EXAMINING FUTURES PRICE CHANGES AND VOLATILITY ON THE TRADING DAY AFTER A LIMIT-LOCK DAY

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This paper examines the effect that price limits have on futures prices by testing what happens to price changes and volatility on the trading day following a limit-lock day. The results show evidence that prices continue to rise on average the day after an up-limit day. In addition, limits appear to influence price volatility for some but not all of the futures contracts. However, since the findings vary across the different commodity futures contracts, it is likely that limits do not directly impact price volatility. © 2000 John Wiley & Sons, Inc. Jrl Fut Mark 20:445–466, 2000

INTRODUCTION

Daily price limits in futures markets set a maximum price range at which trades may occur. Unlike conventional trading halts or "circuit breakers,"

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The Journal of Futures Markets, Vol. 20, No. 5, 445–466 (2000)

where trading is automatically suspended after a price or index reaches a predetermined level, price limits allow trading to continue as long as transaction prices remain within certain limits. Often, however, when prices are at the limits, trading is essentially halted. These limits, which are voluntarily placed by futures exchanges, serve to delay gains and losses that might otherwise occur with large price swings. These delayed gains and losses assist the futures clearing house in ensuring against contract default or at the least in easing the flow of funds.

Besides helping against contract default, it has been argued that price limits may influence price changes and volatility. Research by Ma, Rao, and Sears (1989a) examine the price behavior around price limits. In their paper, they find that prices tend to either stabilize or reverse themselves after reaching a limit. Moreover, limits appear to reduce price volatility in the period after a limit movement. However, Miller (1989) and Lehmann (1989) question their results by stating that volatility is biased downward during the period after a limit move. Lehmann (1989) also adds that because their study does not incorporate a supply and demand model of liquidity, it is difficult to determine whether certain events may be attributed to limits.

In examining the Mini-crash of October 1989, Kuhn, Kuserk, and Locke (1991) find that limits do not help to quell volatility. This finding contrasts the work by Ma, Rao, and Sears (1989b), which shows that price limits appear to be effective in reducing volatility after a limit move. Related work by Kim and Rhee (1997) supports Kuhn et al. (1991) and shows that price limits are not effective in reducing volatility.

Despite the given literature, it remains uncertain whether limits facilitate or harm the trading and price discovery process. They could facilitate price resolution by bringing prices more quickly to their equilibrium prices, but could equally likely harm traders by disrupting the trading process. This disruption could cause greater price and market uncertainty and lead to greater short-term price volatility, while impeding the price discovery process. Given this uncertainty between the supposed and actual effects of price limits on price behavior, this paper studies the impact that price limits have on futures prices on the trading day after a price-limit day.

Examining futures prices for four agricultural commodities, the paper estimates the magnitude and significance of the effect that limits have on prices the trading day after a limit move. The empirical results vary across the grain futures contracts. Futures prices tend to rise the day after an up-limit day for corn and soybean futures but not for oat and wheat futures. In general, there is some evidence of price continuations but no evidence of reversals on the trading day following a limit day. Next, the paper examines whether volatility is affected by the presence of limits. Specifically, does volatility rise, fall, or remain the same after limit days? The findings are mixed, as volatility appears to rise when volatility is measured by the variance of the price changes, yet it falls using the variance of returns. The analysis reveals that although limits are correlated with volatility, whether they cause volatility to rise or fall remains uncertain. Changes in volatility are more likely attributed to imbalances in supply and demand and not tied to limits directly.

The paper is organized as follows. The next section describes the empirical hypotheses of the paper. The third section describes the data, price limits, and transition probabilities of limit days for futures contracts. The approach and empirical results of whether limits affect changes in futures price and price volatility are presented and interpreted in the fourth section. The fifth section summarizes the main results of the paper and concludes.

THE HYPOTHESES

Price limits are a particular type of circuit breaker that can close markets during unstable economic conditions. Although most researchers would agree that price limits help reduce credit and default risk, there are varying views on how price limits may or may not affect the price formation process.[1] Previous research in this area commonly asked whether limits stabilize prices or moderate volatility during times of market stress. This section describes the hypotheses and states the testable propositions.

Do Price Limits Affect Price Behavior?

Unfortunately, there is no generally accepted theory on how price limits influence price behavior. A magnet hypothesis says that prices move more quickly to their limits as prices move closer to their limits. In other words, price limits have a destabilizing effect by causing prices to gravitate to the limit prices. A study by Kuserk, Moriarty, Kuhn, and Gordon (1989) tests whether price movements speed up as prices approach a limit. They examine whether traders quickly close out their daily positions when

[1]Brennan (1986) offers a theory of price limits where the credit risk problem is alleviated because price limits act as partial substitutes for margins. Price limits function similarly to margin accounts by limiting the amount of price exposure risk, and thus capping the size of the daily margin call. Additionally, they offer extra time for the clearing association to collect margins and for traders to free capital so as to meet margin calls.

prices are approaching their daily limits, thus creating a "gravitational" or "magnet" effect to the limits. However, instead of finding evidence of a magnet effect, they find the opposite to be true. Prices actually appear to slow down as they approach the daily limit values. These findings are supported by Arak and Cook (1997). After studying eight years of treasury bond futures data, they present evidence against a magnet effect.

Another hypothesis concerning price changes is the delayed price discovery hypothesis. It says that limits slow down the price discovery process by preventing prices from effectively reaching their equilibrium values. Kim and Rhee (1997) find empirical support for this hypothesis in equity markets. They observe a higher frequency of price continuations than reversals for days after limit days.

This paper asks whether price limits delay price discovery in the major grain futures markets. The null hypothesis is that there is no significant change in price the day after a limit-lock day. In other words, it tests if the expected change in price after a limit-lock day is zero. Generally speaking, the basis for the null hypothesis is that futures prices follow a Martingale, $E[P_{t+1} - P_t | P_t, P_{t-1}, P_{t-2}, \ldots] = 0$, which is a weaker form of the random walk hypothesis. If prices are completely random, equally likely to go up or down, then the expected change in price should be insignificantly different from zero.

A rejection of the null hypothesis would provide evidence of price continuations for up-limit days (down-limit days) when $\Delta P > 0$ ($\Delta P < 0$), and reversals when $\Delta P < 0$ ($\Delta P > 0$). If the null cannot be rejected, it suggests that limits do not affect the price discovery. To distinguish continuations from reversals, the source of a limit move must first be identified. Specifically, limit movements induced by news alter the conditional expected return toward the same direction as the limit move. Therefore, if a limit is hit and trading stops, then, absent additional news reducing the equilibrium price, prices should continue to move in the same direction as the limit on the next trading day. However, if the limit move is caused by traders overreacting to news, then one should expect to observe a price reversal. Thus, for price reversals, one should expect that the conditional expected return the following day will be the opposite of the limit move.

Before proceeding further, it should be clearly stated that a rejection of the hypothesis, that is, evidence of positive or negative expected changes, is not evidence against the Martingale property. The reason is that the Martingale property looks only at expected returns, *not* risk. Once corrected for risk, Harrison and Kreps (1979) find that the Martingale property does indeed hold. Therefore, any observed predictability is likely to stem from not having directly modeled the risk.

Do Price Limits Moderate Volatility?

The market crash of October 1987 spurred the Brady Commission to recommend the use of circuit breakers. Circuit breakers are mechanisms that are used to reduce excess volatility by halting markets. Since their imposition, there have been many studies that test if circuit breakers actually moderate volatility.[2] Although circuit breakers may have short-term effects on prices and volatility by halting trading activity, it is unclear whether limits affect volatility in the long run.

The issues pertaining to circuit breakers also apply to limits, since they sometimes act as circuit breakers and suspend trading. For example, do limits offer a period for markets to "cool down" or do they magnify the panic? Most critics of circuit breakers and price limits believe that they are not effective in moderating excess volatility. Although some price volatility is needed to have a successful futures market, excessive volatility over a short period of time is generally considered undesirable. An overreaction hypothesis says that limits help to quell excessive volatility caused by an overreaction of traders to news. Ma, Rao, and Sears (1989b) find evidence supporting an overreaction hypothesis by analyzing volatility around limit events. However, since the overreaction hypothesis is premised on distinguishing between news and panic situations, it is difficult to test, even ex post. The reason is simply that days of panicked trading are difficult if not impossible to distinguish from dramatic news days. Rather than test the overreaction hypothesis, this paper examines if a limit-lock day affects volatility on the following trading day. This would provide a general test for whether limit events influence price volatility.

FUTURES PRICE DATA AND TRANSITIONAL PROBABILITIES

The Data

The analysis employs daily futures prices from January 2, 1986, to July 20, 1998, for four agricultural futures contracts. Daily information for

[2]See Roll (1989), Miller (1991), and Harris (1997) for a review of circuit breakers, price limits, and market crashes. See also, Kuhn et al., Greenwald and Stein (1991), and Santoni and Lui (1993).

TABLE I

Daily Price Limits, Contract Months and Trading Hours for Corn, Oat, Soybean and Wheat Futures Contracts

Futures Contract	*Daily Price Limit**	*Contract Months*	*Trading Hours*
Corn	10 cents per bushel 12 cents per bushel	December, March, May July, & September	9:30am to 1:15pm
Oat	10 cents per bushel	July, September, December, March, & May	9:30am to 1:15pm
Soybean	30 cents per bushel	September, November, January March, May, July, & August	9:30am to 1:15pm
Wheat	20 cents per bushel	July, September, December March, May	9:30am to 1:15pm

*Daily price limits are set symmetrically above and below the previous day's settlement price. If 3 or more contracts of the same year reach their limits, then price limits are expanded 150% for the next 2 trading days. The price limit for corn futures was increased from 10 cents 12 cents per bushel on March 12, 1992.

corn, oats, soybeans, and wheat contracts—that is, open, close, high, low, and settlement prices; open interest; and volume data—is taken from Chicago Board of Trade (CBT). The data have been collected from the permanent records of the Commodity Futures Trading Commission. Since price limits are lifted just prior to the spot delivery month, data are only for the nearby futures contracts.[3]

The paper examines more than one futures contract to determine how results vary across the different commodity contracts. Table I summarizes each contract's daily price limit, the contract months, and trading hours. Since the corn sample had its price limit increased from 10 to 12 cents on March 12, 1992, the corn sample is divided into two subsamples. The table also indicates the contract months for each commodity, which are ordered according to the beginning of the new contract year. For example, the December contract is the start of a new crop year for corn futures.

Descriptive statistics for each of the futures contracts are presented in Table II. Oat futures have the lowest average price (\$1.55), while soybean futures have the highest price (\$6.33). The column to the right of the average price shows the relative size of the commodity's daily limit to its average price. The limits are found to be between 5 and 6% of its average price. Differences in trading activity can be measured by examining open interest and volume. These two variables illustrate that the markets for corn and soybeans are substantially larger than the markets for oats and wheat.

[3]The nearby futures contracts are those futures contracts that are closest to the spot delivery month, but not the delivery month.

TABLE II

Descriptive Statistics for Corn, Oat, Soybean and Wheat Futures from 1/2/86 to 7/20/98

Futures Contract	*Average Price ($)*	*Ratio of Limit to Average Price**	*Average Volume*	*Average Open Interest*	*Ratio of Volume to Open Interest*
Corn	2.53	0.05	22,228	91,779	0.24
Oat	1.55	0.06	822	5,560	0.15
Soybean	6.33	0.05	19,384	43,350	0.45
Wheat	3.56	0.06	7,003	25,752	0.27

*To calculate this ratio for corn, the latest limit value of $0.12 was used, and not its previous value of $0.10.

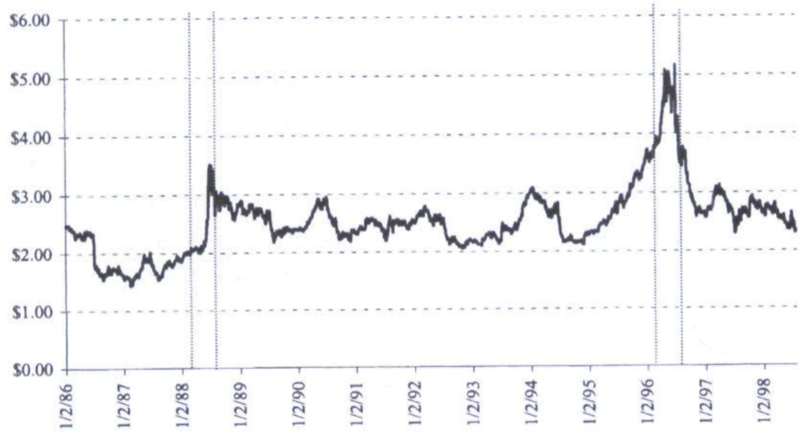

FIGURE 1
Daily corn futures price from January 1986 to July 1998.

Besides price discovery, futures markets are used to transfer risk from hedgers to speculators. If it is true that hedgers tend to hold their positions longer than speculators, then the last column, which displays the ratio of open interest to volume, is a measure of average speculative activity. With speculators moving quickly in and out of various futures positions and avoiding overnight positions, a higher ratio implies greater average speculative activity in one market relative to another. By making this assumption, the table reveals that the market for soybean futures has three times more speculative activity and liquidity than the market for oat futures.

Figures 1 to 4 plot the daily price for each of the futures contract over the entire sample. All four contracts have experienced two periods of high volatility in 1988 and 1996. The episodic periods of volatility are

FIGURE 2
Daily oat futures price from January 1986 to July 1998.

shown by the dotted vertical lines. The first period in 1988 was caused by a severe nationwide draught, which drastically cut supplies. This led oat futures price to soar to its sample high price above \$3.50 and soybean futures price to rise above \$10.00. The second episode in 1996 was mainly initiated by a low carryover of inventory from the previous harvest year and strong export demand. The resulting shortage led spot prices to lie above futures prices. It was during this period that both corn and wheat futures reached their sample high prices. As a result, limits were hit more frequently over these two time periods, between 5 and 10%.

Limit-Bounce and Limit-Lock Days

Daily price limits are set symmetrically above and below the previous trading day's futures settlement price.[4] Table III summarizes the number of up-limit, down-limit, and near-limit days for each of the four agricultural futures contracts. Suppose λ is the limit, the total number of up-limits days are when $H_t - S_{t-1} \geq \lambda$, where H_t is the daily high price and S_{t-1} is yesterday's settlement price. Down-limit days are when $S_{t-1} - L_t \geq \lambda$, where L_t is the daily low price. Limit-lock days are trading days that ends at the limit price. These days have the current day's settlement price greater than or equal to the previous trading day's settlement price plus or minus the standard limit.[5] Those days when limits are hit but trading

[4]The settlement price is a price set by the clearinghouse after trading ends for clearing members to settle all accounts for each open positions. They are also used to calculate margin calls and delivery invoice prices. The close price is the officially labeled price that is recorded at the trading pit in the last moments of trading for that day.

[5]Under CBT price limit rules, price limits are expanded 150% for the next two trading days if three or more futures contracts of the same commodity reach their limits on the same day. For more specific information regarding trading limits, see CBT Rule 1008.01.

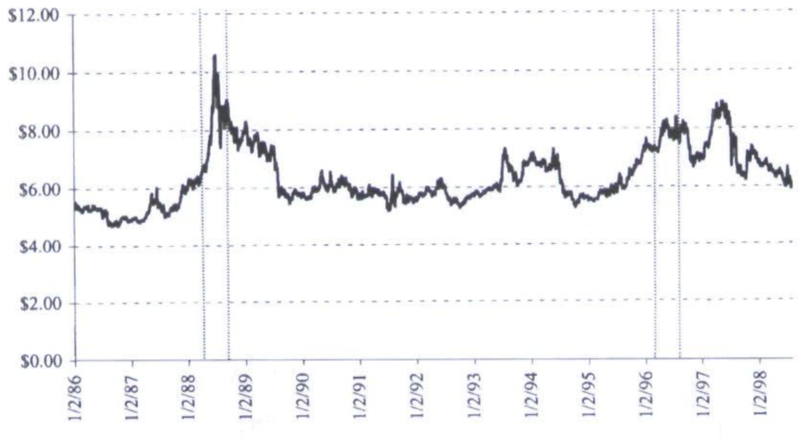

FIGURE 3
Daily soybean futures price from January 1986 to July 1998.

FIGURE 4
Daily wheat futures price from January 1986 to July 1998.

continues within the limits are called limit-bounce days. Near-limit days are days with price changes greater than 75%, but less than 100% of the price limit.[6]

As noted earlier, the paper uses only the nearby futures contract because price limits are lifted two days prior to the start of the spot delivery month. In addition, this study examines only limit-lock days and

[6]The choice of what constitutes a "near-limit" movement is rather subjective. This paper defines a near limit to be price changes between 75 to 100%. To test whether the results are sensitive to this choice, the regressions were rerun using a near limit of prices falling between 90 and 100%. The results of using 90 to 100% were not substantially different from the tests using 75 to 100%. Similar to limits, near limits are rare occurrences. This seems to indicate that price changes on near-limit days may also come from a different price distribution, because as the price approaches a limit, it might gravitate to the limit as it gets closer.

TABLE III

Up and Down Price Limit Movements for Nearby Futures Contracts for Corn, Oat, Soybean and Wheat from 1/2/86 to 7/20/98

			Up Limit Days			*Down Limit Days*		
Futures Contract	*Near-Up Limit Days**	*Near-Down Limit Days**	*Limit-Bounce*	*Limit-Lock*	*Total Limits*	*Limit-Bounce*	*Limit-Lock*	*Total Limits*
Corn	20	26	34	43	77	34	36	70
Oat	28	34	26	45	71	39	43	82
Soybean	24	32	25	23	48	20	27	47
Wheat	14	14	15	15	30	22	8	30
Total	86	106	100	126	226	115	114	229

*A near-limit day is when the price moves more than 75% but less than 100% of the set daily limits. A limit-bounce day is when a limit is hit but prices later move back within the limits. A limit-lock day is when trading suspends for that day after a limit price is reached.

eliminates limit-bounce days. The reason for doing this is that limit-bounce days, by definition, will show prices stabilizing or reversing themselves. Table III indicates that the number of limit-lock days are relatively infrequent and account for fewer than 1% of the total observations.

Transition Probabilities

The occurrence of limit-lock days in futures markets is a rare event, with the number of limits hit for the sample, corn, oat, soybean, and wheat futures contracts occurring less than 1% of the time. However, the probability of hitting a limit on the day following a limit move is greater than a 1%. Therefore, this section examines the sample conditional probabilities of futures price data through the use of computed Markov Chains.

Markov Chains

Let $\{X_t, \text{for } t = 0,1,2, \ldots\}$ be defined as a countable stochastic process, such that when $X_t = i$, the process is in state i at date t. The conditional probability of being in state j tomorrow, given being in state i today, is compactly expressed as $\Pr[X_{t+1} = j|X_t = i] \equiv p_{ij}$. A stochastic process statisfies the Markov property if

$$\Pr[X_{t+1} = j|X_t = i, X_{t-1} = i_{t-1}, X_{t-2} = i_{t-2}, \ldots X_0 = i_0]$$
$$= \Pr[X_{t+1} = j|X_t = i]. \tag{1}$$

This equation says the past and future are statistically independent once the present state is given. In other words, all that is required in determining the probability of being in state j is the present state i; the previous states do not matter.[7]

By using Markov Chains to describe the stochastic process, we can conveniently present the sample transition probabilities from one state to the next. Although the Markov Chains take these probabilities as stationary, it nonetheless provides a rough estimate of what the true conditional probabilities are when the current state is a limit-lock day. The empirical transition matrix is defined as:

$$P \equiv \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ p_{31} & p_{32} & p_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{2}$$

where p_{ij} is the one-step transition probability of going from state i to state j.

Sample Conditional Probabilities

To begin, consider the simple case of a world with only two states for futures price changes, either "limit" or "no limit." In this situation, one could characterize the transition probabilities with a 2-by-2 matrix. A breakdown of contingencies is as follows: p_{11} = Prob(Limit|Limit), p_{12} = Prob(No Limit|Limit), p_{21} = Prob(Limit|No Limit), and p_{22} = Prob (No Limit|No Limit). Each of these sample conditional probabilities is computed by counting the frequency of limit days, given that a limit occurred or did not occurred the day before. These probability values are reported in Table IV.

In Section A of Table IV, the probability of hitting a limit tomorrow, given a limit today, p_{11} ranges from 0.3036 for wheat to 0.3626 for corn. It implies roughly a one-third chance of hitting a limit tomorrow, given that a limit is reached today. These probability values are much larger than the probabilities of hitting a limit tomorrow, given that no limit is reached today, p_{12}, where there is approximately a 1% chance of hitting a limit the next day. In general, these matrices show that there is a much higher probability of hitting a limit the next day, given that today is a limit day, than given that today is not a limit day.

Now consider the case of three states, an "upper limit," "lower limit,"

[7]For a textbook treatment of Markov processes, see Ross (1997).

TABLE IV

Transition Probability Matrices

A. Two States: Limit (L) and No Limit (NL)				
Futures Contract	*Pr(L|L)*	*Pr(NL|L)*	*Pr(L|NL)*	*Pr(NL|NL)*
Corn	0.3626	0.6374	0.0188	0.9812
Oats	0.3258	0.6742	0.0195	0.9805
Soybeans	0.3036	0.6964	0.0125	0.9875
Wheat	0.3043	0.6957	0.0051	0.9949

B. Three States: Upper Limit (UL), Lower Limit (LL), and No Limit (NL)									
Futures Contract	*Pr (UL|UL)*	*Pr LL|UL)*	*Pr (NL|UL)*	*Pr (UL|LL)*	*Pr (LL|LL)*	*Pr (NL|LL)*	*Pr (UL|NL)*	*Pr (LL|NL)*	*Pr (NL|NL)*
Corn	0.2800	0.0732	0.6468	0.1000	0.2683	0.6317	0.0117	0.0097	0.9786
Oats	0.3200	0.1282	0.5518	0.0200	0.1795	0.8005	0.0110	0.0104	0.9786
Soybeans	0.2308	0.0667	0.7026	0.0385	0.2667	0.6949	0.0064	0.0071	0.9865
Wheat	0.2667	0.3750	0.3583	0.0000	0.0000	1.0000	0.0035	0.0025	0.9940

and "no limit." This implies that the transition probabilities are: p_{11} = Prob(Upper Limit|Upper Limit), p_{12} = Prob(Lower Limit|Upper Limit), p_{13} = Prob(No Limit|Upper Limit), p_{21} = Prob (Upper Limit|Lower Limit), p_{22} = Prob (Lower Limit|Lower Limit), p_{23} = Prob (No Limit|Lower Limit), p_{31} = Prob (Upper Limit|No Limit), p_{22} = Prob (Lower Limit|No Limit), and p_{23} = Prob (No Limit|No Limit). These probabilities are summarized in Section B of Table IV.

The table shows that the probability of reaching an upper limit tomorrow, given that today is an upper limit ranges from 0.2308 for soybeans and to 0.3200 for oats. On average, these values are higher than the probabilities of reaching lower limits, given a lower limit today, which ranges from 0.0000 for wheat to 0.2683 for corn. Note that for wheat, $p_{21} = p_{22} = 0$. This outcome is a reflection of wheat having only five down-limit days; see Table II. The one-step transition probabilities of Table IV show the relatively high frequency of limit occurrences given that a limit day has occurred.

Suppose that p_{11} and p_{22} represent the probability of price continuations and that p_{12} and p_{21} represent the probability of price reversals. If this holds, then it appears that, on average, continuations are more likely to occur than reversals. Hence, the results suggest that limits occur more often due to news than the overreaction of traders.

METHODOLOGY AND EMPIRICAL RESULTS

Futures Price Changes and Volatility

To measure whether limits have an affect on price change, the average price change on the trading day after a limit-lock day is estimated.[8] This estimate is later compared to trading day after a near-limit-move day. The specification of the dummy variable regression is given by:

$$\Delta P_t = \beta_0 + \beta_1 D_t^{ul} + \beta_2 D_t^{ll} + \beta_3 D_t^{nul} + \beta_4 D_t^{nll} + e_t, \qquad (3)$$

where $\Delta P_t = P_t - P_{t-1}$ is the change in the settlement price for futures

$D_t^{ul} = 1$ if it is the day that follows an upper limit movement
$= 0$ otherwise,
$D_t^{ll} = 1$ if it is the day that follows an lower limit movement
$= 0$ otherwise
$D_t^{nul} = 1$ if it is the day that follows a near-upper limit movement
$= 0$ otherwise
$D_t^{nll} = 1$ if it is the day that follows a near-lower limit movement
$= 0$ otherwise

This model provides estimates of price changes on post-limit and post-near-limit days, and information pertaining to whether these days are consequential or not. Under this specification, the null hypothesis is $D_t^j = 0$ for $j = ul, ll, nul$, and nll. Similar to Kim and Rhee (1997), the dummy variables of post-near-limit days, *nul* and *nll*, are being used as control variables. By doing so, it will help to identify whether certain effects on prices are connected to limit movements.

The intercept term provides the conditional mean change in futures price excluding post-limit and near post-limit days, since $E(\Delta P_t | D_t^{ul} = D_t^{ll} = D_t^{nul} = D_t^{nll} = 0) = \beta_0$. The mean future price change the day after an upper limit movement is $E(\Delta P_t | D_t^{ul} = 1, D_t^{ll} = D_t^{nul} = D_t^{nll} =$

[8]The dependent variable, changes in futures price, is bounded above and below by limits. Therefore, true price realizations above and below the limits are nonobservable. The censored observations cause ordinary least squares (OLS) estimates to be inconsistent. One approach to obtain consistent parameter estimates is the employ a Tobit model. However, a Tobit model faces the limitation of not being able to simultaneously account for the impact that limits may have on the variance of price changes. An earlier version of this paper ran a heteroscedastic Tobit model that assumes a variance unaffected by limits. Although the Tobit model showed stronger evidence of both limits and near-limit days impacting price changes than the GARCH model, the likely explanation is that the limits affect the variance. This point was made by an anonymous referee. Indeed, this is the case, as shown by the results of the estimates of the variance equation. Therefore, this article only presents the GARCH results. The findings of the Tobit model are available upon request.

$0) = \beta_0 + \beta_1$. The mean future price change the day after a lower limit movement is $E(\Delta P_t | D_t^{ll} = 1, D_t^{ul} = D_t^{nul} = D_t^{nll} = 0) = \beta_0 + \beta_2$. The mean future price change the day after a near-upper limit movement is $E(\Delta P_t | D_t^{nul} = 1, D_t^{ul} = D_t^{ll} = D_t^{nll} = 0) = \beta_0 + \beta_3$. The mean future price change the day after a near-lower limit movement is $E(\Delta P_t | D_t^{nll} = 1, D_t^{ul} = D_t^{ll} = D_t^{nul} = 0) = \beta_0 + \beta_4$.

To estimate these parameters, use a Generalized Autoregressive Conditional Heteroskedastic (GARCH) model.[9] A GARCH model is employed because often in financial markets, periods of large returns (in both directions) tend to be followed by even larger returns. In other words, periods of volatility are frequently observed to be serially correlated. The GARCH model is a suitable technique, since it simultaneously estimates the effect of limits on both the conditional mean and variance equations. For estimation purposes, expanded limit days, where price changes are above the normal price limits are eliminated. Multiple limit days, where the limits have expanded, are removed from the sample because such price phenomena are beyond the given model.[10]

The GARCH model is specified by equation (3) and the following conditional variance equation:

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_3 D_t^{ul} + \alpha_4 D_t^{ll} + \alpha_5 D_t^{nul} + \alpha_6 D_t^{nll} + \sum_{i \neq \text{March}}^{\text{May}} \alpha_i Q_t^i \tag{4}$$

where α_0 is the mean variance, e_{t-1}^2 is the ARCH term representing previous news, σ_{t-1}^2 is the previous forecast of the variance or GARCH term, D_t^j represent post-limit and near post-limit days, and Q_t^i are dummy variables for contracts months. The contract month dummy variables help to control for periodic changes in volatility.[11] Because futures price data exhibits conditional nonnormality, usually in the form of excess kurtosis, a quasi-maximum likelihood (QML) covariance matrix will be estimated;

[9]Engel (1982) first formally introduced ARCH models, and later Bollerslev (1986) generalized the ARCH (GARCH) models to include lagged values of the forecast variance.

[10]The number of observations deleted for the corn, oat, soybean, and wheat samples are 29, 24, 18, and 7, respectively. Deleting observations is one method to remove the problem of successive limit movements. However, removing selected observations exposes the test results to a sample selection bias. Miller (1989) notes the difficulty of the issue of successive limit moves and says that there is no generally accepted resolution.

[11]Anderson (1985) has shown that price volatility varies with seasonality and maturity. This regression corrects for seasonality by using dummy variables for contract months. The maturity effect need not be corrected because the data are not spread over the entire life of the contract, only the nearby futures contracts.

ployed because often in financial markets, periods of large returns (in

TABLE V

A Model of Price Change Volatility on Post-Limit Days, Near-Post-Limit Days, and Contract Months Under A GARCH (1,1) Model

Panel A

Futures Contract	β_0	β_1	β_2	β_3	β_4	*F-statistic*
Corn	−0.0002	0.0241	0.0039	−0.0023	0.0066	0.8425
Sample I	(0.0006)	(0.0133)	(0.0129)	(0.0171)	(0.0086)	(0.6302)
Corn	−0.0002	0.0347	−0.0052	−0.0099	0.0039	1.5709
Sample 2	(0.0006)	(0.0080)**	(0.0225)	(0.0162)	(0.0087)	(0.0744)
Oat	−0.0007	−0.0003	−0.0167	−0.0050	0.0031	0.3017
	(0.0004)	(0.0118)	(0.0093)	(0.0083)	(0.0065)	(0.9954)
Soybean	−0.0002	0.0586	0.0002	−0.0218	0.0185	0.8657
	(0.0010)	(0.0269)*	(0.0365)	(0.0251)	(0.0122)	(0.6158)
Wheat	−0.0004	−0.0304	−0.0702	0.0015	0.0106	0.3994
	(0.0007)	(0.0393)	(0.0378)	(0.0191)	(0.0149)	(0.9798)

This table presents results for a GARCH (1,1) model with on post-limit days and contract months as regressors for the following variance equation:

$$\Delta P_t = \beta_0 + \beta_1 D_t^{ul} + \beta_2 D_t^{ll} + \beta_3 D_t^{nul} + \beta_4 D_t^{nll} + e_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_3 D_t^{ul} + \alpha_4 D_t^{ll} + \alpha_5 D_t^{nul} + \alpha_6 D_t^{nll} + \sum_{i \neq \text{March}}^{\text{May}} \alpha_i Q_t$$

where $\Delta Pt = P_t - P_{t-1}$, e_{t-1}^2 is the ARCH term representing previous news, σ_{t-1}^2 is the previous forecast of the variance or GARCH term, and D_t are the dummy variables that equal 1 on the trading day that follows an upper limit, ul, lower limit, ll, near-upper limit, nul, and near-lower limit, nll, and Q_t are dummy variables that equal 1 for the various futures contract months. The GARCH model is used to test whether the day after limits and the day after limits are almost reached affect price change and volatility. The sample of daily observations, which starts from 1/2/86 and ends on 7/20/98, excludes expanded limit observations. The futures contract for corn is divided into two samples, since there is a change in the price limit from 10 cents to 12 cent in the middle of the sample, on March 12, 1992. The Bollerslev and Wooldridge heteroskedasticity-consistent standard errors are given in parentheses.

see Bollerslev and Wooldridge (1992).[12] The results of the above GARCH (1,1) model are presented in Table V.[13]

The Empirical Effects on Price Changes

This section presents the GARCH estimates of changes in futures price on the day after limit-lock and near-limit days. The estimates for the mean

[12]Because futures price data exhibit conditional nonnormality, usually in the form of excess kurtosis, a quasi-maximum likelihood (QML) covariance matrix will be estimated; see Bollerslev and Wooldridge (1992). Given that the conditional normality of the residuals are not likely to hold, the estimated covariance will not be consistent under normal maximum likelihood. Therefore, this article estimates the Bollerslev and Wooldridge (1992) robust standard errors, which do not alter the parameter values, but furnish a covariance matrix that is consistent under conditional nonnormality.

[13]The results of a GARCH(1,1) model is presented for simplicity, since the results are not considerably altered under alternative GARCH lag specifications.

TABLE V (Continued)

A Model of Price Change Volatility on Post-Limit Days, Near-Post-Limit Days, and Contract Months Under A GARCH (1,1) Model

Panel B

Futures Contract	α_0	α_1	α_2	α_3	α_4	α_5	α_6
Corn	1.19×10^{-5}	0.0429	0.9269	0.0004	−0.0003	0.0004	−0.0001
Sample I	(6.63×10^{-6})	(0.0153)**	(0.0208)**	(0.0003)	(0.0003)	(0.0003)	(0.0002)
Corn	-1.69×10^{-6}	0.0613	0.9475	−0.0005	−0.0006	4.42×10^{-5}	−0.0006
Sample 2	(2.00×10^{-6})	(0.0117)**	(0.0109)**	(0.0002)*	(0.0003)*	(0.0003)	(0.0002)**
Oat	1.24×10^{-5}	0.1086	0.8682	0.0001	−0.0002	−0.0001	−0.0003
	(4.85×10^{-6})**	(0.0175)**	(0.0192)**	(0.0003)	(0.0003)	(0.0002)	(0.0002)
Soybean	-7.28×10^{-6}	0.0663	0.9404	−0.0014	−0.0045	−0.0017	−0.0043
	(1.16×10^{-5})	(0.0109)**	(0.0089)**	(0.0013)	(0.0019)*	(0.0013)	(0.0011)**
Wheat	3.64×10^{-5}	0.0587	0.9212	0.0036	0.0008	−0.0013	0.0001
	(1.20×10^{-5})**	(0.0106)**	(0.01152)**	(0.0016)*	(0.0019)	(0.0008)	(0.0012)

Futures Contract	$\alpha_{January}$	α_{May}	α_{July}	α_{August}	$\alpha_{September}$	$\alpha_{November}$	$\alpha_{December}$
Corn	—	6.24×10^{-6}	4.87×10^{-5}	—	8.79×10^{-7}	—	1.30×10^{-6}
Sample 1	—	(6.19×10^{-6})	(1.65×10^{-5})**	—	(1.12×10^{-5})	—	(7.48×10^{-6})
Corn	—	9.76×10^{-6}	2.41×10^{-5}	—	-1.21×10^{-5}	—	-7.74×10^{-7}
Sample 2	—	(3.68×10^{-6})**	(1.10×10^{-5})*	—	(6.45×10^{-6})	—	(3.38×10^{-6})
Oats	—	1.73×10^{-5}	4.21×10^{-5}	—	7.88×10^{-6}	—	2.48×10^{-6}
	—	(7.39×10^{-6})*	(1.30×10^{-5})**	—	(9.08×10^{-5})	—	(4.98×10^{-6})
Soybeans	-1.00×10^{-5}	5.90×10^{-5}	0.0002	1.22×10^{-5}	-7.54×10^{-5}	2.58×10^{-5}	—
	(1.50×10^{-5})	(1.95×10^{-5})**	(4.54×10^{-5})**	(7.05×10^{-5})	(5.10×10^{-5})	(2.54×10^{-5})	—
Wheat	—	2.04×10^{-5}	1.39×10^{-5}	—	-9.42×10^{-6}	—	3.10×10^{-6}
	—	(1.71×10^{-5})	(1.50×10^{-5})	—	(1.30×10^{-5})	—	(1.21×10^{-5})

**Significance at the 1% level.
*Significance at the 5% level.

equation are reported under Panel A of Table V. The intercept term, measuring average price changes, are not significantly different than zero for all four commodities. In general, the F-statistics, which test the joint significance of the dummy variables, are low. These findings are consistent with futures prices following a Martingale.

The post upper-limit days are statistically significant at the 1% and 5% level for the second corn sample and soybean sample. For the second corn sample, the price is rising on average by 3.5 cents and for the soybean sample, the price is rising by 5.9 cents on days following an upper-limit lock day. The evidence suggests price continuations after limits for corn and soybean futures prices. Although the first corn sample is marginally significant, at the 6% level, the remaining estimates for the upper-limit dummy variables do not reveal a rise in prices on the following trading day.

On the trading day following lower-limit-lock days, there is no strong evidence of either price continuations or reversals after a down limit-lock day. Oats and wheat futures price changes fall on average by 1.7 cents and 7 cents, but their statistical significance is weak at the 7% and 6% levels, respectively. These estimates, consistent with the transition probabilities, suggest that price continuations are more probable after limits than price reversals.[14] The results also agree with Kim and Rhee (1997), who also find evidence of price continuations. The existence of price continuations after limits implies that limits delay price discovery. Last, the control variables, which indicate the day after limits are nearly reached, are insignificant for all the commodities. As noted previously, the insignificance of post near-limit days offer additional support of delayed price discovery.[15]

The Empirical Effects on Price Volatility

This section presents the estimates of the conditional variance equation, equation (4), of the GARCH (1,1) model. The model tests whether futures price volatility is affected by price limits on the trading day after a limit-lock day. Panel B of Table V shows that the ARCH and GARCH terms are all highly significant at the 1% level.[16] Thus, the conditional variance reflects both past news and the previous variance forecast. The May and July contract months show an increase in volatility for the corn, oats, and soybean samples. Hence, futures price volatility varies periodically over the contract year.

Testing the relevancy of post-limit days on volatility, the paper finds that after an up-limit day, volatility falls slightly for the second corn sam-

[14]As stated earlier, these results do not necessarily refute the random walk hypothesis. First, violations of a Martingale are most likely caused by the trade-off between risk and return. Second, the Martingale property is neither a necessary nor a sufficient condition for rational asset prices; see Lucas (1978).

[15]Kim and Rhee (1997) provide evidence of a delayed price discovery process in equity markets. However, unlike equity markets, futures markets do not structurally impose an effort to maintain continuous prices.

[16]The sum of the parameter estimates of previous news and previous variance forecast nearly equals 1. If it is true that $\sigma_1 + \sigma_2 = 1$, then the GARCH model has a unit autoregressive root, integrated of order 1, which is called an IGARCH model. This implies that today's volatility affects forecasts of volatility indefinitely. An IGARCH (1,1) is stationary, but not covariance stationary. The samples for corn and soybeans have the mean parameter insignificantly different than zero. In such cases, volatility follows a Martingale, and thus the model has a stationary distribution; see Nelson (1990). After running coefficient tests of whether $\sigma_1 + \sigma_2 = 1$, the hypothesis is rejected for oats at the 5% level and for wheat at the 1% level. The samples for corn and soybean cannot reject the null of a unit autoregressive root.

ple but rises for the wheat sample. However, post-up-limit days do not appear to impact volatility for the remaining futures contracts. Since none of the control post near-limit-up days significantly influence volatility, it appears that volatility is related with up-limits, and not simply large price movement days.

On the day following down-limit days, volatility tends to fall for the second corn sample and for the soybean sample. Again, some but not all of the contract months do appear to influence volatility. The robustness of these results is lessened due to the high significance of the post near-limit-down days. Since these control variables are significant, it reduces the overall evidence that limits directly affect volatility.

The inconsistency of the findings across agricultural grain futures markets agrees with Ma, Rao, and Sears (1989b). They state that the effect of price limits are likely to vary across different underlying commodities. Thus, market-specific factors could explain these inconsistencies. For example, on days following an upper-price limit, the less active futures contracts exhibit slightly higher average volatility than the more active ones. Because the results may be sensitive to how volatility is defined, the next section reruns the GARCH model using changes in the log price instead of levels.

Estimation Using the Log Price Changes and Volatility

So far the paper has examined the volatility of price changes but not the volatility of log price changes. To examine price returns and its volatility, let $p_t = \ln P_t$, so that $\Delta p_t = \ln P_t - \ln P_{t-1}$. The results of reestimating the above GARCH model using continuously compounded returns are reported in Table VI. The conditional mean equation in Panel A of Table VI shows only slight departures from the results under Panel A of Table V. Both tables report that prices continue to rise after an up-limit day for the second corn sample and soybean sample, but not for the oat and wheat samples. Notice also the absence of significance in the control variables. The difference between the two tables corresponds to a change in significance for the dummy variable that represents post down-limit days for the oat sample from 7% to 5%.

Differences, however, are more substantial for the conditional volatility equation. Panel B of Table VI shows volatility increasing instead of falling on the day after up- and down-limit days for soybeans and wheat futures contracts. The rise in volatility is relatively small and discounted by the fact that the near-limit days are also significant. How limits influ-

TABLE VI

A Model of Return Volatility on Post-Limit Days, Near-Post-Limit Days, and Contract Months under A GARCH (1,1) Model

Panel A

Futures Contract	θ_0	θ_1	θ_2	θ_3	θ_4	*F-statistic*
Corn	0.0003	0.0219	−0.0120	0.0004	−0.0110	3.5895
Sample 1	(0.0003)	(0.0117)	(0.0089)	(0.0075)	(0.0133)	(0.0000)**
Corn	−0.0003	0.0111	−0.0104	−0.0005	0.0011	1.2639
Sample 2	(0.0003)	(0.0021)	(0.0112)	(0.0050)	(0.0032)	(0.2172)
Oat	0.0001	0.0079	−0.0124	−0.0034	−0.0002	1.7433
	(0.0003)	(0.0094)	(0.0053)*	(0.0045)	(0.0038)	(0.0370)*
Soybean	0.0002	0.0115	−0.0068	−0.0012	0.0036	1.7651
	(0.0002)	(0.0038)**	(0.0049)	(0.0034)	(0.0027)	(0.0268)*
Wheat	0.0005	−0.1192	0.0369	−0.0071	−0.0123	2.9307
	(0.0008)	(0.0754)	(0.0223)	(0.0225)	(0.0130)	(0.0001)**

This table presents results for a GARCH (1,1) model with on post-limit days and contract months as regressors for the following variance equation:

$$\Delta p_t = \theta_0 + \theta_1 D_t^{ul} + \theta_2 D_t^{ll} + \theta_3 D_t^{nul} + \theta_4 D_t^{nll} + e_t$$

$$h_t^2 = \omega_0 + \omega_1 v_{t-1}^2 + \omega_2 h_{t-1}^2 + \omega_3 D_t^{ul} + \omega_4 D_t^{ll} + \omega_5 D_t^{nul} + \omega_6 D_t^{nll} + \sum_{i \neq \text{March}}^{\text{May}} \omega_i Q_t$$

where $\Delta p_t = \ln P_t - \ln P_{t-1}$, v_{t-1}^2 is the ARCH term representing previous news, h_{t-1}^2 is the previous forecast of the variance or GARCH term, and D_t are the dummy variables that equal 1 on the trading day that follows an upper limit, ul, lower limit, ll, near-upper limit, nul, and near-lower limit, nll, and Q_t are dummy variables that equal 1 for the various futures contract months. The GARCH model is used to test whether the day after limits and the day after limits are almost reached affect price volatility. The sample of daily observations, which starts from 1/2/86 and ends on 7/20/98, excludes expanded limit observations. The futures contract for corn is divided into two samples, since there is a change in the price limit from 10 cents to 12 cent in the middle of the sample, on March 12, 1992. The Bollerslev and Wooldridge heteroskedasticity-consistent standard errors are given in parentheses.

ence volatility appear sensitive to the definition of volatility used, given these mixed results.

Therefore, unlike Ma, Rao, and Sears (1989a, 1989b), who observe a fall in volatility, I note that volatility rises on the trading day after an upper-limit and lower-limit day for soybeans and wheat futures. Kuserk and Locke (1996) offer the explanation that volatility may rise because traders become locked into their positions overnight after limit-lock days. Traders are left holding positions and forced to unwind them early the next trading day. This additional trading activity, which is being carried over to the next trading days, is one reason why volatility spills over to the next trading day. Furthermore, under all three specifications, oat futures consistently show no support of volatility being affected the day following limit-lock days. This stresses the importance of examining more

TABLE VI (Continued)

A Model of Return Volatility on Post-Limit Days, Near-Post-Limit Days, and Contract Months under A GARCH (1,1) Model

Panel B

Futures Contract	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
Corn	3.55×10^{-5}	0.0682	0.5413	0.0019	0.0002	0.0001	0.0009
Sample I	(1.38×10^{-5})**	(0.0342)*	(0.1550)**	(0.0011)	(0.0002)	(0.0002)	(0.0012)
Corn	4.33×10^{-6}	0.0160	0.9374	0.0001	0.0001	4.04×10^{-5}	-9.99×10^{-7}
Sample 2	(2.20×10^{-6})*	(0.0187)	(0.0255)**	(0.0001)	(0.0002)	(3.21×10^{-5})	(3.72×10^{-5})
Oat	4.04×10^{-5}	0.1522	0.6777	0.0010	1.00×10^{-5}	-2.35×10^{-5}	-2.72×10^{-5}
	(1.01×10^{-5})**	(0.0381)**	(0.0595)**	(0.0009)	(0.0001)	(0.0001)	(0.0001)
Soybean	6.69×10^{-7}	0.0254	0.9483	9.93×10^{-5}	0.0001	0.0001	3.52×10^{-5}
	(3.47×10^{-7})*	(0.0072)**	(0.0085)**	(4.97×10^{-5})*	(5.98×10^{-5})*	(3.93×10^{-5})**	(2.36×10^{-5})
Wheat	0.0002	−0.0059	0.9268	0.0131	0.0058	0.0040	0.0032
	(3.25×10^{-5})**	(0.0028)*	(0.0112)**	(0.0033)**	(0.0029)*	(0.0013)**	(0.0012)**

Futures Contract	$\omega_{January}$	ω_{May}	ω_{July}	ω_{August}	$\omega_{September}$	$\omega_{November}$	$\omega_{December}$
Corn	—	7.54×10^{-6}	4.92×10^{-5}	—	7.37×10^{-5}	—	2.62×10^{-5}
Sample 1	—	(6.89×10^{-6})	(2.17×10^{-5})*	—	(3.29×10^{-5})*	—	(1.13×10^{-5})*
Corn	—	1.71×10^{-6}	9.79×10^{-6}	—	-8.12×10^{-7}	—	3.77×10^{-7}
Sample 2	—	(1.76×10^{-6})	(3.69×10^{-6})**	—	(1.86×10^{-6})	—	(1.93×10^{-6})
Oats	—	2.58×10^{-5}	0.0001	—	5.35×10^{-5}	—	1.17×10^{-5}
	—	(1.02×10^{-5})**	(2.59×10^{-5})**	—	(1.78×10^{-5})**	—	(7.20×10^{-6})
Soybeans	7.38×10^{-7}	1.83×10^{-6}	3.44×10^{-6}	5.89×10^{-6}	-1.04×10^{-6}	1.17×10^{-6}	—
	(5.79×10^{-7})	(6.40×10^{-7})**	(1.05×10^{-6})**	(2.77×10^{-6})*	(2.15×10^{-6})	(9.08×10^{-7})	—
Wheat	—	7.47×10^{-5}	1.07×10^{-5}	—	-3.04×10^{-5}	—	-1.45×10^{-5}
	—	(6.06×10^{-5})	(3.26×10^{-5})	—	(2.85×10^{-5})	—	(2.93×10^{-5})

**Significance at the 1% level.
*Significance at the 5% level.

than one underlying commodity when pursuing any general study regarding the effect that limits have on futures prices.

CONCLUSION

This paper studies what happens to price changes and volatility on the trading day following a limit-lock day. While results vary across the commodities, the estimates show that limits have a significant impact on price changes and volatility on the trading day after a limit-lock day. The findings imply price continuations after an up-limit day for corn and soybean futures. After a down-limit day, there is marginal evidence of price continuations for oats and wheat. There is, however, no evidence that prices reverse themselves after either an up- or a down-limit-lock day. The lack of significance of the near-limit control variables imply that limits do slow down the price discovery process.

The volatility test results are mixed. Using changes in futures price, volatility tends to fall in most cases the day after a limit-lock day. The same result does not hold when using futures price returns. Price return volatility tends to rise slightly after up- and down-limit days for soybeans and wheat futures. Generally, limits do not directly impact price return volatility, since the findings vary across the commodity futures contract. Moreover, the significance of the control variables suggest that limits may not be the sole explanation for the observed changes in price volatility. Whether volatility actually rises or falls is more likely to be explained by factors relating to an imbalance of the supply and demand for liquidity.

Although price limits are rare events, the transition probabilities reveal that if a limit is reached today, the probability of reaching a limit the next day becomes noticeably higher. The transition probabilities appear to indicate that there may be changes in the distribution around multiple limit days. Because consecutive limit days are treated as single limit events, there might be changes in the distribution of prices within consecutive limit days. A development and implementation of a test for such changes in the distribution of price changes would be a challenging new area for further research.

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