Integration and Arbitrage in the Spanish Financial Markets: An Empirical Approach*

ALEJANDRO BALBÁS
IÑAKI R. LONGARELA
ÁNGEL PARDO

Several authors have introduced different ways to measure integration between financial markets. Most of them are derived from the basic assumptions about asset prices, like the Law of One Price or the absence of arbitrage opportunities. Two perfectly integrated markets must give identical prices to identical final payoffs, and a vector of positive discount factors, common to both markets, must exist. If these properties do not hold, the degree to which they are violated can be defined and considered as a measure of integration.

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- *Alejandro Balbás is in the Dpto. de Economia de la Empresa at Universidad Carlos III in Madrid, Spain.*
- *Iñaki R. Longarela is in the Dpto. de Economia de la Empresa at Universidad Carlos III in Madrid, Spain.*
- *Ángel Pardo is in the Dpt. de Economia Financiera y Matematica at Universidad de Valencia in Valencia, Spain.*

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The present paper empirically tests integration measures in the Spanish financial markets. Furthermore, the integration measures are operationalized to analyze the presence of cross-market arbitrage without previously specifying the exact nature of the arbitrage strategy to be used. When the absence of arbitrage holds, several interesting variables are obtained, for instance, state prices or risk-neutral probabilities. When this absence fails, explicit cross-market arbitrage portfolios are provided.

The results of the test yield some evidence about market efficiency and integration outside the United States, and they are surprising for several reasons. First of all, arbitrage opportunities do sometimes appear, and the bid–ask spread and transaction costs seem to be unable to prevent arbitrage profits. Furthermore, the criticisms that are usually raised when empirical papers show the existence of arbitrage opportunities do not apply here because we work with perfectly synchronized high frequency data. On the other hand, different integration measures show a similar evolution along the tested period, although these measures give different information about the markets' efficiency and integration, and they do not necessarily have to be related.

INTRODUCTION

The absence of arbitrage opportunities is a basic assumption common to all asset pricing models, usually characterized by the existence of positive state prices (Ingersoll (1987) or, under the appropriate hypotheses, by the martingale property in dynamic approaches (see Harrison & Kreps 1979; Hansen & Richard 1987; or Back & Pliska 1991).

This absence has been very often tested in studies applying different procedures and aiming at different objectives. Sometimes authors are interested in the financial markets' degree of efficiency. In other situations, the main focus of the analysis is the integration among two or more markets (see, e.g., Chen & Knez 1995; or Kempf & Korn 1996).

When empirical papers analyze financial market efficiency, they usually focus on well-known trading rules or arbitrage portfolios. So, for instance, Hudson et al. (1996) test simple technical trading rules, and Sternberg (1994) or Kamara and Miller (1995) test the existence of violations of put-call parity. Empirical analyses of financial markets integration test the price of specific assets or portfolios in different markets (see, e.g., Harris et al. 1995) or, once again, test concrete arbitrage portfolios (Lee & Nayar 1993), which are specially related to the Law of One Price when spot and future markets are simultaneously involved (Protopapadakis & Stoll 1983; or Kempf & Korn 1996).

Chen and Knez (1995) consider state prices to introduce a new integration measure that is never lower than zero and must vanish in order to guarantee the absence of cross-market arbitrage opportunities. Their measure globally tests the absence of (cross-market) arbitrage, and not just some specific or concrete strategies. The advantages of this new methodology are clear. First, even if different markets give the same price to some specific securities, portfolios or well-known replicas, this is far from being a sufficient condition to guarantee the absence of cross-market arbitrage. Second, computing how often the Chen and Knez measure is greater than zero shows how often a risk-neutral probability measure does not exist in practice and, consequently, how often real data present some deviations with respect to the basic theoretical principles of asset pricing.

Chen and Knez empirically analyzed the NYSE and NASDAQ markets and reported that these markets seem to violate strong-form integration (i.e., cross-market arbitrage may be possible). They also noted that the measure depends crucially on the assumption of a frictionless market and suggested that their discussion should be extended to economies with trading frictions.

Balbás and Muñoz (1998) introduce a new nonnegative measure, which also must vanish and can be computed by solving a dual pair of alternative but equivalent mathematical programming problems. The dual problem provides risk-neutral probabilities, or a proxy for them if arbitrage is possible. The primal problem gives arbitrage portfolios. The measure, **m**, represents attainable relative arbitrage gains; thus, transaction costs can be discounted to analyze whether arbitrage still exists. As Balbás and Muñoz showed, there are some relations between the Chen and Knez measure, **a**, and **m**, but **a** focuses on pricing rules rather than arbitrage gains. As the two measures reflect different things, they may achieve quite different values, and both can be considered in analyzing the integration between markets; see Balbás and Muñoz (1998) for a further discussion.

The present paper applies methods derived from these measures to test the efficiency and degree of integration of Spanish markets and provides further evidence about market efficiency and integration outside the United States. Unlike previous studies, this one considers all the arbitrage strategies available in the market(s) in order to compute the optimal one. Furthermore, it always obtains risk-neutral probabilities, or a proxy for them when arbitrage is possible. Finally, the markets analyzed present some suitable factors such as their computerized trading system, their phenomenal recent growth, and the positive results that have been obtained when other tests of efficiency have been implemented (Lee & Mathur [1999] show that the random walk hypothesis cannot be rejected in the Spanish futures markets).

Some cautions have been incorporated in order to guarantee that agents can really implement the detected arbitrage portfolios. First, the test employs perfectly synchronized high frequency data and two prices (bid–ask) for each asset, and it is considered that investors can sell or buy any security, but the price is larger if they buy. Second, the hypotheses are very weak and are compatible with any "reasonable" asset-pricing model. In fact, the analysis involves pure discount bonds and derivatives on the Spanish index IBEX-35 (European put and call options and future contracts with identical expiration dates and times), but it only makes obvious assumptions about prices at maturity or the expiration date. Third, the transaction costs are estimated and discounted when arbitrage occurs.

Although the test uses only bonds and derivatives on the index, it is also interesting to analyze what happens if the underlying index is incorporated as well. This implies some technical problems because the index is a more complex security. Consequently, a preliminary study presented here uses only two securities: the index and the price of its replica composed of a future and a bond. Computations of the Chen and Knez and the Balbás and Muñoz measures indicate that arbitrage is impossible after transaction costs. Surprisingly, however, both measures show similar evolutions during the period analyzed. As will be shown in the third section of the paper, this could suggest the existence of some relations between the two measures in some restricted contexts; more research about this question would be interesting.

The second analysis involves the same bonds and derivatives and presents computations of the Balbás and Muñoz measure for two different periods. The first one, February–March 1997, can be considered a standard period characterized by stability of financial markets. The second one, October 1997, is characterized by market turmoil, large volatilities, and the effect of the Asian crisis. Analyzing financial market behavior during critical periods can lead to important conclusions about their structure, regulation, and other usual topics (see Kleidon & Whaley 1992).

The main conclusions presented here are surprising. Even in the stable period there are several dates for which there are no positive state prices, positive discount factors, or risk-neutral probabilities. Even after transaction costs are discounted, (cross-market) arbitrage seems to be possible. This conclusion could not have been proved simply by testing concrete strategies. During the convulsive period arbitrage occurs far more often, and the integration measure takes very large values, which means inefficiency on those days.

The paper is organized as follows: the first section discusses methodology along with notation issues, basic assumptions, and some aspects of the integration measures that will be applied. The second section includes a detailed description of markets and data. The third section describes the empirical results, and the fourth analyzes how traders can work in practice to price call and put options. The fifth section concludes and summarizes the paper.

METHODOLOGY

Following the classic model of Ingersoll (1987), this study uses a static pricing approach with a finite number of states of nature. The two-period model is characterized by two dates t_1, t_2 ($t_1 < t_2$) and n different assets denoted by $A^1, A^2, \ldots, A^n$. In order to incorporate the usual bid–ask spread, let v_k and c_k ($k = 1,2, \ldots, n$) be two different values such that $0 \le v_k \le c_k$. So v_k and c_k will respectively represent the bid and ask prices at t_1 for the k^{th} asset. Whenever $v_k = 0$ ($c_k = \infty$), it will be understood that the k^{th} asset cannot be sold (bought). $E_1, E_2, \ldots, E_r$ denote the states of nature at t_2, with the subsequent matrix of payoffs $Z = (z_{ik})$ where $i = 1,2, \ldots, r$ and $k = 1,2, \ldots, n$; that is, z_{ik} will be the payoff for the k^{th} asset in the state of nature E_i. It is assumed that $z_{ik} \ge 0$ since these constraints will always hold in the present empirical analysis.[1]

A portfolio in the feasible set is represented by a vector $x = (x_1, x_2, \ldots, x_n)$ such that $x_k \ge 0$ ($x_k \le 0$) whenever $v_k = 0$ ($c_k = \infty$). As usual, its price at t_1 is $P(x) = p_1x_1 + p_2x_2 + \ldots + p_nx_n$ where $p_k = v_k$ ($p_k = c_k$) if $x_k \le 0$ ($x_k \ge 0$), and its payoffs at t_2 are given by the column vector Zx^t where x^t represents the transpose of x.

For any feasible portfolio x, let x^+ and x^- be the portfolios given by the long and short positions respectively, i.e.,

$$x_k^+ = Max\{x_k, 0\} \textit{ and } x_k^- = Max\{-x_k, 0\}$$

for $k = 1,2, \ldots, n$. The prices of the purchased and sold assets will be denoted by $C(x)$ and $V(x)$, respectively, and they are given by

[1]The constraint $z_{ik} \geqslant 0$ may be avoided in a general framework. However, it simplifies the statement of Theorem 2, and it is fulfilled in the present empirical test. In fact, although the final payoff of a future contract may be negative, securities A^k $k = 1,2,\ldots, n$ will never coincide with this contract. Instead, a concrete security will be the proxy of the index composed of a future and a riskless asset.

$$C(x) = c_1x_1^+ + c_2x_2^+ + \ldots + c_nx_n^+$$

and

$$V(x) = v_1x_1^- + v_2x_2^- + \ldots + v_nx_n^-$$

The relationships $x = x^+ - x^-$ and $P(x) = C(x) - V(x)$ are obvious.

The concept of arbitrage is defined in accordance with Prisman (1986) and Ingersoll (1987).

Definition *The feasible portfolio x is said to be an arbitrage strategy of the second type (henceforth, "arbitrage strategy") if its price at t_1 is negative, and its payoff at t_2 is zero or greater than zero in all the states of nature.*

By readapting the proofs of Theorem 2 in Ingersoll (1987, p. 55) and Theorems 3 and 4 in Balbás and Muñoz (1998), the following results may be established (see also Jouini & Kallal 1995).

Theorem 1 *There are no arbitrage strategies if and only if there exists at least a nonnegative vector of state prices, i.e., a nonnegative vector $d = (d_1, d_2, \ldots, d_r)$ such that for every $k = 1, 2, \ldots, n$*

$$v_k \leq z_{1k}d_1 + z_{2k}d_2 + \ldots + z_{rk}d_r \leq c_k \qquad \blacklozenge$$

The intuition underlying the theorem above seems clear. The sum $z_{1k}d_1 + z_{2k}d_2 + \ldots + z_{rk}d_r$ may be considered as a theoretical price for A^k $k = 1, 2, \ldots, n$ and, consequently, this price must lie within the bid–ask spread to prevent arbitrage.

Theorem 2 *Assume that there are arbitrage opportunities. For each feasible arbitrage strategy x, consider its current price $P(x)$, the price of the sold assets $V(x)$, and the price of the purchased assets $C(x)$. Then the quotients*

$$-P(x)/V(x) \text{ and } -P(x)/[V(x) + C(x)]$$

have a maximum value achieved at the same arbitrage portfolio x^*. $\blacklozenge$

The preceding definition implies that $P(x)$ (the transaction price of the portfolio x, i.e., the net price of both sold and purchased assets) must be negative if x is an arbitrage portfolio. Hence, $-P(x)$ is positive and represents the arbitrage profit. Therefore, the first ratio yields relative arbitrage gains with respect to the price of the sold assets, while the second one provides relative arbitrage gains with respect to the total traded value.

Denote by x^* the portfolio that optimizes both ratios, whose existence is guaranteed by Theorem 2. The measures $\boldsymbol{m}$ and $\boldsymbol{l}$ will be given by

$$\boldsymbol{m} = -P(x^*)/V(x^*) \text{ and } \boldsymbol{l} = -P(x^*)/[V(x^*) + C(x^*)]$$

or zero if no arbitrage opportunities do exist. The inequalities

$$0 \leq \boldsymbol{l} \leq \boldsymbol{m} \leq 1$$

are obvious, and the relationship $\boldsymbol{l} = \boldsymbol{m}/(2 - \boldsymbol{m})$ may be proved (Balbás & Muñoz 1998). Furthermore, x^* although is the optimal solution for two non-linear ratios, this portfolio is easily computed in practice (and, thus, so are $\boldsymbol{m}$ and $\boldsymbol{l}$). In fact, consider two arbitrary portfolios $(y_1, y_2, \ldots, y_n)$ and $z = (z_1, z_2, \ldots, z_n)$ and the linear functions

$$K(y) = c_1 y_1 + c_2 y_2 + \ldots + c_n y_n$$

and

$$W(z) = v_1 z_1 + v_2 z_2 + \ldots + v_n z_n$$

Then, the following linear programming problem is bounded

$$\textit{Max}\ \{W(z) - K(y); \textit{subject to}\ Z(y^t - z^t) \geq 0, W(z) \leq 1, y \geq 0, z \geq 0\}$$

If y^* and z^* solve the problem, it may be proved that $(x^*)^+ = y^*$ and $(x^*)^- = z^*$, from where the equality $x^* = y^* - z^*$ trivially follows (see Balbás and Muñoz ,1998, for further details).[2]

Note that $\boldsymbol{m}$ and $\boldsymbol{l}$ provide information in monetary terms, which allows analysis of markets with frictions because transaction costs can be discounted after computing the (relative) arbitrage profits given by these measures. Furthermore, if the n securities are traded in two or more markets, $\boldsymbol{m}$ and $\boldsymbol{l}$ can represent their integration level. It is clear that the integration level decreases when these measures increase and vice versa. The measures achieve low values when the markets are well integrated.

The integration measure of Chen and Knez (1995) is introduced by means of state prices. Consider that $A^1, A^2, \ldots, A^s$ can be traded in a first market $\boldsymbol{M}_1$, and $A^{s+1}, A^{s+2}, \ldots, A^n$ correspond to a second market $\boldsymbol{M}_2$. Assume that no arbitrage strategies can be found in the market $\boldsymbol{M}_z$

[2]The function K (alternatively W) represents the price of y (alternatively z) if it is bought (sold) at the ask (bid) price. The constraint $Z(y^t - z^t) \geq 0$ will guarantee nonnegative final payoffs, and $W(z) \leq 1$ normalizes the problem (the model seeks relative arbitrage profits) and ensures that it is not unbounded.

(z = 1,2). Then, it is possible to find the set $\mathbf{D}_z \subset R^r$ of nonnegative state prices for $\boldsymbol{M}_z$ (z = 1,2), and the measure of Chen and Knez is given by the infimum value of the set

$$Inf\ \{\|d - d'\|^2 \colon d \in \mathbf{D}_1 \text{ and } d' \in \mathbf{D}_2\}$$

where $\|.\|$ represents the usual Euclidean norm. Denoting this measure by $\boldsymbol{a}$ (or $\boldsymbol{a}$ $(\boldsymbol{M}_1, \boldsymbol{M}_2)$ if necessary), it trivially follows from Theorem 1 that $\mathbf{a} = 0$ if there are no cross-market arbitrage strategies, and the converse may be demonstrated by readapting the proof of Chen and Knez (1995). Therefore, the markets are perfectly integrated if and only if $\boldsymbol{a} = 0$. When the measure $\boldsymbol{a}$ does not vanish but is small enough, the state prices of both markets are "near," and markets are also "near" to perfect integration. If $\boldsymbol{a}$ takes high values then the markets price the securities by applying quite different criteria (the distance between the state prices is "large" and, thus, the distance between the pricing rules is also "large") and the integration level is low. In practice, $\boldsymbol{a}$ may be computed by applying the procedures provided by Chen and Knez (1995).

As was shown in Balbás and Muñoz (1998), the measures $\boldsymbol{m}$ (or $\boldsymbol{l}$) and $\boldsymbol{a}$ are quite different and give complementary information about the integration level, although they have some common elements. It is important to point out that some simple examples may be given such that both measures take very different values and, therefore, the distance between state prices ($\boldsymbol{a}$) and the relative arbitrage profits ($\boldsymbol{m}$) are far from being equivalent criteria. The reason is clear: $\boldsymbol{m}$ computes the markets' discrepancy when they price available assets or portfolios, and $\boldsymbol{a}$ computes discrepancy in the state prices or pricing rules. Of course, state prices are specific criteria for pricing real securities, but quite different state prices can sometimes lead to similar prices for these real securities. Thus, if $\boldsymbol{a}$ and $\boldsymbol{m}$ do not vanish, they reflect similar things only when the portfolio x^*, given by Theorem 2, yields a payoff vector close to the normalized vector

$$(d - d')/\|d - d'\|$$

that provides $\boldsymbol{a}$.

This paper concentrates on Spanish financial markets and examines a riskless pure discount bond, a stock index, future contracts on the stock index and European options on the future contract. Two different situations will be studied. First, the Chen and Knez and the Balbás and Muñoz measures are computed by comparing the current prices of the index and its proxy consisting of the riskless asset and the future contract. The states of nature at t_2 are given by a set of 1,200 equidistant values in the interval $(I_1 - 600, I_1 + 600)$, where I_1 is the value of the stock index at t_1.

The second situation involves all the assets mentioned excluding the index, which has been replaced by its proxy. In this case only the Balbás and Muñoz measure is computed. The first date, t_1, will be arbitrary, while the second, t_2, will coincide with the derivatives expiration date. The states of nature will be finite and described by I_2, the index feasible values at t_2, which will be discrete. It will be assumed that I_2 will move between 60% and 140% of I_1, where I_1 is either the index present value or the average price of the future contract.[3] It is also assumed that the value of I_2 will coincide with either the striking price of some available (call or put) option, or an extreme value $0.6I_1$ or $1.4I_1$. Lemma 3 will guarantee that this assumption does not modify the value of $\boldsymbol{m}$.

As usual, the call (put) options final payoff (at t_2) has been assumed to be determined by $\text{Max}\{I_2 - E, 0\}$ ($\text{Max}\{E - I_2, 0\}$) where E represents the strike price. The shadow asset[4] final payoff will equal I_2, and the bond final payoff will always be 1, and does not depend on I_2. Therefore, if the increasing sequence $E_1 \leq E_2 \leq \ldots \leq E_s$ represents the striking prices for the different options available in the market at t_1, and the inequalities $0.6I_1 \leq E_1$ and $E_s \leq 1.4I_1$ hold, then the following result may be easily proved:

Lemma 3 *The feasible portfolio x has non negative final payoff for any value of I_2 such that $0.6I_1 \leq I_2 \leq 1.4I_1$ if and only if x has nonnegative final payoff when I_2 belongs to the finite set of values $[0.6\, I_1, E_1, E_2, \ldots, E_s\, 1.4I_1\}$.*

The above lemma guarantees that it is possible to work with a finite number of states of nature and apply the results shown in this section. Of course, the states of nature may change (the available options or the value I_1 may change) if t_1 changes. Therefore, $\boldsymbol{m}$ will depend on t_1. Once t_1 has been fixed, the process has two stages. First, the absence of arbitrage opportunities is determined by applying Theorem 1; whenever riskless profits are feasible, a second stage allows identification of the optimum arbitrage portfolio whose existence is guaranteed by Theorem 2.

MARKETS AND DATA

The Spanish financial markets considered here are *the Sistema de Interconexión Bursátil Español* (SIBE), the *Mercado Español de Futuros Fin-*

[3]In this particular empirical analysis, this is a very realistic assumption since $t_2 - t_1$ is never longer than one month.

[4]The shadow asset is the usual replica of the stock index: the future contract and the pure discount bond.

ancieros sobre Renta Variable (MEFF-RV) and the *Mercado de Deuda Anotada* (MDA). MEFF-RV is one of the most important futures and options exchanges in Europe. Specifically, the IBEX-35 stock index future was the most traded contract in the world during 1994 and 1995 (Sutcliffe 1997, p 59).

The IBEX-35 index is a capitalization-weighted index comprising the 35 most liquid Spanish stocks traded in the continuous market (SIBE), and this index is the underlying asset for IBEX-35 Plus future contracts. The IBEX-35 Plus European options are based on the future contract. All derivatives are available only in MEFF-RV. The premium quotation is measured in index points, and each point is equivalent to 1,000 pesetas.[5] The maturity date is the third Friday of the expiration month. It should be emphasized that all derivatives considered take as settlement price at expiration the arithmetic average index value between 16:15 and 16:45 hours on the expiration date, taking a value per minute. Finally, for all the minutes of the same day, the proxy for both the lending and the borrowing rate of interest is the daily interest rate corresponding to the pure discount bond (traded in MDA) whose maturity is closest to the maturity of the derivative contracts.[6]

The trading hours at SIBE are from 10:00 a.m. to 5:00 p.m., while MEFF-RV is open from 10:00 a.m. to 5:15 p.m. In this study, minute-by-minute bid and ask prices for all securities are employed during two intervals: a stable one, from February 24 to March 21, 1997, and a very convulsive period, from October 22 to October 30, 1997. At each minute a two-period model is applied with t_1 being the corresponding minute and t_2 the expiration date. All derivatives have the same expiration date, March 21, 1997, for the first analysis and November 21, 1997, for the second one.

There are two criticisms that usually are raised when empirical papers show the existence of arbitrage opportunities. First, the data must be perfectly synchronized, and second, the transaction costs must be discounted. It is important to point out that the test is implemented with high frequency data and two prices (bid–ask) for each asset, so the ar-

[5]Although the euro has become the European currency since January 1, 1999, and now each index point is equivalent to 10 euros, assets were still quoted in pesetas at the date for which the Spanish markets were tested.

[6]Obviously, borrowing rates are larger than lending rates, so the use of a simple interest rate might be a shortcoming of the study. Anyway, the analysis ignores the spread and takes daily data for several reasons. First, the spread is very low, and information about minute-by-minute borrowing and lending rates was not available. Second, interest rates have little influence on the final number of arbitrage opportunities since they affect only short periods (always shorter than one month and often shorter than two weeks or one week). Third, almost all the detected arbitrage opportunities still work if the lending rate is equal to zero, or the borrowing rate slightly increases.

bitrage strategies are detected under a very restrictive assumption: an investor can sell or buy any portfolio, but the price is larger when he/she buys it. Furthermore, the use of the bid and ask prices of the securities, instead of the last transaction price, eliminates both the nonsynchronous bias and the nontrading effect, so that the highest possible precision is achieved. In order to avoid the difficulties implied by stock market transaction costs, whenever the index is needed to design an arbitrage strategy, it is replaced by its usual replica. Hence, the model works with options, a riskless pure discount bond with maturity at the expiration date, and a shadow asset composed of the future contract and the bond.[7] If F (F', with $F' \geq F$) is the bid (ask) index future price, then $F/(1 + r^*)$ ($F'/(1 + r^*)$) will be the bid (ask) price for the shadow asset, where r^* verifies $1 + r^* = (1 + r)^T$, T verifies $T = t_2 - t_1$, and r being the interest rate from t_1 to t_2, which is obtained from MDA.

EMPIRICAL RESULTS

The first analysis examines a stable period that corresponds to 19 consecutive trading days before the March 1997 expiration date. The payoff matrix for each minute during these days has been computed on the basis of the IBEX-35 stock index. This yields 420 trading minutes per day and 7980 for the whole period.

The level of integration between the IBEX-35 Plus futures contract and its underlying asset is examined first. Although the particular extension of the Chen and Knez (1995) measure used here does take into account the bid and ask future contract prices, to compare $\boldsymbol{a}$ and $\boldsymbol{m}$ the original Chen and Knez measure is used, so the average price is employed in the calculations. Results are given in Figure 1, where these measures are plotted against the corresponding minutes for March 4, 1997, a date that is representative of the general market behavior during this stable interval.

Despite the fact that $\boldsymbol{a}$ is based on state prices and $\boldsymbol{m}$ uses relative profits from an optimum arbitrage portfolio, both measures follow an identical pattern. This finding may merit future research because a relationship between both measures might exist in a restricted context like this. The effect of transaction costs can be discounted by simply subtract-

[7]Basically, the test incorporates futures and options on the same underlying asset, like the analysis implemented by Lee and Nayar (1993), but with a very important difference. Here, the exact nature of the strategy to be used is not specified. On the contrary, the existence of arbitrage is first analyzed. Later, if the arbitrage is possible, the optimal arbitrage portfolio is computed.

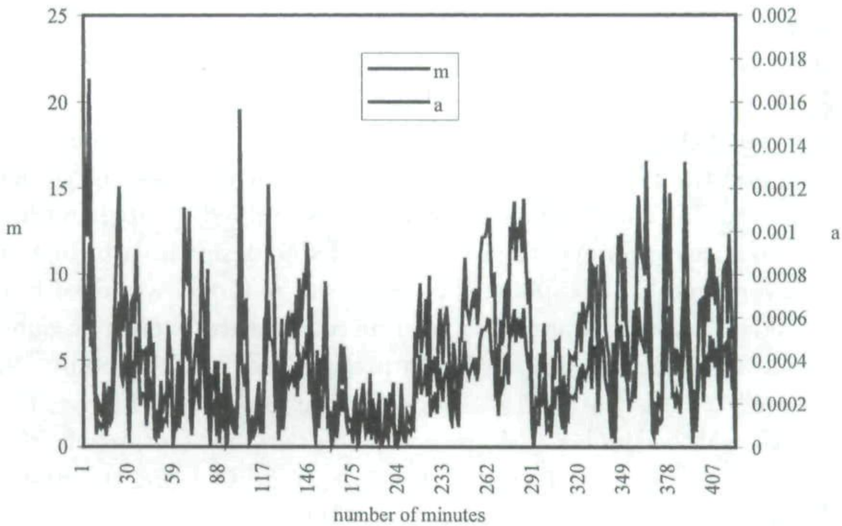

FIGURE 1
Chen and Knez (a) and Balbás and Muñoz (m) measures for the level of integration between the IBEX-35 Plus futures contract and its underlying asset versus time for March 4, 1997.

ing 32.58 basic points from the value of m.[8] Hence, the results justify the conclusion that these two markets are perfectly integrated.

For the same period, this study examines efficiency and integration via the IBEX-35 index (replaced by its proxy), European call and put options on its future contracts and pure discount bonds. The measures m and l have been computed; Table I reports the number of minutes in which arbitrage opportunities were detected along with the maximum value of m for each day. The number of securities for which the bid or the ask price (or both) is available depends on the date (see column 2), and the mean value of this number is 26. Except for the first week, results show that on most of the days riskless profits can be earned, even after transaction costs are considered (see columns 3 and 4). Arbitrage strategies are possible during 1.2 percent of the analyzed minutes although this percentage decreases to 0.46 percent when transaction costs are taken into account.[9] The extremely large values obtained for March 14

[8]Using l as an integration measure allows transaction costs to be discounted. If these costs are proportional to the total traded value ($V(x^*) + C(x^*)$), which may well be assumed in this particular case, and TC is the ratio between the transaction costs and $V(x^*) + C(x^*)$, it follows that $l \leq TC$ must hold in order to guarantee that these markets are integrated. Since $m = 2l/(1 + l)$, m must be less than the quotient $2TC/(1 + TC)$. TC has been estimated in light of the transaction costs associated with the future contract and the average value of the spread that corresponds to the 35 stocks; markets are integrated if m is lower than 32.58 basic points.

[9]In the Spanish options market, transaction costs depend on the trader, but their lowest value is

TABLE I

The first column gives the corresponding day. The second column indicates the maximum number of available securities (securities for which the bid or the ask price is available). The third column gives the number of minutes in which arbitrage opportunities exist if transaction costs (TC) are not considered, while the fourth column takes them into account. The fifth column shows the maximum value of m in basic points. Annualized intraday volatility is given in column 6. All derivatives have as expiration date March 21, 1997.

Day	*Assets*	*Without TC*	*With TC*	*Maximum m*	Volatility
970224	27	0	0	0.00	11.60%
970225	24	0	0	0.00	17.66%
970226	28	1	0	1.52	12.79%
970227	25	0	0	0.00	10.30%
970228	25	1	1	29.63	15.69%
970304	26	0	0	0.00	9.80%
970305	26	10	10	45.01	14.28%
970306	31	2	1	3.53	11.17%
970307	24	4	3	64.05	12.24%
970310	24	10	5	15.78	11.56%
970311	32	26	8	8.33	11.97%
970312	29	5	4	17.58	10.75%
970313	28	2	1	3.45	14.38%
970314	31	6	6	**4704.30**	15.02%
970317	27	6	4	5.40	12.78%
970318	21	12	6	7.34	12.15%
970319	24	1	1	42.27	10.78%
970320	25	6	5	12.92	11.64%
970321	15	4	4	**3025.18**	12.68%
Total/Mean	26	96	59	—	12.59%

and 21 may well be attributable to a serious mispricing caused by a mistake in the premiums offered and demanded (the corresponding arbitrage opportunities last for only one minute). These are unique strategies that

equal to 100 pesetas per contract. These costs amount to 450 pesetas per contract in the futures market. If it is assumed (accordingly the empirical results) an average value of five options and a future contract per arbitrage portfolio, transaction costs are about 1,000 pesetas or, equivalently, one index point. The average price of the sold assets ($V(x^*)$) is close to 6,000 index points. Thus, the ratio between the transaction costs and $V(x^*)$ may be estimated as $1/6000 = 0.0001666$; therefore, m must be greater than 1.66 basic points in order to make up for transaction costs. Anyway, to avoid errors, it has been considered that an arbitrage portfolio overcomes transaction costs when m is greater than 1.8 basic points. This critical value of m is far smaller than 32.58 basic points, the critical value obtained from comparing the index and its proxy. The reason is simple. In this second analysis the effect generated by the bid–ask spread has already been incorporated.

There is another way to discount the transaction costs since each arbitrage strategy may be particularly analyzed. A concrete example will be shown later (see footnote 12), although the global results are not very sensitive with respect to the procedure applied (actually, the number of arbitrage opportunities grows slightly if this alternative method is considered).

TABLE II

The first column gives the minutes in which arbitrage opportunities are detected, clustered in 30 minute intervals. The second column indicates the number of minutes in which arbitrage opportunities exist if there are no transaction costs (TC), while the third column takes them into account. The fourth and fifth column show these minutes as a percentage of all arbitrage opportunities. All derivatives have as expiration date March 21, 1997.

	Number of minutes		*Percentage*	
Interval	*Without TC*	*With TC*	*Without TC*	*With TC*
1001–1030	1	1	1.04%	1.69%
1031–1100	5	2	5.21%	3.39%
1101–1130	6	5	6.25%	8.47%
1131–1200	1	1	1.04%	1.69%
1201–1230	15	3	15.63%	5.08%
1231–1300	9	6	9.38%	10.17%
1301–1330	4	4	4.17%	6.78%
1331–1400	1	0	1.04%	0.00%
1401–1430	1	0	1.04%	0.00%
1431–1500	4	4	4.17%	6.78%
1501–1530	7	3	7.29%	5.08%
1531–1600	9	5	9.38%	8.47%
1601–1630	17	15	17.71%	25.42%
1631–1700	16	10	16.67%	16.95%
Total	96	59		

involve only the options market. Notice that both days are Fridays, and March 21 is the expiration date. Consequently, the high values of $\boldsymbol{m}$ are also a manifestation of the strange expiration date effects pointed out by Day and Lewis (1988).

The number of arbitrage opportunities is grouped into 30-minute intervals in Table II, and the intraday pattern is illustrated in Figure 2. This pattern follows a U-shape from noon on, achieving its minimum in the interval from 13:30 to 14:30, during which no arbitrage opportunities are detected after transaction costs are considered.[10] More than half of the opportunities (51.05% and 55.92% without and with transaction costs, respectively) take place in the last two hours of the trading sessions.

A call for caution is in order here, before jumping to conclusions about efficiency in each Spanish market. Optimum arbitrage strategies are derived whenever state prices do not exist in order to compute the

[10]Is this because of lunch time? No free lunch at lunch time? It probably corresponds to the situation in the United States. Specifically, market makers widen the spread considerably during the lunch hour so that they can have lunch in peace without adverse consequences.

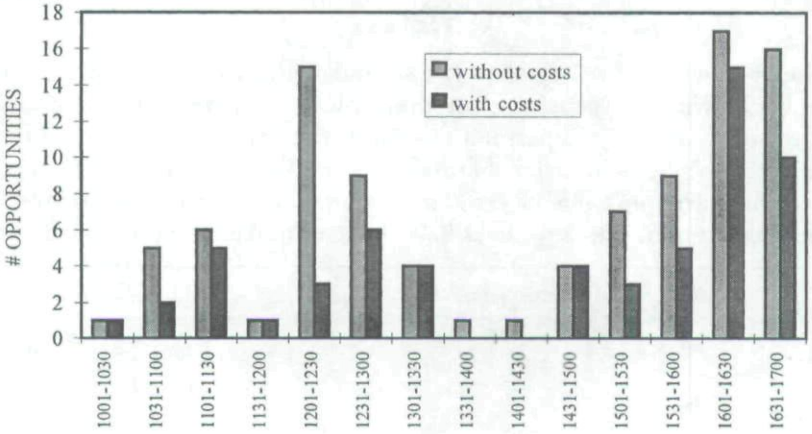

FIGURE 2
Number of arbitrage opportunities grouped into 30-minute intervals.

value of m. Therefore, arbitrage opportunities that are the consequence of an efficiency failure in a particular market (those that require operations in only one market) might also exist. The values of m were therefore computed, after dropping the index proxy in order to check for efficiency in the options market. Only two arbitrage opportunities were detected in this case and they coincide with the ones found for March 14 and 21, giving thus the same values of m. Consequently, these results seem to reveal that option markets have, separately, a high degree of efficiency (Lee and Mathur 1999 show that the random walk hypothesis cannot be rejected in the Spanish futures markets), being cross-market arbitrage (or low degree of integration between markets) the key factor in explaining these deviations during this stable period. Therefore, an idea of Chen and Knez (1995) is clearly illustrated by these empirical results. Although several markets may separately be efficient and Theorem 1 may separately hold, things may be quite different if all the markets are combined in a global one.

In order to analyze efficiency and integration in high volatility periods, the second analysis focuses on the effect of the October 1997 Asian crisis on the Spanish markets. During this convulsive period, serious declines were experienced, leading to extremely large intraday volatilities from October 27 to October 29, 1997 (averaging 48.34% in annual terms). Unprecedented trading volumes were reached in the options market (October 27) and in the stock and futures markets (October 28). Also, the Spanish stock market index experienced both its tenth greatest fall (October 27) and its largest increase (October 29) in the last six years (4.40% and 5.66%, respectively).

TABLE III

The first column gives the day. The second column indicates the maximum number of available securities. The third column gives the number of minutes in which arbitrage opportunities exist if transaction costs (TC) are not considered, while the fourth column takes them into account. The fifth column shows the maximum value of m in basic points. Annualized intraday volatility is given in column 6. All derivatives have as expiration date November 21, 1997.

Day	*Assets*	*Without TC*	*With TC*	*Maximum m*	Volatility
971022	40	2	2	7.21	16.02%
971023	45	6	4	476.19	22.47%
971024	44	4	2	626.78	15.40%
971027	47	11	6	1785.71	27.84%
971028	55	21	18	4199.82	**79.43%**
971029	59	94	79	357.14	37.75%
971030	63	5	4	208.33	38.17%
Total/Mean	50	143	115		33.87%

Because of the abnormal number of limit orders waiting to be traded, the opening of SIBE was delayed and the IBEX-35 stock index was occasionally not available for the days above. This fact forces us to take the IBEX-35 Plus futures contract average price as I_1 (the basis for building the payoffs matrix).

For this period, the main results are given in Table III. The average number of available assets (50) is twice as large as the number for normal trading conditions. Note that the number of minutes in which state prices do not exist is much higher for October 27, 28, and 29 (it accounts for 9.15% out of the overall trading minutes of these days) and also that larger values of m are encountered for the whole period, especially on October 27 and 28. It should be emphasized that 20 out of a total of 143 arbitrage opportunities concern the options market exclusively. Hence, although lack of integration accounts for a larger number of deviations, inefficiency in the options market plays an important role in this unstable period. This is an interesting fact since previous literature (see, for instance, Kleidon & Whaley 1992) suggested that the behavior of financial markets during critical periods must be considered in order to derive important consequences about market structure and regulation.[11]

In order to establish the accuracy of conclusions presented here, it is important to stress again the generality of the study design. All assets

[11]For the convulsive period, the last 15 minutes (from 5:00PM to 5:15PM) were also tested due to the large number of arbitrage opportunities.

quoted in these markets are considered and bid and ask prices are perfectly synchronized. Furthermore, transaction costs are discounted and all possible arbitrage strategies are taken into account when mispricing is detected in the markets. Hence, the results seem to validate the existence of possible riskless profits during stable periods like the one analyzed, and this possibility increases in high volatility situations.

Finally, in order to illustrate how the measures m and l work in practice and provide concrete arbitrage portfolios, the remaining calculations examine an optimum arbitrage strategy that could have been implemented in the October 1997 stock market crash.

The example refers to an arbitrage opportunity for October 29, 1997. In this trading session 59 different assets were traded in all markets considered, and 40 were being quoted at 17:05. Table IV gives best bid and ask prices for all available assets at this moment. There were no state prices in this case, so arbitrage was feasible. m takes a value of 56 basic points, and its corresponding optimum arbitrage strategy gives the following transaction set: long position in 200 units of the bond, buying a call option with a strike price of 6500 and two put options with a strike price of 6300, and selling a call option with a strike price of 6300 and two put options with a strike price of 6400 (all units measured in index points). Note that although some abnormalities may be observed in the put option prices, the optimum strategy is not trivial. Figure 3 plots the resulting total payoff of this portfolio at t_2 (November 21, 1997). It follows that not only are the arbitrage profits provided by m attainable, but additional profits are possible if the stock index value at the expiration date lies between 6300 and 6500.[12]

ON THE BID–ASK SPREAD: PRICING CALL AND PUT OPTIONS BY ARBITRAGE METHODS

The existence of arbitrage has been tested under restrictive assumptions that can be relaxed in many practical situations. First, this study has considered only static models; dynamic arbitrage portfolios have never been analyzed even though previous literature has developed dynamic integration measures (see Balbás et al. 1998). Second, prices are greater

[12]The net profit provided by this portfolio (i.e., $-P(x)$) is equal to 4,250 pesetas (if the interest rate were equal to zero, this profit would be still positive and equal to 3,650 pesetas). Since the portfolio incorporates six options, the total transaction cost is equal to 600 pesetas. Margins are not a problem because they are variable and depend on the total portfolio of options and futures and, therefore, they are lower than the amount of money that must be invested in the riskless security. In practice, traders could have implemented this portfolio on October 29, and the opposite one a few days later, and they would have obtained some money at both dates.

TABLE IV

Best bid and ask prices for all quoted assets on October 29, 1997, at 17:05. Bold type is used for those assets involved in the optimum arbitrage portfolio, and sold or bought units are given in parentheses. All derivatives have as expiration date November 21, 1997.

Call Options			*Put Options*		
Strike price	*Bid price*	*Ask price*	*Strike price*	*Bid price*	*Ask price*
5700	400	N.A.*	5500	65	N.A.
5800	N.A.	800	5600	50	N.A.
6300	**355(1)**	375	5700	85	N.A.
6350	325	345	5800	N.A.	160
6400	295	315	6100	190	210
6450	270	290	6150	200	220
6500	240	**250(1)**	6200	220	240
6550	220	240	6250	235	255
6600	190	210	**6300**	255	**275(2)**
6700	N.A.	198	**6400**	**325(2)**	350
6800	51	N.A.	6450	N.A.	450
6850	15	120	6500	N.A.	400
6900	40	105	6600	350	430
6950	10	N.A.	6700	N.A.	650
7000	33	83	7300	603	N.A.
7050	18	N.A.	7500	800	N.A.
7100	25	50			
7200	N.A.	40	**Asset**	**Bid price**	**Ask price**
7250	10	45	IBEX-35	6375.86	6378.85
7350	N.A.	40	**Bond****	0.997	**0.997(200)**
7400	N.A.	50			
7500	5	N.A.			

*Not available

**The final payoff of one unit of the bond is one index point, and its price equals $1/(1 + r^*)$ where r^* verifies $(1 + r^*) = (1 + r)^T$, $T = t_2 - t_1$, and r is the interest rate from t_1 to t_2

if investors buy assets ($c_k \geq v_k$). Third, the ratio I_2/I_1 has been always assumed to be between 0.6 and 1.4, too large a spread if $t_2 - t_1$ is short enough. So for instance, one can consider that two or three days before the expiration date (before t_2) the ratio will verify $0.8 \leq I_2/I_1 \leq 1.2$, and some hours before t_2 this interval may be far smaller.

These assumptions have clear advantages, because they make it difficult to criticize the fundamental conclusion, i.e., cross-market arbitrage portfolios seem to appear. But it may be important to compute the integration measures after relaxing the hypotheses, and the present section is devoted to a partial development of this idea, concentrating specifically on the stable period and the bid–ask spread. This spread is large and has

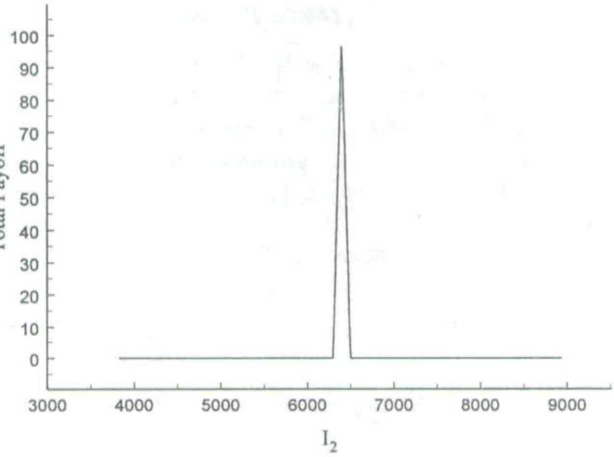

FIGURE 3
Final payoffs for the optimum arbitrage strategy on October 29, 1997, at 17:05.

an important effect in the analysis because it has been never implemented using real transactions prices.

The bid–ask spread has been removed only for the most traded option. Of course, this option (which differs from day to day) often presents the highest and closest-to-one bid/ask ratio; therefore, if another security had been considered, the measure $\boldsymbol{m}$ would have been larger. Anyway, the measure clearly increases, and the basic results are shown in Tables V and VI.

The differences between Tables V and VI and Table I are obvious and prove that market makers' prices often lead to arbitrage. Furthermore, the measure and the arbitrage profits are quite large at many instants, especially when the ask price is used for both buying and selling.

This seems to be a very significant fact, because arbitrage methods are the usual way to price derivative securities in financial theory. Theoretical models (continuously) replicate the derivative to price it, but the empirical results obtained here suggest a simpler and perhaps more realistic procedure. Investors can offer new prices and improve the real bid or ask quotes without any type of risk because hedging (with arbitrage portfolios) would be feasible if new agents accepted these prices. Moreover, the arbitrage portfolio provided by $\boldsymbol{m}$ is static and, therefore, the effect of market imperfections can be easily analyzed. This effect is more complex when a continuous trading or a usual self-financing delta and gamma-neutral strategy is implemented.

This procedure may be illustrated by the arbitrage opportunity for October 29, 1997, provided in the previous section. If the bid–ask spread

TABLE V

The bid-ask spread has been removed for the most traded option, and the bid price is used in both buy and sell operations for this particular asset. The first column gives the day. The second column gives the number of minutes in which arbitrage opportunities exist if there are no transaction costs (TC), while the third column takes them into account. The fourth column shows the maximum value of m in basic points. All derivatives have as expiration date March 21, 1997.

Date	*Without costs*	*With costs*	*Maximum m*
970224	37	37	275
970225	3	3	303
9720226	2	1	18
970227	26	11	6
970228	1	1	32
970304	29	12	463
970305	76	76	719
970306	2	1	4
970307	5	4	64
970310	10	5	16
970311	97	66	1860
970312	10	5	18
970313	140	114	1750
970314	62	49	4803
970317	10	8	5
970318	12	6	7
970319	6	6	249
970320	7	7	28
970321	10	10	4420
Total	545	422	

is removed for the most traded option (the put option with a strike price of 6,400) and both are set equal to the best ask price, profit through arbitrage increases.[13] In this case, the optimum strategy is given by the following transactions: buying for 275 index points two put options and for 430, one put option, with strike prices 6,300 and 6,600, respectively, and selling for 350 three put options with a strike price of 6,400. This combined position leads to a profit of 70 index points (m equals 667 basic points). Again, depending on the index final value, possible arbitrage profits could be obtained at the second date t_2, as shown by the total final payoff illustrated in Figure 4.

[13]That is, the put option with a strike price of 6400 may be valued by static arbitrage methods or, equivalently, traders can improve the ask quote without any sort of risk. Moreover, the hedging portfolio overcomes market imperfections far more easily than usual self-financing dynamic portfolios. It is important to point out that Spanish call and put options may very frequently be priced by this procedure.

TABLE VI

The bid-ask spread has been removed for the most traded option, and the ask price is used in both buy and sell operations for this particular asset. The first column gives the day. The second column gives the number of minutes in which arbitrage opportunities exist if there are no transaction costs (TC), while the third column takes them into account. The fourth column shows the maximum value of *m* in basic points. All derivatives have as expiration date March 21, 1997.

Date	*Without costs*	*With costs*	*Maximum m*
970224	0	0	0
970225	0	0	0
970226	24	23	238
970227	92	75	2591
970228	22	22	234
970304	78	40	183
970305	109	101	515
970306	72	71	500
970307	14	13	1042
970310	10	5	16
970311	152	125	210
970312	44	22	18
970313	402	401	6756
970314	76	61	4704
970317	139	118	1053
970318	36	30	1000
970319	241	241	5417
970320	11	11	2105
970321	33	33	3025
Total	1555	1392	

Once again real quotes show a surprising fact that may be applied by traders in practice. The bid–ask spread is high, and that suggests some new strategies for pricing and investing. At many dates agents could analyze the bid–ask spread for many securities (not only for the most traded option), and offer new prices without assuming any kind of risk. This should lead to smaller spreads. Therefore, this paper's methodology, novel because it tests the markets globally rather than a few specific well-known arbitrage portfolios, may be useful to traders in order to implement new strategies and price derivatives and reduce the spreads.

CONCLUSIONS

This paper has empirically tested the level of integration among different Spanish financial markets by computing some of the integration measures

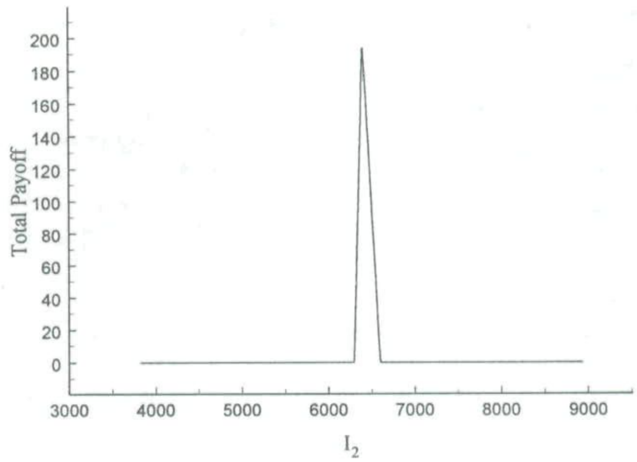

FIGURE 4
Final payoffs for the optimum arbitrage strategy on October 29, 1997, at 17:05 when the bid–ask spread for the most traded option (the put option with a strike price of 6400) is removed and it is assumed that both bid and ask are equal to the best ask price.

that have appeared in the recent literature. These measures are derived from the basic assumptions about pricing assets, like the Law of One Price, or the absence of cross-market arbitrage opportunities, but the paper's methodology is novel in that it tests the markets globally, rather than a few specific well-known arbitrage strategies.

Also, it uses perfectly synchronized high frequency data and always incorporates the bid–ask spread or transaction costs. The hypotheses of the present study are as weak as possible and are compatible with any "reasonable" asset-pricing model.

The results are surprising for several reasons. First, when applied to the index IBEX-35 and its usual replica, different integration measures show similar paths, which means that different criteria lead to similar conclusions about the integration level. Second, and perhaps more important, empirical evidence shows that risk-neutral pricing rules (state prices, discount factors, risk-neutral probability measures, etc.) sometimes fail to hold, so that a potential for riskless arbitrage profits may exist during stable periods, a possibility that significantly increases in high volatility situations.

Traders can use these measures to price different securities, and can frequently improve many real quotes and hedge the position with an arbitrage portfolio. This could be a very significant procedure because it seems to solve some usual problems generated by market imperfections

Finally, the methodology also provides risk-neutral probability measures, or a proxy for them when the absence of arbitrage fails.

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