Lower-boundary Violations and Market Efficiency: Evidence from the German DAX-index Options Market*

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The informational efficiency of the market for options on the German stock index DAX is examined using intraday transactions data. Problems of previous studies on options-market efficiency, arising from dividend estimation and the early-exercise effect, are avoided, because the DAX is a performance index and DAX options are European options. Ex-post and ex-ante tests are carried out to simulate trading strategies that exploit irrational lower-boundary violations of observed option prices. Because the lower-boundary conditions are solely based on arbitrage considerations, the test results do not depend on the assumption that investors use a particular option-pricing model. The investigation shows that ex-post profits are, in general, dramatically reduced when the execution of arbitrage strategies is delayed and/or transaction costs are accounted for. However, arbitrage restrictions,

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which rely on short selling of the component stocks of the index, tend to be violated more often and with higher persistence. An analysis of consecutive subsamples suggests that, over time, traders have been subjected to a learning process when pricing this relatively new instrument. Jrl Fut Mark 20: 405–424, 2000

INTRODUCTION

Since their introduction on the Chicago Board Options Exchange (CBOE) in 1983, index options have developed into popular financial instruments. Their cash-settlement makes them a simple, inexpensive tool for managing the systematic risk of large portfolios and allows investors to convert volatility and/or market-trend expectations into easily implementable investment strategies. The German Options and Futures Exchange (DTB) has been trading a European-type index option on the German DAX index since August 1991. Given the growing importance of index options, the question of whether index options are efficiently priced in this relatively new market is of great importance—both from a theoretical and practical viewpoint. The results of the study should therefore be of major interest for academics interested in market efficiency and practitioners involved in options trading, hedging, or arbitrage operations.

One strategy for testing options-market efficiency is to compare observed transaction prices with those implied by a theoretical option pricing model, such as the Black and Scholes (1973) model. Hedging schemes in continuous time can be devised to generate riskless profits from observed mispricings. A major drawback of this strategy is that it involves a joint test of several hypotheses, namely that (i) the options market is efficient, (ii) the pricing model is valid, (iii) the model parameters are correctly specified, and (iv) the data are synchronized. As an alternative to model-based tests, one can design tests that are solely based on the requirement that there are no systematic arbitrage opportunities in the options market. Instead of assuming a particular pricing model, such "pure arbitrage tests" simply rely on the presumption that investors remove all riskless arbitrage opportunities. These tests have the advantage that they reduce the number of jointly tested hypotheses to only two, namely (i) and (iv).[1] Moreover, they avoid the need to impose restrictions on the stochastic process driving the stock index.[2]

[1]In both strategies one faces the problem of having to rely on appropriately specified testing procedures.

[2]For example, Evnine and Rudd (1985) point out that even if the prices of each individual stock underlying the index are assumed to follow a lognormal diffusion process, as required by the Black-Scholes model, lognormality does not carry over to the index.

Merton (1973) established a lower-boundary condition for the price of a European call option on an individual stock and Galai (1978) modified it to accommodate American calls. A violation of this boundary condition implies an immediate arbitrage opportunity and, thus, irrational option pricing. The lower-boundary constraint can be shown to hold under general assumptions. It is based on the principle of nondominance among financial assets. In the case of index options, a violation of these constrains requires an arbitrageur to "trade the index," that is, to trade the component stocks of the index in the appropriate proportions. Because this is difficult in practice, one expects index options to exhibit, in general, greater and more frequent deviations from rational (equilibrium) prices than stock options.[3]

The objective of this study is to test the efficiency of the German DAX–options market using ex-post and ex-ante tests of lower-boundary violations. The DAX index is particularly suitable for conducting efficiency tests of index-options markets. Because DAX options are European-type options, the ambiguity resulting from studies dealing with American options can be avoided. Therefore, in contrast to most previous studies, which only examined violations for call options, the present analysis considers both calls and puts. The DAX index has the additional advantage that it is a performance index, that is, it accounts for dividend payments from the underlying stocks.[4] Thus, the empirical results of the efficiency tests reported here are not affected by complications arising from the need to predict future dividends or the fact that options are not the European type. A further objective of this study is to examine to what extent market participants have become more rational since the introduction of DAX–index options in 1991. To investigate this, tests for consecutive, annual subsamples are conducted. We find that lower-boundary violations cannot be fully exploited, because they tend to diminish within a short period of time. Moreover, the data suggest that possible market inefficiencies have decreased over the sample period, supporting the hypothesis that traders have gradually learned to better price these new instruments.

The paper proceeds as follows. In the second section we briefly discuss the results of related empirical studies. A brief review of the boundary conditions for European-type options is given in the third section. Technical aspects of the efficiency tests are discussed in the fourth sec-

[3]See Evnine and Rudd (1985). A comprehensive discussion of difficulties with arbitrage trading of index options can be found in Figlewski (1988).

[4]See the section on "The Data" for a description of the DAX index.

tion. The fifth section describes the data and presents the results of the tests. The sixth section concludes.

RELATED STUDIES

Empirical research on lower-boundary violations has primarily been focusing on U.S. markets. For example, Galai (1978) and Bhattacharya (1983) considered options on individual stocks, Evnine and Rudd (1985) on the S&P 100 and the Major Market Index, and Chance (1988) on the S&P 100. Investigations on lower-boundary violations on non-U.S. markets include German (Trautmann 1984, 1987) and Canadian (Halpern & Turnbull 1985) stock options, as well as Finnish (Puttonen 1993) and Swiss (Chesney et al. 1994) index options. However, these studies suffer from complications, which may cast some doubt on their conclusions. One drawback is that most of them exclusively consider call options.[5] This can be explained by the fact that the options examined are typically American style and that the profitability of early exercise for American puts cannot be ruled out, even in the case of no dividend payments until maturity. A second complication arises from the uncertainty associated with the dividend payments. Empirical studies typically abstract from this problem by making the unrealistic assumption that future dividend payments are known with certainty. Taken together, the results of these studies indicate that both the frequency and the magnitude of violations tend to decrease when either intraday data rather than daily closing prices are used or when transaction costs are taken into account. Moreover, there is strong evidence that observed violations of rational option pricing are not easily exploited, because such violations tend to be short lived.[6]

LOWER-BOUNDARY CONDITIONS FOR EUROPEAN STYLE OPTIONS

In the subsequent discussion of the lower-boundary condition it is assumed that options are European style and that no dividends are paid on the stocks composing the index. Both assumptions apply in the present context, because DAX options are European options, and the DAX is a performance index. Throughout the paper, the following notation is adopted:

[5] Exceptions are the studies of Puttonen (1993) and Chesney et al. (1994).

[6] An exception is Puttonen (1993), who, using daily closing prices, finds persistence in lower-boundary violations; i.e., a particular violation observed on one day tends to be followed by a further violation on the next day. However, due to the nonparametric nature of Puttonen's analysis, it is not clear whether or not such violations tend to decrease between two market closings.

$C_t(P_t)$ = market price of a call (put) option at time t.
I_t = level of the underlying index at time t.
K = strike price of the option.
T = expiration time of the option.
$\tau = T - t$: time to maturity of the option (measured in years).
r = risk-free rate, that is, annualized, continuously compounded rate of return on a risk-free security (assumed to be constant until maturity).

In an efficient market, the price of a European call option at time t, C_t, with strike price K and time to maturity τ on an index with value I_t must satisfy the condition

$$C_t \geq \max(0,\ I_t - Ke^{-r\tau}). \tag{1}$$

If condition (1) is violated, that is, $C_t < I_t - Ke^{-r\tau}$, an arbitrage profit can be generated by buying the call (cash flow: $-C_t$), shorting the index (I_t), and lending the amount $Ke^{-r\tau}$ at the risk-free rate. This leads to an immediate cash inflow of $I_t - Ke^{-r\tau} - C_t > 0$. This position is held until expiration, T, where two outcomes are possible. If $I_T > K$, the call is exercised ($I_T - K$), the index is bought and given back to the owner ($-I_T$), and, because the riskless investment has grown to K, the overall terminal cash flow is equal to zero. If, on the other hand, $I_T < K$, the option expires worthless (0), and the index is bought on the market ($-I_T$). In the latter case, the arbitrageur receives $K - I_T > 0$ at expiration. Because this trading strategy results in a positive payoff at time t and a nonnegative payoff at time T, violations of (1) would lead to risk-free profits.

The lower-boundary condition for a European put option is given by

$$P_t \geq \max(0,\ Ke^{-r\tau} - I_t), \tag{2}$$

for which an analogous arbitrage argument can be made. In case of $P_t < Ke^{-r\tau} - I_t$, the arbitrage strategy consists of borrowing $Ke^{-r\tau}$ at the risk-free rate and buying the put and the index. This leads to an instantaneous cash inflow of $Ke^{-r\tau} - I_t - P_t > 0$. Again, the terminal payoff at the close of this position depends on whether $I_T < K$ or $I_T > K$ at the expiration date. If $I_T < K$, the payoff is equal to zero, because the put is exercised ($K - I_T$), the debt is liquidated ($-K$), and the index is sold (I_T). If $I_T > K$, the put expires worthless (0), the index is sold (I_T) and the debt is liquidated ($-K$), resulting in cash inflow $I_T - K > 0$.

It should be noted that there is no institutional framework in Germany for selling stocks short. Consequently, the short-hedge strategy, arising from a violation of the lower-boundary condition for calls (1), can only be implemented by market agents, who already own the component stocks of the index or who can borrow the component stocks of the index.

TEST PROCEDURE

The general design of the tests and the corresponding statistical hypotheses are discussed next. Following Galai (1983) and Jensen (1978), a market is said to be efficient with respect to a given information set if no trader can consistently make risk-adjusted profits that, after accounting for taxes and transaction costs, exceed the risk-free rate. Therefore, testing the efficient-market hypothesis (EMH) in terms of the rational lower boundaries (1) and (2) amounts to testing

$$\varepsilon_t^C = I_t - Ke^{-r\tau} - C_t - \mathcal{T}_t^C \leq 0 \tag{3}$$

and

$$\varepsilon_t^P = Ke^{-r\tau} - I_t - P_t - \mathcal{T}_t^P \leq 0, \tag{4}$$

where ε_t^C and ε_t^P denote the (ex-post) profits from the short and long hedge, respectively, and $\mathcal{T}_t^C$ and $\mathcal{T}_t^P$, the transaction costs involved.

The market-efficiency tests are carried out both under with- and without-transaction-costs scenarios.[7] The transaction costs are assumed to be those of a market maker at the DTB, that is, the least-cost trader.[8] The fee for market makers trading DAX options at the DTB is DM0.40 per contract.[9] The costs for trading the stocks underlying the DAX are assumed to be 0.1% of the index value.[10] The transaction costs in (3) and (4) for trading the stocks underlying the index are assumed to the same

[7]Bhattacharya (1983) points out that financial assets are typically bought at the ask and sold at the bid price, suggesting an implicit source of transaction costs. In general, it is not clear to what extent the bid–ask spread should be included, because, in practice, trades also occur inside the spread. Because the transactions database for options does not include bid and ask prices and the spread for the DAX is difficult to estimate, the bid–ask spreads for both the index and the options are neglected in this study. In doing so, we follow the study of Halpern and Turnbull (1985).

[8]Another source of transaction costs are taxes. Because the tax treatment is very heterogeneous for different market participants, we follow common practice and ignore taxes in this study.

[9]See Deutsche Terminbörse (1992,1993).

[10]This is about half of the lowest discount–broker fee charged to private investors for trading German stocks. For exchange members the transaction costs are substantially lower. Due to the heterogeneity of traders, one can only vaguely estimate the true cost of trading the component stocks of the DAX. Half of the lowest discount–broker fee could be regarded as a crude measure of the transaction costs for the least–cost trader. The qualitative results should be robust with respect to our specification of the costs of trading stocks. In the sample period, the low and high of the DAX were 1413.68 and 2320.22, respectively. Thus, the specified transaction costs for buying or selling the underlying stocks were assumed to be between DM14.14 and DM23.20.

for the long- and the short-hedge strategy. Alternatively, motivated by the missing institutional framework for the short-selling of stocks in Germany, one could assume that the costs of shorting are higher than those for buying the index-component stocks. As noted in Puttonen (1993), if the stocks needed for the short hedge must be borrowed, the arbitrageur may have to pay a premium to the owners of the stocks. This would imply higher transaction costs for shorting, $\mathcal{T}_t^C$, than for buying, $\mathcal{T}_t^P$, the stocks. Because the specification of the transaction costs for shorting the stocks is difficult, we assume that the transaction costs for buying and selling are the same. Therefore, we expect the data to exhibit more frequent and severe violations of the boundary condition for calls than for puts.

The profit-making strategies arising from violations of boundary conditions (1) and (2) assume that an arbitrage scheme can, in fact, be executed at the very prices signaling irrational option pricing. Thus, they can be viewed as strategies implemented under ex-post conditions. One refers to tests under such a scenario as *ex-post tests*. In practice, a certain time period elapses between the inception and the execution of a hedging strategy; and stock and index-options markets may not be perfectly synchronized. Galai (1978) argued, therefore, that market-efficiency tests should be based on *ex-ante tests* implying that a trading strategy is devised at time t, whereas the hedge position is established with prices that are realized at time $t + h$ and yet unknown at time t.

Most empirical studies investigating market efficiency employ daily closing prices and typically lag the execution until the next trading day. Here, a set of shorter execution lags is considered, since lags of one day are rather long—especially for market makers. The use of different lags allows us to get more insight into the pricing process of the market. Under the EMH, profit opportunities should vanish over time. After a mispricing signal, prices are expected to revert to their rational values after some time.

As opposed to an *ex-post* test, where a mispricing signal can be fully exploited without any risk, profits from an *ex-ante* strategy are affected by possible price movements during the execution lag. Therefore, ex-ante payoffs resulting from a mispricing signal can be negative. The risk arising from price movements during the execution lag is referred to as the *immediacy risk* (cf. Kamara & Miller 1995).

EMPIRICAL EVIDENCE FROM THE DAX-OPTIONS MARKET

The Data

Options on the DAX have been trading on the DTB since August 1991 and have, by now, become a highly liquid instrument whose trading volume exceeds the aggregate trading volume of all 41 stock options currently traded on the DTB. As is usual for index options, prices of DAX options are quoted in index points rather than in Deutsch marks (DM). The contract value is DM10 per index point, that is, the payoff at expiration is DM10 $\times$ max(0; $I_T - K$) for an index call and DM10 $\times$ max (0; $K - I_T$) for an index put. Since the options' tick size is 0.1, one tick corresponds to DM1.

During the 1992–1995 sample period, expiration dates were within three months as well as the two immediately following months in the cycle March, June, September, and December. Thus, the maximum time to maturity was nine months. The options expire at the third Friday of the expiration month. The DTB introduces DAX–option series with five different strike prices, each of which are multiples of 25 index points. If on two consecutive days the index closes below (above) the average of the second- and third-lowest (-highest) exercise price of options with at least 10 days to expiration, a new series with a lower (higher) exercise price is introduced.

The DAX is a value-weighted index comprised of the 30 major stocks. As a performance index the DAX is adjusted for stock splits, capital changes, and, most importantly, dividends paid on the component stocks. In contrast to other stock indices, which solely reflect price movements of the underlying stocks, dividends on the 30 component stocks are assumed to be reinvested on the date a stock goes ex-dividend and, thus, leave the index level unaffected. Therefore, dividends paid until the expiration date of the option need not be considered when pricing DAX options. During the floor-trading hours (10:30 AM to 01:30 PM) the German Stock Exchange computes the index every minute at an accuracy of one-hundredth of an index point.

The options data set consists of all transactions in the period from February 3, 1992, to September 29, 1995. Each record is time-stamped up to centiseconds and contains the strike price, expiration month, transaction price, and contract size. All option transactions are paired with the most recent update of the DAX.[11] Only options traded between 10:30 AM to 1:30 PM, that is, when floor trading of stocks takes place, are included.[12]

[11]To check whether the results are affected by this procedure for matching options and index data, we divided the sample into two subsamples: The first subsample corresponds to options traded within the first half minute after the index is updated, and the second to options traded in the second half of the minute following the most recent update of the DAX. It turns out that there are only minor differences between the results for the two subsamples. Below, we present the results for the full sample.

[12]Trading hours at the DTB are from 9:30 AM to 4:00 PM.

As a proxy for the risk-free rate daily data for interbank bid and offer rates for overnight, 1-, 3-, and 6-months maturities are used. The midpoint of the bid–ask spread, which is typically very narrow, is used for simplicity. Interest rates matching the time to maturity of the options are obtained by simple linear interpolation.[13] Since interest rates for maturities longer than six months are not available, options with more than six months to expiration are excluded from the sample, making the longest term to expiration 182 days.

The options data set had been screened for data errors arising from mistrades. According to the so-called mistrade rule, the DTB compares the transaction price of every trade to the fair value, as implied by the Black and Scholes (1973) option pricing model. If the price of a transaction deviates substantially from the fair option value, the trade is defined to be a mistrade.[14] In case of a mistrade, both contract partners are informed by the exchange, so that it can be reversed by making an opposite transaction. Unfortunately, the DTB data neither indicate such mistrades nor the offsetting counter trade. Applying the DTB's mispricing criteria (see footnote 14) mistrades are identified by checking for an offsetting transaction within the next 10 trading days and excluded (together with the offsetting transaction) from the analysis. In total, 862 call and 7521 put transactions have been identified as mistrades, after applying the exclusion criteria just discussed. The much higher value for the puts is somewhat surprising. The percentage of mistrades with respect to the number of transactions amounts to 0.27% for calls and 2.37% for puts. After the filtering procedure, 632,870 trade records—323,074 for calls and 309,796 for puts—are remaining, each matched with the contemporaneous index value and interest rate.

Efficiency Tests

The efficiency of the DAX–options market is tested via ex-ante tests, in which observed lower-boundary violations (i.e., $\varepsilon_t^C > 0$ in (3) or $\varepsilon_t^P > 0$ in (4)) are viewed as mispricing signals initiating the formation of a hedge position. Four different time lags between the occurrence of a mispricing signal and the realization of the hedge are considered, namely $h = 1, 10, 60, 210$ minutes. While the three shorter lags, that is, $h = 1, 10, 60$, give

[13] The potential bias induced by this approximation should be negligible, since the sensitivity of options prices with respect to the interest rate is quite low for options with less than one year to maturity; see, for example, Cox and Rubinstein (1985, p. 275).

[14] The DTB defines a trade to be a mistrade, if one of the following two conditions hold: (i) the absolute deviation exceeds DM15, or (ii) the absolute deviation exceeds 150% of the maximum spread, but at least 30% of the fair option value.

rise to intraday delays—except for signals arising in the last minutes of a trading day—lag $h = 210$ implies a delay until the first trade of the next trading day and is simply for the sake of convenience referred to as $h = 210$.[15] The hedge is constructed at prices observed at the first transaction after delay h. If there is no matching transaction for the remainder of the current and the subsequent trading day, no hedge is constructed.

The results for distinct lags should be interpreted with care. They cannot be viewed as independent evidence, because the time periods admitted for portfolio construction are overlapping, that is, the time period corresponding to a given lag is in fact a subperiod of the period belonging to the next lower lag.

Test Results Without Transaction Costs

Results of the efficiency tests without transaction costs are summarized in Table I. The upper part of the table (labeled with "Lag 0") contains the outcomes of the ex-post tests, whereas the results of the ex-ante tests for the different execution lags are given in the lower part.

The results of the ex-post tests are discussed first. Rows (a) and (b) in Table I relate to the excess of the options prices over their lower-boundary values, that is, $-\varepsilon_t^C$ in (3) and $-\varepsilon_t^P$ in (4), respectively. Means and standard deviations are given in these two rows. Two-sided test (against zero) are carried out for the means of ε_t^C and ε_t^P. A conventional t-test may be misleading, because the empirical distributions of ε^C and ε^P have been found to be substantially skewed. Therefore, the modified t-test of Johnson (1978), denoted by t^J, is applied.[16] Rows (c) through (g) relate to the mispricing signals for calls ($\varepsilon_t^C > 0$) and puts ($\varepsilon_t^P > 0$). Rows (c) to (f) indicate the absolute numbers, percentages, means, and standard deviations of these signals. No test is performed here, because the mispricing signals are positive by definition. In row (g) we report a crude measure of the total ex-post profit (measured in 1000 index points or equivalently, DM10,000), which we define as the sum of all mispricing

[15]Because the length of a trading day never exceeds 210 minutes in the sample, a minimum lag of (at least) 210 minutes corresponds to a lag until the beginning of the next trading day.

[16]The modified t statistic allows for asymmetry in the distribution of a random variable and is given by $t^J = (\bar{x} + \mu_3/6\sigma^2 n + \mu_3\bar{x}^2/3\sigma^4)\,(s^2/n)^{-1/2}$, where $\bar{x}$ and s^2 denote the sample mean and the sample variance; σ^2 is the variance of X; μ_3 is the third central moment of X; and n is the sample size. Under the null hypothesis of a zero population mean, t^J is Student t distributed with $n - 1$ degrees of freedom. In practice, μ_3 and σ^2 are replaced by their sample counterparts $\hat{\mu}_3$ and s^2. If the sample distribution is symmetric, $\hat{\mu}_3$ equals zero and the modified t statistic t^J reduces to the conventional t statistics $t = \sqrt{n}\bar{x}/s$.

TABLE I

Lower-boundary Efficiency Tests without Transaction Costs

		Calls				*Puts*			
		Sample Year (Number of Transactions) [Average Volume per Transaction]				*Sample Year (Number of Transactions) [Average Volume per Transaction]*			
		1992 (68074) [53.05]	*1993 (88151) [53.10]*	*1994 (87404) [45.58]*	*1995 (79445) [47.41]*	*1992 (71075) [50.73]*	*1993 (80882) [52.58]*	*1994 (85516) [46.60]*	*1995 (72323) [49.01]*
Lag		Ex-post Tests							
0	(a) mean (ε)	−16.81‡	−16.53‡	−26.71‡	−17.89‡	−18.22‡	−19.11‡	−29.03‡	−18.85‡
	(b) std(ε)	12.20	14.27	20.32	14.44	13.96	13.76	20.91	14.75
	(c) #signals	1368	5566	2063	1640	2831	267	748	580
	(d) % of opt	2.01	6.31	2.36	2.06	3.98	0.33	0.87	0.80
	(e) mean (signal)	2.09	2.56	2.12	1.68	2.01	1.26	1.47	1.68
	(f) std(signal)	2.33	2.41	1.98	1.76	2.49	1.12	2.95	1.70
	(g) total profit	151.67	756.56	199.34	130.62	288.68	17.69	51.24	47.76
		Ex-ante Tests							
1	(a) #hedges	1126	4857	1754	1426	25.10	229	660	501
	(b) % of signals	82.31	87.26	85.02	86.95	88.66	85.77	88.24	86.38
	(c) % > 0	74.33	74.33	74.57	67.46	66.14	60.26	63.48	60.48
	(d) mean(profit)	1.05‡	1.66‡	1.14‡	0.74‡	0.76‡	0.22†	0.13	0.20
	(e) std(profit)	2.90	2.93	2.86	2.33	2.72	1.65	2.86	3.25
	(f) total profit	62.72	428.09	91.13	50.03	96.78	2.65	4.00	4.91
10	(a) #hedges	1072	4727	1665	1376	2422	222	562	481
	(b) % of signals	78.36	84.93	80.71	83.90	85.55	83.15	75.13	82.93
	(c) % > 0	69.78	70.21	67.09	56.76	62.35	47.75	52.85	53.64
	(d) mean(profit)	0.87‡	1.51‡	0.83‡	0.33‡	0.56‡	−0.22*	−0.22	−0.19
	(e) std(profit)	2.98	3.01	2.65	2.58	2.84	1.89	3.17	3.54
	(f) total profit	49.47	378.98	62.99	21.53	68.81	−2.57	−5.76	−4.48
60	(a) #hedges	872	4205	1308	1186	2146	195	429	389
	(b) % of signals	63.74	75.55	63.40	72.32	75.80	73.03	57.35	67.07
	(c) % > 0	68.35	62.14	59.56	46.29	52.94	37.44	34.97	49.36
	(d) mean(profit)	0.77‡	0.99‡	0.45‡	−0.05	0.16†	−0.72‡	−1.13‡	−0.48‡
	(e) std(profit)	3.15	3.18	2.91	3.12	2.85	2.35	3.47	3.55
	(f) total profit	35.62	221.03	26.83	−2.81	17.42	−7.38	−22.59	−9.15
210	(a) #hedges	501	3384	608	704	1518	148	215	213
	(b) % of signals	36.62	60.80	29.47	42.93	53.62	55.43	28.74	36.72
	(c) % > 0	51.50	49.02	48.03	34.52	53.03	25.00	38.14	46.95
	(d) mean(profit)	−0.06	0.00	0.08	−1.12‡	0.26‡	−1.21‡	−1.18‡	−1.17‡
	(e) std(profit)	3.83	3.25	3.37	4.29	3.30	2.93	3.99	3.71
	(f) total profit	−1.59	0.00	2.22	−37.38	20.02	−9.42	−11.82	−12.21

Note: Significance of the mean against zero at levels 10, 5, or 1%, based on a two-sided modified *t*-test, is indicated by *, †, and ‡, respectively.

signals within a year ((c) times (e)), multiplied by the average volume per transaction in that year.

It follows from row (a), that is, mean(ε), that options traded, on average, about DM165 to DM191 above their rational lower-boundary values in the years 1992, 1993, and 1995. In 1994 this value is almost DM300 for the calls and more than DM300 for the puts. As is indicated by the significance tests, on average, option prices traded significantly above their lower bounds in the four sample years considered. The minimum and the maximum size of the mean mispricing signals is DM12.60 for puts and DM25.60 for calls, both occurring in 1993. In 1992 and 1995 this value is about the same for both option types. In the remaining years, however, it is substantially higher for calls. A decreasing magnitude of the average signals over the years cannot be stated. In general, the total ex-post profit is substantially higher for calls than for puts. An exception is the year 1992, where the total profit is almost twice as high for puts than for calls. Ex-post profit opportunities are highest in 1993, where the total profit from lower-boundary violations of calls amounts to more than 756,000 index points, or equivalently, DM7,560,000.

The ex-ante tests associated with positive execution lags are considered next. For each execution lag, h, six entries are reported per column:

(a) Number of hedges constructed, assuming an execution lag of at least h minutes

(b) Proportion of mispricing signals, for which hedge portfolios are constructed

(c) Percentage of positive positions

(d) Mean value of the ex-ante hedges (significance of t^J-statistic is indicated by symbols, see above)

(e) Standard deviation of the hedge values

(f) Total ex-ante profit (in 1000 index points), based on the average trading volume (i.e., (a) times (d), multiplied by the average trading volume)

The number of hedges constructed from observed mispricing signals varies between about 82 and 89% for the 1-minute lag and decreases to sometimes less than 30%, if execution is delayed until the next trading day. Moreover, the portion of negative hedge payoffs decreases with increasing lag, suggesting that profit opportunities decrease steadily due to market reactions. In general, for the one-day lag, more than half of all the positions produce losses. The decrease of the mean and the total profits with increasing execution lag supports the EMH. Even a delay of the execution by a single trade leads in every subperiod to a substantial drop in the mean and the total profits from both the long- and the short-hedge strategies.

The significance tests indicate that the mean profits for the one-trade lag are in most cases positive and highly significant. However, for the long-hedge strategy, the mean profit is insignificant (at standard significance levels) for the last two years of the sample. When increasing the lag to 10 minutes, the mean profits from long-hedge positions become insignificantly negative in the last three sample years; whereas the mean profits from short-hedge positions remain significantly positive in each subperiod. For the 1 hour lag, long-hedge (mean) losses further decrease, as compared to the 10-minute lag; and the t^J-tests indicate that the insignificant losses for the 10-minute lag turn to significant losses in the case of a 1-hour lag. The first insignificant mean short-hedge profit occurs for an execution lag of 1 hour in 1995. Finally, assuming that the hedge strategies are executed at the prices observed at the first trade of the following trading day, the short-hedge strategy yields insignificant profits in the first three and a significant loss in the most recent sample year; whereas the long-hedge strategy would have, in general, entailed highly significant losses. The highest total loss (more than DM370,000) is obtained for an investor applying the long-hedge strategy in 1995. For the one-day lag the only significantly positive profits were possible for investors exploiting lower-boundary violations of puts in the first year of the sample, where the total profit is higher than DM200,000.

The significance patterns in Table I indicate that the mean profits for calls and puts tend to be insignificant for different sets of lags. For calls, (highly) significantly positive mean profits for shorter lags tend to become insignificant as the execution lag increases. The means for puts, on the other hand, turn from being (marginally significantly or insignificantly) positive to being insignificant and partly negative. The reason is that for both calls and puts there is a tendency for the mean to decrease as the execution lag increases. However, the mean profits for calls start from a higher level than those for puts. These phenomena, together with the fact that we employ a two-sided test to test the means against zero, lead to the different patterns for the two option types.

Test Results With Transaction Costs

The effects of considering transaction costs—as they are described in the fourth section—are evident from Table II. A comparison of the means of ε^C and ε^P with those in Table I indicates that the mean transaction costs

TABLE II

Lower-boundary Efficiency Tests with Transaction Costs

		Calls				*Puts*			
		Sample Year (Number of Transactions) [Average Volume per Transaction]				*Sample Year (Number of Transactions) [Average Volume per Transaction]*			
		1992 (68074) [53.05]	*1993 (88151) [53.10]*	*1994 (87404) [45.58]*	*1995 (79445) [47.41]*	*1992 (71075) [50.73]*	*1993 (80882) [52.58]*	*1994 (85516) [46.60]*	*1995 (72323) [49.01]*
Lag		Ex-post Tests							
0	(a) mean (ε)	−18.80‡	−18.78‡	−29.22‡	−20.40‡	−20.19‡	−21.36‡	−31.54‡	−21.35‡
	(b) std(ε)	12.17	14.32	20.32	14.44	13.93	13.79	20.90	14.74
	(c) #signals	462	2384	670	343	1032	47	105	126
	(d) % of opt	0.68	2.70	0.77	0.43	1.45	0.06	0.12	0.17
	(e) mean (signal)	2.31	2.38	1.76	1.86	1.90	0.90	2.11	1.61
	(f) std(signal)	2.70	2.41	2.03	2.00	3.29	0.91	6.95	2.14
	(g) total profit	56.61	301.26	53.74	30.24	99.47	2.22	10.32	9.94
		Ex-ante Tests							
1	(a) #hedges	344	2004	546	272	893	37	87	104
	(b) % of signals	74.46	84.06	81.49	79.30	86.53	78.72	82.86	82.54
	(c) % > 0	61.34	66.82	47.80	50.37	47.48	18.92	21.84	38.46
	(d) mean(profit)	0.27	1.07‡	−0.40†	−0.15	−0.43‡	−1.66‡	−2.54‡	−1.68‡
	(e) std(profit)	3.97	3.20	3.78	3.07	3.29	2.12	5.40	4.53
	(f) total profit	4.93	113.85	−9.95	−1.93	−19.48	−3.23	−10.30	−8.56
10	(a) #hedges	326	1951	522	259	862	36	77	96
	(b) % of signals	70.56	81.84	77.91	75.51	83.53	76.60	73.33	76.19
	(c) % > 0	51.23	63.51	39.46	40.93	42.69	8.33	20.78	25.00
	(d) mean(profit)	−0.13	0.83‡	−0.70‡	−0.66‡	−0.70‡	−2.81‡	−3.06‡	−2.73‡
	(e) std(profit)	4.06	3.31	2.98	3.27	3.27	2.39	5.74	4.64
	(f) total profit	−2.25	85.98	−16.65	−8.10	−30.61	−5.32	−10.98	−12.84
60	(a) #hedges	266	1733	451	203	766	31	71	87
	(b) % of signals	57.58	72.69	67.31	59.18	74.22	65.96	67.62	69.05
	(c) % > 0	48.87	52.74	29.27	29.56	36.81	9.68	5.63	18.39
	(d) mean(profit)	−0.47*	−0.04	−1.36‡	−1.53‡	−1.29‡	−2.87‡	−4.32‡	−3.05‡
	(e) std(profit)	4.33	3.50	3.23	3.60	3.19	2.60	6.02	3.81
	(f) total profit	−6.63	−3.68	−27.96	−14.72	−50.13	−4.68	−14.29	−13.01
210	(a) #hedges	175	1501	213	153	564	24	46	59
	(b) % of signals	37.88	62.96	31.79	44.61	54.65	51.06	43.81	46.83
	(c) % > 0	38.86	31.25	32.86	12.42	42.20	29.17	0.00	0.00
	(d) mean(profit)	−1.37‡	−1.79‡	−1.72‡	−3.20‡	−1.12‡	−1.94‡	−5.15‡	−4.18‡
	(e) std(profit)	5.15	3.61	3.67	3.81	3.83	3.52	7.17	3.61
	(f) total profit	−12.72	−142.66	−16.70	−23.21	−32.05	−2.45	−11.04	−12.09

Note: Significance of the mean against zero at levels 10, 5, or 1 percent, based on a two-sided modified *t*-test, is indicated by *, †, and ‡, respectively.

for constructing a hedge portfolio range from about DM20 to DM26. One important effect of allowing for transaction costs is the substantial decrease in the number of mispricing signals. In each case more than half of all mispricing signals reported in Table I disappear after transaction costs are accounted for. A comparison of the means of the mispricing signals in both tables shows, however, that the trading costs increase the magnitude of the mean signal in some cases, but the differences are rather small. The overall effect from the means and the number of signals is reflected in the total profits, which, compared to the case of no transaction costs, drop substantially.

As can be seen from the ex-ante section of Table II, the introduction of trading costs tends to lower both the portion of profitable hedges and the mean profits. Even for the executions lagged by a single trade, most of the previously significant profits turned into losses. In this case, the t^J statistic indicates that only investors applying the short-hedge strategy in 1993 encountered significantly positive profit opportunities. All other without-transaction-cost profits either become insignificant (in 1992 and 1995 for the short-hedge strategy) or turn into highly significant losses (in 1994 for the short- and, in each subsample, for the long-hedge strategy). Similar to the without-transaction-cost case, increasing the lag in portfolio construction, on the average, decreases profits or increases losses; that is, significant gains tend to become insignificant and ultimately turn to significant losses. If a 1-hour execution lag is assumed, there are no significant profit opportunities in the market. Increasing the execution lag to the next trading day results, for both strategies and each subsample, in highly significant losses that can amount to more than 140,000 index points (DM1,400,000) for the long-hedge strategy in 1993.

Up to now the analysis has focused predominantly on the average size of the mispricing signals and the resulting profits. More distributional characteristics are revealed by plotting medians as well as lower and upper quartiles of the mispricing signals and the corresponding ex-ante hedge profits in dependence of the execution lag. Figures 1 and 2 describe the without- and with-transaction-cost scenarios, respectively. The quartiles of the mispricing signals (labeled "Lag 0") indicate that most signals are rather small. The upper quartiles never exceed 4 index points or, equivalently, DM40. The upper quartiles associated with the short-hedge strategy tend to be higher than those with the long-hedge strategy.

The plots show that the medians and quartiles of the ex-ante profits exhibit a strong tendency to decrease quite rapidly as the execution lag increases. Even if the execution of the strategies is delayed by only one

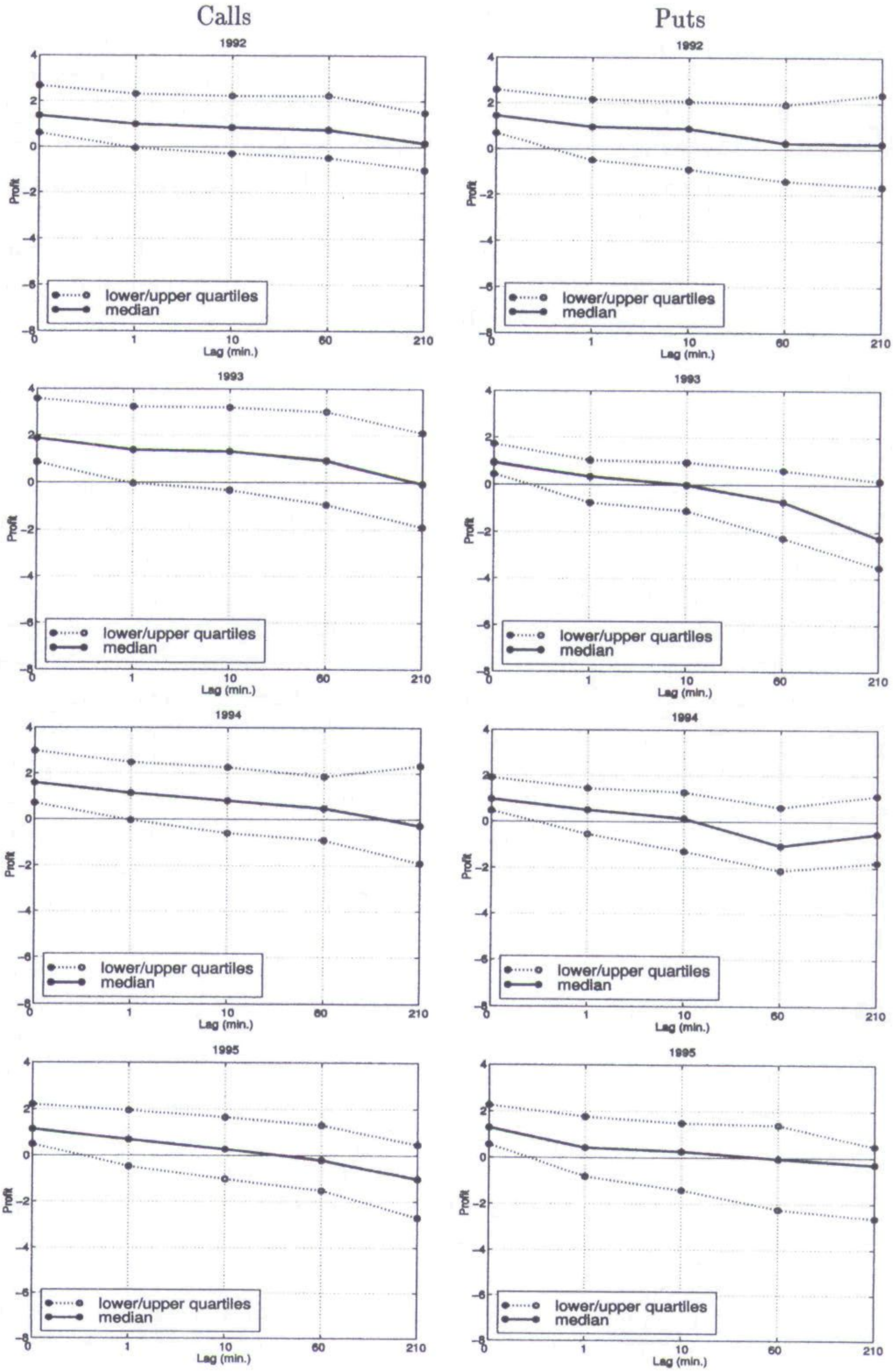

FIGURE 1
Median and quartiles of ex-post and ex-ante hedge profits from lower-boundary violations, without transaction costs.

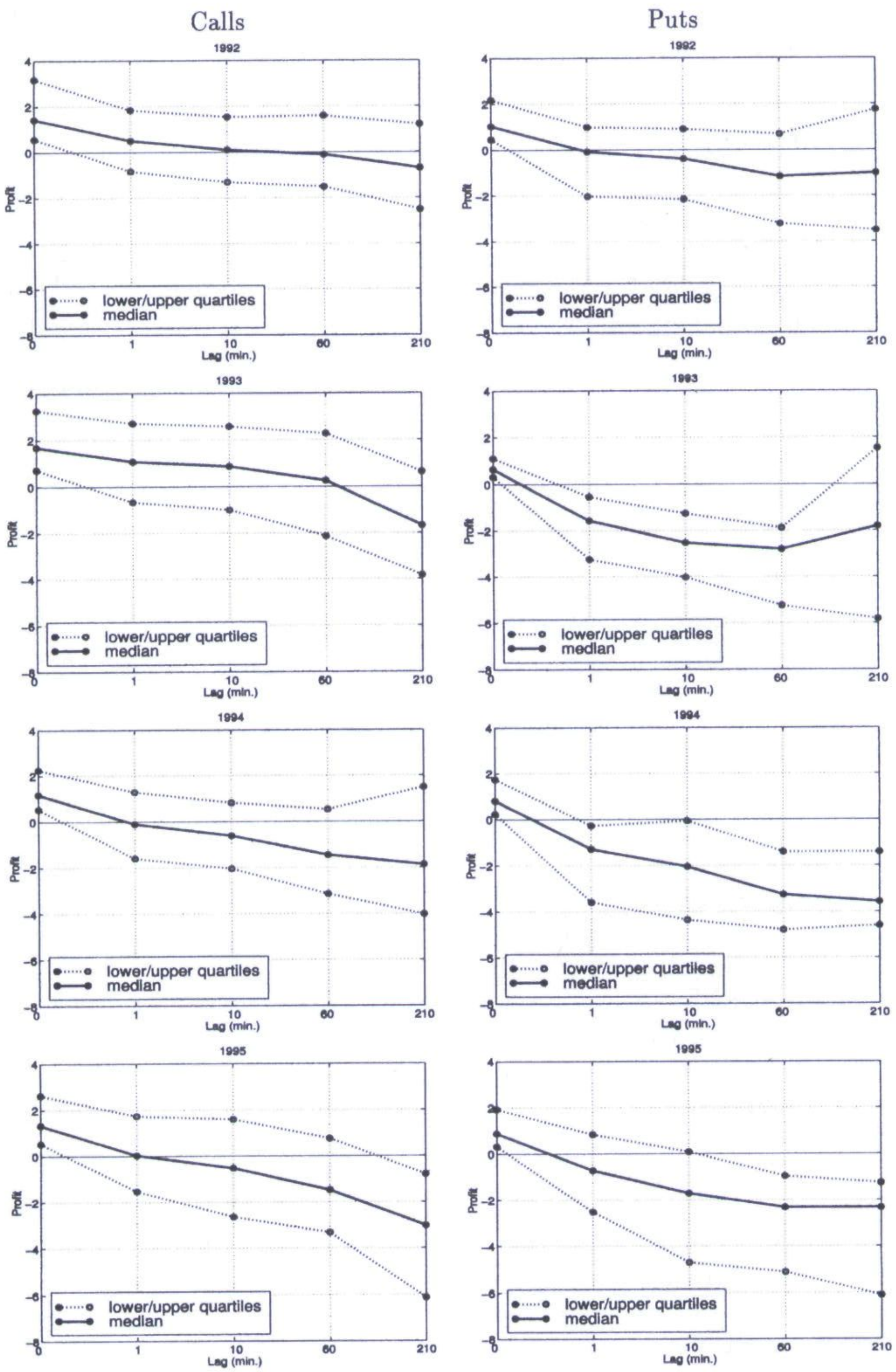

FIGURE 2
Median and quartiles of ex-post and ex-ante hedge profits from lower-boundary violations, with transaction costs.

trade, high losses occur frequently—especially for an investor applying the long-hedge strategy.

The results from the ex-ante tests give rise to several conclusions. First, even in the absence of transaction costs, lower-boundary violations, which are observed ex-post, cannot be fully exploited ex-ante, because prices tend to quickly move above the lower boundary. Thus, arbitrage opportunities dissipate quite rapidly, and the profit potential depends highly on the execution delay. Second, mispricing signals for calls, initiating the short-hedge strategy, seem to prevail for a longer period than those for puts. Because the long-hedge strategy is more easily implemented than the short-hedge strategy—the latter requires shorting the stocks underlying the index—the practical realization of short-hedge profits is questionable. Third, the efficiency of the DAX options market has improved over time. This follows from the ex-ante tests (see Tables I and II), which indicate that mean profits (losses) tend to be lower (higher) for the last two than for the first two years of the sample. This evidence is supported by the statistical significance of the ex-ante tests without transaction costs. Execution delays removed profit opportunities more slowly for the first two sample years than for the last two years, for which mean profits become insignificant (or significantly negative) more rapidly.

CONCLUSION

The efficiency of the German DAX–options market has been examined using transactions data over a period of 44 months. Rational lower-boundary violations have been analyzed on an ex-post and ex-ante basis. Ex-post violations of the lower-boundary condition are found for both calls and puts, but the portion of violating transactions varies over the years. Except for 1992, calls tend to violate lower-boundary conditions more often than puts. The ex-ante efficiency tests indicate that lower-boundary violations cannot be fully translated into hedge profits, because they tend to decrease quickly or vanish. Rather than being riskless, ex-ante profits are subject to immediacy risk. Therefore, profits reflect a compensation for the risk being taken.

There is some evidence that profit opportunities have decreased over the sample period, suggesting that traders gradually learned how to better price DAX options. When taking transaction costs into account, it is found that more than half of the mispricing signals, which are observed in the case of zero transaction costs, correspond to "violations" within the transaction-cost bounds. When increasing the execution lag of the hedge strategy, profit opportunities tend to decrease quickly. This suggests that the DAX–options market was efficient during the sample period. However, mispricing signals for calls prevailed longer than for puts. Given Germany's restrictions on short-selling, the implementation of a hedging strategy motivated by lower-boundary violations of calls is difficult, as it involves shorting the index.

To summarize, the empirical findings on lower-boundary violations during the period from February 1992 to September 1995 do not present any strong evidence against the efficiency of the DAX–options market. However, the results suggest that arbitrage efforts were undermined by institutional restrictions.

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