
TIME SERIES VOLATILITY OF COMMODITY FUTURES PRICES

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This article examines the pattern of volatility over time of a series of commodity futures prices, and focuses in particular on the futures price variability as the maturity date of the futures contract approaches. In a rational expectations model of asymmetric information, the article provides conditions under which the Samuelson hypothesis—that the variability of futures prices increases as maturity approaches—will be true. © 2000 John Wiley & Sons, Inc. *Jrl Fut Mark* 20:127–144, 2000

INTRODUCTION

The purpose of this article is to examine the pattern of volatility over time of a series of commodity futures prices. In particular, we wish to see whether the futures price variability increases or decreases as the maturity date of the futures contract approaches. According to Samuelson (1965), futures price variability will increase as the maturity date approaches. In

*This article has benefited from comments made by seminar participants at the University of British Columbia and The Ohio State University. Part of this paper was written during the ESSEM Gerzenzee Summer Symposium, July 1997.

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order to examine this question further, our paper follows the multiperiod model of hedging in a futures market outlined in Anderson and Danthine (1983), which is a rational expectations model of a futures market with three trading dates.

Samuelson (1965, 1976) argued that we would expect a negative relationship between maturity and futures price volatility, since a piece of information released when there is a long time to maturity will have little effect on futures prices, but the same information released just before maturity will have a large effect. On the empirical side early work by Rutledge (1976) and Grauer (1977) found evidence that contradicted the Samuelson hypothesis. Subsequently Anderson (1985), Milonas (1986), Grammatikos and Saunders (1986), Kenyon et al. (1987), Leistikow (1989), Khoury and Yourougou (1993), and Galloway and Kolb (1996) have all found support for the Samuelson hypothesis in agricultural commodity futures. For example, Khoury and Yourougou (1993) examine the maturity effect in daily futures prices for six commodity futures on the Winnipeg Commodity Exchange over the period March 1980 to March 1989. They find that the average normalized monthly variance of daily futures price changes typically falls as the contracts get close to maturity.

Anderson and Danthine (1983) examine the time series volatilities in futures prices and clarify the meaning of the Samuelson hypothesis in a rational expectations model of a futures market with diverse information. They argue that whether the volatility of futures prices rises or falls as maturity approaches depends upon the amount of uncertainty that has been resolved. If a great deal of uncertainty has been left unresolved as maturity approaches, then the pattern of time series volatilities will increase, but if much underlying uncertainty has already been resolved then futures prices will tend to stabilize before maturity.¹ They examine this issue in a three-trade-date rational expectations model in which producers observe individual supply shocks before date 2. The date 2 prices then aggregate this diverse information, so that some uncertainty is resolved before the delivery date. The Anderson and Danthine articles is a rational expectations model with diverse information in which the separate pieces of information are aggregated into fully revealing prices.² In the current article we consider a rational expectations model of asymmetric information, rather than diverse information at date 2, and in our model not all of the information will be reflected in date 2 prices.

¹Leistikow (1990) criticizes the Anderson and Danthine paper as not being in the spirit of the Samuelson model, since Anderson and Danthine assume that there is a time pattern to the resolution of the uncertainty.

²This is a simplification of the differential information models outlined in Grossman (1977), Danthine (1978) and Bray (1981)

In this model, at the first date producers make production decisions, the output from which will be sold on the spot market at the third trading date. The underlying uncertainty in this model is due to the demand for the output in the spot market being random. The spot market only opens at date 3, and all remaining uncertainty is resolved at this date. The asset produced in our model is a perishable commodity, and producers can therefore be thought of as farmers, and the time line from date 1 to date 3 represents a harvest period.

At the first date all farmers not only make production decisions but can also hedge their output by buying or selling futures contracts that mature at date 3. In addition there is a second market in futures contracts at date 2, called the retrade market, which opens after a subset of farmers observe some information about the likely realization of the spot demand at date 3. These informed farmers may wish to retrade futures contracts on the basis of their information, and in the rational expectations equilibrium that we construct, some of this information will be reflected in date 2 prices. The retrade equilibrium is not fully revealing because of the actions of noise traders in the date 2 market. These same noise traders also participate in the initial futures market, which ensures that the date 1 price is not deterministic.

At the third trading date, the commodity is harvested and sold on the spot market. Futures contracts are fulfilled and farmers realize their wealth levels. The third trading date is referred to alternatively as the maturity date, the delivery date or the cash settlement date. By specifying that the futures markets are commodity futures in which trade occurs prior to the opening of the spot market, we do not get involved with the arbitrage arguments between the futures markets and the underlying asset markets that exist in index futures markets (Fremault, 1990).

MODEL

We follow the multiperiod futures model of Anderson and Danthine (1983), which allows an initial futures position to be revised within the cash market holding period. Each harvest or cash market holding period extends from the farmers' production input decision to the sale of this harvests' output on the spot market. The commodity is repeatedly harvested and we do not allow storage of the commodity between harvests by assuming that the commodity is perishable, and therefore can be thought of as an agricultural commodity.

At the start of each harvest period an initial futures market opens and trade takes place in futures contracts. Farmers must decide how

much of the commodity to produce and the number of futures contracts to buy or sell. In addition, noise traders also buy and sell futures contracts at this initial date. Following the release of information about the likely realization of spot prices for that particular harvest to a subset of farmers, a second futures market opens called the retrade market, where farmers and noise traders can revise their holdings of futures contracts. Finally the commodity is harvested, and sold on the spot market, and the futures contracts are delivered.

The reason that within each period there are two futures markets is that we will allow there to be information revealed to some farmers about the realization of the final price, after the initial futures trades. This information causes farmers to reallocate their futures holdings at the re-trade date.

There are large numbers of three types of traders in the futures markets. The first and second types are called informed and uninformed farmers. Both these types form a group of rational agents who must allocate their wealth between the futures contract and an alternative asset in order to maximize the utility of terminal wealth, W_3 .³ The proportion of informed farmers is λ .⁴ The third type of traders is called noise traders who have price inelastic demands, and whose behavior is random and exogenous to the model.

At the beginning of harvest period farmers choose the quantity of an input z which is related to the non-random output of the commodity y through a production function which we write as

$$y = 2\alpha z^{1/2} \quad (1)$$

The production function is non-stochastic so we can use Result 1 from Anderson and Danthine (1983) and consider the futures and output decisions separately.⁵ However, although output is known, spot market demand when the commodity is harvested is assumed to be random, and consequently farmers may wish to use the futures markets to reduce the risks of their input decisions. Farmers choose the number of futures contracts f_1 to buy or sell at the initial date. A long position in futures, when the farmer buys futures, is represented by $f_1 < 0$. If the farmer sells

³Instead of distinguishing between informed and uninformed farmers, we could introduce another group of participants called speculators and assume that they have access to the information about spot price realisations.

⁴The proportion of informed and uninformed farmers in the model could be an endogenous variable, such that free entry to the ranks of the informed ensures that the expected utilities of the informed and the uninformed are equal as in Grossman and Stiglitz (1980).

⁵Kamara (1992) examines the conditions for this separability even when the production function is stochastic.

futures contracts, and thereby agrees to deliver a quantity of the commodity at harvest time at a price agreed at the start of the cash holding period, we say the farmer holds a short position in futures, $f_1 > 0$. The equilibrium futures price p_1 is determined to ensure the initial futures market clears.

The initial futures price is a random variable, since the behavior of noise traders introduce a random element into the excess demand for futures contracts because they satisfy their demands, x_1 , irrespective of the prices of the futures contract. To justify the inclusion of these noise traders we note the findings of Lauterbach and Monroe (1989) and Ma et al. (1992), who find evidence of noise trading in futures markets. We assume x_1 is normally distributed $N(Ex_1, v_{x_1})$. If $x_1 > 0$, then noise traders buy futures contracts. If $x_1 < 0$, then noise traders sell futures contracts.

At the start of the cash holding period all farmers have the same information set. After the initial input and futures decisions, the informed farmers observe a piece of information about the likely realization of final demand in the spot market. This information may cause them to revise their holdings of futures contracts, which they are allowed to do at the second futures market. At this retrade date farmers issue new demands, f_2 , and a new equilibrium futures price p_2 is found to clear the market. Again the behavior of the noise traders in the second futures market means that supply is random, $x_2 \sim N(Ex_2, v_{x_2})$.

On both trading occasions, futures trades only takes place at equilibrium prices. At the retrade date, the uninformed farmers will be able to infer something about the information observed by the informed farmers from their revisions. The retrade futures price will be a rational expectations equilibrium which will reflect the information held by the informed.

Following the harvest, the spot price is random due to a random component in the commodity demand, ε . Equating supplies and demands we write the equilibrium random spot price as

$$p_3 = \theta - bny + \varepsilon \quad (2)$$

where there are n farmers and aggregate supply is ny . θ and b are demand parameters, and θ is an unknown intercept at the initial futures market, but prior beliefs are $\theta \sim N(0, v_\theta)$, $\varepsilon \sim N(u, v_\varepsilon)$, and $cov(\varepsilon, \theta) = 0$. So the unconditional distribution of p_3 is $N(u - bny, v)$, where $v = v_\theta + v_\varepsilon$. The value of θ can be observed by the group of informed farmers after the initial futures market has closed, but before the retrade futures market has opened, so that some uncertainty is resolved by date 2, the remaining uncertainty concerning ε is resolved at date 3, the delivery date.

The Retrade Futures Market Equilibrium

The objective of each farmer is to maximize the expected utility of terminal wealth W_3 where the utility function is specified as

$$U = -\exp\{-\gamma W_3\} \quad (3)$$

and terminal wealth is given by

$$W_3 = W_2 + (p_2 - p_3)f_2 + p_3y \quad (4)$$

and

$$W_2 = W_1 + z + (p_1 - p_2)f_1 \quad (5)$$

Terminal wealth is given by the value of output sold in the final spot market y plus the payoff to closing out the retrade futures position f_2 . In turn wealth from the initial period is the inherited wealth W_1 plus the profits made on the initial futures contracts f_1 , which reflects the fact that futures contracts are market-to-market. We assume there is unlimited borrowing and lending at a zero rate of interest. At the retrade date an informed farmer who has observed θ , will choose f_2^I to maximize expected utility EU_2^I

$$\begin{aligned} EU_2^I = & -\exp\{-\gamma[W_2 + (p_2 - Ep_3^I)f_2] \\ & - \gamma Ep_3^I y + \frac{\gamma^2}{2} v_e(y - f_2)^2\} \end{aligned} \quad (6)$$

where $Ep_3^I = u + \theta - bny$. This yields the demand function

$$f_2^I = -\frac{Ep_3^I - p_2}{\gamma v_e} + y \quad (7)$$

which is the standard result that a risk averse farmer hedges his entire output and then speculates in the futures market to obtain his desired risk-return trade-off. Similarly the uninformed farmer will choose f_2^U to maximize EU_2^U where the distribution of p_3 is conditioned upon the equilibrium futures price p_2 . This yields the demand function

$$f_2^U = -\frac{E(p_3|p_2) - p_2}{\gamma v_{p_3|p_2}} + y \quad (8)$$

Market equilibrium satisfies

$$\lambda f_2^I + (1 - \lambda)f_2^U = x_2 \quad (9)$$

Adapting Grossman and Stiglitz (1980) we may write the date 2 futures price function as

$$p_2 = u - bny + \frac{c_2}{c_1}w - \frac{y - Ex_2}{c_1} \quad (10)$$

where

$$w = \theta + \frac{\gamma v_\varepsilon}{\lambda} (x_2 - Ex_2)$$

$$c_1 = \frac{\lambda}{\gamma v_\varepsilon} + \frac{(1 - \lambda)}{\gamma v_{p_3|p_2}} \text{ and } c_2 = \frac{\lambda}{\gamma v_\varepsilon} + \frac{(1 - \lambda)}{\gamma v_{p_3|p_2}} \cdot \frac{v_\theta}{v_w}$$

$$v_w = v_\theta + \frac{\gamma^2 v_\varepsilon^2}{\lambda^2} v_{x_2}$$

$$E(p_3|p_2) = u - bny + \frac{v_\theta}{v_w} w$$

$$v_{p_3|p_2} = v_\varepsilon + v_\theta - \frac{v_\theta^2}{v_w}$$

Substituting the conditional mean and variance into the uninformed demands (8), it can be seen that the market clearing condition (9) is satisfied and the futures price eq. (10) is a rational expectations equilibrium. The time series volatility of futures prices between date 2 and cash settlement at date 3 from (10) and (2) is given by

$$\text{var}(p_3 - p_2) = v + \left(\frac{c_2}{c_1}\right)^2 v_w - 2\frac{c_2}{c_1} v_\theta \quad (11)$$

THE INITIAL FUTURES MARKET EQUILIBRIUM

We now solve the futures positions of both the informed and the uninformed farmers at the initial date. Consider first the decision of the informed at the beginning of the period. Substituting the optimal demands f_2^I from (7) back into the expected utility function (6) we obtain the indirect expected utility function EV_2^I

$$EV_2^I = - \exp\left(-\gamma W_2 - \gamma p_2 y - \frac{[Ep_3 - p_2]^2}{2v_\varepsilon}\right) \quad (12)$$

where W_2 is given by eq. (5). To find the informed farmers' initial expected utility function it is necessary to integrate (12) over the retrade price, conditional on the information, and then integrate over the information. The necessary integration can be performed, to obtain the expected utility function for the informed farmers at the start of the period.

Differentiating this expected utility function with respect to f_1 , and setting the result equal to zero, enables us to solve for the optimal quantity of initial futures contracts demanded by the informed as

$$f_1^I = - \left[\frac{u - (1 + 2bn\alpha^2)p_1 - \gamma \frac{2\alpha^2 p_1 - Ex_2}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_\theta\right)^2}{\gamma v - \frac{\gamma}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_\theta\right)^2} \right] + 2\alpha^2 p_1 \quad (13)$$

and the optimal output is a function of the initial futures price, $y = 2\alpha^2 p_1$ (Danthine, 1978). Eq. (13) shows the initial optimum demands for the commodity by the informed farmers, who know they will observe some information in the future. We can carry out a similar computation for the uninformed farmers. Substituting the optimal f_2^U from (8) back into EU_2^U , and gives the indirect utility function EV_2^U . Integrating this expression over the conditional distribution of the retrade futures price, and then over the marginal distribution of the information yields the initial expected utility of the uninformed farmers, EU_1^U . This can be differentiated with respect to f_1^U , to solve for the optimal demands of initial futures contracts by the uninformed farmers. It is shown in Black and Tonks (1990, 1999) that the demands by the informed and uninformed farmers are the same because, before observing the information, both sets of farmers have the same prior beliefs, and the specification of the utility function as exhibiting constant absolute risk aversion, means that there are no income effects. The fact that the demands of the informed and uninformed for futures contracts in the initial period are equal, means that from the market clearing conditions, the initial futures price distribution may be written as

$$p_1 = \frac{\left[u + \frac{\gamma Ex_2}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_\theta\right)^2 + \gamma x_1 \left[v - \frac{1}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_\theta\right)^2 \right] \right]}{[1 + 2\alpha^2(\gamma v + bn)]} \quad (14)$$

The time series variance of futures prices from the initial date to the retrade date is obtained from (10) and (14). Write the difference in futures prices as

$$p_2 - p_1 = u + \frac{c_2}{c_1}w + \frac{Ex_2}{c_1} - \frac{[1 + 2\alpha^2(1/c_1 + bn)]}{[1 + 2\alpha^2(\gamma v + bn)]}$$

$$\left[u + \gamma x_1 v - (x_1 - Ex_2) \frac{\gamma}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_0 \right)^2 \right] \quad (15)$$

and the time series variance of futures prices between period 1 and 2 is

$$\text{var}(p_2 - p_1) = \left(\frac{c_2}{c_1} \right)^2 v_w + \frac{[1 + 2\alpha^2(bn + 1/c_1)]^2}{[1 + 2\alpha^2(\gamma v + bn)]^2} \gamma^2$$

$$\left[v - \frac{1}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_0 \right)^2 \right]^2 v_{x_1} \quad (16)$$

In the next section we will compare the values of $\text{var}(p_2 - p_1)$ from eq. (16) and $\text{var}(p_3 - p_2)$ from eq. (11), and see under what conditions they conform to the Samuelson hypothesis.

PATTERN OF TIME SERIES VOLATILITY

We now answer the question posed at the beginning of this paper. What happens to the time series volatility of futures prices as the maturity of the futures contract approaches? The conclusion of Anderson and Danthine (1983) is that the futures prices are more volatile when large amounts of uncertainty are resolved, but are stable when only small amounts of uncertainty are resolved. We now distinguish between the amount of uncertainty and the informational efficiency in a market. Whether or not futures price variance decreases or increases as maturity approaches depends on not only the quantity of uncertainty that may be potentially resolved, but also on the informational efficiency of the futures market, which enables the resolution of the uncertainty to be incorporated into prices. In Anderson and Danthine (1983), all of the idiosyncratic production shocks are aggregated into date 2 prices because their rational expectations equilibrium is fully revealing. The resolution of uncertainty modeled in this article occurs through information being acquired by only a subset of farmers. Their actions then result in some of the information being incorporated into retrade futures prices. But the extent of the incorporation of this private information depends on the percentage of farmers who have access to this information: The more farmers who are informed, the more information that will be reflected in prices, and hence uncertainty will be resolved more quickly. The implication is that the more farmers who are informed the more likely it is that $\text{var}(p_2 - p_1)$ will be greater than $\text{var}(p_3 - p_2)$.

Chart 1: $v_\theta = 0.1$; $v_\varepsilon = 0.9$; $v_{x1} = v_{x2} = 0.1$

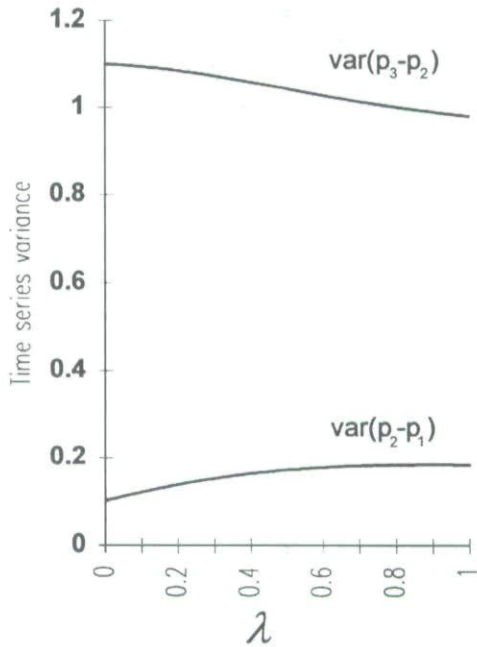


CHART 1.

Theorem 1 $\text{var}(p_3 - p_2)$ will be greater than $\text{var}(p_2 - p_1)$ [the Samuelson hypothesis holds] if

$$v - \frac{c_2}{c_1} v_\theta > \frac{[1 + 2\alpha^2(bn + 1/c_1)]^2}{[1 + 2\alpha^2(bn + \gamma v)]^2} \gamma^2 \left[v - \frac{1}{\text{var}(p_3 - p_2)} \left(v - \frac{c_2}{c_1} v_\theta \right)^2 \right]^2 v_{x1} \quad (17)$$

The theorem is proved by simply comparing the relative the values of $\text{var}(p_2 - p_1)$ and $\text{var}(p_3 - p_2)$ from eqs. (11) and (16).

To aid the interpretation of these conditions, we plot the patterns of time series volatilities in a number of charts. Each chart shows the time series variance as a function of the percentage of informed farmers in the market, which determines the informational efficiency of the market. In each chart we set the risk aversion parameter $\gamma = 1$; the production

Chart 2: $v_\theta = 0.9$; $v_\varepsilon = 0.1$; $v_{x1} = v_{x2} = 0.1$

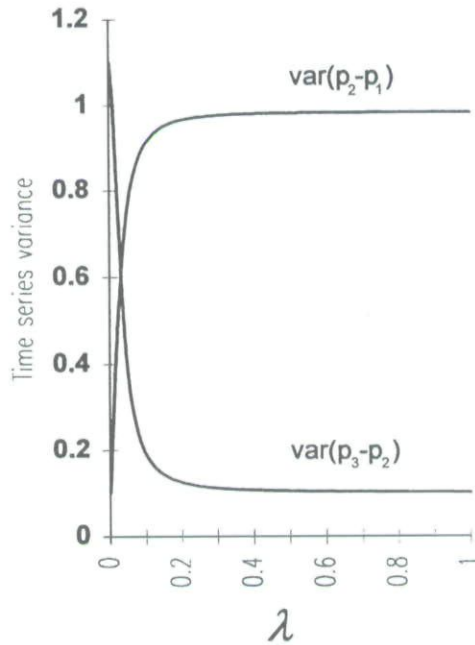


CHART 2.

function parameter $\alpha = 0.01$, the slope of the final demand function $\beta = 1$; and the number of farmers $n = 10,000$. Chart 1 illustrates a situation where the Samuelson hypothesis holds for all values of λ . Recall that the relative values of v_θ and v_ε determine the importance of the signal relative to the final demand uncertainty. A high value v_θ means that a large amount of demand uncertainty can be resolved through the signal at the retrade date, whereas a high value for v_ε means that most of the demand uncertainty can only be resolved at the final date. Chart 1 computes the values of $\text{var}(p_3 - p_2)$ and $\text{var}(p_2 - p_1)$ from eqs. (11) and (16), and sets $v_\theta = 0.1$ and $v_\varepsilon = 0.9$, with $v_{x1} = v_{x2} = 0.1$. Hence v_θ is low relative to v_ε so most of the uncertainty is not resolved until date 3, and in this case $\text{var}(p_3 - p_2)$ is greater than $\text{var}(p_2 - p_1)$ irrespective of the value of λ . Note that the earlier variance $\text{var}(p_2 - p_1)$ is a non-decreasing function of λ , because the higher is λ the more of the information contained in the signal will get into time 2 prices.

Chart 3: $v_\theta = 0.1$; $v_\varepsilon = 0.9$; $v_{x1} = v_{x2} = 10$

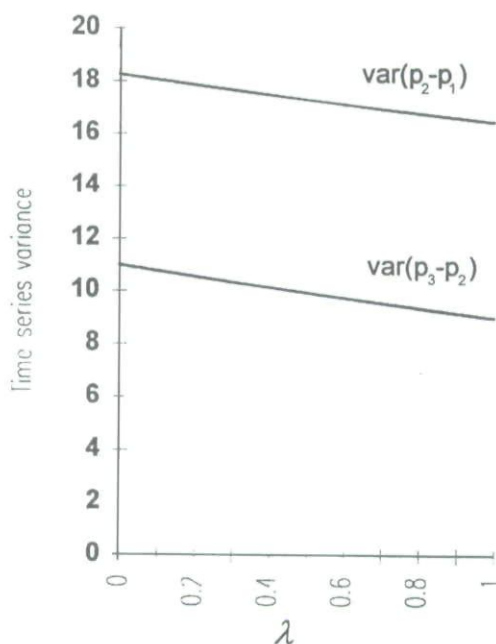


CHART 3.

In order to more meaningfully analyze the condition in eq. (17), it is instructive to consider the extreme values of $\lambda = 0$ when no farmers are informed, and $\lambda = 1$ when all farmers are informed.

Proposition 1 When $\lambda = 0$, the Samuelson Hypothesis is satisfied [$\text{var}(p_3 - p_2) > \text{var}(p_2 - p_1)$] if $\gamma^2 v v_{x1} \leq 1$

Proof. From the definitions in the Appendix

$$c_1|_{\lambda=0} = \frac{1}{\gamma v}, \frac{c_2}{c_1}|_{\lambda=0} = 0, \text{var}(p_3 - p_2)|_{\lambda=0} = v(1 + \gamma^2 v v_{x2})$$

so condition (17) becomes

$$1 + 2\gamma^2 v v_{x2} + \gamma^4 v^2 v_{x2}^2 (1 - \gamma^2 v v_{x1}) > 0$$

and a sufficient condition for the Samuelson hypothesis to be validated is $\gamma^2 v v_{x1} \leq 1$.||

Chart 4: $v_\theta = 0.9$; $v_\varepsilon = 0.1$; $v_{x1} = v_{x2} = 10$

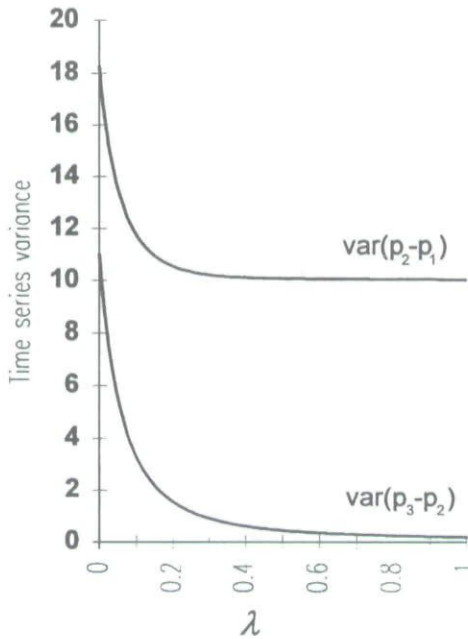


CHART 4.

The situation of $\lambda = 0$, when no one is informed, is equivalent to no uncertainty resolution before the settlement date, because, although v_θ is greater than zero, no one has access to this information. In which case $\text{var}(p_2 - p_1)$ is likely to be small even if v_θ is large, because effectively there is no uncertainty resolved irrespective of the relative values of v_θ and v_ε . Chart 2 reverses the relative values of v_θ and v_ε from Chart 1, and has $v_\theta = 0.9$ and $v_\varepsilon = 0.1$, with $v_{x1} = v_{x2} = 0.1$ in both cases. It can be seen that with these parameter values, at $\lambda = 0$, the Samuelson hypothesis holds, even though there is a lot of uncertainty potentially resolved in the signal, but because no one sees the signal—the market is informationally inefficient—the uncertainty remains unresolved. As λ increases in Chart 2, the information in the signal is revealed through market prices at time 2, and at higher values of λ the Samuelson hypothesis is violated since $\text{var}(p_2 - p_1)$ is greater than $\text{var}(p_3 - p_2)$.

Turning to the situations where the Samuelson hypothesis does not hold, from the conditions in (17) and Proposition 1, we may note that if

farmers are very risk averse (high γ), or if the noise traders in the initial futures market are very volatile (high v_{x_1}), then the initial price volatility is so large that even though little spot price uncertainty is resolved at date 2, the time series variance will fall as maturity approaches.

Corollary 1 *When $\lambda = 0$, a sufficient condition for the Samuelson hypothesis to be violated is*

$$\frac{(1 + \gamma^2 v v_{x_2})^2}{\gamma^6 v^3 v_{x_2}^2} < v_{x_1}$$

This case is represented in Chart 3 in which the parameter values are the same as in Chart 1 except now the variance of the noise traders is much higher, $v_{x_1} = v_{x_2} = 10.0$. It can be seen that the early initial price variance is higher than the variance at maturity. For all values of λ , irrespective of the amount of demand uncertainty that may be resolved at the retrade date, the noise trader uncertainty is so large that the Samuelson hypothesis is violated and the time series variance of futures prices actually declines as the maturity date approaches. Similarly in Chart 4, which has the same parameter values as Chart 2, except for $v_{x_1} = v_{x_2} = 10.0$, the higher noise trader variance means that even at $\lambda = 0$, $\text{var}(p_2 - p_1)$ is greater than $\text{var}(p_3 - p_2)$.

Turning to the extreme of full information, when all the farmers are informed, which is equivalent to a public announcement of the signal, the Samuelson hypothesis will be violated whenever the amount of information revealed is high relative to the remaining demand uncertainty.

Proposition 2 *When $\lambda = 1$, then $\text{var}(p_3 - p_2) < \text{var}(p_2 - p_1)$ if $v_\theta > v_e$*

Proof. From the definitions in the Appendix

$$c_1|_{\lambda=1} = \frac{1}{\gamma v_e}, \frac{c_2}{c_1}|_{\lambda=1} = 1, \text{var}(p_3 - p_2)|_{\lambda=1} = v_e(1 + \gamma^2 v_e v_{x_2})$$

Condition (17) becomes

$$v_e - v_\theta < \frac{[1 + 2\alpha^2(bn + \gamma v_e)]^2}{[1 + 2\alpha^2(bn + \gamma v)]^2} \gamma^2 \left[\frac{v_\theta + \gamma^2 v v_e v_{x_2}}{1 + \gamma^2 v_e v_{x_2}} \right]^2 v_{x_1}$$

Hence a sufficient condition for the Samuelson hypothesis to be violated is $v_\theta > v_e$

This condition is equivalent to the one identified by Anderson and Danthine (1983): A large value for v_θ relative to v_e , suggests that a large

amount of uncertainty is resolved at date 2, and given that $\lambda = 1$, all of this private information is incorporated into prices. In that case $\text{var}(p_2 - p_1)$ will be relatively high and the time series variance of futures prices will fall as maturity approaches, in contradiction to the Samuelson hypothesis. This situation is confirmed again in charts 2 and 4, in which the variance of the signal is high relative to the variance of the remaining demand uncertainty, and both charts have parameter values as $v_\theta = 0.9$ and $v_\varepsilon = 0.1$. In both charts and in common with the assertion of Andersen and Danthine (1983), it is demonstrated that at $\lambda = 1$, when the values v_θ is high relative v_ε the Samuelson hypothesis is violated. However, in contrast to Andersen and Danthine (1983), Chart 2 illustrates that with only a small percentage of the farmers informed the time series volatility of futures price actually satisfies the Samuelson hypothesis but, as the percentage of informed farmers increases, the uncertainty resolved in the signal at date 2 gets increasingly reflected into date 2 prices, which become more volatile, so that the Samuelson hypothesis is violated after some λ^* , which in Chart 2 is at 0.05.

CONCLUSIONS

This paper has examined the time series pattern of volatility of commodity futures prices, in a three-trading-date rational expectations equilibrium model. In this model some uncertainty was resolved at the date 2 trade, by allowing informed farmer to have access to information about the likely realization of the date 3 spot price. We identified conditions under which the Samuelson hypothesis (that the variability of futures prices increases as maturity approaches) will be true, and the conditions under which it will be violated. Anderson and Danthine (1983) argued that whether the Samuelson hypothesis holds or not depends upon the resolution of uncertainty over the lifetime of the futures contract. They argued that, if a large amount of uncertainty is resolved early in the life of the futures contract, then the Samuelson hypothesis will be violated and price variability will decrease as maturity approaches.

We have refined this condition to show that this statement is not independent of the informational efficiency of the retrade futures market. There are three components to the uncertainty in this model. As can be seen from eq. (2), the output price contains two of these sources of uncertainty (θ and ε). The final uncertainty component in each of the trading periods is the realization of the noise trades. If a large percentage of the final output uncertainty is resolved at the retrade date, due to a realization of θ , when v_θ is high, and if the retrade market is informationally

efficient so that the information revealed in the private signal is reflected in the retrade futures prices, then the Samuelson hypothesis will be violated. However, even if v_θ is high so that a large percentage of the final output uncertainty could be potentially resolved at the retrade date, but if the retrade market is informationally inefficient, because only a small percentage of the farmers are informed (low λ), then the information θ will not get into prices and the Samuelson hypothesis will hold.

APPENDIX

From the definitions in (10) we can rewrite c_2/c_1 as

$$\frac{c_2}{c_1} = \frac{\lambda^2 v_\theta + \lambda(v_\theta + v_\varepsilon)\gamma^2 v_\varepsilon v_{x_2}}{\lambda^2 v_\theta + \lambda v_\theta \gamma^2 v_\varepsilon v_{x_2} + \gamma^2 v_\varepsilon^2 v_{x_2}} \quad (\text{A1})$$

and also from (10), rearrange the definition of c_1 as

$$c_1 = \frac{\lambda^2 v_\theta + \lambda \gamma^2 v_\varepsilon v_\theta v_{x_2} + \gamma^2 v_\varepsilon^2 v_{x_2}}{\gamma v_\varepsilon [\lambda^2 v_\theta + \gamma^2 v_\varepsilon v_\theta v_{x_2}]} \quad (\text{A2})$$

From (10) we have

$$v_{p_2} = \Omega^2 [v_\theta \lambda^2 + \gamma^2 v_\varepsilon^2 v_{x_2}] \quad (\text{A3})$$

where

$$\Omega \equiv \frac{c_2}{c_1 \lambda} = \frac{\lambda v_\theta + (v_\theta + v_\varepsilon) \gamma^2 v_\varepsilon v_{x_2}}{\lambda^2 v_\theta + \lambda v_\theta \gamma^2 v_\varepsilon v_{x_2} + \gamma^2 v_\varepsilon^2 v_{x_2}} \quad (\text{A4})$$

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