Empirical Performance of Alternative Pricing Models of Currency Options

GHULAM SARWAR*
TIMOTHY KREHBIEL

This article examines the out-of-sample pricing performance and biases of the Heston's stochastic volatility and modified Black-Scholes option pricing models in valuing European currency call options written on British pound. The modified Black-Scholes model with daily-revised implied volatilities performs as well as the stochastic volatility model in the aggregate sample. Both models provide close and similar correspondence to actual prices for options trading near- or at-the-money. The prices generated from the stochastic volatility model are subject to fewer and weaker aggregate pricing biases than are the prices from the modified Black-Scholes model. Thus, the stochastic volatility model may provide improved estimates of the measures of option price sensitivities to key option parameters that may lead to more effective hedging and speculative strategies using currency options. © 2000 John Wiley & Sons, Inc. Jrl Fut Mark 20:265–291, 2000

*Correspondence author, Department of Finance, Insurance and Real Estate, St. Cloud State University, St. Cloud, MN 56301-4498.

- *Ghulam Sarwar is an Associate Professor in the Department of Finance, Insurance and Real Estate at St. Cloud State University in St. Cloud, Minnesota.*
- *Timothy Krehbiel is an Associate Professor in the Department of Finance at Oklahoma State University in Stillwater, Oklahoma.*

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INTRODUCTION

Previous studies indicate that the modified Black-Scholes model exhibits systematic biases in pricing currency options (Bodurtha & Courtadon, 1987a,b; Shastri & Tandon, 1986a; Tucker, 1985; Melino & Turnbull, 1987, 1990).[1] Specifically, option pricing errors, the difference between the model-predicted premia and actual premia, are systematically related to interest rates, volatility, option's strike price, and time to expiration. Further, implied volatility of the Black-Scholes model bears a nonlinear, often a U-shaped, relationship with the option's moneyness, a pattern referred to as the "smile effect" in the literature. Because the observed biases in the predicted prices of currency and stock options have little changed by using models that incorporate stochastic interest rates and jump processes, many investigators argue that such biases stem from the constant variance assumption of the Black-Scholes model (Johnson & Shanno, 1987; Chesney & Scott, 1989; Hull & White, 1987; Wiggins, 1987; Melino & Turnbull, 1990). Consequently, stochastic volatility option pricing models have been developed to allow for the impact of changing volatility on option prices.

Volatility risk stems from temporal volatility of the volatility of the underlying asset's return. When the volatility of an asset's return is stochastic, a riskless hedge cannot generally be formed from only one option and the underlying asset because the stochastic differential for the option contains two sources of risk, namely, the stochastic component of price changes and the stochastic component of the volatility of price changes (Wiggins, 1987). Although a riskless hedge can be formed from the stock and two options, such a hedge does not yield a unique option pricing function because volatility of price changes, or a known function of volatility, is not traded (Scott, 1987). Equilibrium asset pricing models are required to derive a unique option pricing function. The resulting valuation equation explicitly involves the price of volatility risk, the volatility of volatility parameter, and the correlation between the asset price and its volatility (Hull & White, 1987; Johnson & Shanno, 1987; Wiggins, 1987; Scott, 1987; Bakshi & Chen, 1997). The volatility variation parameter controls the kurtosis of spot returns, whereas the correlation of volatility with the spot return produces skewness. Thus, the stochastic volatility option pricing models attempt to improve the pricing of options by

[1]The modified Black-Scholes model for European currency options refers to either the model of Biger and Hull (1983) or the model of Garman and Kohlhagen (1983). In these models, the spot exchange rate replaces the stock price in the standard Black-Scholes model and the foreign interest rate enters as an additional variable.

allowing for a non-zero skewness and for higher kurtosis than is allowable in the log-normal distribution of the Black-Scholes model.

In the models of Hull and White (1987), Scott (1987), Wiggins (1987), and Knoch (1992), the price of volatility risk is assumed to be zero and thus removed from the valuation equation. Further, the correlation of volatility with the spot return is zero in Hull and White (1987), Knoch (1992), Stein and Stein (1991), Ball and Roma (1994), and Scott (1987). Although the assumptions of zero price of volatility risk and zero correlation simplify the option valuation with stochastic variance, they have little empirical support. Merville and Pieptea (1989), Christie (1982), and Lamoureux and Lastrapes (1993) all provide evidence that the volatility risk is priced. Melino and Turnbull (1990) and Tucker (1985) find a negative correlation between the volatility of currency returns and currency exchange rates. Further, Beckers (1983); French, Schewert, and Stambaugh (1987); and Christie (1982) support the notion of a negative relationship between a stock price and its volatility.

Using Fourier inversion methods, Heston (1993), Bakshi and Chen (1997), and Bakshi, Cao, and Chen (1997) provide closed-form solutions for valuing currency and stock options under stochastic volatility without imposing the restrictions of zero correlation and zero price of volatility risk. This study examines the out-of-sample performance of the stochastic volatility option pricing model of Heston (1993) applied to the valuation of British pound options traded at Philadelphia Stock Exchange. Option values from Heston's stochastic volatility model and values from the modified Black-Scholes model are compared to observed option premiums. Out-of-sample pricing biases are then compared for these models. Pricing biases related to option strike price, time to maturity, volatility, and interest rate differential are considered.

The present study is similar in overall focus to that of Bakshi, Cao, and Chen (1997). Our study differs from theirs in at least three respects. First, the volatility risk premium is estimated explicitly herein rather than internalized in parameter estimates as in Bakshi, Cao, and Chen (1997). Although an explicit estimate of volatility risk premium is not required to implement the stochastic volatility model, the internalized (hidden) values of volatility risk premium in parameters provide little clue to the size and statistical significance from zero of the risk premium. Second, the present study estimates the parameters of volatility and exchange rate processes, which are required inputs of the model, using seemingly unrelated regressions. Such parameters are implied out of observed option prices using a nonlinear least square procedure in Bakshi, Cao, and Chen (1997). Whereas their estimation approach may be theoretically more plausible than ours, the implied parameter approach is computationally intensive and provides highly implausible parameter values in their study. Although the relative efficiency of the two approaches remains to be seen, our approach may be viewed as a simple alternative to estimating the model parameters. Third, the present study uses British pound currency options, whereas Bakshi, Cao, and Chen (1997) utilize S&P 500 index options. Bakshi, Cao, and Chen suggest that their index option results may not hold for currency options.

THE MODEL

Following Heston (1993) and Knoch (1992), assume that the currency exchange rate and volatility of the exchange rate obey the stochastic processes:

$$dS = \mu S dt + \sqrt{V} S \, dZ_1 \tag{1}$$

$$d\sqrt{V} = -\beta\sqrt{V}\, dt + \delta dZ_2 \tag{2}$$

where S is the spot exchange rate, $\sqrt{V}$ is the volatility (standard deviation) of the exchange rate, μ, β, and δ, and are fixed constants, and dZ_1 and dZ_2 are Wiener processes. The process dZ_1 has a correlation ρ with process dZ_2. Eq. (1) proposes a conditional lognormal diffusion process for the exchange rate dynamics subject to a stochastic volatility that follows an arithmetic Ornstein-Uhlenbeck process. Applying Ito's lemma to (2), the variance V follows the square-root process:

$$dV = \kappa[\theta - V]dt + \sigma\sqrt{V}\, dZ_2 \tag{3}$$

where $\kappa = 2\beta$, $\theta = \delta^2/2\beta$, and $\sigma = 2\delta$. Assuming that the domestic and foreign interest rates, r and r^* respectively, are constant, the fundamental partial differential equation (PDE) is:

$$\begin{aligned} &0.5(VS^2C_{SS}) + \rho\sigma\, VSC_{SV} + 0.5(\sigma^2VC_{VV}) + (r - r^*)SC_S \\ &+ \{\kappa[\theta - V] - \lambda(S, V, t)\}C_v - rC + C_t = 0 \end{aligned} \tag{4}$$

where C denotes the call option price, the subscripts on C denote the partial derivatives, and $\lambda(.)$ is the price of volatility risk. Heston (1993) and Knoch (1992) show that the solution to the price of a European currency call option, with an exercise price of K and time to expiration of τ, can be expressed:

$$C(S, V, t) = S\exp(-r^*\tau)P_1 - K\exp(-r\tau)P_2 \quad (5)$$

where P_1 and P_2 are unknown probabilities analogous to the cumulative normal probabilities under the Black-Scholes model. The cumulative probabilities P_j can be obtained using the Fourier inversion formulas as:

$$P_j(x, V, T; \ln(K)) = (½) + (1/\pi)\int \mathrm{Re}[\{e^{-i\phi\ln(K)} f_j(x, V, T; \phi)\}/i\phi]d\phi;\ j = 1, 2 \quad (6)$$

where $x = \ln(S)$, Re denotes the real part, and the integral extends from zero to infinity. The exact solution for the characteristic function $f_j(.)$ is:

$$f_j(x, V, t; \phi) = \exp\{C(T - t; \phi) + D(T - t; \phi)V + i\phi x\} \quad (7)$$

where

$$C(\tau; \phi) = r\phi i\tau + (a/\sigma^2)\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[(1 - ge^{d\tau})/(1 - g)]\}$$

$$D(\tau; \phi) = ((b_j - \rho\sigma\phi i + d)/\sigma^2)[(1 - e^{d\tau})/(1 - ge^{d\tau})]$$

$$g = [b_j - \rho\sigma\phi i + d]/[b_j - \rho\sigma\phi i - d]$$

$$d = [(\rho\sigma\phi i - b_j)^2 - \sigma^2(2\mu_j\phi i - \phi^2)]^{½}$$

$$\tau = T - t$$

$$\mu_1 = 1/2,\ \mu_2 = -½,\ a = \kappa\theta,\ b_1 = \kappa + \lambda - \rho\sigma,\ b_2 = \kappa + \lambda.$$

Given the above parameter values, the present study solves the integral in eq. (6) using numerical integration methods.

DATA DESCRIPTION

The estimation of probabilities in Heston's model requires the spot exchange rate, option's exercise price and expiration date, domestic and foreign interest rates, volatility estimates, price of volatility risk, and parameter estimates of the spot exchange rate and volatility processes. The study uses daily closing transaction prices for European currency call options written on British pound during January 1993 to October 1995. Most investigators have used call options in previous option pricing studies. The dollar/British pound exchange rates are selected because, among the British pound, Canadian dollar, Japanese yen, German mark, and French franc, the British pound has the largest volatility in monthly volatilities during the sample period.[2] Because volatility risk stems from the temporal volatility of the volatility of the underlying asset's return (Heston, 1993), the volatility risk may be more pronounced in British pound options than in other currency options during the sample period. In addition, Bodurtha and Courtadon (1987b) and Shastri and Tandon (1986b) show that the modified Black-Scholes model performs much worse in pricing high interest-rate currency options, such as options on the British pound, than in pricing low-interest-rate currency options. These observations suggest that the potential impacts of stochastic volatility on option prices, and in turn on estimated volatility risk premium, may be more pronounced in options written on British pound than in other currency options.

The closing transaction option and currency prices are obtained from the transaction surveillance report of the Philadelphia Stock Exchange. The report lists for each option contract the date and time of the trade; maturity, exercise price, and option price; number of contracts traded; opening, high, low, and closing option prices; and the prevailing spot exchange rates at the time of the trade.[3] All the mid-month and month-end European call options written on British pound are extracted from the transaction data for the period January 1993 to October 1995, providing a total of 879 observations.[4]

To exclude uninformative options records from the sample, the call options that fell in the following categories are discarded:

i. Options with time to expiration of less than 5 calendar days.

ii. Options violating the European boundary conditions, that is, $C < Se^{-r^{*}\tau} - Ke^{-r\tau}$.

[2]The volatility of volatility estimate is the standard deviation of the monthly standard deviations of the actual daily spot exchange rates during the period January 1993 to October 1995. The monthly standard deviation is the standard deviation of the daily exchange rates during each of the thirty-four months in the sample. The standard deviation of these thirty-four monthly standard deviations, expressed in U.S. dollars, is the volatility of volatility estimate, which is 0.00532 for British pound, 0.00528 for Japanese yen, 0.0050 for German mark, 0.0048 for French franc, and 0.0018 for Canadian dollar. The British pound also exhibits the largest volatility of volatility in both 1993 and 1994, whereas it shows the second smallest volatility of volatility (second to Canadian dollar) in 1995.

[3]The trading hours of the Philadelphia Stock Exchange for standardized currency options run from 2:30 AM to 2:30 PM Eastern time. The daily closing option price represents the price of the last trade of an option contract during the twelve-hour trading session. The last option trade does not often correspond to the closing time of the market, and could occur anytime during the trading hours. When only a single trade for a specific option contract occurs during an entire trading session, the opening, high, low, and closing option prices have the same value and trading time. The actual trading time reported for closing option prices in our sample ranged from 2:41 AM to 2:29 PM.

[4]The European options comprised 19%, or 1957 options, of the 10,300 American and European British pound options traded during the period January 1993 to October 1995. Of the 1957 European options, 879 are call options whereas 1078 are put options. Overall, call options accounted for 62% of the American and European options traded during the sample period.

iii. Options with a premia of less than or equal to 0.02 cents.

Criterion (i) is justified because the implied volatility of options with small time to maturity behaves erratically. Criterion (ii) removes options violating the boundary conditions for European options. Criterion (iii) eliminates options that are very thinly traded and are not representative of the market. These criteria removed twelve options from the sample, leaving 867 call options for the analysis.

The yield on the U.S. Treasury bills with maturity closest to the option maturity date is used for the domestic interest rates. The yield is based on the average of the bid and asked discounts on the Treasury bills. The T-bill data are obtained from the Federal Reserve Bank of New York.

Similar data on yields of British Treasury bills are not available. Following Bodurtha and Courtadon (1987a), the interest rate parity (IRP) relationship is used to estimate the implied British risk-free rate. The interest rate parity relationship can be written:

$$[(F - S)/S] = [(1 + r)/(1 + r^*)] - 1 \tag{8}$$

where F is the futures (forward) exchange rate of the U.S. dollar to British pound, S is the spot exchange rate, r is the U.S. T-bill rate, and r^* is the British risk-free rate. At any given time t, the futures exchange rate that has a maturity date most closely matched to that of the concurrent T-bill is used in the interest rate parity relationship. The futures exchange rate data is supplemented by the forward exchange rates in a few cases where the futures rates with maturities matching with those of the T-bills are unavailable. The futures exchange rates are obtained from the Futures Industry Institute and the forward exchange rates are hand collected from the *Wall Street Journal*. Given the observed values of futures (forward) exchange rate, spot exchange rate, and U.S. T-bill rate in eq. (8), the corresponding implied British risk-free rates are estimated.

METHODS FOR ESTIMATING THE MODEL'S PARAMETERS AND PRICING PERFORMANCE

Computation of Implied Volatility

The volatility of exchange rate in both the modified Black-Scholes model and Heston's model is approximated by the implied volatility (standard deviation). Empirical evidence appears to favor an implied standard deviation (ISD) estimate over a historical estimate as a predictor of ex-post volatility (Chirac & Manaster, 1978; Merville & Pieptea, 1989). Although the true volatility is not option dependent, the ISD estimates are option dependent. The regression-based iteration approach of Whaley (1982) is employed to smooth out option-specific errors. The approach uses only the options that share a common maturity, and finds the volatility value that minimizes the sum of squared errors between the model prices and actual prices. The iteration process for finding the implied volatility is based on the regression model:

$$c_j - c_j(\sigma_0) + \sigma_0 (\partial c_j/\partial \sigma_0|\sigma_0) = \sigma(\partial c_j/\partial \sigma_0|\sigma_0) + e_j \qquad (9)$$

where c_j are the actual option premia, $c_j(\sigma_0)$ are the predicted option premia using σ_0 as an estimate of volatility, and $(\partial c_j/\partial \sigma_0|\sigma_0)$ is the option vega conditional on σ_0. The regression (9) yields an estimate of σ, which is then used in another iteration of the regression until the implied volatility estimate changes less than a stipulated amount in absolute value (0.001 in this study). Following Whaley (1982), the ISD is estimated at time $t-1$ using only options that share a common maturity. This implied volatility based on previous day's option prices is then used as an input to compute the current day's model-based option prices. Such a daily-revised estimate is theoretically more plausible than contemporaneous ISD estimate because it does not use information that might be unavailable to option traders at the time of the option transactions.

Because the European call options written on British pound are not traded daily in our sample, the estimation of implied volatility using Whaley's approach evolved as follows. To price a call option with maturity τ at time t, European call options sharing maturity τ at time $t-1$ are first sought and used to estimate implied volatility. In the absence of such call options, European put options sharing maturity τ at time $t-1$ are used to derive implied volatility.[5] When such put options are also not traded at time $t-1$, the relation of British riskless interest rate to the U.S. T-bill rate is evaluated to choose either the American puts or American calls for estimating implied volatility. If the British rate is higher (lower) than the U.S. rate at time $t-1$, then American put (call) options sharing maturity τ at time $t-1$ are used to derive implied volatility. Because rational option holders will not exercise an American put (call) option before ma-

[5]Whereas we are estimating the premiums for call options, the data used in the study may be viewed as a mixture of call and put options because implied volatilities are calculated in part using maturity-comparable put options. If put-call parity relationship held at every instant, then examining the pricing behavior of put options would be pointless because it will provide no information that is not already reflected in the comparable call options and the currency exchange rate, as pointed out by Easley, O'Hara, and Srinivas (1998). This may explain why previous investigators have focused only on call options.

turity when the British riskless rate is higher (lower) than the U.S. riskless rate, an American option on British pound should have the same value as its European counterpart, thus justifying the use of American options to derive the implied volatility for European options.[6]

The above procedure requires parameters of the volatility process, volatility risk premium, and option's vega in order to estimate the ISD for the stochastic volatility model. The currency option's vega for the Heston model is:

$$\partial c/\partial\sigma = Se^{-r^{*}\tau}\,\partial P_1/\partial\sigma - K\,e^{-r\tau}\,\partial P_2/\partial\sigma \tag{10}$$

where for $j = 1,2$:

$$\partial P_j/\partial\sigma = (1/\pi)\int \mathrm{Re}[\{D_j(\cdot)\,\exp\{-i\phi\ln(K) + C_j(\cdot) + D_j(\cdot)V + i\phi x\}\}/i\phi]d\phi \tag{11}$$

All the variables in eqs. (10) and (11) are defined as before under the Heston model. Given parameters of the volatility process and volatility risk premium (discussed in the next subsection), the ISD for the stochastic volatility model is estimated using the above procedures. The resulting ISD estimates using option prices at time $t-1$ are then used as an input to compute Heston's model-based option prices at time t.

Estimating the Price of Volatility Risk

It is well known in the literature that, when volatility is constant, the hedge ratio of the Black-Scholes model can be used to construct a riskless hedge portfolio. However, when volatility is random and a marketwide volatility effect exists, a hedge portfolio formed from one option and the underlying stock would not be riskless and would not yield a return equal to the riskless interest rate (Wiggins, 1987). Merton (1976) demonstrates that, when the underlying stock returns experience jumps, the hedge portfolio will not be riskless. Hull and White (1987) point out that Merton's mixed jump-diffusion process for stock prices is a special type of stochastic volatility problem. Heston (1993) argues that the excess return

[6]It is worth noting that the one-period lagged implied volatility using either European or American options is needed only for those days when European British pound call options are traded. Further, American call options are rarely used to estimate implied volatility because the implied British riskless rate is higher than the U.S. riskless rate in much of our study period. The use of put options to estimate the implied volatility of corresponding call options assumes that the put-call parity relationship holds in our sample.

on the hedge portfolio, the difference between the hedge portfolio's return and the riskless interest rate, can serve as a measure of the premium (price) for volatility risk.

Following the conceptual approaches of Wiggins (1987) and Heston (1993), the present study measures the premium for volatility risk as the excess actual return on the hedge portfolio. The hedge portfolio is based on the hedge ratio of the modified Black-Scholes model where the implied volatility is revised daily. The modified Black-Scholes model with daily-revised volatilities can be viewed as a less expensive approximation of the stochastic volatility model as argued by Chesney and Scott (1989). The value of the hedge portfolio at time t, Q_t, is

$$Q_t = H_t S_t - C_t \tag{12}$$

where H_t is the hedge ratio, S_t is the spot exchange rate, and C_t is the price (premium) of currency option. The hedge ratio is defined as

$$H_t = \partial C/\partial S = e^{-r^*\tau} N(d) \tag{13}$$

where $d = \{\ln(S/K) + (r - r^* + (V/2))\tau\}/\sqrt{V\tau}$

V = volatility of currency exchange rate

K = option's strike price

r = domestic risk-free interest rate

$e^{-r^*\tau}$ = foreign risk-free interest rate

τ = the time to expiration of the option

$N(d)$ = the cumulative normal distribution function

$\ln(.)$ = natural logarithm

Because the hedge ratio in (13) varies with S, τ, and V, it is readjusted frequently to maintain the hedge position of the portfolio. Specifically, the value of the hedge portfolio at time $t+i$ is estimated as

$$Q_{t+i} = H_t S_{t+i} - C_{t+i} \tag{14}$$

In forming the hedge portfolio Q_{t+i}, the option price C_{t+i} refers to that particular option at time $t+i$ that has the same exercise price as the option in C_t, so that the two options differ only in time to expiration by i days. Such an option structure is required to track changes in the value of the hedge portfolio Q_t over time; otherwise, the composition of portfolios Q_t and Q_{t+i} would differ and the hedge portfolio's return in (15), below, would be meaningless.

The return on the hedge portfolio, R, is estimated:

$$R_{t+i} = (Q_{t+i} - Q_t)/Q_t \tag{15}$$

The premium for volatility risk, λ, is estimated:

$$\lambda_{t+i} = \text{annual } R_{t+i} - \text{annual } r_{t+i} \tag{16}$$

where r_{t+i} is the t-bill rate at time $t+i$ and is estimated as the yield on the one-month U.S. Treasury bill.

Although the option pricing formulas are predicated on continuous rebalancing, through time, of the hedge portfolios, a continuous rebalancing scheme is not possible in empirical investigations. Whaley (1982), Bhattacharya (1983), Chiras and Manaster (1978), and Leland (1985) show that continuous rebalancing of the hedge portfolio can be reasonably well-approximated by discrete rebalancing.

ESTIMATING THE PARAMATERS OF VOLATILITY PROCESS

The volatility parameters are estimated using the seemingly unrelated regression (SUR) method on the time series of implied volatilities and exchange rates. The stochastic processes of spot exchange rates (S) and volatility of exchange rates (V) are earlier specified in eqs. (1) and (2), respectively:

$$dS = \mu S dt + \sqrt{V} S\, dZ_1 \tag{1}$$

$$d\sqrt{V} = -\beta\sqrt{V}\, dt + \delta dZ_2 \tag{2}$$

Assuming t is 1 (one day or 1/360 year), discrete-time versions of processes (1) and (2) can be written:

$$\Delta\log(s_t) = \mu + \sqrt{V_t} u_t; \qquad \mu_t \sim N(0,1) \tag{17}$$

$$\sqrt{V_t} = \alpha\sqrt{V_{t-1}} + \phi e_t; \qquad e_t \sim N(0,\phi^2) \tag{18}$$

where $\alpha = (1/1+\beta)$, $\phi =$ standard deviation of $e_t = \delta(1/1+\beta)$, and $\Delta S/S = \Delta\log(S)$. The daily-updated implied volatilities of the modified Black-Scholes model derived using only near- or at-the-money options are used for V. These options are used because Bakshi, Cao, and Chen (1997) show that, for a given maturity category, the Black-Scholes and stochastic volatility option pricing models yield virtually identical implied volatilities for at-the-money stock index options. Eqs. (17) and (18) can be rewritten:

$$\Delta\log(s_t)/\sqrt{V_t} = \mu/\sqrt{V_t} + u_t;\ u_t \sim N(0,\ 1) \tag{19}$$

$$\sqrt{V_t} = \alpha\sqrt{V_{t-1}} + \phi e_t; \qquad e_t \sim N(0,\ \phi^2) \tag{20}$$

The equations are estimated as a SUR system without allowing for intercepts in the equations. The correlation between estimated u_t and e_t is the estimate for parameter ρ in eq. (7). The estimated variance of e_t is used for ϕ^2. Given α and ϕ, the estimates for β and δ are recovered:

$$\beta = (1/\alpha) - 1; \qquad \delta = \phi(1 + \beta)$$

Given β and δ, the estimates of original parameters are recovered:

$$\kappa = 2\beta,\ \theta = \delta^2/2\beta,\ \text{and}\ \sigma = 2\delta$$

The variances of κ, θ, and σ are derived from the variances of β and δ using the large-sample approximation procedure of Kmenta (1971, p. 444):

$$\text{if } A = f(B_1,\ B_2, \ldots,\ B_k),\ \text{then}$$

$$\mathrm{Var}(A) = \Sigma[\partial f/\partial B_k]^2\ \mathrm{Var}(B_k) + 2\Sigma[\partial f/\partial B_k]\mathrm{Cov}(B_j,\ B_k)$$

where Var(.) denotes the variance and Cov (.) represents the covariance. The procedure indicates that $\mathrm{var}(\beta) = (-1/\alpha^2)^2{*}\mathrm{var}(\alpha)$; $\mathrm{var}(\kappa) = 4{*}\mathrm{var}(\beta)$; and $\mathrm{var}(\sigma) = 4{*}\mathrm{var}(\delta)$. Similarly, other variances are derived.

METHODS FOR EVALUATING THE MODEL'S PRICING PERFORMANCE

The pricing accuracy is initially evaluated by comparing the relative mean error, mean absolute error, and root mean squared error (RMSE) of the Heston's model with those of the modified Black-Scholes model. Following Melino and Turnbull (1990), Bakshi, Cao, and Chen (1997), and Chesney and Scott (1989), the pricing biases of the stochastic volatility and modified Black-Scholes models are investigated using the cross-sectional regressions:

$$C = \alpha_0 + \alpha_1 C^* + e_1 \tag{21}$$

$$C - C^* = \gamma_0 + \gamma_1(S - K)/K + \gamma_2 T + \gamma_3 V + \gamma_4(r - r^*) + e_2 \tag{22}$$

where C denotes the observed option price, C^* the model-predicted price, K the exercise price, S the spot exchange rate, T the maturity of the option, $r(r^*)$ the U.S. (U.K.) interest rate, V the estimated volatility, and e_1 and e_2 are the disturbance terms. In eq. (21), the model price provides an unbiased estimate of the actual option price if $\alpha_0 = 0$ and $\alpha_1 = 1$, a hypothesis tested using the Wald statistic. Eq. (22) is designed to examine whether the pricing errors exhibit the strike price bias, time-to-maturity bias, volatility bias, and interest rate differential bias. The equation is structured to identify if the prediction errors are systematically related to the fundamental inputs of the currency option pricing model. A positive value of γ_1 indicates that, for call options, the model underprices (overprices) an option that is in (out of) the money. A positive value of γ_2 suggests the maturity bias. A positive value of γ_3 points to the existence of the volatility bias. If γ_4 is positive, the degree of mispricing the options is an increasing function of the interest rate differential. If $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$, then the pricing errors are independent of the exercise price, time to maturity, volatility, and interest rate differential. The Wald statistic to test the joint hypothesis of a zero value for all the parameters in regression (22) is asymptotically distributed as $\chi^2(5)$.

Eq. (21) is estimated using the ordinary least squares (OLS) method. Because the residuals of eq. (22) may be heteroscedastic owing to the clustering of option observations across cross sections as pointed out by Hilliard et al. (1991) and Scott (1987), this equation is estimated using the generalized method of moments with correction for heteroscedasticity.

Most investigators estimate eq. (22) as a cross-sectional regression using either the OLS method or the generalized method of moments (Melino & Turnbull, 1990; Nandi, 1996; Chesney & Scott, 1989; Hilliard et al., 1991; Shastri & Tandon, 1986a; Whaley, 1982). Such an estimation approach makes no distinction between the time series and cross sectional properties of the options data even though the option observations are actually clustered in time and across cross sections. The approach implies that, on a given day, each traded option (a cross section) is distinct and that the same exact option is not traded over time. Whaley (1982) and Nandi (1996) recognize that the pricing biases from eq. (22) may vary by the option's time-to-maturity and moneyness classes (cross sections), and thus estimate separate OLS regressions for each of the several such classes defined for their sample data. Their approach suggests an inter-class distinction of option observations.

The present study attempts to capture the cross-sectional and time-series properties of the sample options as well as the possible inter-class

TABLE I

Descriptive Summary Statistics of the Sample Data on British Pound Currency Options, 1993–1995

Variable	*Mean*	*Standard Deviation*	*Minimum*	*Maximum*
Call price (cents/unit)	1.786	2.13	0.030	14.35
Spot exchange rate ($/pound)	1.54	0.054	1.417	1.650
Exercise exchange rate ($/pound)	1.57	0.074	1.475	2.10
Implied volatility	0.089	0.025	0.010	0.261
Years to expiration	0.205	0.171	0.016	0.991
Annualized U.S. riskless rate (%)	4.07	1.126	2.08	7.15
Annualized British riskless rate (%)	4.35	1.159	1.714	8.46

Note: The sample has 867 options.

variation in pricing biases in a single pooled cross-section time series regression equation. To implement such an approach, an index variable is created using the option's time-to-maturity and moneyness classes. A scale variable differentiates the time-to-maturity classes by assigning 1 to options with maturity less than 30 days, 2 to options with maturity between 30 and 90 days, 3 to options with maturity between 90 and 180 days, and 4 to options with maturity more than 180 days. A second scale variable distinguishes the moneyness classes by assigning 5 to in-the-money options, 6 to at- or near-the-money options, and 7 to out-of-the-money options. The product of the two scale variables creates a cross-sectional index variable, providing twelve possible cross sections for the sample data. The number of observations under each of the twelve cross sections varied in the sample data, thereby yielding an unbalanced pooled cross-section time-series data structure that is estimated alternatively using the error components model and dummy variable model. The former model assumes that the intercepts are drawn from a random sample, whereas the latter model assumes that the intercepts are fixed parameters. The models also assume that each cross section has its own, distinct intercept but all cross sections share the same slope coefficients. Both procedures are used because the choice between fixed and random effects is unclear a priori for the sample data.

RESULTS

Descriptive statistics of sample currency options are presented in Table I. The call premium, implied volatility, and the exercise and spot exchange rates are all subject to significant fluctuations during the sample period.

The high volatility of volatility values clearly violate the constant volatility assumption of the Black-Scholes option pricing model. As expected, the U.S. riskless rate has lower mean as well as lower volatility than the implied British riskless rate.

Because the stochastic volatility of asset's returns affects the structure of the distribution of returns (Heston, 1993), the distributional properties of the exchange rates and exchange rate returns are shown in Table II. The daily exchange rate returns ($\Delta\log(s)$) display a small negative mean and negative skewness but consistently display positive excess kurtosis. Similar pattern holds for the distribution of exchange rate changes. Heston shows that the excess kurtosis in price and return distributions stems in part from the stochastic volatility of returns. He also demonstrates that the correlation between the asset return and volatility of asset return, estimated at -0.14 in the present study, affects the skewness of return distributions. Because a non-zero correlation estimate cannot result from volatilities that are constant over time, the stochastic volatility feature of exchange rates must have contributed to the high skewness and excess kurtosis values of return distributions shown in Table II. Given such skewness and kurtosis values, it is not surprising that the Kolmogrov statistics as well as the Jarque-Bera test indicate that the distributions of exchange rates and exchange rate returns are not normal. Such a finding is at variance with the assumption of a normal distribution of returns in the modified Black-Scholes model.

Volatility Risk Premium

Table III presents tests of the significance of volatility risk premium. The hedge portfolio that provides estimates of volatility risk premium is rebalanced, on average, after every 3.2 business days. A daily rebalancing of the hedge portfolio is not possible due to the non-availability of two consecutive options throughout in our daily data that share the same exercise price but differ in time to expiration by one day. Previous investigators have used rebalancing intervals of one week (Whaley, 1982), two weeks (Bhattacharya, 1983), one month (Chiras & Manaster, 1978), and two months (Leland, 1985). Boyle and Emanuel (1980) argue that the systematic risk of hedge portfolio is expected to be small over short rebalancing intervals such as ours than over long rebalancing intervals such as those in Chiras and Manaster (1978) and Leland (1985).

The annualized volatility risk premium, estimated as the difference between the return on a hedge portfolio and the risk-free rate, averaged

TABLE II

Descriptive Statistics of the Distribution of U.S. Dollar–British Pound Exchange Rate, 1993–1995[a]

Statistic	S^b	*Log(S)*	*Δ(S)*	*ΔLog(S)*
Mean	1.54	0.43	−0.0001	−0.0002
Skewness	0.77	0.60	−0.37	−0.28
Excess kurtosis	1.86	1.34	3.12	2.55
Kolmogorov statistics[c] (W-statistics)	0.94*	0.95*	0.96*	0.96*
Jarque-Bera test[d]	196.1*	85.24*	345.7*	229.19*

[a]These descriptive statistics are for 867 observations.
[b]The letter S denotes the spot exchange rate.
[c]H_0: the data set is a sample from a normal distribution. An asterisk (*) indicates the rejection of the null hypothesis at the 1 percent significance level.
[d]The critical chi-square value is 9.21. The null hypothesis is that the data set is a sample from a normal distribution. The test statistic is: $LM = n[(g^2/6) + (k^2/24)]$, where LM denotes the Lagrange Multiplier, n is the number of observations, and g and k are coefficients of skewness and kurtosis, respectively.

TABLE III

Tests for the Non-Zero Mean of Volatility Risk Premium, 1993–1995

	Mean	*S.D.*	*n*	*t-test*[a] *(normal)*	*Modified t-test*[b] *(non-normal)*
Volatility Risk Premium (λ)	−0.029	0.39	867	−2.19*	−2.20*

[a]$t = \sqrt{n}\, x/s$, where x is the mean value, s is the standard deviation (S.D.) of x, and n is the total number of observations. An asterisk on the t-value implies that the null hypothesis of zero mean is rejected at the 1% significance level.
[b]$t_{John} = n[x + (\mu/6s^2n) + \mu x^2 + /3\, s^4]/s$, where μ is the third central moment. The modified t-value, adapted from Whaley (1982), is the t_{John} value.

−2.9%.[7] Both a t-test and a non-normal t- test suggest that this estimated volatility risk premium is statistically different from zero at the 1% significance level. The finding that the risk premium is non-zero is consistent with those of Lamoureux and Lastrapes (1993), Merville and Pieptea (1989), and Melino and Turnbull (1990). These studies, however, employed only indirect methods for examining the significance of volatility risk, and thus failed to suggest the sign and size of the volatility risk premium.

A negative volatility risk premium implies that the systematic risk of the hedge portfolio is lower, on average, than that of the commonly used empirical proxy of the riskless asset, the Treasury bill. Wiggins (1987) demonstrates that the sign of volatility risk premium depends on the beta

[7]This estimate of volatility risk premium is somewhat biased because investment costs, such as transaction costs and interest costs, are not subtracted from the returns of currency option-hedge portfolios.

TABLE IV

Seemingly Unrelated Regression Estimates of Exchange Rate and Volatility Processes

	Independent Variables		
Dependent Variables	$1/\sqrt{V_t}$	$\sqrt{V_{t-1}}$	System R^2
$\Delta\log(S_t)/\sqrt{V_t}$	0.0022 (5.48)	—	0.89
$\sqrt{V_t}$	—	0.965 (84.34)	0.89

Notes: The *t*-statistics are given in parenthesis. The above results were based on the model:

$$\Delta\log(S_t)/\sqrt{V_t} = \mu\ 1/\sqrt{V_t} + u_t,\ u_t \sim N(0, 1) \quad (1)$$

$$\sqrt{V_t} = \gamma\ \sqrt{V_{t-1}} + \phi e_t,\ e_t \sim N(0, \phi^2) \quad (2)$$

where S denotes the British pond to dollar spot exchange rate, V_t is the implied volatility (variance), and log represents the natural logarithm. Eqs. (1) and (2) are estimated as a system of seemingly unrelated regressions (SUR).

coefficient of the hedge portfolio. Thus, a negative volatility risk premium translates into a negative beta for the currency option hedge portfolio, suggesting that the returns on the currency hedge portfolio move in opposite direction to returns on the market portfolio. Whaley (1982) reports also a negative beta for his stock hedge portfolios. Because a negative beta asset is conceptually like a fire insurance policy as pointed out by Brigham and Gapenski (1996), the currency option hedge portfolio of the present study provides an excellent hedge for uncertain currency returns when the market returns turn out to be quite dismal.

The finding that the volatility risk premium is negative contradicts the assumption of zero price of volatility risk in models of Hull and White (1987), Johnson and Shanno (1987), and Scott (1987), and the resulting derivations of option pricing formulas in those models.

Parameter Estimates

Parameter estimates of the volatility and exchange rate processes are reported in Tables IV and V.[8] The model as a whole, shown in Table IV,

[8]These parameters are estimated using the data on implied volatility and exchange rates over the entire sample period January 1993 to October 1995. To test the stability of parameters during the sample period, we sequentially updated the data each week starting with the data from the first quarter and then estimated the parameters for all resulting subperiods. A pair-wise F-test to examine the equivalence of estimated parameters across the subperiods suggested no statistically significant difference among the parameters. Thus, the use of parameters that are estimated using only data available to traders at the time of option transactions would not have affected the results reported in the text in any material way.

TABLE V

Parameter Estimates of the Stochastic Volatility Model, 1993–95[a]

Parameter	*Estimates*	*Standard Error*
Drift term of the $\sqrt{V}$ process (β)	0.036	0.0119
Mean reversion of the V process (κ)	0.072	0.023
Volatility of volatility for the $\sqrt{V}$ process (δ)	0.022	0.0002
Volatility of volatility of the V process (σ)	0.044	0.0005
Long-run mean volatility, daily (θ)	0.006	0.0003
Long-run mean volatility, annual	0.114	0.057
Correlation of ΔS and ΔV (ρ)	−0.142	0.015[b]
Mean of volatility risk premium (λ), annual	−0.029	−2.19[c]

[a]These parameter estimates and their standard errors, except ρ and λ, were recovered from SUR-based parameter estimates in Table IV.
[b]The value is the probability significance level, p-value, at which the null hypothesis of zero correlation was rejected.
[c]This is the calculated t-value for the null hypothesis of zero mean of the volatility risk premium.

appears to be well specified, judging from the system R^2 and t-ratios for the estimated parameters. All parameter estimates in Table V are different from zero at the 1% significance level. The mean reversion estimate, κ, suggests that shocks to volatility in the British pound/U.S. dollar exchange rates are short-lived, and there is a tendency for volatility to revert quickly to the mean level of volatility. The half-life of a shock to volatility, $\ln(2)/(\kappa+\lambda)$, is approximately nine days. Melino and Turnbull (1990) report the half-life of a shock to Canada–U.S. currency exchange rate as one week, whereas Chesney and Scott (1989) estimate the mean half-life of a shock to U.S. dollar/Swiss franc exchange rate as 32.6 days.

The correlation between the innovations to volatility and the level of exchange rates is negative (-0.14), suggesting that depreciation of the British pound tends to precede periods of high volatility. Melino and Turnbull (1990) and Tucker (1985) also report negative correlation estimates. The negative, significant correlation is strongly at variance with the assumption of zero correlation in models of Stein and Stein (1991), Hull and White (1987), and Knoch (1992). As discussed by Heston (1993), the negative correlation creates a fat left tail and a thin right tail, which decrease the prices of out-of-the-money options relative to in-the-money options.

Out-of-Sample Pricing Performance

The pricing performance of the stochastic volatility model and the modified Black-Scholes model is compared in Table VI using the models' relative mean error, mean absolute error, and RMSE. Both models yield vir-

TABLE VI

Pricing Errors as Differences between Actual Prices and Predicted Prices of the Black-Scholes Model and Stochastic Volatility Model, by Moneyness Classes, 1993–1995

Statistic	*All Options*	*In-the-Money*	*Near-the-Money & At-the-Money*	*Out-of-the-Money*
		Black-Scholes Model		
Relative Mean Error (%)	−4.0	−15.0	−12.0	9.0
Mean Absolute Error (%)	19.0	17.0	20.0	18.0
RMSE (cents)	0.36	0.96	0.21	0.21
		Stochastic Volatility Model		
Relative Mean Error (%)	−7.0	−10.0	−13.0	−4.0
Mean Absolute Error (%)	21.0	20.0	19.0	24.0
RMSE (cents)	0.37	1.03	0.20	0.24
N	867	88	443	336

Notes: In-the Money: (Spot/Exercise) ≥ 1.02
Near- or At-the-Money: 0.98 < Spot/Exercise < 1.02
Out-of-the-Money: Spot/Exercise ≤ 0.98
Pricing Error: Actual Price- Model Predicted Price
Relative Mean Error = $(1/N)\ \Sigma$ [(Actual-Predicted)/Actual]
Absolute Mean Error = $(1/N)\ \Sigma$[(|Actual-Predicted|)/Actual]
RMSE = $[(1/N)\Sigma(\text{Actual-Predicted})^2]^{0.5}$

tually identical values for the three measures of pricing errors for near- or at-the-money options, a result consistent with Heston's observation that, for at-the-money options, the Black-Scholes model and stochastic volatility model produce identical option prices. Also, the two models show similar pricing performance in the aggregate sample: The aggregate RMSE and mean absolute errors are 0.36 cents and 0.19%, respectively, for the modified Black-Scholes model, compared to their respective values of 0.37 cents and 0.21% for the stochastic volatility model. However, the modified Black-Scholes model performs somewhat better than the stochastic volatility model in pricing in-the-money and out-of-the-money options when judged using mean absolute errors. An F-test on the two RMSE values (0.36 vs. 0.37) suggests no significance difference between the two values at the 1% level.

The pricing performance of the two models is further compared using regression tests of the degree of association between the actual prices and model-based prices. The tests in Table VII suggest that the Black-Scholes model with daily-revised volatilities is somewhat better than the stochastic volatility model in predicting the actual option prices. The Wald test, however, rejects the null hypothesis that model prices are unbiased estimate of actual prices for both models. Chesney and Scott

TABLE VII

Regression Tests for the Relation between Actual Call Prices and Predicted Call Prices from the Black-Scholes and Stochastic Volatility Models, by Moneyness Classes, 1993–1995

	All Options	*In-the-Money*	*Near-the-Money & At-the-Money*	*Out-of-the-Money*
		Black-Scholes Model		
Intercept (α_0)	0.02	0.09	−0.10	0.10
	(1.68)	(0.34)	(−0.64)	(5.87)
Slope (α_1)	0.87	0.90	0.91	0.90
	(72.29)	(26.02)	(130.13)	(44.36)
R^2	0.88	0.88	0.95	0.82
Wald-Test	27.35*	2.06	40.23*	34.54*
(H_0: $\alpha_0 = 0$, $\alpha_1 = 1$)				
		Stochastic Volatility Model		
Intercept (α_0)	0.28	3.53	0.09	0.52
	(6.38)	(8.01)	(3.92)	(13.3)
Slope (α_1)	0.77	0.74	0.89	0.79
	(53.60)	(8.17)	(80.15)	(16.35)
R^2	0.86	0.84	0.94	0.81
Wald-Test	79.0*	103.0*	54.04*	81.92*
(H_0: $\alpha_0 = 0$, $\alpha_1 = 1$)				

Notes: An asterisk on a value indicates the rejection of the null hypothesis at the 5% level. The *t*-values are given in the parenthesis. The above results were based on the regression model:
$C = \alpha_0 + \alpha_1 C^* + e$
C = actual call price
C^* = predicted call price

(1989) also report that the Black-Scholes model with daily-revised implied volatilities outperforms their stochastic volatility model for currency options on Swiss franc. So far, the present results fail to suggest that the gains from incorporating the stochastic volatility feature in the model are worth the additional complexity and implementation costs of the stochastic volatility model.

The marginally better performance of the modified Black-Scholes model, compared to the stochastic volatility model, in regression-based tests is a surprising result as one may expect the opposite to hold because of the large number of additional structural parameters of the stochastic volatility model. However, additional parameters do not necessarily imply a better out-of-sample pricing performance of the stochastic volatility model. As Bakshi, Cao, and Chen (1997) point out, the presence of more parameters may actually cause out-of-sample overfitting of the stochastic volatility model, so that the model is penalized if the additional parameters do not improve its structural fitting.

These results on the comparative pricing performance of the two models may also be attributed to two other factors. First, traders and market-makers are generally believed to be using some variants of the Black-Scholes model with implied volatilities that are revised daily, as pointed out by Nandi (1996) and Chesney and Scott (1989). Such variants of the Black-Scholes model may be as good an approximation to the true underlying currency option pricing model as is Heston's stochastic volatility model. Second, the true underlying volatility process may be approximated as well by the structure of daily-revised volatilities implied by the Black-Scholes model as by the mean-reverting volatility process of Heston's model.

Turning to tests of pricing biases, the prices generated from the stochastic volatility model are subject to fewer and weaker biases than those generated from the Black-Scholes model, as shown in Table VIII. In the aggregate sample, the stochastic volatility model exhibits only the moneyness bias, whereas the Black-Scholes model displays the moneyness, volatility, and interest-rate differential biases. The direction of moneyness bias differs between the two models, however. Both models suffer from the moneyness and volatility biases for near- or at-the-money options. Also, both models are free of the four pricing biases for in-the-money options. Hence, the major difference between the pricing biases of the two models comes from out-of-the-money options. The stochastic volatility model displays the moneyness, maturity, and volatility biases for these options, whereas the Black-Scholes model exhibits the volatility and interest rate differential biases. Although the volatility bias is common to both models, its direction differs between the two models. The stochastic volatility model tends to underprice out-of-the-money options when volatility is high and overprice options when volatility is low. The results are the exact opposite for the Black-Scholes model. These opposing pricing biases for out-of-the-money options in turn affect the aggregate pricing biases between the two models. For instance, the stochastic volatility model tends to underprice in-the-money calls and overprice out-of-the-money calls in the aggregate sample; the exact opposite pricing pattern holds for the Black-Scholes model.

One noticeable finding of the present study is the absence of interest-rate differential bias in prices derived from the stochastic volatility model, suggesting that the payoffs from relaxing the constant interest rate assumption in Heston's model are likely to be minimal. Bakshi, Cao, and Chen (1997) also report that adding the stochastic interest rate feature to their stochastic volatility model does not significantly improve the pricing performance of the model for short- and medium-term equity index

TABLE VIII

Regression Tests of the Biases of the Black-Scholes and Stochastic Volatility Models in Pricing British Pound Currency Options, 1993–1995

	All Options	*In-the-Money*	*Near-the-Money & At-the-Money*	*Out-of-the-Money*
	Black-Scholes Model			
Intercept	0.09	0.04	0.09	0.23
	(1.50)	(0.08)	(2.78)	(4.83)
S-K/K	−1.26	−4.47	−2.59	−0.21
	(−3.70)	(−0.91)	(−2.80)	(−0.66)
T	0.06	0.25	0.10	0.05
	(0.72)	(0.43)	(1.26)	(1.20)
$r - r^*$	−0.04	−0.23	−0.01	−0.05
	(−1.74)	(−1.17)	(−1.02)	(−2.63)
V	−1.58	−0.38	−2.22	−1.80
	(−2.88)	(−0.08)	(−4.74)	(−3.56)
R^2	0.04	0.03	0.06	0.07
Wald-Test	31.06*	3.48	58.43*	40.27*
(H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)				
	Stochastic Volatility Model			
Intercept	0.15	1.01	0.17	−0.05
	(1.04)	(0.66)	(3.00)	(−0.33)
S-K/K	4.18	−6.17	−5.74	13.40
	(3.98)	(−0.37)	(−3.62)	(11.71)
T	0.01	0.65	0.005	0.82
	(0.06)	(0.34)	(0.04)	(3.25)
r-r^*	0.08	0.09	−0.02	0.02
	(1.18)	(0.15)	(−0.63)	(0.33)
V	−2.48	−13.50	−3.54	3.39
	(−1.41)	(−0.86)	(−4.42)	(1.86)
R^2	0.04	0.03	0.07	0.17
Wald-Test	5.02*	1.84	60.69*	249.04*
(H_0: $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$)				

Notes: The above results were derived using the regression model:
$C\text{-}C^* = \alpha_0 + \alpha_1\, S\text{-}K/K + \alpha_2\, T + \alpha_3\, r\text{-}r^* + \alpha_4\, V + e$, where C is the actual call price and C^* is the model predicted call price. The *t*-values are given in parenthesis. An asterisk (*) on a value indicates the rejection of the null hypothesis at the 5% level.

options. Merton (1976) suggests that stochastic volatilities result in part from variable interest rates. Thus, the nonexistence of interest-rate differential bias supports Merton's conjecture, suggesting that the stochastic volatility process of the model may have captured the impact of variable interest rates on currency option prices.

Results of aggregate pricing biases using the dummy variable model and error components model are presented in Table IX. Both models attempt to capture the separate impacts of time-series and cross-sectional properties of the sample data in regression parameters as well as the

TABLE IX

Tests of the Aggregate Pricing Biases Using Dummy Variable and Error Components Regression Models, 1993–1995

	Black-Scholes Model		*Stochastic Volatility Model*	
	Dummy Variable Model	*Error Components Model*	*Dummy Variable Model*	*Error Components Model*
S-K/K	−1.23	−1.23	4.08	4.11
	(3.66)	(3.68)	(3.84)	(3.88)
T	0.05	0.05	0.004	0.004
	(0.70)	(0.69)	(0.018)	(0.017)
*r-r**	−0.05	−0.05	0.08	0.08
	(1.76)	(1.79)	(1.22)	(1.20)
V	−1.60	−1.60	−2.49	−2.49
	(2.89)	(2.85)	(1.4)	(1.4)
R^2	0.045	0.043	0.047	0.044
LR-Test (χ^2-Test)	1.49	—	2.45	—
(Ho: no fixed effects)	(0.95)		(0.87)	
F-Test	0.24	—	0.40	—
(Ho: no fixed effects)	(0.95)		(0.87)	
LM-Test (χ^2-Test)	—	0.75	—	0.48
(Ho: no random effects)		(0.38)		(0.48)

Notes: The absolute *t*-values of the coefficients and probability values (*p*-values) of the three tests are given in parenthesis. The LR denotes the Likelihood Ratio test and LM represents the Lagrange Multiplier test.

distinct influences of option's time-to-maturity and moneyness classes on pricing biases. Such data features are not captured in the estimated pricing biases of Table VIII. The Likelihood ratio test, Lagrange Multiplier test, and F-test all indicate that cross-sectional random or fixed effects in intercepts do not exist. These findings fail to undermine the normal practice in previous studies of estimating the pricing biases using the ordinary least squares method. The pooled cross-section time-series approach of Table IX may uncover differential cross-sectional impacts in options data from other markets.

In sum, the stochastic volatility model exhibits, compared to the Black-Scholes model, fewer aggregate pricing biases as well as different impacts of these biases. Thus, the stochastic volatility model provides some improvement over the Black-Scholes model in eliminating some of the well-known systematic pricing biases of the Black-Scholes model. Such an improvement may provide better estimates of the measures of option price sensitivities, such as option delta, vega, rho, and gamma, which may lead to more effective hedging or speculative strategies using currency options.[9] For instance, currency hedging and speculative strategies established using the delta-neutral and gamma-neutral portfolios are sensitive to other option parameters such as the volatility of exchange rates or the domestic interest rates. Because aggregate option pricing errors from the stochastic volatility model are free of the volatility and interest-rate biases, these hedging and speculative strategies may be less subject to spurious sensitivity to changing volatilities and interest rates when they are based using option prices predicted from the stochastic volatility model than from the modified Black-Scholes model. Such an outcome may lead to better estimates of the risk exposures that are being sought or minimized by traders.

The pricing biases of the stochastic volatility model of this study differs from those of other stochastic volatility models of previous studies. Chesney and Scott (1989) uncover the moneyness, maturity, and volatility biases in Swiss franc option prices generated from a stochastic volatility model. On the other hand, Melino and Turnbull (1990), examining Canadian dollar options, find only the interest rate bias in their stochastic volatility model. Examining S&P 500 index options, Nandi (1996) reports the moneyness, maturity, trading volume, and bid–ask spread biases in prices derived from the Heston's model. Similarly, Bakshi, Cao, and Chen (1997), also examining S&P 500 index options, find that the aggregate pricing errors of their stochastic volatility model have moneyness, maturity, volatility, bid–ask spread, and interest-rate term-structure related biases. In contrast, the present study uncovers only the moneyness bias in the aggregate sample; no bias for in-the-money options; moneyness and volatility biases for near- or at-the-money options; and moneyness, volatility, and maturity biases for out-of-the-money options. Such differences in reported pricing biases may arise because these studies used either different option commodities or different versions of the stochastic volatility model.

CONCLUSIONS

This study investigates whether an option pricing model that incorporates the stochastic volatility feature can improve the out-of-sample pricing of currency options. Option values from Heston's stochastic volatility option pricing model and values from the modified Black-Scholes model are compared to option premiums. Out-of-sample pricing performance and

[9]Delta, vega, and rho measure the sensitivities of the option's price to changing exchange rates, volatilities, and domestic interest rates, respectively. Gamma gauges the sensitivity of the estimated delta to changing exchange rates. Vega is sometimes known as kappa, lambda, or sigma as well.

biases are then compared for these models. Pricing biases related to option strike price, time to maturity, volatility, and interest rate differential are considered. Empirical analyses are conducted using the daily data on European call options written on British pound from the Philadelphia Stock Exchange for the period January 1993 to October 1995.

The modified Black-Scholes model with daily-revised implied volatilities performs as well as the stochastic volatility model in the aggregate sample. Also, prices generated from both models provide close and similar correspondence to actual prices for options trading near- or at-the-money. The present results may extend to other currency options markets with similar structure of volatilities.

The prices generated from the stochastic volatility model are subject to fewer and weaker aggregate pricing biases than are the prices of the modified Black-Scholes model. The only bias that exists for the stochastic volatility model in the aggregate sample is the moneyness bias. In contrast, the prices from the modified Black-Scholes model exhibits the moneyness bias, volatility bias, and interest-rate differential bias. Further, the direction of moneyness bias differs between the two models. Consequently, the main advantage of the stochastic volatility model over the modified Black-Scholes model for the present sample of options is in eliminating some of the well-known pricing biases of the Black-Scholes model. Such an advantage may provide improved estimates of the measures of option price sensitivities to key option parameters that may lead to more effective hedging or speculative strategies using currency options.

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