PRODUCTION AND HEDGING UNDER KNIGHTIAN UNCERTAINTY

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This article introduces Knightian uncertainty into the production and futures hedging framework. The firm has imprecise information about the probability density function of spot or futures prices in the future. Decision-making under such scenario follows the "max-min" principle. It is shown that inertia in hedging behavior prevails under Knightian uncertainty. In a forward market, there is a region for the current forward price within which full hedge is the optimal hedging policy. This result may help explain why the one-to-one hedge ratio is commonly observed. Also inertia increases as the ambiguity with the probability density function increases. When hedging on futures markets with basis risk, inertia is established at the regression hedge ratio. Moreover, if only the futures price is subject to Knightian uncertainty, the utility function has no bearing on the possibility of inertia. © 2000 John Wiley & Sons, Inc. Jrl Fut Mark 20: 397–404, 2000

INTRODUCTION

This article introduces Knightian uncertainty into the futures hedging framework. Knightian uncertainty allows ambiguity in the probability

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density function of the unknowns. Decision making under such scenario follows the "max-min" principle. Whereas the traditional probabilistic approach requires transaction costs to generate inertia in agents' actions (see, for example, Lence, 1996), "ambiguity aversion" underlying the Knightian uncertainty leads to the same results without additional assumptions. Herein the Knightian approach is applied to a hedger's production and futures hedging decisions.

Under Knightian uncertainty, inertia in hedging behavior is found. However, the inertia is not established in the "hedge or not hedge" decision; instead, it prevails in the "full hedge or not" dimension. More specifically, it is found that, when hedging with forward contracts, the producer is much more likely to adopt the traditional one-to-one hedge ratio under Knightian uncertainty. Probabilistic approach finds full hedging to be optimal only when the current forward price equals to the expected spot price in the future. Knightian uncertainty introduces a region for the current forward price within which full hedge is the optimal hedging policy. This result may help explain why the one-to-one hedge ratio is commonly observed even if the transaction cost is very small. Also, inertia increases as the ambiguity with the probability density function increases.

When hedging on futures markets with basis risk, inertia is established at the regression hedge ratio (that is, the ratio of the covariance between spot and futures prices to the variance of the futures price), provided certain conditions are satisfied. If only the futures price is subject to Knightian uncertainty, the utility function has no bearing on the possibility of inertia. The result is a generalization of Lence (1995). When the spot price is also subject to Knightian uncertainty, inertia prevails for some utility functions. Moreover, the specification of the utility functional form helps determine the prevalence of the inertia. For other utility functions, the regression hedge ratio is not optimal.

BASIC FRAMEWORK

Consider a one-period model from time 0 (the current time) to time 1 (the future). At time 0, the producer/hedger has to decide how much to produce and how much to trade forward. Production is non-stochastic and output becomes available at time 1. Forward contract provides a given price, f_0, for delivery at time 1. Let Q denote the output level and let x denote the amount of output sold on forward contracts. The producer's profit at time 1 is:

$$\pi = p_1(Q - x) + f_0x - c(Q),$$

where p_1 is the price prevailing at time 1 and c(.) is the cost function incurred at time 0. It is assumed that c(.) is increasing and strictly convex.

The producer has a von Neumann-Morgenstern utility function, u(.), assumed to be increasing and strictly concave. Optimal production level and forward sales are chosen so as to maximize the expected utility

$$Eu(\pi) = E[u(p_1(Q - x) + f_0x - c(Q))].$$

To introduce Knightian uncertainty, assume that the probability density function of p_1 cannot be precisely specified. With probability θ the probability density function of p_1 is g(.) with a bounded support $[p_L, p_H]$ and with probability $1 - \theta$ the probability density function can take any form with the same bounded support.

Decision making under Knightian uncertainty follows a max-min principle. That is, when the probability density is unknown the producer will assume the worst possible outcome. The optimal action is then chosen to maximize the resulting expected utility (Dow and Werlang, 1992). Let m(.) be an arbitrary probability density function with bounded support $[p_L, p_H]$. The objective function for the producer/hedger is as follows:

$$\underset{\{Q,\, x\}}{\text{Maximize}} \quad \underset{\{m\}}{\text{Minimize}} \quad \theta\, E_g[u(\pi))] + (1 - \theta)\, E_m[u(\pi)]$$

where E_g(.) and E_m(.) denote expectation operators when the density function of p_1 is g(.) and m(.), respectively.

OPTIMAL HEDGING DECISION

To solve the max-min problem, consider first the case $Q - x > 0$. Herein the worse outcome occurs at a degenerate density function: $h(p_L) = 1$ and zero elsewhere. Thus,

$$\begin{aligned} Eu_L(\pi) = {} & \theta\, E_g[u(p_1(Q - x) + f_0x - c(Q))] \\ & + (1 - \theta)\, u(p_L(Q - x) + f_0x - c(Q)). \end{aligned}$$

Similarly, when $Q - x < 0$, the degenerate density function, $h(p_H) = 1$ and zero elsewhere, generates the worse outcome. The expected utility is then

$$Eu_H(\pi) = \theta\, E_g[u(p_1(Q - x) + f_0x - c(Q))] + (1 - \theta)\, u(p_H(Q - x) + f_0x - c(Q)).$$

Full hedge (that is, $Q = x$) is the optimal solution if and only if the following two conditions are met: (i)$\partial Eu_L(\pi)/\partial x \geq 0$ when evaluated at $x = Q$; (ii)$\partial Eu_H(\pi)/\partial x \leq 0$ when evaluated at $x = Q$. Throughout the article, it is assumed that the second order condition is always satisfied. Given that c(.) is increasing and strictly convex and u(.) is increasing and strictly concave, Feder, Just, and Schmitz (1980) validated the assumption when the probability density function can be precisely specified.

After algebraic manipulation, condition (i) leads to

$$u'(f_0Q - c(Q))[\theta E_g(f_0 - p_1) + (1 - \theta)(f_0 - p_L)] \geq 0.$$

Similarly, condition (ii) implies the following:

$$u'(f_0Q - c(Q))[\theta E_g(f_0 - p_1) + (1 - \theta)(f_0 - p_H)] \geq 0.$$

Combining the above two conditions, Proposition 1 can be established.

Proposition 1: Full hedge is the optimal policy when the current forward price f_0 satisfies the property, $\theta E_g(p_1) + (1 - \theta)p_L \leqslant f_0 \leqslant \theta E_g(p_1) + (1 - \theta)p_H$..

Thus, there is a region for the current forward price within which full hedge is the optimal policy. Moreover, the region expands as p_L decreases, p_H increases. If one applies the range, the difference between the maximum and the minimum, to measure volatility, the result indicates that the more volatile the spot price is, the more likely full hedge will prevail.

The full hedge region also expands as θ decreases. A decrease in θ implies that the producer/hedger is less certain of the probability density function; and hence corresponds to an increase in ambiguity. Corollary 1 summarizes these results.

Corollary 1: Full hedge is more likely to be the optimal policy when the spot price is more volatile or when the producer/ hedger is more ambiguous about the probability density function.

As a special case, when $\theta = 1$, full hedge is the optimal policy only if $f_0 = E_g(p_1)$. This result is well known in the literature, see for example Feder, Just, and Schmitz (1980).

Proposition 2: The forward sale is less than the output, $x < Q$, when the current forward price is low, that is, $f_0 \leq \theta E_g(p_1) + (1 - \theta)p_L$. The forward sale will exceed the output, $x > Q$, when the current forward price is high, that is, $f_0 \geq \theta E_g(p_1) + (1 - \theta)p_H$.

These results are intuitive. Lower forward prices depress incentives to sell forward, whereas higher forward prices promote forward sales. Following FJS (1980), it can be easily shown that the optimal output level satisfies the condition, $f_0 = c'(Q)$. Thus, Knightian uncertainty has no effect on the production decision.

FUTURES MARKETS

If the producer engages in futures trading instead of forward contracting, (s)he has to liquidate her/his short position at time 1 by buying back the same amount of futures contracts. Profits at time 1 become

$$\pi^0 = p_1 Q + (f_0 - f_1)x - c(Q),$$

where f_1 is the futures price at time 1. When $f_1 = p_1$, that is, the case of no basis risk, futures trading is identical to forward contracting. The results from the previous section are directly applicable.

In the presence of basis risk, Batlin (1983) showed that the optimal output decision depends not only on the current futures price but also on the density function of the future spot price. Let $E(f_1) = f_0$ and assume that, conditional on f_1, $p_1 = \alpha + \beta f_1 + \varepsilon$ where f_1 and ε are stochastically independent. Under these assumptions, Lence (1995) showed that $-\beta$ is the optimal hedge ratio (that is, x/Q) regardless of the specification of the utility function. An extension of this result to the Knightian uncertainty framework is considered next.

Suppose that only the futures price is subject to Knightian uncertainty, whereas the conditional density of the spot price can be precisely specified as proposed in Lence (1995). Then by decomposing the joint expectation into marginal and conditional expectations,

$$Eu(\pi^0) = E_\varepsilon\, E_f\{u[(\alpha + hf_0 + \varepsilon + (\beta - h)f_1)Q - c(Q)]\},$$

where $h = -x/Q$ is the hedge ratio and E_f and E_ε denote expectations with respect to f_1 and ε, respectively. It is assumed that, with probability ϕ the probability density function of f_1 is $k(.)$ with a bounded support $[f_L, f_H]$ and with probability $1 - \phi$ it can take any functional form with the same bounded support.

Let $n(.)$ be an arbitrary probability density function with bounded support $[f_L, f_H]$. The objective function for the producer/hedger is the following:

$$\underset{\{Q,\, x\}}{\text{Maximize}} \quad \underset{\{n\}}{\text{Minimize}} \ \phi \mathrm{E}_{\varepsilon} E_k[u(\pi^0))] + (1 - \phi) E_{\varepsilon} E_n[u(\pi^0)]$$

where $\mathrm{E}_k(.)$ and $\mathrm{E}_n(.)$ denote expectation operators when the density function of f_1 is k(.) and n(.), respectively.

If $\beta - h > 0$, the worse scenario occurs when $n(f_L) = 1$ and zero elsewhere. The objective function becomes

$$Eu_L(\pi^0) = \phi \mathrm{E}_{\varepsilon} E_k[u(\pi^0))]$$
$$+ (1 - \phi) E_{\varepsilon}[u([(\alpha + hf_0 + \varepsilon + (\beta - h) f_L) Q - c(Q)])].$$

Similarly, if $\beta - h < 0$, the worse outcome is obtained when $n(f_H) = 1$ and zero elsewhere. The objective function is then

$$Eu_H(\pi^0) = \phi \mathrm{E}_{\varepsilon} E_k[u(\pi^0))]$$
$$+ (1 - \phi) E_{\varepsilon}[u([(\alpha + hf_0 + \varepsilon + (\beta - h) f_H) Q - c(Q)])].$$

Thus, $h = \beta$ is the solution if and only if (i)$\partial \mathrm{Eu_L}(\pi^0)/\partial h \geq 0$ when evaluated at $h = \beta$ and (ii)$\partial \mathrm{Eu_H}(\pi^0)/\partial h \geq 0$ when evaluated at $h = \beta$.

Applying the stochastic independence property between ε and f_1, after algebraic manipulation, conditions (i) and (ii) lead to the following two inequalities:

$$E_{\varepsilon}\{u'[(\alpha + \beta f_0 + \varepsilon) Q - c(Q)]\}[\phi \mathrm{E}_k(f_0 - f_1)$$
$$+ (1 - \phi)(f_0 - f_L)] Q \geq 0;$$
$$E_{\varepsilon}\{u'[(\alpha + \beta f_0 + \varepsilon) Q - c(Q)]\}[\phi \mathrm{E}_k(f_0 - f_1)$$
$$+ (1 - \phi)(f_0 - f_H)] Q \leq 0.$$

Note that u(.) is a strictly increasing function. The following result is immediate.

Proposition 3: The optimal hedge ratio is β when the current futures price f_0 satisfies the property, $\phi \mathrm{E}_k(f_1) + (1 - \phi) f_L \leq f_0 \leq \phi \mathrm{E}_k(f_1) + (1 - \phi) f_H$.

By assumption, conditional on f_1, $p_1 = \alpha + \beta f_1 + \varepsilon$ where f_1 and ε are stochastically independent. Let Cov(.,.) and Var(.) denote the covariance and variance operators, respectively. Then,

$$\mathrm{Cov}(p_1, f_1) = E[\mathrm{Cov}(p_1, f_1 | f_1)] + \mathrm{Cov}(E(p_1 | f_1), f_1) = \beta \, \mathrm{Var}\,(f_1).$$

In other words, β is the ratio of the covariance between spot and futures prices to the variance of the futures price despite that Knightian uncertainty prevails in the futures price. It is also commonly referred to as the regression hedge ratio. Proposition 3 establishes the inertia of the regression hedge ratio. There is a region for the current futures price within which the regression hedge ratio is the optimal policy. The region is independent of the utility functional form. Moreover, the region expands as the volatility or the ambiguity of the futures price increases.

Proposition 4: The optimal hedge ratio is smaller than (when the current futures price f_0 is smaller than $\phi E_k(f_1) + (1 - \phi)f_L$. It is larger than β when f_0 is larger than $\phi E_k(f_1) + (1 - \phi)f_H$.

IMPRECISE CONDITIONAL SPOT PRICE DISTRIBUTION

Propositions 3 and 4 were established with the assumption that the conditional spot price distribution is known, whereas Knightian uncertainty prevails in the futures price. Now, suppose that the conditional density function of the spot price is also imprecise. That is, $p_1 = \alpha + \beta f_1 + \varepsilon$ with probability λ and the conditional density can take any form with a bounded support $[p_L, p_H]$ with probability $1 - \lambda$. Thus, the new objective function is

$$\lambda\ E_\varepsilon\ E_f\{u[(\alpha + hf_0 + \varepsilon + (\beta - h)f_1)Q - c(Q)]\}$$
$$+ (1 - \lambda)E_f\ \{u[(p_L - h(f_1 - f_0))Q]\}$$

after the worst scenario for the spot price is taken into account. Applying a similar analysis to the one performed above, Proposition 5 follows.

Proposition 5: The regression hedge ratio, β, is the optimal hedge ratio if the following two conditions are met:

$$\lambda\Delta_L + (1 - \lambda)\Omega \geq 0;$$
$$\lambda\Delta_H + (1 - \lambda)\Omega \leq 0,$$

where

$$\Delta_L = E_\varepsilon\{u'[(\alpha + \beta f_0 + \varepsilon)Q - c(Q)]\}[\phi E_k(f_0 - f_1)$$
$$+ (1 - \phi)(f_0 - f_L)]Q,$$
$$\Delta_H = E_\varepsilon\{u'[(\alpha + \beta f_0 + \varepsilon)Q - c(Q)]\}[\phi E_k(f_0 - f_1)$$
$$+ (1 - \phi)(f_0 - f_H)]Q, \text{ and}$$

$$\Omega = \theta E_k[u'((p_L + \beta(f_1 - f_0))Q - c(Q))(f_1 - f_0)] + (1 - \theta)u'((p_L + \beta(f_L - f_0))Q - c(Q))(f_L - f_0).$$

For some utility functions, the two inequalities can be established and, hence, inertia of the regression hedge ratio prevails. However, the occurrence of the inertia depends upon the utility functional form (in contrast to the results in Propositions 1 and 3). In addition, the two inequalities may not hold for certain utility functions. For these cases, β is actually not the optimal hedge ratio.

CONCLUSIONS

This article introduces Knightian uncertainty into the production and hedging framework. It shows that there is a region for the current futures price within which full hedge is the optimal policy for forward contracting. For futures trading, inertia is established at the regression hedge ratio. The inertia becomes more prevalent when the spot (or futures) price volatility increases or when the hedger's ambiguity increases.

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