Price Limits, Margin Requirements, and Default Risk

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This article investigates whether price limits can reduce the default risk and lower the effective margin requirement for a self-enforcing futures contract by considering one more period beyond Brennan's (1986) model to take into account the spillover of unrealized residual shocks due to price limits. The results show that, when traders receive no additional information, price limits can reduce the margin requirement and eliminate the default probability at the expense of a higher liquidity cost due to trading interruptions. Consequently, the total contract cost is higher than of that without price limits. When traders receive additional signals about the equilibrium price, we find that the optimal margin remains unchanged with or without the imposition of price limits, a result that is in conflict with Brennan's assertion. Hence, we conclude that price limits may not be effective in

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INTRODUCTION

The use of price limits in futures markets has generated a great deal of discussion since the global market crash of 1987. Several researchers have tried to examine the impact and effectiveness of price limits, either empirically or theoretically. In essence, price limits are designed to reduce the total cost for market participants by serving as a price-stabilization mechanism to assure the proper operation of futures markets. Their impact on the operation of markets, however, is still under debate.

Advocates and supporting evidence of price limits suggest that they prevent extreme price movements and provide a cool-off period in the events of overreaction (e.g., Anderson, 1984; Arak & Cook, 1997; Greenwald & Stein, 1991; Ma, Rao, & Sears, 1989a, 1989b). Furthermore, since price limits constrain the price change in a trading period, they may serve to reduce the potential default risk in futures contracts (Brennan, 1986; Moser, 1990).

Opponents to price limits contend that the imposition of price limits, instead of stabilizing price changes, may impede the price-discovery process by preventing the price from reaching its equilibrium level effectively. The view that price limits serve nothing, but merely slow down the price adjustment process, also has many proponents and supporting evidence (e.g., Fama, 1989; Figlewski, 1984; Kim and Rhee, 1997; Lehmman, 1989; Meltzner, 1989; Miller, Malkiel, & Hawke, 1987; Telser, 1981). It also is argued that price limits may impose additional risks on market participation because they prohibit mutually beneficial trades at prices outside the limits (e.g., Ackert & Hunter, 1994). A final argument against price limits is that they may have a "magnet effect" that acts to draw the price closer to a limit. This is because traders, for fear of losing liquidity and being locked in a position, would rush to protect themselves through active trading. As a result, when the price is close to a limit, the trading volume will be heavy and the price limit rule will serve as a magnet that further pulls the price even closer to the limit (e.g., Harris, 1997; Kuhn, Kuserk, & Locke, 1989; Lee, Ready, & Seguin, 1994; Subrahmanyam, 1994).

In spite of the above debates on the impact of price limits on the price process, some researchers have tried to examine whether the use of price limits, in conjunction with the margin requirements, can improve the efficiency of the futures markets.[1] Telser (1981) and Figlewski (1984) point out that price limits cannot substitute for margins since they, though lengthening the time it takes to adjust to a new equilibrium, do not reduce the size of price change. On the other hand, Ackert and Hunter (1994) argue that price limits can decrease the margin that brokers and exchanges require, since repressing prices reduces the probability of default resulting from unfavorable price movements and, thus, lowers the risks.[2]

Using a "one-period" model, Brennan (1986) shows that price limits, in conjunction with margins, may be useful in controlling the default risk.[3] More specifically, Brennan shows that it may be optimal to run some risk of trading interruption due to price limits because they may help reduce the default risk, lower the margin requirement, and decrease the total cost for market participants. In Brennan's model, price limits do not affect the underlying generating process for the equilibrium futures prices. Under the same assumption, we show that tradings following a limit move will reflect unrealized shocks carried over from previous trading periods. Consequently, the probability for consecutive-limit hits in the same direction is higher,[4] and the losing party of the futures contract will have more incentives to renege. Hence, additional liquidity costs and reneging costs exist in the following periods. Since Brennan's model only considers the cost incurred in one period, and ignores the costs in the following periods accompanied by the spillover of residual shocks due to price limits, the effect of price limits may have been overstated.

In this article, we consider a two-period model, which takes into account the possibility of consecutive limit moves, to investigate whether price limits can lower the margin requirement, reduce the default risk, and eventually decrease the total cost for a completely self-enforcing contract. Our results, based on numerical examples following Brennan's setting, suggest that price limits do help reduce the optimal margin requirement when the traders receive no additional information. However, the total-contract cost rises, rather than falls, due to the higher liquidity cost

[1]Kodres and O'Brien (1994) investigated a different aspect of price limits. They showed that price limits may be Pareto improving in the presence of a certain form of market incompleteness.

[2]In a different dimension, there is a rich literature on the margin setting (see, for example, Edwards & Neftci, 1988; Gay, Hunter, & Kolb, 1986; Longin, 1999) and the impact of margin on trading activities (see, for example, Adrangi & Chatrath, 1999; Fishe & Goldberg, 1986; Fishe, Goldberg, Cosnell, & Sinha,, 1990).

[3]Although Brennan's model has two periods and three dates, it is, however, in essence a single-period model because the representative trader decides whether to renege, conditional on the knowledge of limit hit at the first period, to minimize the total expected loss for both periods.

[4]For example, Kodres (1988) pointed out that the positive serial correlation for price changes in currency futures seems to be induced by price limits.

of limit hits. Additionally, if the traders receive additional information about the unobserved equilibrium price when a limit is triggered, the optimal effective margin remains unchanged with or without price limits. Our results differ from those of Brennan's one-period model in which price limits restrict the immediate realized loss when a limit is first triggered, thereby reducing the probability for the losing party to renege. Since the unrealized shocks will be reflected in the following trading periods and entail higher default risk and higher liquidity cost in a more realistic multiperiod framework, we conclude that price limits may not be effective in improving the performance of futures contracts. As a further investigation, we examine the case where price limits, as alleged by several studies, restrain the market from overreaction following a limit move. Our numerical results show that only when price limits change both the mean and variance of the return-generating process can they serve as a partial substitute for margin requirement.

The remainder of this paper is organized as follows. In the next section, we characterize the price behavior under price limits. A review of Brennan's model is given in the section entitled "Models," and then it is extended to two periods to incorporate the effect of unrealized residual shocks. "Numerical Examples" demonstrates numerical results under normally distributed price changes, and is followed by the conclusion.

THE PRICE BEHAVIOR UNDER PRICE LIMITS

We begin by noting that under a price-limit rule, the price during each trading day (period) cannot be above the previous *settlement price* plus an up limit, or below the previous settlement price minus a down limit. More formally, the *observed* futures price, Z_t, follows the relationship:

$$Z_t = \begin{cases} Z_{t-1} + L & \text{if } P_t \geq Z_{t-1} + L \\ P_t & \text{if } Z_{t-1} - L < P_t < Z_{t-1} + L \\ Z_{t-1} - L & \text{if } P_t \leq Z_{t-1} - L, \end{cases} \tag{1}$$

where P_t is the equilibrium futures price at time t and L is the maximum daily limit imposed on the absolute change in futures price in a trading day. Thus, the true futures price is observed (i.e., $Zt = P_t$) only when it falls within the range, $(Z_{t-1} - L, Z_{t-1} + L)$; otherwise, it is observed as equal to the limit price if it is outside the range. By subtracting Z_{t-1} from both sides of eq (1), we obtain:

$$Z_t = \begin{cases} L & \text{if } P_t - Z_{t-1} \geq L \\ P_t - Z_{t-1} & \text{if } -L < P_t - Z_{t-1} < L \\ -L & \text{if } P_t - Z_{t-1} \leq -L, \end{cases} \tag{2}$$

where z_t is the observed daily futures price change at time t, i.e., $z_t = Z_t - Z_{t-1}$. Hence, the occurrence of a limit price depends on the magnitude of $Q_t \equiv P_t - Z_{t-1}$, rather than that of $r_t = P_t - P_{t-1}$, the true futures price change. We refer to Q_t as the *pseudo true price change*, and decompose it into two terms:

$$\begin{aligned} Q_t &= P_t - Z_{t-1} \\ &= (P_t - P_{t-1}) + (P_{t-1} - Z_{t-1}) \\ &= r_t + e_{t-1}, \end{aligned}$$

where $e_s = P_s - Z_s$ denotes a spillover term from trading day s, which also can be viewed as an unrealized residual shock being carried over to future trading days. Clearly, z_t is never equal to the true price change unless both t and $(t - 1)$ are non-limit days.

To help understand the effect of price limits on price behavior, assume that the true price change follows an independent normal distribution with mean zero and variance $\sigma^2_{r_t}$, i.e., $r_t \sim \mathcal{N}(0,\sigma^2_{r_t})$. Suppose further that time $t - 1$ is not a limit day and an up limit is hit at time t, then potentially the price change at time $t + 1$ is $Q_{t+1} = r_{t+1} + r_t - L$, whose conditional mean is given as the following:

$$\begin{aligned} E(r_{t+1} + r_t - L | r_t \geq L) &= E(r_{t+1}|r_t \geq L) + E(r_t|r_t \geq L) - L \\ &= \sigma_{r_t} \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)} - L \\ &\geq 0 = E(r_{t+1}), \end{aligned} \tag{3}$$

where $\alpha_1 = L/\sigma_{r_t}$. $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and distribution functions, respectively. This indicates that the expected price change following an up limit will increase. Likewise, the expected price change following a down limit will decrease.

The conditional variance of $r_{t+1} + r_t - L$ is given as follows:

$$\begin{aligned} &Var(r_{t+1} + r_t - L | r_t \geq L) = Var(r_{t+1}|r_t \geq L) + Var(r_t|r_t \geq L) \\ &= \sigma^2_{r_{t+1}} + \sigma^2_{r_t}\left(1 - \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)}\left(\frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)} - \alpha_1\right)\right) \\ &\geq \sigma^2_{r_{t+1}}. \end{aligned} \tag{4}$$

This shows that the volatility after an up limit will increase because it also reflects unrealized residual shocks from the previous trading period [i.e., the second term of the eq (4)]. Clearly, the price change following an up (a down) limit move will have a higher (lower) expected return and higher variance. Hence, if the up (down) limit is triggered at time t, a higher probability for time $t + 1$ to hit the up (down) limit also can be expected, i.e.,

$$Pr(r_{t+1} \geq L | r_t \geq L) \geq Pr(r_{t+1} \geq L | r_t < L),$$

$$Pr(r_{t+1} \leq -L | r_t \leq -L) \geq Pr(r_{t+1} \leq -L | r_t > -L).$$

Consequently, we would expect a higher probability of trading interruptions and a higher liquidity cost resulting from price limits in the period following a limit move.

MODELS

This section first gives a brief review of Brennan's one-period model, and then extends the model to two periods to take into account the effect of unrealized shocks due to price limits.

A Brief Review of Brennan's Model

The Basic Model

Suppose that a representative risk-neutral trader enters a futures contract at time 0 and deposits an initial margin, m, with his broker. The price at time 0, P_0, is given and is not subject to price limits. At time 1, the position must be settled. The trader will have an incentive to renege if the expected default benefits exceed the expected default costs. Let Π be the probability that the broker will not take legal action, and if he does, it will be unsuccessful. In addition, let $\mathcal{Y}$ be the sum of the expected reputation and legal costs the trader must bear as a result of reneging. Then the trader in a short position will have an incentive to renege if $\Pi[P_I - P_0 - m] > \mathcal{Y}$. Hence, there will be an incentive for one of the parties to renege whenever the absolute price change exceeds the "effective margin," $M = m + \Pi^{-1}\mathcal{Y}$, i.e.,

$$|P_1 - P_0| > M.$$

For a financial contract to survive in a competitive financial market, the contract must be designed to minimize the total cost of trading for market participants. The effective margin represents the least margin requirement for a contract to be self-enforcing, but it may not be the optimal margin that minimizes the total contract cost. Without price limits, the contract cost is composed of the cost of capital and the cost of reneging. Let the cost of capital be kM, where k is the unit cost of margin per unit of time. The cost of reneging is assumed to be proportional to the probability of reneging, i.e., $\beta P_r(|r_1| > M)$. Hence, the total cost in the absence of price limits is

$$C_1^{NL}(M) = kM + \beta P_r(|r_1| > M).$$

With price limits, the cost contains three components: the cost of margin, the cost of reneging, and the liquidity cost due to trading interruptions. Below we analyze how cost differs in the presence of price limits when no additional information about equilibrium futures price is available.

No Additional Information

Suppose that a limit, L, is imposed on the price change so that no trades may occur at time 1 at prices above $P_0 + L$, or below $P_0 - L$. When a limit is triggered at time 1 and the losing party receives no additional information, his consideration will shift to his expected position at time 2 since he cannot trade at time 1. The trader will have an incentive to renege if his expected loss at time 2, conditional on the information of limit hit at time 1, exceeds the effective margin. Hence, a necessary and sufficient condition for neither party of a trade to renege is that the expected loss is not greater than the size of the effective margin for both parties, i.e.,

$$|E(P_2 - P_0|P_1 - P_0 < -L)| \leq M, \tag{6}$$

$$|E(P_2 - P_0|P_1 - P_0 > L)| \leq M. \tag{7}$$

Assume that the futures price change is symmetrically distributed and the futures price follows a martingale process such that $E(P_t|P_{t-1}) = P_{t-1}$. Then, for the contract to be completely self-enforcing, the margin must be set such that

$$M \geq E(r_1 + r_2|r_1 \geq L),$$

where $r_t = P_t - P_{t-1}$ is the true price change at time t. Since margin is costly, the right hand side (RHS) in (8) defines an optimal self-enforcing contract-margin level as a function of the price limit, $M(L)$.[5] Note that here the condition that $L < M$ is required because if the condition is violated (i.e., $L \geq M$), a price-limit rule has no effect on the incidence of reneging since it would always be possible to observe directly whether price change is more than M.

Under an effective price-limit rule, reneging occurs for a positive price change if and only if

$$r_1 \geq L$$

and

$$E(r_1 + r_2 | r_1 \geq L) > M.$$

Note that for a self-enforcing contract the condition $\partial M/\partial L \geq 0$ in (8) is required in order for a price-limit rule to reduce the effective margin. While price limits reduce the default risk to market participants, there is liquidity cost associated with their use. Clearly, the tighter the daily limits, the more often the trading is interrupted, thereby causing greater losses in liquidity to traders. Hence, exchanges face a trade off between the default risks and liquidity costs associated with extreme price movements in setting price limits. Assume that the cost of price limits is proportional to the probability that a limit is triggered, then the cost of price limits at time 1 can be written as $\alpha P_r(|r_1| \geq L)/P_r(|r_1| \leq L)$.

Incidentally, if $L < M$, but the condition (8) is violated, reneging occurs whenever a limit is hit. Besides, the default probability will not only be decreasing in L, but also will be higher than it would have been without price limits, since reneging occurs whenever $|r_1| > L$, instead of $|r_1| > M$. Therefore, the optimal self-enforcing contract is also the cost-minimizing contract. Thus, the cost of the optimal contract for the representative trader at time 1, $C_1^{PL}(M,L)$, is given by

$$C_1^{PL}(M,L) = kM + \alpha \frac{P_r(|r_1| \geq L)}{P_r(|r_1| \leq L}. \tag{9}$$

It is worth noting that, with price limits, the cost due to the occurrence of reneging is eliminated completely. This is because when the trader receives no additional information about the true equilibrium price when a limit is hit, the exchange always can choose a limit that is small enough such that the expected loss, conditional on the knowledge of a limit hit, is smaller than the specified margin, as formulated by the condition (8).

[5]Note that condition (8) defines a self-enforcing contract only for a risk-neutral trader .

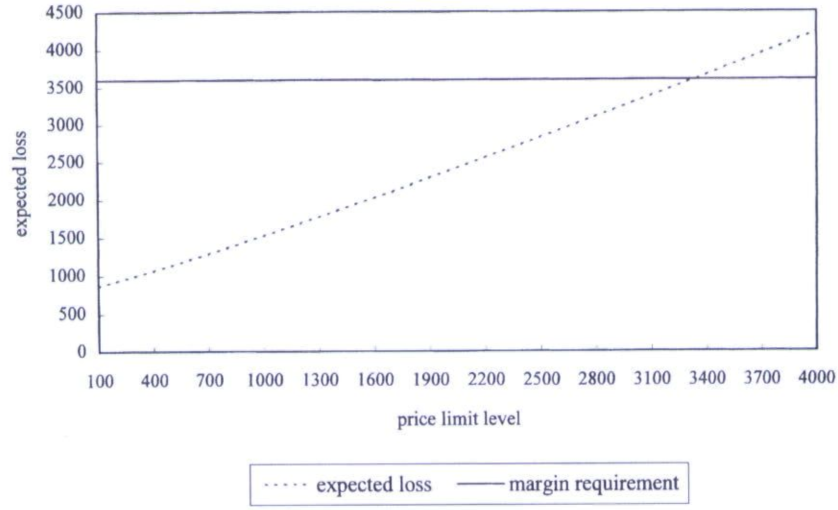

FIGURE 1

Expected Loss of a Futures Contract Conditional on the Knowledge of Limit Hit Time 1. This figure draws the expected loss of a futures contract conditional on the knowledge of limit hit at time 1 as a function of the level of price limit under normality assumption (the dashed line). The solid line represents a given margin.

Figure 1 plots the expected loss as a function of the price limit under normality assumption. It indicates that the expected loss is a linear function of the limit. Hence, unless the margin is set too low, exchanges always can find a limit so that the expected loss is smaller than the margin. However, if the trader receives additional information about the true equilibrium futures price when a limit is hit, then the story may be different because the precision of the information will affect the trader's conditional expectation about the amount of loss. The effectiveness of price limits with additional information is analyzed in the following.

With Additional Information

Now, suppose that the trader is able to observe a signal Y_t, a random variable that is correlated with the change in the equilibrium futures price, r_t. Such a signal may be derivable from the spot market for the underlying commodity, from the markets for other futures contracts, or from other sources. Then reneging occurs for a positive price change if, and only if,

$$r_1 \geq L$$

and

$$E(r_1 + r_2|r_1 \geq L, Y_1) > M.$$

Unlike the previous case in which the conditional expected loss is fixed once a limit is specified, the expected loss with the availability of additional information is unknown because it is conditional on the level of the signal Y_1. It is possible that a certain level of the signal will cause the conditional loss to exceed the margin. If the joint distribution of (r_t, Y_t) is symmetric about the origin, the probability of reneging is given by

$$2P_r(r_1 \geq L,\ Y_1 \geq Y_1^*(M,L)),$$

where $Y_1^*(M,L)$ denotes a critical level of the information beyond which reneging will occur. Hence, the total cost for the representative trader at time 1, $C_1^{PL}(M,L)$, becomes:

$$C_1^{PL}(M,L) = kM + \alpha \frac{P_r(|r_1| \geq L)}{P_r(|r_1| \leq L)} + 2\beta P_r(r_1 \geq L,\ Y_1 \geq Y_1^*(M,L)). \qquad (10)$$

The precision of the additional information can be characterized by the correlation coefficient between the signal Y_t and the equilibrium price change r_t. Without assuming a specific distribution for the future price change, solving for the above optimization problems [(9) and (10)] would not be possible. Even if a specific distribution, say the normal distribution, is assumed, finding analytical solutions for the optimization is still difficult. Therefore, Brennan (1986) uses some numerical examples to examine if price limits are useful.

Based on the above setting, Brennan (1986) shows that price limits may serve as a partial substitute for margin requirements in ensuring contract performance, but their effectiveness of price limits deteriorates as precise information about the unobserved equilibrium price is obtained. Price limits can alleviate the default problem because they can hide the information from the losing party about the extent of his losses. Knowing that the adverse price movement exceeds the limit, but not knowing exactly how much will be lost, the trader is forced to form a conjecture about the size of his losses. In this sense, price limits act to create noises when the trader is forming expectation about the unobserved equilibrium futures price. Consequently, situations exist in which reneging would have occurred in the absence of price limits, but is avoided if price limits exist.

Nevertheless, unrealized shocks accumulate in successive trades when price limits are triggered, and the delayed price movements will raise the trading-halt risk in the following trading periods. In addition, the increase in the probability of limit move at time 2 suggests that the conditional cumulative expected loss would be even greater than that at time 1, thereby raising the default risk at time 2. Hence, although the immediate cost at time 1 is lowered with price limits, additional cost is transmitted to the next trading period. Thus, the effectiveness of price limits will be weakened, and whether it remains beneficial in a multiperiod framework is doubtful.

Brennan's model, despite having two periods and three dates, is essentially a *one-period* model because the total cost is calculated based on a representative trader's single-period decision. Hence, a two-period model that incorporates the spillover of unrealized shocks is investigated in the following section.

The Two-Period Model

We now consider a two-period model[6] for which the optimal combination of price limits and margin is chosen to minimize the sum of costs at both periods 1 and 2. Our two-period setting allows us to consider the spillover effect from limit moves and can serve as the lower bound measure for the cost increment in the multiperiod framework. If price limits prove to be cost increasing for the two-period model, one can conclude that price limits are necessarily useless for the more realistic multi-period model.[7]

Without price limits, the total cost is the following:

$$\begin{aligned} C^{NL}(M,L) &= C_1^{NL}(M,L) + C_2^{NL}(M,L) \\ &= 2kM + 2\beta(Pr(r_1 > M) + Pr(r_1 < M,\ r_1 + r_2 > M)). \quad (11) \end{aligned}$$

Note that the cost of capital is assumed doubled for simplicity, and the cost of reneging now includes the cost of reneging at time 2. The probability of reneging at time 2 is the joint probability that the trader does not renege at time 1 and the cumulative price changes in two periods exceed the margin.

With price limits, the total cost for two periods is much more difficult to derive because the decision is more "path-dependent" and the probabilities of limit hit and reneging at the second period are more complicated. In the following, we first present the basic model with no additional

[6]Alternatively, our model can be viewed as a three-period, four-date model in terms of Brennan's framework.

[7]It would be extremely difficult to set up a multiperiod model because, in the presence of consecutive limit moves, the costs associated with reneging and liquidity involve multiple integrals.

information, and then extend the model to incorporate additional information.

No Additional Information

Similar to the one-period case, for the contract to be completely self-enforcing in the two-period model, the margin must be set such that the following condition holds:

$$M \geq E[r_1 + r_2 + r_3 | r_1 \geq L, r_1 + r_2 \geq 2L]. \tag{12}$$

Given the level of price limit L, the above condition (12) specifies a larger effective margin than that specified in the one-period case (8). The RHS in (12) defines the optimal self-enforcing contract margin level as a function of the price limit $M(L)$.

As in the one-period model with price limits, it is possible to find a combination of margin and price limit such that the contract is completely self-enforcing in the two-period model if the trader does not receive additional information about the equilibrium price. Thus, the total cost over the two periods is composed of the cost-of-capital and the cost-of-trading interruption. The cost-of-trading interruption is proportional to the probabilities of limit hit at time 1 and time 2. Due to the spillover of unrealized shocks, the probability of an "up" limit hit at time 2 is the following:

$$\begin{aligned} A_2 &\equiv P_r(z_2 = L) \\ &= P_r(r_2 \geq L) + P_r(r_1 \geq L, r_2 < L, r_1 + r_2 \geq 2L) \\ &= \int_L^{\infty} f(r_2)dr_2 + \int_L^{\infty} \int_{2L-r_1}^{L} f(r_1,r_2)dr_2dr_1, \end{aligned} \tag{13}$$

where, roughly, $f(\cdot)$ denotes the probability density function (pdf) or the joint pdf for the true price changes. The first term of the above equation corresponds to the standard limit-hit probability, while the second term corresponds to the probability of additional limit hit at time 2 due to spillover of shocks from the previous trading period. Thus, the probability of limit move for both directions at time 2 is approximately twice the above probability. Hence, the cost of the optimal contract for the representative trader, $C^{PL}(M,L)$, is given by

$$\begin{aligned} C^{PL}(M,L) &= C_1^{PL}(M,L) + C_2^{PL}(M,L) \\ &= 2kM + \alpha\left(\frac{2P_r(r_1 \geq L)}{1 - 2P_r(r_1 \geq L)} + \frac{2A_2}{1 - 2A_2}\right). \end{aligned} \tag{14}$$

With Additional Information

If the trader receives additional information about the true price, an additional term of reneging cost will be present. Since the decision of reneging depends on the level of margin, the following two cases need to be considered (see Appendix A for a summary of all situations under which reneging will occur).

First, if $M > 2L$, then the short position will renege at time 2 if the following holds for the up-limit case:

$$E(P_3 - P_0|z_2 = L, r_1 + r_2 < M) \geq M.$$

That is, the trader will renege if the expected accumulated losses exceed the margin, given that an up limit is triggered at time 2 and the trader does not renege at time 1. Hence, the probability of reneging is given as follows:

$$\begin{aligned} B_2 &= P_r(z_1 = L, Y_1 < Y_1^*, z_2 = L, Y_2 \geq Y_{21}^*) \\ &+ P_r(z_1 < L, z_2 = L, Y_2 \geq Y_{22}^*). \end{aligned}$$

The first term of the above equation can be expressed as:

$$\begin{aligned} &P_r(z_1 = L, Y_1 < Y_1^*, z_1 = L, Y_2 \geq Y_{21}^*) \\ &= \int_L^\infty \int_{2L-r_1}^\infty \int_{-\infty}^{Y_1^*} \int_{Y_{21}^*}^\infty f(r_1,r_2,Y_1,Y_2)dY_2dY_1dr_2dr_1, \end{aligned}$$

while the second term can be written as:

$$P_r(z_1 < L, z_2 = L, Y_2 \geq Y_{22}^*) = \int_{-\infty}^{L} \int_L^\infty \int_{Y_{22}^*}^\infty f(r_1,r_2,Y_2)dY_2dr_2dr_1,$$

where Y_{21}^* and Y_{22}^* are critical signals above which reneging occurs for some positive price change. Y_{21}^* and Y_{22}^* are given by solving $E[r_1 + r_2 + r_3|r_1 \geq L, r_1 + r_2 \geq 2L, Y_{21}] = M$ and $E[r_1 + r_2 + r_3|r_1 < L, r_2 \geq L, Y_{22}] = M$, respectively.

Second, if $M \leq 2L$, i.e., the margin is smaller than twice the limit, the probability of reneging is:

$$B_2 = P_r(r_1 \geq L, Y_1 < Y_1^*, r_1 + r_2 \geq M)$$
$$+ P_r(M - L \leq r_1 < L, r_2 \geq L) + P_r(r_1 < M - L, r_2 \geq L, Y_2 \geq Y_{23}^*)$$
$$+ P_r(r_1 < L, r_2 < L, r_1 + r_2 \geq M)$$
$$= \int_L^{\infty} \int_{-\infty}^{Y_1^*} \int_{M-r_1}^{\infty} f(r_1,r_2,Y_2) dr_2 dY_1 dr_1 + \int_{M-L}^{L} \int_L^{\infty} f(r_1,r_2) dr_2 dr_1$$
$$+ \int_{-\infty}^{M-L} \int_L^{\infty} \int_{Y_{23}^*}^{\infty} f(r_1,r_2,Y_2) dY_2 dr_2 dr_1 + \int_{-\infty}^{L} \int_{M-r_1}^{L} f(r_1,r_2) dr_2 dr_1,$$

where Y_{23}^* is given by solving $E[r_1 + r_2 + r_3 | r_1 < M - L, r_2 \geq L, Y_{23}] = M$. Hence, the contract cost at time 2, $C_2^{PL}(M,L)$, is the following:

$$C_2^{PL}(M,L) = kM + \alpha \left(\frac{2A_2}{1 - 2A_2} \right) + 2\beta B_2.$$

M and L should be set to minimize the total contract cost at both time 1 and time 2, which can be represented as the following optimization problem:

$$\min_{M,L} C^{PL}(M,L) = C_1^{PL}(M,L) + C_2^{PL}(M,L).$$

Notice that, because of the spillover of unrealized shocks, both the liquidity cost and reneging cost increase following a limit move in the two-period model. Consequently, imposing price limits may not be able to lower the total cost.

Price Limits Have a Cool-Off Effect

So far we have assumed that price limits only serve to delay the price from reaching its equilibrium level, but do not affect the underlying equilibrium-price-generating process. Since investors are given additional time to process relevant information under price limits, it is possible that price limits will cool off the market and aid in the price-resolution process. Consequently, the volatility after a limit move might decrease rather than increase. If price limits have an effect on the underlying price-generating process when a limit is hit, the story may be different. According to our discussion in "The Price Behavior Under Price Limits," we know that the potential price change following an up limit move, Q_{t+1}, has the following conditional mean and variance under normality assumption:

$$E(Q_{t+1}|r_t \geq L) = \sigma_{r_t} \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)} - L$$

$$Var(Q_{t+1}|r_t \geq L) = \sigma^2_{r_{t+1}} + \sigma^2_{r_t}\left(1 - \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)}\left(\frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)} - \alpha_1\right)\right).$$

If part of the volatility is not fundamental, but "transitory," and can be eliminated by introducing price limits, the conditional variance $Var(Q_{t+1})$ may become smaller. In addition, as alleged by some price-limit proponents, price limits might reduce the extreme price movement in the same direction by pulling the price back. The conditional mean also might differ. If the mean and variance for the potential price change take the following form:

$$E(Q_{t+1}|r_t \geq L) = \theta + \sigma_{r_t} \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)} - L$$

$$Var(Q_{t+1}|r_t \geq L) = \delta\sigma^2_{r_{t+1}} + \sigma^2_{r_t}\left(1 - \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)}\left(\frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)} - \alpha_1\right)\right),$$

then certain values of θ and δ might reduce the spillover effect, and further lower the margin requirement and the total contract costs. A negative value for θ and a value smaller than 1 for δ imply that the price-limit rule has a cool-off effect. Otherwise, if $\theta > 0$ and $\delta > 1$, then the price-limit rule can be said to have a "magnet" effect. Presumably, if the price-limit rule has a cool-off effect, the default probabilities at time 1 and time 2 will be lowered, while if it has a magnet effect, the probabilities of reneging at time 1 and time 2 will be raised, in which case price limits will not be effective.

NUMERICAL EXAMPLES

In this section, we present numerical examples for normally distributed price changes to determine the effects of price limits on contract costs and margin requirements in the two-period model. We use the same set of parameter values as in Brennan (1986). Specifically, σ_{r_t} is taken as 1000 and the following values are used for the parameters of the cost function: $k = 0.02\%$, $\alpha = 1$, and $\beta = 3$. Three different levels of the extra-market signals, measured by ρ that represents the correlation between the signal and the equilibrium price change, are analyzed. They are 0.1, 0.50, and 0.96. A larger ρ is associated with a more-accurate signal. Contract costs for each of the three levels of signals are computed

TABLE I

Minimum Costs of a Futures Contract Under Various Margin Levels Without Additional Information

	Without Price Limits				*With Price Limits*						
	Pr(Reneging) (%)					*Pr(Reneging)* (%)			$Pr(\lvert z_t \rvert \geq L)$ (%)		
Margin	$t = 1$	$t = 2$	*Sum*	*Total Cost*	L^*	$t = 1$	$t = 2$	*Sum*	$t = 1$	$t = 2$	*Total Cost*
2400	1.6400	8.172	9.812	5.866	783.08	0	0	0	43.358	52.279	2.8210
2600	0.932	6.192	7.124	4.602	826.51	0	0	0	40.852	48.906	2.6879
2800	0.511	4.574	5.085	3.663	865.75	0	0	0	38.663	46.001	2.6022
3000	0.270	3.300	3.570	2.985	901.80	0	0	0	36.704	43.447	2.5484
3200	0.137	2.326	2.463	2.512	935.34	0	0	0	34.961	41.167	2.5173
3400	0.067	1.605	1.672	2.196	966.87	0	0	0	33.361	39.106	2.5028
3600	0.032	1.085	1.117	1.998	996.76	0	0	0	31.888	37.224	2.5011[b]
3800	0.015	0.719	0.733	1.887	1025.31	0	0	0	30.522	35.490	2.5094
4000	0.006	0.467	0.473	1.837	1052.74	0	0	0	29.246	33.881	2.5257
4200	0.003	0.298	0.301	1.830[a]	1079.23	0	0	0	28.049	33.881	2.5486
4400	0.001	0.186	0.187	1.854	1104.93	0	0	0	26.919	30.972	2.5770
4600	4.0e-4	0.114	0.115	1.897	1129.96	0	0	0	25.849	29.647	2.6000

This table presents the two-period minimum costs of a futures contract with and without price limits for various margin levels where the market participants have no additional information other than the observed futures prices. Under price limits, the minimum cost for a given margin is obtained by choosing an optimal-limit level that minimizes the contract cost over two periods. The same set of parameter values as in Brennan (1986) are used. L^* denotes the optimal-price-limit level for a given margin. Pr(Reneging) (%) and $Pr(\lvert z_t \rvert \geq L)$ (%) denote the probability of reneging and that of limit hit at time t, respectively. [a] and [b] refer to the minimum costs with and without price limits, respectively.

for margins at intervals of 200, and for limits at intervals of 100 for each margin. The specific functional forms for each of the costs and the associated cost components are presented in Appendix B. The two-period optimization problem is solved numerically, and only the results of the cost-minimizing limit for each margin are reported. The next subsection presents the results for the case where price limits only serve to delay price reaction and have no "real" effect. The section following then presents the case where price limits have a cool-off effect and change the price parameters following limit hits.

Price Limits Only Delay Price Reaction

No Additional Information

Table I presents the results for the simplest case for which the trader receives no additional information. The results show that the optimal

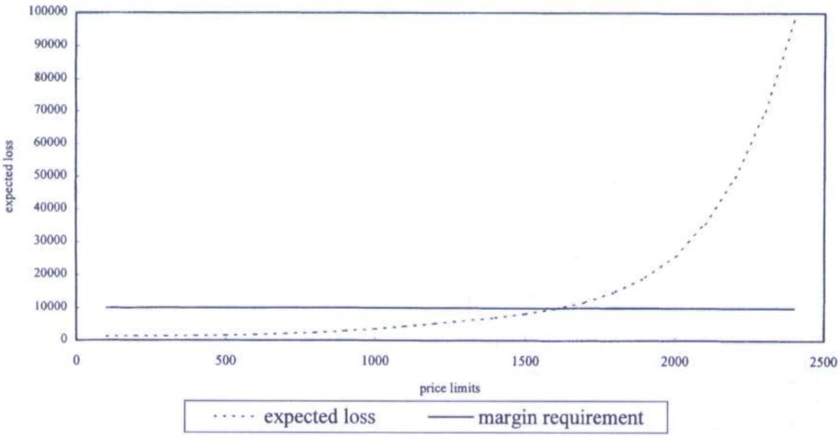

FIGURE 2

Expected Loss of a Futures Contract Conditional on the Knowledge of Limit Hits at Both Time 1 and Time 2. This figure draws the expected loss of a futures contract conditional on the knowledge of limit hits at both time 1 and time 2 as a function of the level of price limit under normality assumption (the dashed line). The solid line represents a given margin.

contract without price limits requires a margin of 4200 and has a total cost of 1.830, but with a limit of 996.76, it requires only a margin of 3600 and has a cost of 2.501. Price limits reduce the optimal margin requirement from 4200 to 3600. However, the corresponding optimal limit, 996.76, is only about 28% of the required margin. As a result, the probability of limit hit is as high as 30% for both periods, thereby causing the contract cost to be much higher than without price limits.[8] Figure 2 plots the conditional expected loss as a function of price-limit level. While the conditional loss is not a linear function of price limit in the two-period model, it still is possible to find a critical limit that makes the contract default free for any given margin. Hence, the reneging cost can be eliminated completely at the expense of higher costs of trading interruptions.

With Additional Information

Table II presents the results when the trader has additional information. The results show that without price limits, the default probability at time 2 is larger than that at time 1 for each margin, which is expected because at time 2 the cumulative price changes have a larger variance. However,

[8]Note that the optimal margin requirement of 4200 for the two-period model is higher than that of 3200 in Brennan's model. Part of the increase is because we do not incorporate margin call as an additional variable in our model. We conjecture that not much insight can be gained with such complication.

TABLE II

Minimum Costs of a Futures Contract Under Various Margin Levels With Additional Information

	Without Price Limits				*With Price Limits*						
	Pr(Reneging) (%)					*Pr(Reneging)* (%)			$Pr(\lvert z_t \rvert \geq L)$ (%)		
Margin	*t = 1*	*t = 2*	*Sum*	*Total Cost*	*L**	*t = 1*	*t = 2*	*Sum*	*t = 1*	*t = 2*	*Total Cost*
					Panel A: $\rho_t = 0.1$						
2400	1.640	8.172	9.812	5.866	800	0	3.172	3.172	42.371	50.735	4.3112
2600	0.932	6.192	7.124	4.602	900	0	0.583	0.583	36.812	43.570	2.8266
2800	0.511	4.574	5.085	3.663	900	0	0.446	0.446	36.812	43.570	2.6979
3000	0.270	3.300	3.570	2.985	1000	0	0.441	0.441	31.731	37.101	2.5853
3200	0.137	2.326	2.463	2.512	1100	0	0.330	0.330	27.133	31.329	2.3560
3400	0.067	1.605	1.673	2.196	3100	1.0e-4	1.711	1.702	0.194	0.194	2.2193
3600	0.032	1.085	1.117	1.998	3300	6.5e-9	1.143	1.143	0.097	0.097	2.0133
3800	0.015	0.719	0.733	1.887	3500	8.8e-15	0.749	0.749	0.047	0.047	1.8953
4000	0.006	0.467	0.473	1.837	3700	1.2e-51	0.482	0.482	0.022	0.022	1.8413
4200	0.003	0.298	0.301	1.830[a]	3900	0	0.305	0.305	0.010	0.010	1.8325[b]
4400	0.001	0.186	0.187	1.854	4100	0	0.189	0.189	0.004	0.004	1.8548
4600	4.0e-4	0.114	0.115	1.898	4300	0	0.116	0.116	0.002	0.002	1.8979
					Panel B: $\rho_t = 0.5$						
2400	1.640	8.172	9.812	5.866	1200	1.0e-3	7.033	7.034	23.014	26.238	5.1314
2600	0.932	6.192	7.124	4.602	1300	9.8e-5	5.289	5.289	19.360	21.795	4.2033
2800	0.511	4.574	5.085	3.663	1400	8.3e-6	3.923	3.923	16.151	17.958	3.4929
3000	0.270	3.300	3.570	2.985	1500	6.2e-7	2.870	2.870	13.361	14.679	2.9610
3200	0.137	2.326	2.463	2.512	3100	0.194	2.341	2.535	0.194	0.194	2.5564
3400	0.067	1.605	1.673	2.196	3300	0.097	1.623	1.720	0.097	0.097	2.2218
3600	0.032	1.085	1.117	1.998	3500	0.047	1.096	1.143	0.047	0.047	2.0122
3800	0.015	0.719	0.733	1.887	3700	0.022	0.725	0.746	0.022	0.022	1.8937
4000	0.006	0.467	0.473	1.837	3900	0.010	0.470	0.480	0.010	0.010	1.8402
4200	0.003	0.298	0.301	1.830[a]	4100	0.004	0.299	0.303	0.004	0.004	1.8318[b]
4400	0.001	0.186	0.187	1.854	4300	0.002	0.187	0.189	0.002	0.002	1.8544
4600	4.0e-4	0.114	0.115	1.897	4500	0.001	0.115	0.115	0.001	0.001	1.8977
					Panel C: $\rho_t = 0.96$						
2400	1.640	8.172	9.812	5.866	1200	1.242	6.695	7.937	23.014	26.238	5.5832
2600	0.932	6.192	7.124	4.602	1300	0.677	4.943	5.620	19.360	21.795	4.3684
2800	0.511	4.574	5.085	3.663	1400	0.354	3.613	3.967	16.151	17.958	3.5152
3000	0.270	3.300	3.570	2.985	1500	0.178	2.636	2.814	13.361	14.679	2.9333
3200	0.137	2.326	2.463	2.512	3100	0.188	2.359	2.546	0.194	0.194	2.5571
3400	0.067	1.605	1.673	2.196	3300	0.093	1.627	1.720	0.097	0.097	2.2219
3600	0.032	1.085	1.117	1.998	3500	0.047	1.096	1.143	0.047	0.047	2.0123
3800	0.015	0.719	0.733	1.887	3700	0.021	0.726	0.746	0.022	0.022	1.8936
4000	0.006	0.467	0.473	1.837	3900	0.009	0.471	0.480	0.010	0.010	1.8400
4200	0.003	0.298	0.301	1.830[a]	4100	0.004	0.299	0.303	0.004	0.004	1.8317[b]
4400	0.001	0.186	0.187	1.854	4300	0.002	0.187	0.189	0.002	0.002	1.8543
4600	4.0e-4	0.114	0.115	1.897	4500	0.001	0.115	0.115	0.001	0.001	1.8976

This table presents the two-period minimum costs of a futures contract with and without price limits for various margin levels where the market participants observe a signal that is correlated to the equilibrium futures prices; the precision of the signal is represented by the correlation coefficient between the signal and futures price at time *t*, denoted ρ_t. Panels A through C present the results for three precision levels of the signal (ρ_t = 0.1, 0.5, and 0.96). Under price limits, the minimum cost for a given margin is obtained by choosing an optimal-limit level that minimizes the contract cost over two periods. The same set of parameter values as in Brennan (1986) are used. L^* denotes the optimal-price-limit level for a given margin. Pr(Reneging) (%) and $Pr(\lvert z_t \rvert \geq L)$ (%) denote the probability of reneging and that of limit hit at time *t*, respectively. [a] and [b] refer to the minimum costs with and without price limits, respectively.

with the imposition of price limits, the total default probability of the *optimal* contract is never lowered for any level of information precision. As reported in Table 2, the cost-minimizing default probabilities with price limits at time 2 are 0.305% for low precision of information ($\rho = 0.1$) and 0.299% for high precision of information ($\rho = 0.5$ and 0.6). Both of these default probabilities are higher than those without price limits, 0.298%. The results indicate that price limits not only fail to reduce the probability of reneging, but also exacerbate the problem of contract enforcement in the two-period setting.

In addition, the least cost combination of margin and limit is (4200, 3900) for low precision of information and (4200, 4100) for high precision of information. Note that the optimal margin level, 4200, is the same as that without price limits, and cannot be lowered at any level of information precision. The minimum total cost with price limits is 1.8325 (for $\rho = 0.1$), 1.8318 (for $\rho = 0.5$), and 1.8317 (for $\rho = 0.96$). These costs are slightly greater than the cost without price limits, 1.8303. This example indicates that price limits are useless in either reducing the contract cost or lowering the margin requirement. The results also suggest that the signal precision has little impact on the effectiveness of price limits in the two-period setting, since, no matter how precise the signal is, unrealized shocks will accumulate and be realized in the following days. Consequently, the optimal margin remains the same regardless of the precision of the information.

Since financial futures precise information about the equilibrium futures price can be obtained generally from its spot counterpart, we expect that the price-limit rule can neither decrease default risk nor substitute for margin-to-control contract costs in the two-period model. This contradicts Brennan's (1986) finding that "price limits may serve as a partial substitute for margin requirements in ensuring contract performance." However, for many agriculture futures and many of the new futures being introduced (e.g., catastrophe, weather, and bankruptcy futures), knowledge of the spot price hardly is sufficient to valuing these product. Price-limit rule still may be effective in decreasing default risk and substituting for margin-to-control contract costs in our model.

EXPANDING PRICE LIMITS

In high volatility markets, the exchange may invoke expanded price limits to avoid the possibility of limit hits on successive days. Given our framework, if the exchange expands price limits (say 150% of normal values), the probability of limit hit at time 2 will be lowered, thereby causing the liquidity cost to be lowered. However, the "ambiguity effect" of price limits also is reduced, which, in turn, causes the default probability to increase. Hence, a priori it is not clear whether expanding limits is beneficial. Based on the same numerical setting, Table III presents the results for the case where volatility increases by 20% and the limits following a limit hit are expanded by 50%. Overall, the results show that price limits remain useless in reducing margin requirements when precise information is available (i.e., $\rho_t \geq 0.5$). Surprisingly, however, the numerical results indicate that, for the case of little additional information (i.e., $\rho_t \geq 0.1$), expanding price limits causes the optimal combination of margin and contract cost to decrease from (4800, 2.1567) to (3800, 1.8879). The reason is that when precise information is not available, price limits induce greater volatility spillover to next trading day when a limit is triggered. Hence, if the second-day limits are expanded while leaving the original margin level of 4800 intact, a much higher default cost will incur. As a result, Panel A of Table III indicates that the optimal price limit for the margin of 4800 has to be tightened to 1900 (and the second-day limit 2850) to reduce the second day's default probability. However, the total contract cost increases to 2.0767 because of substantial increase in limit-hit probability. Due to the significant reduction of price-limit level, Panel A of Table III shows that the optimal margin can be lowered further to 3800, where the corresponding limit is 1500.[9]

Price Limits Have a Cool-Off Effect

Our analyses thus far have been based on the assumption that price limits only serve to delay the price-discovery process. As the return-generating process might be affected by the imposition of price limits, below we consider two cases where the mean and variance parameters change following a limit hit.

Table IV presents the results for the case when price limits reduce the volatility by 25% following a limit hit, but do not affect the expected price change. That is, all parameter values remain the same as the previous example except that $\theta = 0$ and $\delta = 0.75$. The results in Table IV show that the optimal combination of margin and limit is (3200, 1500) for $\rho = 0.1$ and (4200, 4100) for $\rho = 0.5$ and 0.96. The results indicate

[9]It is clear that there is always a trade-off between liquidity cost and the default cost. In Panel A of Table III, the reneging probability is replaced with a higher limit-hit probability. Our results that price limits are effective in the absence of precise information could be due to the parameter values we chose. It is not necessarily always the case that the reduction in default cost is higher than the increase in liquidity cost for other parameter values.

TABLE III

Minimum Costs of a Futures Contract Under Various Margin Levels: The Case of Expanding Limit Following a Limit Move

	Without Price Limits				*With Price Limits*						
	Pr(Reneging) (%)					*Pr(Reneging)* (%)			$Pr(\lvert z_t\rvert \geq L)$ (%)		
Margin	$t = 1$	$t = 2$	*Sum*	*Total Cost*	L^*	$t = 1$	$t = 2$	*Sum*	$t = 1$	$t = 2$	*Total Cost*
					Panel A: $\rho_t = 0.1$						
3400	0.046	4.340	9.600	3.7599	1300	0	0	0	27.866	15.117	1.9244
3600	0.267	3.300	3.570	3.2249	1400	0	0	0	24.335	11.801	1.8954
3800	0.154	2.470	2.624	2.8321	1500	0	0	0	21.130	9.095	1.8879[b]
4000	0.086	1.821	1.907	2.5534	1500	0	0	0	21.130	9.095	1.9679
4200	0.047	1.323	1.370	2.3649	1600	0	0	0	18.242	6.920	1.9775
4400	0.025	0.948	0.973	2.2463	1700	0	0	0	15.658	5.196	2.0005
4600	0.013	0.670	0.683	2.1813	1800	0	0	0	13.361	3.850	2.0343
4800	6.3e-3	0.467	0.473	2.1567[a]	1900	0	0	0	11.335	12.394	2.0767
5000	3.1e-3	0.321	0.324	2.1622	1900	0	0	0	11.335	2.814	2.1567
5200	1.5e-3	0.218	0.220	2.1898	5100	1.1e-3	0.218	0.219	2.1e-3	1.0e-7	2.1895
5400	6.8e-4	0.146	0.147	2.2335	5300	5.0e-4	0.146	0.147	0.001	1.0e-7	2.2333
5600	3.1e-4	0.097	0.097	2.2885	5500	2.3e-4	0.097	0.097	4.6e-4	6.2e-10	2.2885
					Panel B: $\rho_1 = \rho_2 = \rho = 0.5$						
3400	0.046	4.340	9.600	3.7599	1000	9.8e-7	0.205	0.205	40.466	29.474	2.6669
3600	0.267	3.300	3.570	3.2249	1000	1.2e-7	0.029	0.029	40.466	29.474	2.6519
3800	0.154	2.470	2.624	2.8321	1000	1.4e-10	0.006	0.006	40.466	29.474	2.6205
4000	0.086	1.821	1.907	2.5534	1200	1.5e-9	0.171	0.171	31.731	19.122	2.3868
4200	0.047	1.323	1.370	2.3649	4100	0.032	1.320	1.352	0.063	2.1e-4	2.3564
4400	0.025	0.948	0.973	2.2463	4300	0.017	0.947	0.964	0.034	1.4e-4	2.2417
4600	0.013	0.670	0.683	2.1813	4500	8.8e-3	0.669	0.678	0.018	8.3e-5	2.1794
4800	6.3e-3	0.467	0.473	2.1567[a]	4700	4.5e-3	0.467	0.472	0.009	8.8e-6	2.1558[b]
5000	3.1e-3	0.321	0.324	2.1622	4900	2.2e-3	0.321	0.323	4.4e-3	3.9e-7	2.1618
5200	1.5e-3	0.218	0.220	2.1898	5100	1.1e-3	0.218	0.219	2.1e-3	1.2e-7	2.1897
5400	6.8e-4	0.146	0.147	2.2335	5300	5.0e-3	0.146	0.147	0.001	1.0e-7	2.2334
5600	3.1e-4	0.097	0.097	2.2885	5500	2.3e-4	0.097	0.097	5.0e-4	6.2e-10	2.2885
					Panel C: $\rho_1 = \rho_2 = \rho = 0.96$						
3400	0.046	4.340	9.600	3.7599	1100	0.158	0.135	0.293	40.466	29.474	2.8776
3600	0.267	3.300	3.570	3.2249	1100	0.089	0.525	0.614	35.932	23.888	2.8120
3800	0.154	2.470	2.624	2.8321	3700	0.102	2.459	2.561	0.205	0.205	2.8022
4000	0.086	1.821	1.907	2.5534	3900	0.057	0.907	0.964	0.115	5.4e-4	2.5375
4200	0.047	1.323	1.370	2.3649	4100	0.031	1.321	1.352	0.063	2.5e-4	2.3566
4400	0.025	0.948	0.973	2.2463	4300	0.017	0.947	0.964	0.034	4.5e-5	2.2422
4600	0.013	0.670	0.683	2.1813	4500	8.7e-3	0.669	0.674	0.018	2.0e-5	2.1792
4800	6.3e-3	0.467	0.473	2.1567[a]	4700	4.4e-3	0.467	0.471	0.009	1.5e-6	2.1557[b]
5000	3.1e-3	0.321	0.324	2.1622	4900	2.2e-3	0.321	0.323	4.4e-3	3.9e-7	2.1618
5200	1.5e-3	0.218	0.220	2.1898	5100	1.1e-3	0.218	0.219	2.1e-3	1.2e-7	2.1897
5400	6.8e-4	0.146	0.147	2.2335	5300	4.9e-4	0.146	0.147	0.001	1.0e-7	2.2334
5600	3.1e-4	0.097	0.097	2.2885	5500	2.2e-4	0.097	0.097	4.6e-4	6.2e-10	2.2885

This table presents the two-period minimum costs of a futures contract with and without price limits for various margin levels. It is assumed that the price limit is expanded by 50% following a limit move. Panels A through C present the results for three precision levels of the signal (ρ_t = 0.1, 0.5, and 0.96). Under price limits, the minimum cost for a given margin is obtained by choosing an optimal-limit level that minimizes the contract cost over two periods. The same set of parameter values as in Brennan (1986) are used. L^* denotes the optimal-price-limit level for a given margin. Pr(Reneging) (%) and $Pr(\lvert z_t\rvert \geq L)$ (%) denote the probability of reneging and that of limit hit at time t, respectively. [a] and [b] refer to the minimum costs with and without price limits, respectively.

TABLE IV

Minimum Costs of a Futures Contract Under Various Margin Levels With a Reduction in Volatility Following Limit Moves

	Without Price Limits				*With Price Limits*						
	Pr(Reneging) (%)					*Pr(Reneging) (%)*			*Pr($\|z_t\| \geq L$) (%)*		
Margin	*t = 1*	*t = 2*	*Sum*	*Total Cost*	*L**	*t = 1*	*t = 2*	*Sum*	*t = 1*	*t = 2*	*Total Cost*
					Panel A: $\rho_t = 0.1$						
2400	1.640	8.172	9.812	5.866	1100	0	0.062	0.062	27.133	30.846	1.8093
2600	0.932	6.192	7.124	4.602	1200	0	0.064	0.064	23.014	25.658	1.7162
2800	0.511	4.574	5.085	3.663	1300	0	0.076	0.076	19.360	21.795	1.6771
3000	0.270	3.300	3.570	2.985	1400	0	0.060	0.060	16.151	17.958	1.6332
3200	0.137	2.326	2.463	2.512	1500	0	0.042	0.042	13.361	14.679	1.6206[b]
3400	0.067	1.605	1.673	2.196	1600	0	0.022	0.022	10.960	11.499	1.6241
3600	0.032	1.085	1.117	1.998	1700	0	0.082	0.082	8.913	9.170	1.6810
3800	0.015	0.719	0.733	1.887	1700	0	0.021	0.021	8.913	9.655	1.7407
4000	0.006	0.467	0.473	1.837	1700	0	0.019	0.019	8.913	9.655	1.8093
4200	0.003	0.298	0.301	1.830[a]	3900	0	0.305	0.305	0.010	0.010	1.8325
4400	0.001	0.186	0.187	1.854	4100	0	0.188	0.188	0.004	0.004	1.8547
4600	4.0e-4	0.114	0.115	1.897	4300	0	0.116	0.116	0.002	0.002	1.8978
					Panel B: $\rho_t = 0.5$						
2400	1.640	8.172	9.812	5.866	1000	5.1e-4	0.510	0.510	31.731	36.824	2.2627
2600	0.932	6.192	7.124	4.602	1000	4.3e-5	0.156	1.156	31.731	36.824	2.1656
2800	0.511	4.574	5.085	3.663	1100	4.1e-6	0.128	0.128	27.133	30.846	2.0842
3000	0.270	3.300	3.570	2.985	1100	3.0e-7	0.034	0.034	27.133	30.846	2.0356
3200	0.137	2.326	2.463	2.512	1100	2.0e-8	0.011	0.011	27.133	30.846	2.0137
3400	0.067	1.605	1.673	2.196	1200	1.3e-9	0.006	0.006	23.014	25.658	2.0070
3600	0.032	1.085	1.117	1.998	1700	9.9e-11	0.605	0.605	8.913	9.256	1.9422
3800	0.015	0.719	0.733	1.887	1900	5.0e-12	0.317	0.317	5.743	5.871	1.8799
4000	0.006	0.467	0.473	1.837	3900	1.1e-2	0.470	0.480	0.010	0.010	1.8403
4200	0.003	0.298	0.301	1.830[a]	4100	4.7e-3	0.295	0.300	0.004	0.004	1.8319[b]
4400	0.001	0.186	0.187	1.854	4300	1.8e-3	0.187	0.189	0.002	0.002	1.8544
4600	4.0e-4	0.114	0.115	1.897	4500	7.1e-4	0.115	0.115	0.001	0.001	1.8977
					Panel C: $\rho_t = 0.96$						
2400	1.640	8.172	9.812	5.866	1100	1.242	4.581	5.823	27.133	30.846	4.6897
2600	0.932	6.192	7.124	4.602	1200	0.676	1.050	1.726	23.014	25.658	3.7114
2800	0.511	4.574	5.085	3.663	1300	0.354	2.568	2.922	19.360	21.200	3.0901
3000	0.270	3.300	3.570	2.985	1500	0.178	2.030	2.208	13.361	14.679	2.6236
3200	0.137	2.326	2.463	2.512	1600	0.086	1.425	1.511	10.960	11.499	2.2885
3400	0.067	1.605	1.673	2.196	1700	0.040	0.461	0.501	8.913	9.255	2.0769
3600	0.032	1.085	1.117	1.998	1800	0.018	0.354	0.372	7.186	7.398	1.9500
3800	0.015	0.719	0.733	1.887	1900	0.008	0.265	0.272	5.743	5.871	1.8825
4000	0.006	0.467	0.473	1.837	3900	0.009	0.471	0.480	0.010	0.010	1.8400
4200	0.003	0.298	0.301	1.830[a]	4100	0.004	0.299	0.303	0.004	0.004	1.8317[b]
4400	0.001	0.186	0.187	1.854	4300	0.002	0.187	0.189	0.002	0.002	1.8543
4600	4.0e-4	0.114	0.115	1.897	4500	0.001	0.115	0.115	0.001	0.001	1.8976

This table presents the two-period minimum costs of a futures contract with and without price limits for various margin levels. It is assumed that there is a 25% reduction in volatility following a limit move. Panels A through C present the results for three precision levels of the signal (ρ_t = 0.1, 0.5, and 0.96). Under price limits, the minimum cost for a given margin is obtained by choosing an optimal-limit level that minimizes the contract cost over two periods. The same set of parameter values as in Brennan (1986) are used. L^* denotes the optimal-price-limit level for a given margin. Pr(Reneging) (%) and Pr($|z_t| \geq L$) (%) denote the probability of reneging and that of limit hit at time t, respectively. [a] and [b] refer to the minimum costs with and without price limits, respectively.

that when the price-limit rule has the real effect in reducing volatility, it can reduce margin, default probability, and contract cost only in the case of low-information precision. When information precision is high, the numerical results show that the optimal margin remains the same as that without price limits, and the limit rule neither can reduce the contract cost nor lower the optimal margin.

In addition to its impact on volatility, the price-limit rule also may have an impact on expected price changes. We consider an example for which the expected price change shifts backward by 100 [i.e., $\eta = -100$ (100) for an up (a down) limit move] and the volatility is reduced by 25% (i.e., $\delta = 0.75$). Table V shows that the optimal combination of margin and limit is (3200, 1500) for $\rho = 0.1$ and (4000, 3900) for $\rho = 0.5$ and 0.96. The margins are lower than without price limit, and so are their default probabilities and contract costs. That is, at $\eta = -100$ and $\delta = 0.75$, the cool-off effect is large enough to wipe out the spillover effect of price limits. The results show that when we assume that price limits can cool off the market and have a real effect in changing both the mean and variance of the price parameters, they can be effective in reducing the margin requirement, default risks, and contract costs.

However, whether price limits do have a cool-off effect on the volatility and expected returns following limit moves is an empirical issue. Empirical investigation of this issue, however, is not easy because price limits distort the observed-price process. If empirical evidence supports the cool-off effect of price limits, then price limits may be useful; otherwise, price limits seem to be useless.

CONCLUSION

This article investigates whether the imposition of price limits can reduce the default risk and lower the effective margin requirement for a self-enforcing futures contract by considering one more period beyond Brennan's (1986) one-period model to take into account the spillover of unrealized shocks due to price limits.

Overall, we find that when price limits do not change the underlying futures price process, but only delay the price to reach its equilibrium level, price limits may not be effective. Although price limits can reduce the margin requirement when traders have no additional information about the equilibrium futures price, the total contract cost increases because of a much higher liquidity cost due to trading interruptions. When traders receive additional signals about the equilibrium price, we find that price limits fail to obscure the expected loss occurring to the traders in

TABLE V

Minimum Costs of a Futures Contract Under Various Margin Levels with a Reduction in Volatility and Expected Price Change Following Limit Moves

	Without Price Limits				*With Price Limits*						
	Pr(Reneging) (%)					*Pr(Reneging) (%)*			*Pr($\lvert z_t \rvert \geq L$) (%)*		
Margin	$t = 1$	$t = 2$	*Sum*	*Total Cost*	L^*	$t = 1$	$t = 2$	*Sum*	$t = 1$	$t = 2$	*Total Cost*
					Panel A: $\rho_t = 0.1$						
2400	1.640	8.172	9.812	5.866	1100	0	0.139	0.139	27.133	30.846	1.8327
2600	0.932	6.192	7.124	4.602	1200	0	0.109	0.109	23.014	25.658	1.7288
2800	0.511	4.574	5.085	3.663	1300	0	0.080	0.080	19.360	21.795	1.6626
3000	0.270	3.300	3.570	2.985	1400	0	0.056	0.056	16.151	17.958	1.6274
3200	0.137	2.326	2.463	2.512	1500	0	0.039	0.039	13.361	14.679	1.6166[b]
3400	0.067	1.605	1.673	2.196	1600	0	0.027	0.027	10.960	11.499	1.6264
3600	0.032	1.085	1.117	1.998	1700	0	0.008	0.008	8.913	9.169	1.6427
3800	0.015	0.719	0.733	1.887	1700	0	0.017	0.017	8.913	9.655	1.7273
4000	0.006	0.467	0.473	1.837	1700	0	0.018	0.018	8.913	9.655	1.7979
4200	0.003	0.298	0.301	1.830[a]	3900	0	0.243	0.243	0.010	0.010	1.8014
4400	0.001	0.186	0.187	1.854	4100	0	0.149	0.149	0.004	0.004	1.8547
4600	4.0e-4	0.114	0.115	1.897	4300	0	0.090	0.090	0.002	0.002	1.8851
					Panel B: $\rho_t = 0.5$						
2400	1.640	8.172	9.812	5.866	1000	2.0e-4	0.273	0.273	31.731	36.824	2.1211
2600	0.932	6.192	7.124	4.602	1000	1.5e-5	0.075	0.075	31.731	36.824	2.1018
2800	0.511	4.574	5.085	3.663	1100	1.3e-6	0.062	0.062	27.133	30.846	1.9542
3000	0.270	3.300	3.570	2.985	1100	8.7e-8	0.015	0.015	27.133	30.846	1.9109
3200	0.137	2.326	2.463	2.512	1500	9.9e-9	0.586	0.586	13.361	14.000	1.8899
3400	0.067	1.605	1.673	2.196	1600	4.8e-10	0.471	0.471	10.960	11.369	1.8470
3600	0.032	1.085	1.117	1.998	1700	2.3e-11	0.435	0.435	8.913	9.169	1.8563
3800	0.015	0.719	0.733	1.887	3700	0.022	0.587	0.609	0.022	0.022	1.8247
4000	0.006	0.467	0.473	1.837	3900	0.010	0.377	0.387	0.010	0.010	1.7936[b]
4200	0.003	0.298	0.301	1.830[a]	4100	0.004	0.237	0.242	0.004	0.004	1.8010
4400	0.001	0.186	0.187	1.854	4300	0.002	0.147	0.149	0.002	0.002	1.8344
4600	4.0e-4	0.114	0.115	1.897	4500	0.001	0.089	0.090	0.001	0.001	1.8850
					Pane C: $\rho_t = 0.96$						
2400	1.640	8.172	9.812	5.866	1000	0.921	2.355	3.276	31.731	36.824	3.6221
2600	0.932	6.192	7.124	4.602	1000	0.492	2.017	2.508	31.731	36.824	3.3185
2800	0.511	4.574	5.085	3.663	1300	0.252	2.056	2.308	19.360	20.807	2.7766
3000	0.270	3.300	3.570	2.985	1500	0.124	1.642	1.766	13.361	14.000	2.4000
3200	0.137	2.326	2.463	2.512	1600	0.059	1.147	1.206	10.960	11.369	2.1343
3400	0.067	1.605	1.673	2.196	1700	0.027	0.795	0.822	8.913	9.169	1.9697
3600	0.032	1.085	1.117	1.998	1800	0.012	0.547	0.559	7.186	7.343	1.8762
3800	0.015	0.719	0.733	1.887	3700	0.019	0.588	0.607	0.022	0.022	1.8240
4000	0.006	0.467	0.473	1.837	3900	0.008	0.378	0.386	0.010	0.010	1.7931[b]
4200	0.003	0.298	0.301	1.830[a]	4100	0.004	0.238	0.241	0.004	0.004	1.8007
4400	0.001	0.186	0.187	1.854	4300	0.001	0.147	0.148	0.002	0.002	1.8343
4600	4.0e-4	0.114	0.115	1.897	4500	6.0e-4	0.089	0.0890	0.001	0.001	1.8849

This table presents the two-period minimum costs of a futures contract with and without price limits for various margin levels. It is assumed that there is a reduction in volatility by 25% and a reduction in expected price change by 100 following a limit move. Panels A through C present the results for three precision levels of the signal (ρ_t = 0.1, 0.5, and 0.96). Under price limits, the minimum cost for a given margin is obtained by choosing an optimal-limit level that minimizes the contract cost over two periods. The same set of parameter values as in Brennan (1986) are used. L^* denotes the optimal-price-limit level for a given margin. Pr(Reneging) (%) and Pr($\lvert z_t \rvert \geq L$) (%) denote the probability of reneging and that of limit hit at time t, respectively. [a] and [b] refer to the minimum costs with and without price limits, respectively.

the presence of large price movements, thereby fail to lower the effective margin. Consequently, the total contract cost over the whole period also increases with the imposition of price limits. Of course, when we assume price limits can cool off the market and have a real effect on the underlying price process, they may serve, for certain parameter values, as a partial substitute for margin requirement.

APPENDIX A: POSSIBLE RENEGING CONDITIONS IN THE TWO-PERIOD MODEL

This appendix presents all possible situations at times 1 and 2 in which the default decision is made. The yes(t) in the last column indicates the occurrence of reneging at time t.

I. If $M > 2L$

$t = 1$		$t = 2$		renege
$r_1 \geq L$	$Y_1 \geq Y_1^*$			yes(1)
	$Y_1 < Y_1^*$	$r_1 + r_2 \geq 2L$	$Y_2 \geq Y_{21}^*$	yes(2)
			$Y_2 < Y_{21}^*$	no
		$r_1 + r_2 < 2L$		no
$r_1 < L$		$r_2 \geq L$	$Y_2 \geq Y_{22}^*$	yes(2)
			$Y_2 < Y_{22}^*$	no
		$r_2 < L$	$r_1 + r_2 < M$	no

II. If M □ $2L$

$t = 1$		$t = 2$			renege
$r_1 \geq L$	$Y_1 \geq Y_1^*$				yes(1)
	$Y_1 < Y_1^*$	$r_1 + r_2 \geq M$			yes(2)
		$r_1 + r_2 < M$			no
$r_1 < L$		$r_2 \geq L$	$r_1 + L \geq M$		yes(2)
			$r_1 + L < M$	$Y_2 \geq Y_{23}^*$	yes(2)
				$Y_2 < Y_{23}^*$	no
		$r_2 < L$	$r_1 + r_2 \geq M$		yes(2)
			$r_1 + r_2 < M$		no

APPENDIX B: COST FUNCTIONS UNDER NORMALLY DISTRIBUTED PRICE CHANGES

This appendix presents the formulas for the cost components in the two-period model under the assumption that price changes follow a normal distribution. These formulas are used in the Numerical Examples section.

No Additional Information

If price changes are normally distributed, i.e., $r_t \sim N(0, \sigma_{r_t}^2), t = 1, 2, 3$, and traders receive no additional information about the equilibrium price, the margin required for the completely self-enforcing contract is $M = E[r_1 + r_2 | r_1 \geq L] = \sigma_{r_1}\lambda_1(\alpha_1)$ and the probability of a positive limit move, $P_r(r_1 \geq L)$, is $\Phi(-\alpha_1)$, where

$$\alpha_1 = \frac{L}{\sigma_{r_1}},\ \lambda_1(\alpha_1) = \frac{\phi(\alpha_1)}{1 - \Phi(\alpha_1)},$$

$$\delta_1(\alpha_1) = \lambda_1(\alpha_1)(\lambda_1(\alpha_1) - \alpha_1);\ \phi(\cdot) \text{ and } \Phi(\cdot)$$

are standard normal density and distribution function, respectively. However, in the two-period model, the margin must be set at $M = E[r_1 + r_2 + r_3 | r_1 \geq L, r_1 + r_2 \geq 2L]$ to ensure the contract to be default-free. $E[r_1 + r_2 + r_3 | r_1 \geq L, r_1 + r_2 \geq 2L]$ is given by

$$F\left(\frac{2L}{\sigma_{r_{12}}}, \frac{L}{\sigma_{r_1}}, \rho_{12}\right) \frac{E[r_1 + r_2 + r_3 | r_1 \geq L, r_1 + r_2 \geq 2L]}{\sigma_{r_{12}}}$$

$$= \phi\left(\frac{2L}{\sigma_{r_{12}}}\right)(1 - \Phi(A)) + \rho_{12}\phi\left(\frac{L}{\sigma_{r_1}}\right)(1 - \Phi(B)),$$

with

$$\sigma_{r_{12}} = \sqrt{\sigma_{r_1}^2 + \sigma_{r_2}^2},\ \rho_{12} = \frac{\sigma_{r_1}}{\sigma_{r_{12}}},\ A = \frac{\dfrac{L}{\sigma_{r_1}} - \rho_{12}\left(\dfrac{2L}{\sigma_{r_{12}}}\right)^2}{\sqrt{1 - \rho_{12}^2}},$$

$$B = \frac{\dfrac{2L}{\sigma_{r_{12}}} - \rho_{12}\left(\dfrac{L}{\sigma_{r_1}}\right)^2}{\sqrt{1 - \rho_{12}^2}},\ r_{12} = r_1 + r_2$$

and $F(.,.,.)$ is the the standard-bivariate-distribution function with the lower bounds $2L$ and L, respectively,

$$F\left(\frac{2L}{\sigma_{r_{12}}}, \frac{L}{\sigma_{r_1}}, \rho_{12}\right) = (2\pi\sigma_{r_1}\sigma_{r_{12}}\sqrt{(1 - \rho_{12}^2)})^{-1} \int_{2L} \int_{L}$$

$$\exp\left(-\frac{1}{2(1 - \rho_{12}^2)}\left(\left(\frac{r_1}{\sigma_{r_1}}\right)^2 - 2\rho_{12}\left(\frac{r_1}{\sigma_{r_1}}\right)\left(\frac{r_{12}}{\sigma_{r_{12}}}\right) + \left(\frac{r_{12}}{\sigma_{r_{12}}}\right)^2\right)\right)dr_1 dr_{12}.$$

In addition, the probability of a positive limit move, A_2, can be written as

$$A_2 = \Phi(-\alpha_2) + \Phi\left(\frac{\sigma_{r_1}\lambda_1(\alpha_1) + \sigma_{r_2}\lambda_2(\alpha_2) - 2L}{\sqrt{\sigma_{r_1}^2(1-\delta_1(\alpha_1)) + \sigma_{r_2}^2(1 - \delta_2(\alpha_2))}}\right)$$

$$\Phi(-\alpha_1)\Phi(\alpha_2),$$

where

$$\alpha_2 = \frac{L}{\sigma_{r_2}},\ \lambda_2(\alpha_2) = \frac{-\phi(\alpha_2)}{\Phi(\alpha_2)},\ \delta_2(\alpha_2) = \lambda_2(\alpha_2)(\lambda_2(\alpha_2) - \alpha_2).$$

With Additional Information

Suppose that traders receive additional information about the equilibrium futures price and the additional signal is assumed to be of the form $Y_t = r_t + \varepsilon_t$, where $cov(r_t,\varepsilon_t) = 0$, $cov(r_t,\varepsilon_k) = 0$, $cov(\varepsilon_t,\varepsilon_k) = 0$, $\varepsilon_t \sim N(0,\sigma_{\varepsilon_t}^2)$, and t, $k = 1, 2, 3$. Conditional on the signal Y_t, $r_t|Y_t \sim N(b_tY_t,\sigma_{\eta_t}^2)$, where $\sigma_{\eta_t}^2 = (1 - \rho_t^2)\sigma_{r_t}^2$ and $b_t = \rho_t^2 = \sigma_{r_t}^2/(\sigma_{r_t}^2 + \sigma_{\varepsilon_t}^2)$. Then, using the properties of truncated normal distribution, it can be shown that

$$E(r_1 + r_2|r_1 \geq L, Y_1) = b_1Y_1 + \sigma_{\eta_1}\lambda_1(q_1),$$

where

$$q_1 = \frac{L - b_IY_I}{\sigma_{\eta_1}} \text{ and } \lambda_1(q_1) = \frac{\phi(q_1)}{1 - \Phi_{(q_1)}}.$$

It follows that the critical signal level above which reneging occurs for a positive limit move, Y_1^*, is given by

$$b_IY_1^* + \sigma_{\eta_1}\lambda_1\left(\frac{L - b_IY_1^*}{\sigma_{\eta_1}}\right) = M.$$

The probability that trading will be halted due to a limit move, $P_r(|r_1| \geq L$, is $2\Phi(-\alpha_1)$ and the probability that reneging will occur, given Y_1^* and L, is

$$2\int_{L}^{\infty}\Phi\left(\frac{r_1 - Y_1^*}{\sigma_{\varepsilon_1}}\right)d\Phi\left(\frac{r_1}{\sigma_{r_1}}\right).$$

Hence, the contract cost at time 1, $C_1(M,L)$ can be written as

$$C_1(M,L) = kM + \frac{2\alpha\Phi(-\alpha_1)}{2\alpha\Phi(\alpha_1) - 1} + 2\beta\int_{L}^{\infty}\Phi\left(\frac{r_1 - Y_1^*}{\sigma_{\varepsilon_1}}\right)d\Phi\left(\frac{r_1}{\sigma_{r_1}}\right).$$

Now we consider the contract cost at time 2. If $M > 2L$, it follows from the properties of truncated normal distribution that the probability of reneging for a positive-price-limit move at time 2, B_2, can be written as

$$B_2 = \int_{L}^{\infty}\int_{2L-r_1}^{\infty}\int_{-\infty}^{Y_1^*-r_1}\int_{Y_{21}^*-r_2}^{\infty}\phi(r_1)\phi(r_2)\phi(\varepsilon_1)\phi(\varepsilon_2)d\varepsilon_2 d\varepsilon_1 dr_2 dr_1$$
$$+ \int_{-\infty}^{L}\int_{L}^{\infty}\int_{Y_{22}^*-r_2}^{\infty}\phi(r_1)\phi(r_2)\phi(\varepsilon_2)d\varepsilon_2 dr_2 dr_1.$$

On the other hand, if $M \leq 2L$, the probability of reneging is

$$B_2 = \int_{L}^{\infty}\int_{-\infty}^{Y_1^*-r_1}\int_{M-r_1}^{\infty}\phi(r_1)\phi(r_2)\phi(\varepsilon_1)dr_2 d\varepsilon_1 dr_1$$
$$+ \left(\Phi(\alpha_1) - \Phi\left(\frac{M-L}{\sigma_{x_1}}\right)\right)\Phi(-\alpha_2)$$
$$+ \int_{-\infty}^{M-L}\int_{L}^{\infty}\int_{Y_{23}^*-r_2}^{\infty}\phi(r_1)\phi(r_2)\phi(\varepsilon_2)d\varepsilon_2 dr_2 dr_1$$
$$+ \Phi\left(\frac{\sigma_{r_1}\lambda_2(\alpha_1) + \sigma_{r_2}\lambda_2(\alpha_2) - M}{\sqrt{\sigma_{r_1}^2(1 - \delta_2(\alpha_1)) + \sigma_{r_2}^2(1 - \delta_2(\alpha_2))}}\right),$$

with

$$\lambda_2(\alpha_1) = \frac{-\phi(\alpha_1)}{\Phi(\alpha_1)};\ \delta_2(\alpha_1) = \lambda_2(\alpha_1)(\lambda_2(\alpha_1) - \alpha_1).$$

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