PRICING EURODOLLAR FUTURES OPTIONS USING THE BDT TERM STRUCTURE MODEL: THE EFFECT OF YIELD CURVE SMOOTHING

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This article focuses on pricing Eurodollar futures options using the single-factor Black, Derman, and Toy (1990) term structure model with particular emphasis on yield curve smoothing. Of the various approaches, the maximum smoothness forward rate approach developed by Adams and van Deventer (1994), cubic yield spline, and linear interpolation are used to produce finely spaced binomial trees. We compare the pricing accuracy associated with the use of yield curve smoothing techniques within the BDT framework. The findings provide the first supporting evidence that using a forward rate curve with maximum smoothness together with a time-varying volatility structure improves best the performance of the BDT model. The em-

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pirical results are found to be robust across factors affecting the option price such as time-to-expiration, moneyness, and trading volume.
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INTRODUCTION

This article provides empirical evidence on the effectiveness of forward rate curve smoothing in pricing Eurodollar futures options using the Black, Derman, and Toy (1990; henceforth BDT) term structure model. The BDT model has been popular among practitioners partly for its ease of calibration and for its straightforward solutions. The model uses two sets of inputs, a *yield* and *volatility curve*, in pricing interest rate sensitive claims. This study emphasizes that using a better representation of the yield and volatility curves can enhance the performance of the BDT model, which then yields more precise estimated option prices.

The original BDT model requires a binomial tree with finely spaced steps to obtain accurate solutions for option values.[1] This can be achieved by choosing actual interest rates with the shortest-maturity increments available. Smoothing the initial yield curve to reduce the interval size can increase model accuracy even further. The article makes a contribution to the literature by providing evidence of the assertion that the use of finely spaced binomial trees in the BDT model yields more precise estimated option prices.

To price Eurodollar futures options more accurately using the BDT model, one needs to obtain rates with shorter maturities than the original interval size of three months. In the literature, yield curve smoothing is used to derive shorter maturity interest rates from the observed market data. Of the various approaches, we use the maximum smoothness forward rate approach developed by Adams and van Deventer (1994), the cubic yield spline approach of McCulloch (1975), and the linear interpolation to produce finely spaced binomial trees. We compare the pricing accuracy associated with the use of yield curve smoothing techniques within the BDT framework. The results indicate that the Eurodollar fu-

[1]The BDT model was initially applied to pricing Treasury bonds and Treasury bond options. There are several differences between the application of the BDT model to T-Bond options and Eurodollar futures options. T-Bond options are of European style and can only be exercised at expiration. Eurodollar futures options are of American style and can be exercised prior to expiration. Thus, at each node of the binomial trees generated in the BDT model, possibility of early exercise is handled by comparing the payoffs from exercising and not exercising. In pricing T-Bond options, it is necessary to take into account the coupons paid over the life of the option. There are no intermediate payments associated with the underlying Eurodollar futures that need to be taken into account when building the binomial trees in pricing Eurodollar futures options.

tures options are more accurately priced when the forward rate curve with maximum smoothness is used.

The short term interest rate driving the changes in the entire term structure is the fundamental variable in the BDT model. Therefore, accurate estimation of the short rate volatility process is crucial for pricing interest rate options. The BDT model is originally implemented using the traditional historical volatility, which is a simple description of the evolution of volatility that places constant weights on past observations. However, Black et al. (1990) also suggest that using time-dependent instead of constant forward rate volatilities would enhance the performance of the model. The second contribution of the article is to use a time-varying volatility structure in a discrete-time econometric approach, consistent with the BDT model. We compare estimated option prices with the actual values and discuss the empirical findings taking into account the factors that affect the option price. The estimation results are classified according to the type of option (i.e., call or put), trading volume, time-to-expiration, and moneyness. The significance of these factors for the valuation of Eurodollar futures options is tested. The increase in accuracy of the estimated option prices is measured by using the mean absolute pricing error. The empirical results, based on 1,159 estimated prices, confirm that binomial trees of expected future short rates with finely spaced steps are needed to obtain more accurate estimates for actual option prices.

THE SINGLE-FACTOR BDT MODEL

The BDT model assumes that changes in short-term interest rates follow a lognormal probability distribution, which implies that the short rate process in continuous time can be written as

$$d \ln r_t = (\beta_{0,t} + \beta_{1,t} \ln r_t)\, dt + \sigma_t\, dZ_t \tag{1}$$

where $\ln r_t$ is the natural logarithm of the short-term interest rate at time t; $(\beta_{0,t} + \beta_{1,t} \ln r_t)$ is the drift function of the short rate process where $\beta_{0,t}$ and $\beta_{1,t}$ are time-varying parameters that must be estimated; σ_t is the conditional volatility at time t; and Z_t is a standard Brownian motion at time t so that dZ_t is a normally distributed random variable with zero mean and variance of dt.

The evolution of the short rate process presented in (1) can be approximated via the binomial representation:

$$\ln r_{t+\Delta} - \ln r_t = \begin{Bmatrix} (\beta_{0,t} + \beta_{1,t} \ln r_t)\Delta + \sigma_t\sqrt{\Delta}; \text{ probability } 0.5 \\ (\beta_{0,t} + \beta_{1,t} \ln r_t)\Delta - \sigma_t\sqrt{\Delta}; \text{ probability } 0.5 \end{Bmatrix} \quad (2)$$

where $\Delta \ln r_t = \ln r_{t+\Delta} - \ln r_t$ is the change in the log-interest rate where Δ is the length of the time interval and $\beta_{1,t}$ is treated as a measure of the speed of mean reversion in log-rate levels. Equation (2) yields the *sensitivity formula*, which shows how the volatility of log-interest rate changes in the BDT model is determined by the spread between the two possible log-interest rates at time $t + \Delta$:

$$(\ln r^{up}_{t+\Delta} - \ln r^{down}_{t+\Delta})/2 = \sigma_t\sqrt{\Delta}. \quad (3)$$

The BDT model was initially implemented using the traditional historical volatilities that do not vary through time, although time-varying volatilities would more closely match the true underlying volatility structure. The use of time-varying volatilities may lead, however, to a non-recombining tree. It can be shown that the binomial tree will recombine when the critical condition given in eq. (4) between the time-varying volatility, σ_t, and the mean reversion parameter, $\beta_{1,t}$, is satisfied[2]:

$$\beta_{1,t+\Delta} = \left(\frac{\sigma_{t+\Delta} - \sigma_t}{\sigma_t\Delta}\right). \quad (4)$$

The continuous-time limit of this expression,

$$\lim_{\Delta\to 0} \beta_{1,t+\Delta} = \left(\frac{\sigma_{t+\Delta} - \sigma_t}{\Delta}\right)\left(\frac{1}{\sigma_t}\right),$$

implies $\beta_{1,t} = (\sigma_t'/\sigma_t)$ where $\sigma_t' = d\sigma_t/dt$. This expression can be rewritten as $\beta_{1,t} = (d \ln \sigma_t/dt)$, which yields in discrete time $\sigma_t = \sigma_{t-1}e^{\beta_{1,t}}$ or $\sigma_t^2 = \sigma_{t-1}^2 e^{2\beta_{1,t}}$. Thus, in order to have a recombining binomial tree, the mean reversion parameter, $\beta_{1,t} = \sigma_t'/\sigma_t$, must be a function of the short rate volatility, σ_t, and its derivative with respect to time, σ_t'.

The BDT model uses the local expectations hypothesis (LEH), which is consistent with no-arbitrage. Applied to the valuation of interest rate derivatives, the LEH yields the *valuation* formula:

$$P_t(T) = \frac{E_t[P_{t+\Delta}(T)]}{1 + r_t} = \frac{0.5P_{t+\Delta}(T)^{up} + 0.5P_{t+\Delta}(T)^{down}}{1 + r_t} \quad (5)$$

where $P_t(T)$ is the price of a T-maturity claim at time t, which is assumed

[2]Bali (1999) tests the empirical performance of continuous time models of the short term interest rate. One of his econometric specifications, which produces accurate predictions of the short rate volatility, is very similar to the volatility model used here with the condition given in eq. (4).

after Δ-period to move up to $P_{t+\Delta}(T)^{\mathrm{up}}$ or down to $P_{t+\Delta}(T)^{\mathrm{down}}$ with equal probability of 0.5. With the help of an iterative procedure, the valuation and sensitivity formulas [eqs. (5) and (3)] are used to determine the evolution of the short rate tree.[3]

YIELD CURVE SMOOTHING

Yield curve smoothing has developed because the term structure of coupon bonds is incomplete in that not all maturities are offered. The idea is that the discount bond prices should vary smoothly with maturity. In pioneering work, McCulloch (1975) uses polynomial spline functions to fit the observed prices of U.S. Treasury securities. Vasicek and Fong (1982) note the essential exponential nature of the term structure, and adapt the exponential form to fit bond price data. Both studies select a cubic as the lowest odd order form with continuous derivatives.

While the spline approaches to fitting the term structure have considerable advantages, their practical disadvantages have led many sophisticated financial market participants to use simpler approaches such as linear yield curve smoothing. Buono, Gregory-Allen, and Yaari (1992) provide a detailed survey of previous approaches to yield curve smoothing. Ferguson and Raymar (1998) compare the results of several models that fit a term structure to market data.

Adams and van Deventer (1994; hereafter AvD) introduce a mathematical measure of smoothness, and show that the yield curve with the smoothest possible forward rate function, consistent with the observable data, is related to, but significantly different from, the popular *cubic spline approach* to the smoothing of yields and discount bond prices. The yield curve that produces the smoothest possible forward rates consistent with given zero-coupon bond prices has a quartic forward rate function. This contrasts with the cubic polynomial used to fit either yields or discount bond prices in the cubic spline approach.

AvD point out two principal problems associated with the use of cubic yield (or price) splines. The cubic spline approach as by McCulloch (1975) produces forward rate curves that are not "smooth" in that the forward rate curve is not twice differentiable at the knot point. In addition, the forward rate curves associated with a cubic spline-based yield curve tend to be volatile, particularly on the right-hand side of the yield curve, to such a degree that their use can lead to implausible forward rate

[3]For further information on the implementation of the BDT model, see Ricthken (1996), Tuckman (1996), and Rebonato (1998).

curves. Shea (1985) concludes that the same critiques apply to the exponential splines used by Vasicek and Fong (1982).

AvD attempt to remedy the two problems of cubic spline smoothing with a new approach that addresses both problems directly. They derive the curve in such a way that the forward rate curve is the smoothest curve of any of the family of curves that are continuous, twice differentiable, and consistent with the observable data. They show that the smoothest possible forward rate curve consists of a quartic forward rate function that is fitted between each knot point.

The term structure $f(T)$, $0 \leq T \leq \tau$, of forward rates that satisfies the maximum smoothness criterion is given by:

$$\min \int_0^{\tau} [f''(s)]^2 \, ds \tag{6}$$

while fitting the observed prices $\mathrm{P(T_1)}$, $\mathrm{P(T_2)}$, . . . , $\mathrm{P(T_m)}$ of Eurodollar futures with maturities $\mathrm{T_1}$, $\mathrm{T_2}$, . . . , $\mathrm{T_m}$ is a fourth-order spline given by[4]:

$$f(T) = a_i + b_i\mathrm{T} + c_i\mathrm{T}^2 + d_iT^3 + e_iT^4 \text{ for } T_{i-1} < T < T_i,$$
$$i = 1, 2, \ldots, m + 1 \tag{7}$$

where $0 = T_0 < T_1 < T_2 < \ldots < T_m < T_{m+1} = \tau$. Estimation of the coefficients in equation (7) is described in Adams and van Deventer (1994).

THE DATA AND ESTIMATION

Daily data on the three-month Eurodollar spot rates, futures, and futures options are obtained from the Futures Industry Institute. For each of 42 futures expiration dates between March 14, 1988 and June 15, 1998, prices of options on Eurodollar futures are obtained. Table I presents the descriptive statistics of the 1,159 daily option prices as well as the factors affecting these prices. The median price of Eurodollar futures options is 23 basis points, i.e., 0.23, while the median time-to-expiration is six months. The options used in this study are out-of-the money by on average 24 basis points.

[4]In their original study, the smoothest forward rate curves are produced by a fourth-degree polynomial with the cubic term missing. However, in our analysis, we employ a more general version of the smoothing polynomial used in Adams and van Deventer (1994). Our specification given in eq. (7) introduces an additional parameter that must be estimated. The only difference between the two specifications used here and in AvD is the additional coefficient attached to the cubic term.

TABLE I

Descriptive Statistics Eurodollar Futures Options
(March 14, 1988–June 15, 1998)

		Variables Affecting Option Price		
	Market Price of Option[a] *(points of 100%)*	*Trading Volume*[b] *(contracts)*	*Time to Expiration (months)*	*Moneyness*[c] *(basis points)*
Mean	0.33	1,086	7.30	−23.83
Median	0.23	500	6.00	−22.00
Maximum	3.08	20,500	18	312
Minimum	0.01	100	3.00	−394
Standard Deviation	0.39	1,669	4.14	97.39
Number of Obs.	1,159	1,159	1,159	1,159

[a]Minimum price fluctuation, 1 tick, is 0.01, i.e. 1 basis points. With the face value of $1 million, the dollar value of a tick is $25. In order to find the option premium, the price expressed in basis points is multiplied by 25.
[b]In order to reduce any pricing bias that may arise from limited liquidity in the market, options with daily trading volume of less than 100 are excluded from the analysis.
[c]Moneyness is defined as either P − X, if the option is a call, or X − P, if the option is a put, where X is the exercise price and P is the price of Eurodollar futures. Negative and positive values denote out-of-the-money and in-the-money, respectively.

TABLE II

Descriptive Statistics Daily Eurodollar Spot and Futures Rates
(July 1, 1987–June 30, 1998)

	$r_{0,0}$	$r_{3,6}$	$r_{6,9}$	$r_{9,12}$	$r_{12,15}$	$r_{15,18}$	$r_{18,21}$
Mean	6.12%	6.17%	6.32%	6.50%	6.70%	6.90%	7.07%
Median	5.81	5.83	5.99	6.14	6.37	6.58	6.76
Maximum	10.63	11.18	11.21	11.1	10.72	10.39	10.37
Minimum	3.13	3.01	3.05	3.35	3.7	3.99	4.23
Standard Deviation	1.85	1.81	1.76	1.70	1.65	1.58	1.52
Number of Obs.	2,782	2,782	2,782	2,782	2,782	2,782	2,782

Notes: $r_{0,0}$ refers to the spot Eurodollar rate and $r_{i,i+3}$ refers to the rate inferred by the price of Eurodollar futures expiring in i months. All rates are expressed in annualized percentages.

This table presents the descriptive statistics of the daily Eurodollar futures rates. These rates and time varying volatility extimates of these rates are used to construct the forward rate and price trees within the BDT framework.

The data set for the Eurodollar spot and futures rates covers the period from July 1, 1987 to June 30, 1998, giving a total of 2,782 daily observations. Table II presents summary statistics of the daily Eurodollar spot and futures rates. The annualized unconditional average levels of the three-month Eurodollar rates vary from 6.12 % with a standard deviation of 1.85 % (for the spot) to 7.07 % with a standard deviation of 1.52 % (for the rate on the 18-month deferred futures).

It is a widely accepted idea that pricing interest-rate-sensitive claims is sensitive to the volatility model used.[5] Previous research on pricing interest rate derivatives employs either a standard historical volatility or implied volatility (see, for example, Black et al., 1990; Kalotay, Williams, and Fabozzi, 1993; and Mathis and Bierwag, 1999). We use a time-varying volatility structure that satisfies all the assumptions of the BDT model.

Chan, Karolyi, Longstaff, and Sanders (1992), Bali (1999), and others use a discrete-time econometric specification to estimate the parameters of the continuous-time interest rate models. In this article, the volatility of interest rate changes is estimated using a similar econometric approach in a discrete-time BDT framework:

$$\ln r_t(T) - \ln r_{t-1}(T) = \beta_{0,t}(T) + \beta_{1,t}(T) \ln r_{t-1}(T) + \varepsilon_t(T) \quad (8a)$$

$$E(\varepsilon_t(T)|\mathcal{T}_{t-1}) = 0,\ E(\varepsilon_t^2(T)|\mathcal{T}_{t-1}) \equiv \sigma_t^2(T) \doteq \sigma_{t-1}^2(T)e^{2\beta_{1,t}(T)} \quad (8b)$$

where $T = 1, 2, \ldots, 6$ is time to maturity in three-month intervals or $T = 1, 2, \ldots, 18$ is time to maturity in one-month intervals. $\mathcal{T}_{t-1}$ is the information set available at time $t - 1$ so that $E(\ldots|\mathcal{T}_{t-1})$ represents the conditional expectations operator at time $t - 1$; $\beta_{0,t}(T) + \beta_{1,t}(T) \ln r_t - 1(T)$ is the time-varying conditional mean; and $\sigma_t^2\ (T)$ is the time-varying conditional variance of unexpected log-interest rate changes. The current volatility of log-interest rate changes is modeled to be a function of the time-dependent mean-reversion parameter $\beta_{1,t}(T)$ as well as the last period's volatility.

The econometric specification of the BDT model is quite different from the standard volatility estimators (e.g., historical, moving average, and implied volatility) because of its crucial assumption about future volatilities. The conditional mean and variance of the log-interest rate given in eqs. (8a) and (8b) are estimated using ordinary least squares (OLS), and then the estimate of $\beta_{1,t}(T)$ together with $\sigma_{t-1}^2\ (T)$ is used to compute $\sigma_t^2\ (T)$. Finally, the volatility $\sigma_t^2\ (T)$ that satisfies the critical condition of the BDT model is employed in the sensitivity formula in order to estimate Eurodollar futures options.

The volatility estimation procedure that we use differs from its predecessors in an important respect. The earlier studies compute the variance of interest rate levels (or changes) using a fixed sample period assuming that the volatility parameters are independent of time. We use a rolling regression procedure and generate one-step-ahead forecasts by

[5] Bali and Karagozoglu (1999) show that different volatility estimators have different effects on the pricing of interest rate sensitive options by the single-factor BDT model.

estimating the time-varying drift and diffusion parameters. The volatility of daily changes in the three-month and one-month spot and futures rates is estimated using 250 daily observations on a rolling basis.[6]

EMPIRICAL RESULTS

The empirical results demonstrate the effect of yield curve smoothing on pricing interest rate options within the BDT framework. To begin with, the iterative procedure in the BDT model is repeated to construct the short rate tree for each of the Eurodollar futures expiration dates between March 14, 1988 and June 30, 1998. Corresponding to these futures expiration dates, there are a total of 1,657 Eurodollar futures options. In order to reduce any pricing bias that may arise from limited liquidity in the market, we eliminate the options with daily trading volume of less than 100 from our analysis. Therefore, we price 1,159 Eurodollar futures options using different yield curve smoothing techniques. We categorize the estimation results according to factors that affect the option price, which are type of option (i.e., call or put), trading volume, time-to-expiration, and moneyness.

Option prices are estimated using four different sets of forward rates in order to illustrate the effect of yield curve smoothing on pricing interest rate options. The first set is the original market data for the three-month forward rates, and the second, third, and fourth sets are derived from the actual market rates using the maximum smoothness forward rate approach of AvD (1994), the cubic yield spline approach of McCulloch (1975), and the linear interpolation, respectively. Using the smoothed one-month rates, the expected future short rate and price trees are regenerated. Then the prices of Eurodollar futures options are re-estimated using finely spaced binomial trees.

In order to evaluate the pricing accuracy associated with the use of different smoothing polynomials, we utilize the mean absolute error (MAE), which is calculated as:

$$\mathrm{MAE} = \frac{1}{n} \sum_{i=1}^{n} |\Phi_i - \hat{\Phi}_i| \tag{9}$$

where Φ_i and $\hat{\Phi}_i$ are the actual and estimated option prices, respectively.

Mean absolute pricing errors are reported in Table III for each factor

[6]The time-varying parameters of the BDT model are estimated using the maximum likelihood estimation technique with the Berndt-Hall-Hall-Hausman algorithm.

TABLE III

The Effect of Yield Curve Smoothing on Estimated Prices of Eurodollar Futures Option[a] Comparison of Mean Absolute Pricing Errors (March 14, 1988–June 15, 1998)

		Mean Absolute Pricing Errors for Finely Spaced Trees With 1-Month Intervals Using:			
	# of obs.	*Max. Smoothness (AvD)*	*Cubic Spline*	*Linear spline*	*Coarse Trees With 3-month Intervals*
		Panel A: Option Type			
All options	1,159	0.0629	0.0748	0.0795	0.1096
Call options	568	0.0558	0.0678	0.0716	0.0902
Put options	591	0.0698	0.0816	0.0872	0.1282
		Panel B: Time-to-Expiration			
3 months	369	0.0295	0.0385	0.0420	0.0456
6 months	199	0.0601	0.0669	0.0739	0.1090
9 months	328	0.0900	0.1048	0.1017	0.1542
12 months	140	0.0985	0.1168	0.1307	0.1711
15 months	95	0.0950	0.1188	0.1252	0.1766
18 months	28	0.0623	0.0821	0.0765	0.1143
		Panel C: Moneyness			
−400 to −226	23	0.0857	0.0922	0.0974	0.1370
−225 to −176	38	0.1050	0.1191	0.1280	0.1510
−175 to −126	95	0.0867	0.0982	0.1046	0.1227
−125 to −76	162	0.0597	0.0707	0.0718	0.1063
−75 to −25	240	0.0417	0.0548	0.0559	0.0895
−25 to +25	264	0.0325	0.0381	0.0418	0.0711
+26 to +75	195	0.0595	0.0717	0.0815	0.1119
+76 to +125	78	0.1215	0.1451	0.1551	0.1841
+126 to +175	34	0.1611	0.1819	0.1893	0.2094
+176 to +225	19	0.1681	0.1926	0.1925	0.2173
+226 to +400	11	0.0654	0.0877	0.0897	0.1404
		Panel D: Trading Volume			
100 to 200	266	0.0835	0.0949	0.1015	0.1454
201 to 300	132	0.0765	0.0919	0.0969	0.1335
301 to 400	106	0.0645	0.0747	0.0818	0.1198
401 to 600	143	0.0615	0.0730	0.0762	0.1073
601 to 1000	157	0.0597	0.0709	0.0755	0.1046
1001 to 2000	187	0.0524	0.0671	0.0697	0.0824
2001 to 5000	132	0.0373	0.0468	0.0501	0.0666
5001 to 20500	36	0.0249	0.0324	0.0364	0.0561

[a]Mean absolute pricing errors are expressed in points of 100%. For example, the pricing error for the AvD method, 0.0629, indicates an error of 6.29 basis points with a dollar value of 157.25.

affecting the option price. The smaller MAE values obtained from the smoothing techniques indicate that options are more accurately priced when finely spaced binomial trees are used. Panel A of Table III shows that when all factors are taken into consideration and a coarse binomial tree is employed, the mean absolute pricing error is 0.1096 for the entire sample. With the use of AvD smoothing technique, the pricing error reduces to 0.0629, indicating a gain of \$116.75 per contract on average.[7] Among the three alternative smoothing techniques, the AvD is found to be superior to both cubic and linear splines. The MAE values for the AvD, cubic, and linear smoothing methods are found to be 0.0629, 0.0748, and 0.0795, respectively. According to these figures, using a more finely spaced tree with the AvD technique of maximum smoothness yields \$29.75 and \$41.50, per contract, improvements over the cubic yield spline and the linear interpolation, respectively. The results indicate that the use of yield curve smoothing within the BDT model has positive effects for predicting the prices of Eurodollar futures options. This positive effect is more pronounced if the maximum smoothness technique of AvD is used.

Panel B of Table III provides evidence that the relative performance of the BDT model is sensitive to time-to-expiration of the Eurodollar futures options that are being priced. The results confirm that using finely spaced trees produces more accurate option prices as time-to-expiration shortens. The improved accuracy of estimated option prices when time-to-expiration shortens is attributed to the pricing bias due to bid-ask spreads and liquidity. Near-term options are more liquid than the longer-term ones, which in the case of long-term options yields higher observed prices compared to their theoretical values. The mean absolute errors for the AvD technique are found to be consistently smaller than those for the cubic and linear spline methods across different maturities.

Moneyness is defined as $P - X$ for call options and $X - P$ for put options, where P is the futures price, and X is the exercise price of the option on the futures. Panel C of Table III shows that, as expected, at-the-money options (where at-the-money refers to moneyness ranging from -25 to $+25$ basis points) are priced more accurately by the BDT model than either in- or out-of-the-money options. As the option is either more in- or more out-of-the money, the forecasting ability of the model decreases. This is also attributed to pricing bias introduced by the bid-ask spreads and liquidity, given that there is less trading in in- or out-of-

[7]Prices of Eurodollar futures options are quoted in points of 100%. Minimum price fluctuation, 1 tick, is 0.01, i.e. 1 basis points. With the face value of \$1 million, the dollar value of a tick is \$25. In order to find the option premium, the price expressed in basis points is multiplied by 25.

the-money options than in at-the-money options. The results indicate that smoothing the forward rate curve improves the predictive power of the BDT model in pricing Eurodollar futures options. The AvD technique outperforms both cubic and linear spline methods for all levels of moneyness. However, the improvement in pricing accuracy is more evident for deep in-the-money options. For pricing options that are in-the-money by 175 to 225 basis points, using the AvD technique results in a $61.25 gain over the cubic spline method.

The effect of an increase in trading volume on the explanatory power of the BDT model is demonstrated in Panel D of Table III. The pricing accuracy of all three smoothing techniques improves with the trading volume. Because of the higher bid-ask spreads charged by market makers, observed option prices are expected to be higher than their theoretical values when there is less market liquidity. For different levels of trading volume, the AvD technique consistently outperforms both the cubic spline and the linear interpolation.

We test the significance of factors (option type, time-to-expiration, moneyness, and trading volume) affecting the valuation of Eurodollar futures options. Table IV presents the OLS results of regressing the absolute pricing error, $|\varepsilon|$, on time-to-expiration, moneyness, and trading volume.[8] Panel A of Table IV shows how the predicted power of the BDT model is affected by time-to-expiration, absolute moneyness, and trading volume for all types of option (calls or puts). Panels B and C demonstrate the results for call and put options separately. In every instance, the coefficients of the right-hand side variables are statistically significant at the 5 % level, with signs compatible with our previous findings. In other words, the regression results provide supporting evidence of our earlier empirical results that, as trading volume increases, or time-to-expiration or moneyness decrease, the BDT model performs better, i.e., the absolute pricing error diminishes. In addition, these factors affecting the option price are more significant when binomial trees with finely spaced steps are used within the BDT framework.

CONCLUSION

This article shows that reducing the interval size of binomial trees within the BDT framework produces more accurate estimated prices of Eurodollar futures options. We build a tree with one-month steps rather than

[8]Since the AvD technique outperforms the cubic and linear splines in pricing interest rate options, the regressions for 1-month intervals presented in Table IV use the absolute pricing errors of the AvD method as the dependent variable.

TABLE IV

Testing the Significance of Factors Affecting the Valuation of Eurodollar Futures Options[a] (March 14, 1988–June 15, 1998)

	Panel A $\lvert\varepsilon\rvert = \alpha_0 + \alpha_1\lvert M\rvert + \alpha_2 V + \alpha_3 T$				
All options	α_0	α_1	α_2	α_3	R^2
1-month intervals	0.0183	0.0015	-5.83×10^{-6}	0.0531	0.1596
	(2.15)*	(10.07)*	(−2.68)*	(7.43)*	
3-month intervals	0.0443	0.0013	-7.27×10^{-6}	0.0476	0.1512
	(1.99)*	(9.52)*	(−2.33)*	(7.25)*	
	Panel B $\lvert\varepsilon\rvert = \alpha_0 + \alpha_1\lvert M\rvert + \alpha_2 V + \alpha_3 T$				
Call options	α_0	α_1	α_2	α_3	R^2
1-month intervals	0.0565	0.0019	-8.00×10^{-6}	0.0568	0.4034
	(2.49)*	(16.06)*	(−2.38)*	(7.72)*	
3-month intervals	0.0798	0.0019	-1.01×10^{-6}	0.0534	0.3733
	(3.39)*	(15.09)*	(−1.98)*	(7.03)*	
	Panel C $\lvert\varepsilon\rvert = \alpha_0 + \alpha_1\lvert M\rvert + \alpha_2 V + \alpha_3 T$				
Put options	α_0	α_1	α_2	α_3	R^2
1-month intervals	0.0137	−0.0005	-1.76×10^{-5}	0.0488	0.1714
	(3.66)*	(−2.73)*	(−2.72)*	(3.98)*	
3-month intervals	0.1475	−0.0003	-1.64×10^{-5}	0.0423	0.1633
	(4.61)*	(−2.27)*	(−2.17)*	(4.06)*	

Notes: t-statistics are given in parentheses where * denotes significance at the 5% level. $\lvert\varepsilon\rvert$ is the absolute pricing error, V is the trading volume, T is the time-to-expiration, M and $\lvert M\rvert$ are moneyness and absolute moneyness, respectively.
[a]Regressions for 1-month intervals use the absolute pricing errors of the AvD method as the dependent variable.

original three-month steps to illustrate the improvement in pricing Eurodollar futures options achieved using the maximum smoothness forward rate, the cubic yield spline, and the linear interpolation. The maximum smoothness technique of Adams and van Deventer results in economically significant pricing improvement over the cubic and linear smoothing methods, \$29.75 and \$41.50 respectively. We show that adopting the AvD smoothing technique results in a \$116.75 gain over pricing with coarse binomial trees. The results indicate that using a time-varying volatility structure as well as the forward rate curve with maximum smoothness improves best the performance of the BDT model. The empirical findings are also robust across factors affecting the option price, including time-to-expiration, moneyness, and trading volume.

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