrix};$$

$$\tilde{\Sigma}(s_2) = \begin{bmatrix} 3.6154E\text{-}05 & 3.3271E\text{-}05 \\ 3.3271E\text{-}05 & 3.0936E\text{-}05 \end{bmatrix};$$

$$\tilde{\Sigma}(s_3) = \begin{bmatrix} 8.3846E\text{-}05 & 6.2631E\text{-}05 \\ 6.2631E\text{-}05 & 5.1494E\text{-}05 \end{bmatrix}$$

$$\tilde{P} = \begin{bmatrix} 0.7790 & 0.0481 & 0.6691 \\ 0.1558 & 0.8534 & 0.0044 \\ 0.0652 & 0.0985 & 0.2867 \end{bmatrix}; \tilde{\Xi} = \begin{bmatrix} 0.4190 \\ 0.4768 \\ 0.1041 \end{bmatrix}$$

LR linearity test: 291.85 {0.0000}

	$\bar{R}^2$	*ARCH(1)*	*ARCH(4)*	*RESET*
Futures equation	0.9994	{0.2328}	{0.4381}	{0.5602}
Spot equation	0.9995	{0.2920}	{0.7360}	{0.3653}

Notes: Tildes denote estimated values obtained using the expectation-maximization algorithm for maximum likelihood discussed in the text, while figures in parentheses are estimated standard errors; μ_f and μ_p refer to the *M*-dimensional vector μ for the futures price equation and the spot price equation, respectively; *P* and Ξ denote the $M \times M$ transition matrix and the *K*-dimensional ergodic probabilities vector, respectively. The LR linearity test tests the hypothesis that the true model is a linear VECM against the alternative of MSMH-VECM, and is distributed as $\chi^2(q)$ with *q* equal to the number of restrictions under which the two models are identical. $\bar{R}^2$ is the adjusted coefficient of determination, calculated as discussed in the text (see footnote 17). ARCH(j) is a test statistic for autoregressive conditional heteroskedasticity up to order j (Engle, 1982), and RESET is a second-order RESET test constructed as discussed in the Notes to Table IV. Figures in braces denote *p*-values.

Next, the authors applied the conventional "bottom-up" procedure designed to detect Markovian shifts in order to select the most adequate

(1991, Chapter 4, pp. 134–139). This test is a powerful test for detecting nonlinearity in mean and has no power in rejecting the linearity hypothesis if the series in question is only nonlinear in variance. As one would expect, using the third moment test, the authors could reject strongly the linearity hypothesis applied to the estimated u_t for both the S&P 500 and the FTSE 100, suggesting the characterization of a M-regime p-th order MS-VECM for Δy_t of the form previously discussed.[18] However, for any MS-VECM estimated, including the MS(3)-VECM(1) selected by the bottom-up procedure for both the S&P 500 and the FTSE 100, the implicit assumption that the regime shifts affect only the drift term of the VECM was found to be inappropriate. In fact, the authors checked the relevance of conditional heteroskedasticity by estimating a MS-VECM, where the Gaussian innovation is allowed to be heteroskedastic, $\varepsilon_t \sim NID(0, \Sigma(s_t))$. The authors then tested the hypothesis of no-regime dependence in the variance–covariance matrix using a LR test ($LR1$) of the type suggested by Krolzig (1997, p. 136), in addition to constructing a LR test for the null hypothesis of no-regime dependent $\mu(LR2)$. The results, reported in Table V, indicate very strong rejection of the null of no-regime dependence under both $LR1$ and $LR2$, clearly suggesting a MS-VECM that allows for shifts in both μ and Σ, MSMH(3)-VECM(1) is the most appropriate model within its class in the present application.

Thus, the authors estimated a bivariate MSMH(3)-VECM(1) for Δy_t of the form

$$(\Delta y_t - \mu(s_t)) = A(By_{t-1} - \beta) + \Lambda_1(\Delta y_{t-1} - \mu(s_t)) + \varepsilon_t$$
$$\varepsilon_t \sim NID(0, \Sigma(s_t)) \quad s_t = 1, 2, 3 \tag{7}$$

using the expectation-maximization algorithm for maximum-likelihood estimation previously mentioned.[19] The results, reported in Table VI, are very encouraging on a number of fronts. The estimation yields plausible estimates of all coefficients for both VECMs estimated, and all coeffi-

existence of nonlinearity in mean (results not reported but are available upon request). However, since the third moment test also has power against 'hybrid' alternative hypotheses where nonlinearity enters both mean and variance, this finding does not discard the possibility of nonlinearity both in mean and variance in the true, unknown data-generating process. Further theoretical research is warranted on designing linearity tests that may enable the identification of specific nonlinear stochastic models.

[18]Essentially the bottom-up procedure consists of starting with a simple, but statistically reliable, Markov-switching model by restricting the effects of regime shifts on a limited number of parameters and check the model against alternatives. In such a procedure, most of the structure contained in the data is not attributed to regime shifts, but explained by observable variables, therefore being consistent with the general-to-specific approach to econometric modeling. For a comprehensive discussion of the bottom-up procedure, see Krolzig (1997).

[19]One may argue that the lag length of unity that was selected by the bottom-up procedure may be too low, given that the authors used a lag length of five in estimating the linear VECM. Nevertheless, estimation of a MSMH-VECM with higher-order lags became computationally more demanding with several autoregressive parameters found insignificant, but did not change qualitatively any of the results reported below. Further, given that the diagnostic tests reported in Table VI and the graphs of the residuals from the MSMH(3)-VECM(1) did not indicate either misspecification or residual serial correlation, and that the coefficients of determination were found to be very high, the authors decided to stick to the more parsimonious models.

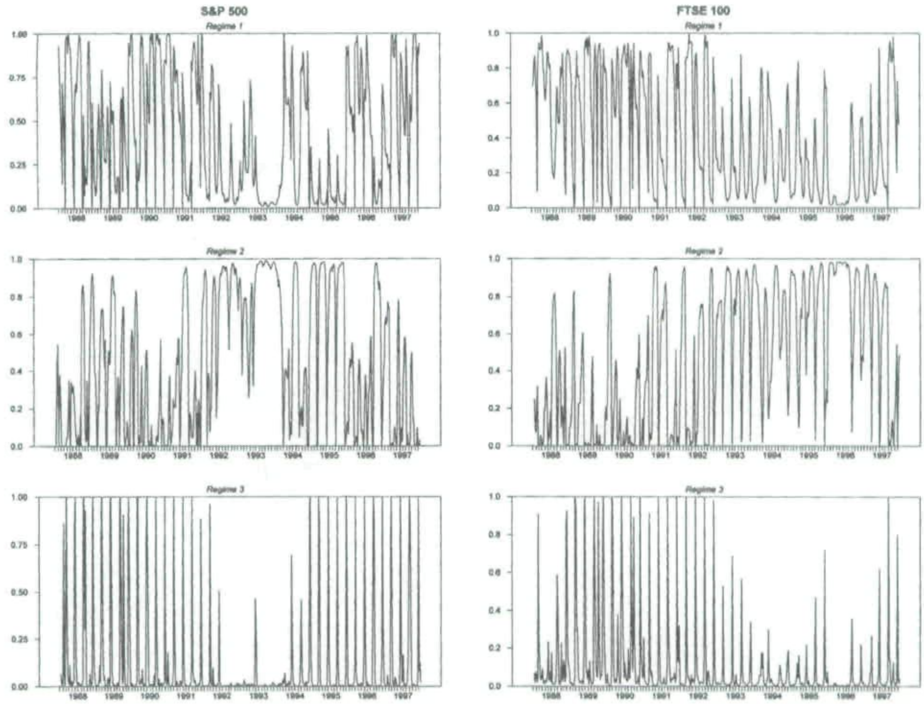

FIGURE 2
MSH-VECM smoothed transition probabilities.

cients, including the adjustment coefficients in A, were found to be strongly statistically significantly different from zero. Also, while the values of $\mu(s_t)$ are relatively small in absolute size, the impact effect of the regime shifts is rather heavy, especially in the futures price equation for the S&P 500 and in the spot price equation for the FTSE 100. The impact effect of regime shifts also appears to be heavy on the variance–covariance matrix $\Sigma(s_t)$. Furthermore, the authors computed a LR test statistic for linearity, which essentially tests the hypothesis that the true model is a linear VECM against the alternative of the MSMH-VECM reported in Table VI. Even by invoking the upper bound of Davies (1977), the linearity hypothesis is rejected extremely strongly, with a p-value of virtually zero, providing convincing evidence of the need of employing a regime-switching process that allows for shifts in μ and Σ in the econometric modeling of the data under examination.

In Figure 2, the authors also report the smoothed transition transition probabilities: for both the S&P 500 and the FTSE 100, Regimes 1 and 2 seem to characterize a very large fraction of the movements of the spot price and the futures price, with each regime being somewhat persistent but with a rather large number of switches over the sample; one obvious interpretation of these changes in the intercept term suggested by the model is that they may reflect regime shifts in factors underlying the Cost of Carry model, such as interest rates or dividends. Regime 3 is much less persistent and is likely to pick up outliers that do not fall within either Regime 1 or Regime 2, being relatively more important in the first half of the sample.

In Figure 3, the authors then graph the standardized residuals, the smoothed residuals, and the one-step prediction errors from the estimated MSMH-VECM. The graphs provide strong visual evidence of no residual serial correlation in any of the residuals series plotted.[20] Having obtained the prediction errors, the authors could then compute a coefficient of determination ($\bar{R}^2$), which was adjusted both for the bias towards preferring a larger model relative to a smaller one, as well as for the fact that the model allows for regime-dependent heteroskedasticity, for each equation of the MSMH-VECM, reported in Table VI; the $\bar{R}^2$ is comfortingly very high in each case.[21] Further, as a final check of the adequateness of their model, the authors calculated some test statistics for autoregressive conditional heteroskedasticity (Engle, 1982) and some second-order RESET tests on the residuals series, also reported in Table VI. In each case, these diagnostics tests were found to be statistically insignificantly different from zero at conventional significance levels, providing further evidence that the MSMH-VECM model proposed here captures fairly well the nonlinearity present in the data.[22,23]

[20]The difference is concerned with the weighting of the residuals. Loosely speaking, the smoothed residuals are the closest to the sample residuals from a linear regression model; however, they overestimate the explanatory power of the Markov-switching model due to the use of the full-sample information covered in the smoothed-regime vector. The standardized residuals are conditional residuals. The one-step prediction errors are based on the predicted regime probabilities. Unfortunately, many conventional diagnostic tests, such as standard residual serial correlation tests, may not have their conventional asymptotic distribution when the residuals come from Markov-switching models and therefore are not reported here. See, for example, Krolzig (1997) for a discussion of residual based model checking in this context.

[21]In this case, the $\bar{R}^2$ is equal to

$$\bar{R}^2 = 1 - \frac{T - 1}{T - M(M - 1 + K + K(K + 1)/2) - Kp - 1}(1 - R^2)$$

where K_p is the number of regressors in each equation, and $R^2 = 1 - \sigma_e^2/\sigma_{dep}^2$ with σ_e^2 and σ_{dep}^2 being the sample variance of the estimated prediction errors and of the dependent variable, respectively.

[22]The authors also calculated BDS test statistics on the MSMH-VECM residuals of the type reported in Table IV, in each case recording statistically insignificant statistics.

[23]Although the empirical analysis has been based upon examination of post-1987 data in order to convince the reader that the presence of nonlinearity in the dynamic relationship under investigation is genuine rather generated by the 1987 crash, to investigate the robustness of the model proposed, the authors also examined data for a longer sample from 1984 to 1997, as well as the sample using data from 1984 up to the 1987 crash. The results from using data for the whole sample from 1984

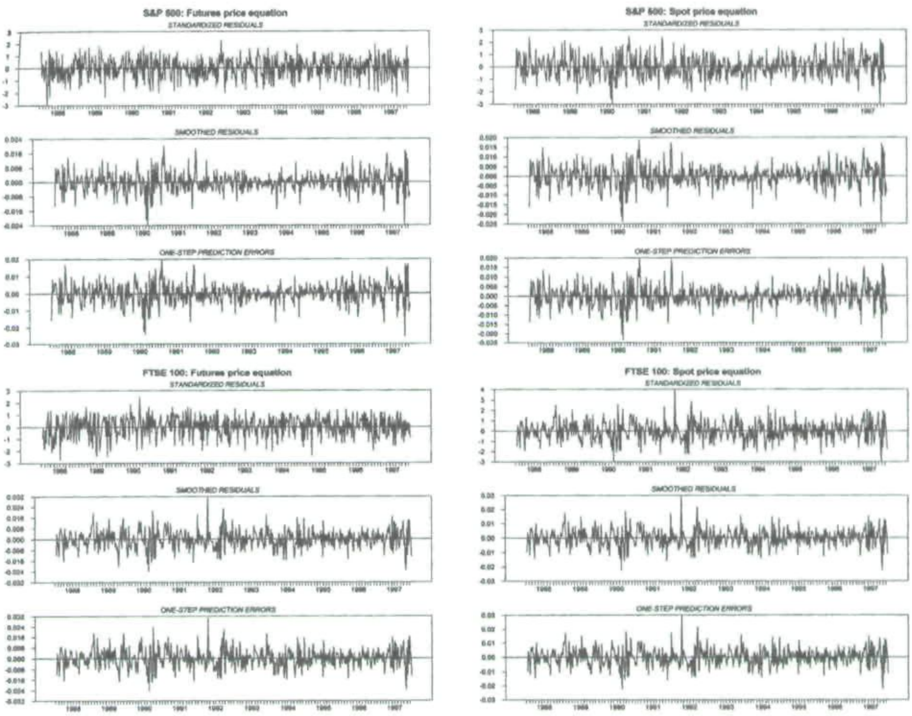

FIGURE 3
MSH-VECM estimated errors.

CONCLUSIONS

This article has re-examined the dynamic relationship between spot and futures prices in stock index futures markets using post-1987 crash data at weekly frequency for the S&P 500 and the FTSE 100 indices, and employing nonlinear, Markov-switching vector equilibrium correction models. The empirical results provide strong evidence in favor of a cointegrating relationship between spot and futures prices with a cointegrating vector, which implies mean reversion of the basis and is consistent with the predictions of the Cost of Carry model. Nevertheless, a linear-vector-equilibrium-correction model was rejected strongly when tested against a Markov-switching-vector-equilibrium-correction model that allowed for three regimes in the mean of the equilibrium correction model, as well as in the variance–covariance matrix.

to 1997 suggested massive evidence in favor of nonlinearity, and the MSMH(3)-VECM(1) was found to fit the data well, although one regime was essentially dominated by the outlier of the 1987 crash (especially strong for the S&P 500). Using data for the pre-1987, however, produced sensible results that are very much in line with the results obtained using post-1987 data, again yielding a MSMH(3)-VECM(1) (results not reported to conserve space but are available from the authors upon request).

The empirical model ultimately proposed therefore, is consistent with the large theoretical literature focusing on the dynamic relationship between spot and futures prices in the spirit of the Cost of Carry model, as well as with the increasingly growing empirical literature stressing the existence of important nonlinearities in both spot and futures prices movements.

While these results aid the profession's understanding of the behavior of futures prices and spot prices over the post-crash period, they should be viewed as a tentatively adequate characterization of the data that appears to be superior to linear-equilibrium-correction modeling in a number of respects, but which nevertheless may be capable of improvement. In particular, one may gain further insights into the dynamic relationship under investigation by developing more-complex nonlinear-equilibrium-correcting systems of equations involving, for example, other financial, as well as economic variables, which may affect spot prices and futures prices movements. In addition, one may attempt modeling the error term of the Markov-switching model as a process containing autoregressive conditional heteroskedasticity, which may be done by extending the Markov-switching-autoregressive-conditional-heteroskedasticity model to cointegrated vector autoregressions and vector equilibrium corrections.[24] These challenges remain on the agenda for future research.

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[24]The Markov-switching-autoregressive-conditional-heteroskedasticity (often termed SWARCH) model was introduced by Hamilton and Susmel (1994) for modeling volatility in a stationary context—specifically, Hamilton and Susmel examined weekly stock-return data. The extension of these models to cointegrated vector autoregressions and vector equilibrium corrections requires, however, further developments in econometric theory, particularly with regard to model specification, estimation, and inference.

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