THE RELATIONSHIP BETWEEN INDEX OPTION MONEYNESS AND RELATIVE LIQUIDITY

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Previous research has implicitly assumed, or even suggested, that the relationship between option moneyness and liquidity is quadratic with liquidity maximized for at-the-money options. This study investigated the nature of the relationship between moneyness and three liquidity proxies for options on the Standard & Poor's (S&P) 100 and S&P 500 indexes. With bid–ask spreads, volume and time between quotes as liquidity proxies, statistical analysis rejected the hypothesis of a simple quadratic relationship between moneyness and liquidity in these markets. Although liquidity was maximized near the money, liquidity did not decrease symmetrically as option strikes moved deeper in the money or deeper out of the money. © 2000 John Wiley & Sons, Inc. Jrl Fut Mark 20:971–987, 2000

The authors thank Charles J. Corrado, John S. Howe, Bradford D. Jordan, Susan D. Jordan, Avraham Kamara, Carolyn E. Miller, Richard Startz, and seminar participants at the University of Missouri and Wichita State University for helpful comments and suggestions. Any remaining errors are the responsibility of the authors.
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Received January, 1997; Accepted April, 2000

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The Journal of Futures Markets, Vol. 20, No. 10, 971–987 (2000)

INTRODUCTION

Liquidity is recognized as a desirable market property. When liquidity is low, traders face *liquidity risk,* the risk of adverse price movements from the time they decide to trade until the transaction is completed. Consequently, liquidity risk is particularly important when a trader is attempting simultaneous transactions in several markets because a delay of even a few seconds can be critical (Grossman & Miller, 1986). Because liquidity risk can differ across option strikes, liquidity risk is an important factor for hedgers and speculators to consider in formulating hedging and trading strategies.

Despite the importance of liquidity risk, there has been little, if any, research focused exclusively on the nature of relative liquidity for index options. This study fills a gap in the existing literature by examining the nature of the relationship between option moneyness and liquidity. This analysis used intradaily data for options on the Standard & Poor's 100 index (OEX) and the Standard & Poor's 500 index (SPX) to investigate the nature of the relationship between moneyness and liquidity. The three liquidity proxies were bid–ask spreads, volume, and time between quotes.

This study rejected the hypothesis that the relationship between option moneyness and liquidity is quadratic with the greatest liquidity occurring at the money. With both the volume and the time between quotes as liquidity proxies, liquidity was maximized near the money but did not decrease symmetrically as options moved in the money or out of the money. With bid–ask spreads as a liquidity proxy, a slightly different result was found. Although the relationship between moneyness and liquidity is described well by a quadratic function for OEX options, a cubic function better describes this relationship for SPX options.

This article is organized as follows. The next section contains the motivation for the study. The third section is a review of some of the previous empirical research on market liquidity and a description of the liquidity proxies. In the fourth section, the empirical methods are outlined along with a data description, a discussion of the sample selection, and a definition of the variables used in statistical tests. The fifth section contains the results of statistical tests that we used to examine the symmetric relationship between moneyness and liquidity. Test results are also presented for quadratic and cubic relationships. A brief summary and concluding remarks close the article.

MOTIVATION

Liquidity cost is an important factor in constructing hedges. The greater the market liquidity is, the faster arbitraguers are able to enforce no-arbitrage prices. In turn, this leads to increased market efficiency and decreased hedging costs.

Because it is well known that trading volume and open interest are concentrated in at-the-money options, hedgers could reasonably believe that these options are the most liquid. Furthermore, if hedgers assume that liquidity falls symmetrically as option strikes move away from at-the-money strikes, they may be overestimating the cost of one hedging strategy relative to another. In particular, they may be overestimating the hedging cost of using in-the-money options and, as a result, be less willing to hedge with them.

This is potentially important to hedgers because in-the-money options can provide a significant hedging benefit. The benefit arises as follows. The deeper the option is in the money, the more the option price changes as the price of the underlying asset changes. As a result, fewer options are needed to offset a dollar price change in the underlying asset. In addition, a hedge constructed with in-the-money options requires less frequent rebalancing than a hedge constructed with at-the-money options. Therefore, a hedge constructed with in-the-money options could cost less to maintain over the life of the hedge.

No previous study has focused exclusively on the nature of the relationship between option moneyness and liquidity. Some previous studies have implicitly assumed (Jameson & Wilhelm, 1992) or suggested (Kamara & Miller, 1995) that the relationship between option moneyness and proxies for liquidity is approximately quadratic, maximized for at-the-money options. Because Kamara and Miller also documented an asymmetric relationship between moneyness and liquidity, it is possible that the relationship between moneyness and liquidity is not as simple as conventional wisdom suggests.

RELEVANT LITERATURE CONCERNING LIQUIDITY PROXIES

Although liquidity per se is unobservable, several proxies have become the de facto standards for measuring market liquidity. These proxies are bid–ask spread, volume, and time between quotes.

The bid–ask spread is a popular measure of liquidity because the bid–ask spread can be viewed as the price the market maker demands for providing liquidity services and immediacy of execution (Amihud & Mendelson, 1986). The market maker is compensated for trading on demand by selling the asset at a premium and buying the asset at a discount. When the spread is wide, market makers are charging more to provide liquidity.

Using the bid–ask spread divided by the asset price as a liquidity proxy, Vijh (1990) reported that options listed on the Chicago Board Options Exchange (CBOE) had a wider relative bid–ask spread than their underlying stocks. However, Vijh did not study the impact on liquidity as option strikes move away from at-the-money strikes.

Jameson and Wilhelm (1992) investigated option liquidity indirectly by investigating the bid–ask spread and the cost of hedge rebalancing for options market makers. Jameson and Wilhelm used the absolute value of moneyness as a proxy for the market maker's expected holding period in Ho and Stoll's (1983) bid–ask spread model. As Jameson and Wilhelm predicted, there is a positive statistical relationship between moneyness and the bid–ask spread. However, using an absolute value explicitly assumes that bid–ask spread and, thus, liquidity are symmetric for in-the-money and out-of-the-money options.

Relative volume is an intuitive measure of liquidity because it measures thin trading and the ability to sell an asset quickly. If an asset has low trading volume relative to similar assets, a trader might have difficulty finding a counterparty. Thus, the investor might need to either sell at a discount or buy at a premium.

Because the time to transact is not directly observed, the volume and the time between quotes are used as proxies to measure the investors' ability to have their trades processed quickly. Kamara and Miller (1995) investigated the relationship between moneyness and the time between quotes. They found that the relationship between moneyness and liquidity is asymmetric and reported that the time between quotes is shortest for at-the-money options. However, they also reported that deep-in-the-money options have time between quotes similar to at-the-money options. Kamara and Miller's results suggest that there is a relationship between moneyness and liquidity, but it is not quadratic. Day and Lewis (1988) also presented evidence of a relationship between moneyness and liquidity when the liquidity proxies are either the time between quotes or the volume.

EMPIRICAL METHODS

Liquidity Proxy Definitions

Let S represent the underlying index level and X represent the exercise price. Define the moneyness of a call option as

$$M = \frac{S - X}{X} \tag{1}$$

and define the moneyness of a put option as

$$M = \frac{X - S}{X} \tag{2}$$

M is positive for in-the-money call or put options, negative for out-of-the-money call or put options, and zero for call or put options that are exactly at the money.

Moneyness can also be examined by a discrete moneyness class variable, *MCLASS* defined as follows. *MCLASS* is set to zero for the in-the-money exercise price closest to the underlying index level. For successive in-the-money strikes, *MCLASS* increases by one. For successive out-of-the-money strikes, *MCLASS* decreases by one. For example, suppose there are 10 different strike prices spanning 45 index points. If so, *MCLASS* will range from -5 (representing the deepest out-of-the-money option) to 4 (representing the deepest in-the-money option).

As noted previously, three common liquidity proxies are the bid–ask spread, volume, and length of time between quote updates. The bid–ask spread is generally measured in two ways. The first measure, called the *dollar bid–ask spread (BASPREAD),* measures the cost of immediacy of execution paid to the market maker. This liquidity proxy is calculated simply as

$$BASPREAD = \text{ask price} - \text{bid price} \tag{3}$$

The second measure, called the *relative bid–ask spread (RELSPREAD),* divides the cost of immediacy of execution by the option cost. It is calculated as

$$RELSPREAD = \frac{BASPREAD}{0.5(\text{ask price} + \text{bid price})} \tag{4}$$

Because the empirical results are qualitatively similar for these data, the only results reported are those using the dollar bid–ask spread. However, results are reported with two relative market activity measures, *RELVOL* and *RELTRADE,* to verify that outliers do not unduly influence empirical tests.

One relative market activity measure is relative volume. This relative volume variable is defined as $RELVOL_\kappa$, where κ is the intradaily moneyness class. With $VOLTOT_\kappa$ defined as the total contracts traded during each trading day, $RELVOL_\kappa$ is calculated by

$$RELVOL_\kappa = \frac{VOLTOT_\kappa}{\Sigma_\kappa VOLTOT_\kappa} \tag{5}$$

It is possible that a few large trades throughout a sample could distort the true nature of the relative liquidity across option classes. Therefore, a liquidity proxy that is less likely to be influenced by a few large trades is also included. This variable is called $RELTRADE_\kappa$, where κ denotes the intradaily moneyness class. With $TRADETOT_\kappa$ defined as the total number of trades for each moneyness class during the trading day, $RELTRADE_\kappa$ is calculated as

$$RELTRADE_\kappa = \frac{TRADETOT_\kappa}{\Sigma_\kappa TRADETOT_\kappa} \tag{6}$$

Both $RELVOL_\kappa$ and $RELTRADE_\kappa$ are calculated for each option class κ with intradaily data. That is, as the index level wanders during the day, the moneyness class can change to reflect the actual intradaily moneyness level. Therefore, the volume and number of trades for a particular option can appear in different moneyness classes during the trading day if there is a wide intradaily range in the value of the underlying index.

Finally, a security with frequent quote revisions has traders who are inquiring frequently and market makers who are closely monitoring the security and standing ready to trade at the latest quotes. It is not necessary for the quoted price to change when a new quote is posted. It is possible that an inquiry from a potential trader could cause the market maker to repost a standing quote. Then, this quote is recorded by the CBOE and captured on the data tape. Therefore, the third liquidity proxy is the time between quote observations, $QTTIME_\kappa$. Given a quote, the variable $QTTIME_\kappa$ is computed by the calculation of the elapsed time from the previous quote for the same option with the same strike and days to expiration. In the data sample used in this analysis, the median times between quotes for OEX calls and puts were 8 and 9 seconds, respectively. However, for SPX calls and puts, the median times between quotes were one minute, eighteen seconds and two minutes, twenty-eight seconds, respectively.

In this study, it was implicitly assumed that the posted quotes were good until updated. Effective July 1989, Rule 8.51 of the *CBOE Constitution and Rules* (1996), commonly called the *10-up rule,* requires that at all times other than the opening rotation, all market makers at a trading station must trade up to 10 contracts at either the bid or the ask price. Because the sample in this study was from a time period after the 10-up rule was instituted, one can be confident that the quotes in this study sample were sound.[1]

[1]Hemler and Miller (1997) examined this critical assumption in a sample before the 10-up rule was instituted. They reported that before the rule was passed, the CBOE encouraged, but did not legally require, market markers to stand by their quotes for one to five contracts.

TABLE I

Trading Dates Included in the Sample and Days Until the Nearest Option Expiration Date

Trading Date	*Days Until the Nearest Option Expiration Date*
November 1, 1990	15
November 9, 1990	7
November 19, 1990	25
November 28, 1990	23
December 6, 1990	15
December 11, 1990	10
December 24, 1990	20
January 3, 1991	15
January 11, 1991	7
January 21, 1991	20
January 29, 1991	18

Note: From November 1, 1990 through January 31, 1991, options expired on three dates: November 16, 1990; December 21, 1990; and January 18, 1991.

Data Description and Sample Selection

During 1990, options on the OEX ranked first in trading volume for options listed at the CBOE, whereas options on the SPX ranked second. The major difference between these index options is that the OEX options have American-style exercise and the SPX options have European-style exercise. The early exercise premium embedded in an American-style OEX option must be nonnegative. All else equal, this means that an OEX option should sell for more than the European-style SPX option.

Traders could prefer OEX options because of the early exercise option. Therefore, the market microstructure (including liquidity) of the OEX option could differ from that of the SPX option. If, however, traders prefer to purchase SPX options because of the lower cost, this is yet another reason to expect liquidity differences between the OEX and SPX options. Accordingly, both OEX and SPX options are included in this study.

Intradaily trade and quote data for put and call options on the SPX and OEX were used in this study. Over the period November 1, 1990 through January 31, 1991, 11 trading days were selected from the Berkeley Options Database. The 11 days in the sample and the corresponding number of days until the nearest option expiration date appear in Table I. Using only 11 days of the period helped to limit sample sizes in some tests. Thus, the significance of parameter estimates is attributed to eco-

TABLE II

Descriptive Statistics

Expiration Month	*M Strike Price*	*M Moneyness*	*M Bid–Ask Spread*	*Volume*	*Contracts per Trade M*	*Contracts per Trade Mdn*	*Contracts per Trade Max*	*Number of Quotes*
			Panel A. OEX Calls					
First	305	−0.003	0.14	3,419,543	18.1	10	2,152	399,094
Second	309	−0.014	0.24	678,277	21.3	10	5,000	100,498
Third	300	0.022	0.67	81,803	25.7	5	2,000	18,196
Fourth	297	0.039	0.85	11,594	31.0	5	550	4,011
			Panel B. OEX Puts					
First	299	−0.016	0.13	3,199,768	17.9	10	2,556	409,167
Second	294	−0.035	0.20	806,182	16.8	6	2,500	126,348
Third	295	−0.034	0.42	102,534	13.1	5	2,199	20,348
Fourth	303	−0.011	0.35	12,870	11.7	3	450	3,124
			Panel C. SPX Calls					
First	302	0.075	0.65	306,495	44.1	17	2,500	109,079
Second	304	0.069	0.71	243,950	85.0	25	5,306	67,581
Third	305	0.061	0.80	134,341	78.7	30	2,400	88,717
Fourth	324	0.009	0.75	20,976	73.6	50	530	19,617
			Panel D. SPX Puts					
First	331	0.022	0.54	280,247	35.3	10	1,854	69,284
Second	330	0.023	0.64	156,212	34.1	8	2,000	41,166
Third	330	0.018	0.71	132,383	31.8	7	2,400	45,039
Fourth	324	−0.008	0.74	30,478	40.8	9	2,300	13,314

Note: Mean strike price, mean moneyness, mean bid–ask spread, volume, mean median and maximum contracts per trade, and number of quotes by expiration month for Standard & Poor's 100 Index (OEX) and Standard & Poor's 500 Index (SPX) calls and puts, November 1, 1990 through January 31, 1991. Moneyness is defined as $(S - X)/X$ for calls and $(X - S)/X$ for puts, where X is the strike and S is the index level.

nomic relationships rather than the small standard errors that are characteristic of large sample sizes.

To ensure data quality, the following screens were employed. Quote records were required to test the relationship between the bid–ask spread and moneyness and the relationship between time between quotes and moneyness. Quote observations were discarded if the bid–ask spread was greater than five or less than zero or if the bid or ask price was zero. Trade records were also necessary to test the relationship between volume and moneyness. Trade records were discarded if the trade price or volume was zero.

Table II presents a summary of the data used in this study. Several items are of interest. First, note the range of the number of contracts traded. Because the range was so large, it was possible that simply using volume in a moneyness class to proxy for liquidity could skew the statistical relationship between moneyness and liquidity. In addition, although the number of quotes and trades was significantly larger for the OEX than for the SPX, the number of contracts per trade for the OEX was half that for the SPX. This suggests that the OEX market could contain more speculators than the SPX market. Finally, the number of trades and quotes were more evenly distributed across deferred contracts for the SPX than for the OEX. This might be associated with hedgers trying to offset some long-term cash index variability.[2]

MONEYNESS AND LIQUIDITY: EMPIRICAL RESULTS

Tests for a Symmetric Relationship

The investigation into the nature of the relationship between moneyness and liquidity began with testing for a symmetrical relationship.

Variable Definitions and Hypotheses Tests

The liquidity proxies for the symmetry tests were the relative volume, relative number of trades, and time between quotes. The bid–ask spread or relative bid–ask spreads were not used in these tests because these variables were related to the price of the option. Because in-the-money options were more expensive than out-of-the-money options, there was no reason to expect to observe symmetry in these variables.

The tests for a symmetric relationship between moneyness and liquidity use three separate regressions where the dependent variable is a liquidity proxy. As previously defined, these dependent variables are $RELVOL_{\kappa}$, $RELTRADE_{\kappa}$, and $QTTIME_{\kappa}$. The independent variables are a set of binary regressors reflecting a moneyness measure that compares the most recent underlying index level to the strike price. This moneyness class variable is *MCLASS*, as previously defined. A mapping of *MCLASS* to a set of binary independent variables is performed as follows: $X_0 = 1$ if $MCLASS = 0$ and $X_0 = 0$ otherwise; $X_1 = 1$ if $MCLASS = -1$, and $X_1 = 0$ otherwise; $X_2 = 1$ if $MCLASS = 1$, and $X_2 = 0$ otherwise; and so on for all values of *MCLASS*. Under this mapping method, indepen-

[2]If speculators turn their positions more frequently than hedgers, additional evidence that the OEX could contain relatively more speculators than the SPX is shown in the volume open-interest ratio. At the end of January 1991, open interest in the OEX was 815,787 contracts, and total volume for the month of January 1991 was 6,964,248 contracts. In the SPX, the respective numbers were 455,818 and 675,754. The volume open-interest ratio was 8.5 for the OEX and 1.5 for the SPX. (CBOE Market Statistics, 1991)

dent variables for in-the-money options are even numbers (i.e., X_0, X_2, X_4), and out-of-the-money options are odd numbers (i.e., X_1, X_3, X_5).

With the variable assignments and a liquidity proxy provided, the following regression equation is estimated to test the null hypothesis of a symmetric relationship between moneyness and liquidity.[3]

$$\text{liquidity proxy} = \sum_{i=0}^{K} \beta_i X_i + \varepsilon \tag{7}$$

Because the binary regression is run without an intercept, the slope coefficients represent the mean of the dependent variable for each moneyness class. This technique allows an F value to be calculated to test the hypothesis that the coefficients are equal for moneyness classes equidistant from $MCLASS = 0$. That is, these are tests of individual hypotheses that $\hat{\beta}_0 = \hat{\beta}_1$, $\hat{\beta}_2 = \hat{\beta}_3$, $\hat{\beta}_4 = \hat{\beta}_5$, and so forth. If parameter estimates for the matching in-the-money and out-of-the-money options are significantly different from each other, this is evidence against the null hypothesis of a symmetric relationship between moneyness and liquidity.

Binary Regression Results

Table III contains a summary of the results of the binary regressions. Panel A presents the results for the OEX call and put options, and Panel B presents the results for the SPX call and put options. Each panel contains the comparison F statistics and their associated p values.

The regression results using *RELVOL* as a dependent variable suggest that there were significant differences in the relative number of contracts traded for corresponding moneyness classes. For example, the OEX calls had significant differences in the relative number of contracts traded in the first, second, third, and fourth strikes that bracketed an at-the-money strike. The OEX puts exhibited significant differences for all but the eighth and ninth strikes away from at the money. Similar though less pervasive results are shown for the SPX calls and puts.

In the sample of OEX and SPX calls, the out-of-the money options had higher relative volumes than the in-the-money options. In agreement with findings from previous research, the relative number of contracts traded generally decreased as the option strikes moved either deeper in

[3]With the range of strike prices given in the sample, at most 20 independent variables were estimated. In addition, there were days when trades did not occur for deep in-the-money and deep out-of-the-money options, thereby reducing the number of observations for the relative volume estimates for some moneyness classes. As a result, the parameter estimates for the relative number of contracts and the relative dollar volume can sum to more than one.

TABLE III

F Statistics from Binary Regressions

	Liquidity Proxy $= \sum_{i=0}^{K}\beta_i X_i + \varepsilon$ Dependent Variable					
Number of Strikes From At the Money	*RELVOL*		*RELTRADE*		*QTTIME*	
	Calls	*Puts*	*Calls*	*Puts*	*Calls*	*Puts*
	Panel A. Options on the OEX					
1	5.11 (.0243)	7.46 (.0065)	0.52 (.4729)	3.51 (.0615)	0.32 (.5724)	0.62 (.4323)
2	14.94 (.0001)	16.04 (.0001)	5.48 (.0197)	7.92 (.0051)	77.8 (.0001)	49.0 (.0001)
3	11.35 (.0008)	30.55 (.0001)	2.95 (.0866)	9.13 (.0026)	472.1 (.0001)	438.6 (.0001)
4	9.09 (.0027)	18.42 (.0001)	0.95 (.3297)	4.81 (.0287)	878.6 (.0001)	1511.5 (.0001)
5	3.25 (.0721)	7.68 (.0058)	0.27 (.6065)	2.43 (.1195)	856.9 (.0001)	1004.9 (.0001)
6 or 7	3.18 (.0751)	5.70 (.0173)	0.08 (.7730)	1.04 (.3082)	476.2 (.0001)	872.2 (.0001)
8 or 9	0.06 (.8102)	1.54 (.2153)	0.006 (.9368)	0.14 (.7129)	66.9 (.0001)	273.8 (.0001)
Observations	420	561	420	561	85,840	85,339
	Panel B. Options on the SPX					
1	6.84 (.0092)	2.17 (.1415)	2.99 (.0846)	1.76 (.1854)	0.21 (.6430)	0.20 (.6570)
2	0.94 (.3337)	0.03 (.8662)	2.55 (.1110)	4.64 (.0316)	11.42 (.0001)	0.27 (.6051)
3	2.81 (.0946)	1.80 (.1801)	4.36 (.0375)	5.23 (.0225)	36.89 (.0001)	14.38 (.0001)
4	7.05 (.0082)	8.13 (.0045)	4.20 (.0411)	6.39 (.0118)	91.42 (.0001)	32.89 (.0001)
5	9.39 (.0023)	1.00 (.3189)	1.70 (.1932)	0.58 (.4475)	171.5 (.0001)	86.8 (.0001)
6 or 7	1.40 (.2366)	11.63 (.0007)	0.29 (.5911)	5.84 (.0159)	264.4 (.0001)	116.9 (.0001)
8 or 9	1.27 (.2608)	4.89 (.0274)	0.03 (.8717)	4.49 (.0345)	307.8 (.0001)	148.0 (.0001)
10 or 11	0.03 (.8740)	0.43 (.5105)	0.02 (.8791)	0.77 (.3821)	427.1 (.0001)	124.5 (.0001)
12 or 13	0.03 (.8643)	0.62 (.4326)	0.02 (.8887)	2.88 (.0900)	312.3 (.0001)	59.4 (.0001)
Observations	454	631	454	631	51,091	29,202

Note: This table presents a set of comparison *F* values for a test of a null hypothesis that the mean relative volume or time between quotes is the same for the in-the-money and out-of-the-money moneyness classes equidistant from at-the-money. These *F* values were generated from a binary regression with the intercept constrained to zero. In these binary regressions, the independent variables corresponding to the moneyness class have a value of 1 if the observation is in that moneyness class and a value of 0 otherwise. Three sets of results are presented for Standard & Poor's 100 Index (OEX) and Standard & Poor's 500 Index (SPX) calls and puts corresponding to three different dependent variables: *RELVOL* (relative trading volume, *RELTRADE*, (relative number of trades), and *QTTIME*, (time between quotes).

the money or deeper out of the money. However, the near-money options, not the at-the-money options, had the highest relative volume. For example, in the sample for SPX calls, the first in-the-money strike had 15.7% of the relative volume, the first out-of-the-money strike had 23.8%, the second out-of-the-money strike had 20.1%, and the third out-of-the-money strike had 16.3%. Thus, results consistent with other studies were found in that there was a relationship between moneyness and volume. However, this relationship was not symmetric about the at-the-money options.

It is possible that a few large trades caused this asymmetry. With the dependent variable measuring the relative number of trades, *RELTRADE*, the asymmetric relationship between moneyness and liquidity for the near-money options was confirmed. However, these tests did not reject the null hypothesis of a symmetric relationship for far-money options. This latter result was consistent with a few large trades resulting in a rejection of the symmetric relationship for volume alone. However, an asymmetric relationship for the volume and number of trades existed for near-money options.

Stronger results were obtained for the regressions using the time between quotes as the dependent variable. For the OEX call and put options, significant differences existed for all corresponding strike prices except for the pair one strike away from at the money. For the SPX calls, significant differences existed between the time between quotes at all corresponding strikes except one strike away from at the money. For the SPX puts, significant differences existed for all except those one and two strikes away from at the money.[4]

Tests for a Quadratic Relationship

Additional symmetry tests were conducted through an examination of whether the relationship between liquidity and moneyness is quadratic in nature. In these tests, however, moneyness was not the discrete variable *MCLASS*. Instead, moneyness, M, was given by Equations 1 and 2. When volume, contracts per trade, and time between quotes were regressed on the continuous measure of moneyness, M, adjusted $\bar{R}^2$s were less than .10, and parameter estimates were insignificant at traditional confidence levels. Therefore, the investigation continued with an examination of the relationship between the continuous moneyness measure, M, and a liquidity proxy based on the bid–ask spread.

Regression Model

As defined in Equations 1 and 2, the independent variable measures the distance an option's strike is from at the money. The dependent variable, *BASPREAD*, is the dollar value of the bid–ask spread. The following ordinary least squares regression was estimated:

$$BASPREAD = \alpha + \beta_1 M + \beta_2 M^2 + \varepsilon \qquad (8)$$

[4]Sensitivity tests were conducted with smaller random samples to ensure that the strong significance was not an artifact of the large sample sizes. The results were similar for the smaller samples.

TABLE IV

Tests for a Quadratic Relationship in Standard & Poor's 100 Index (OEX) and Standard & Poor's 500 Index (SPX) Options

$(BASPREAD = \alpha + \beta_1 M + \beta_2 M^2 + \varepsilon)$

Option	*Observations*	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\bar{R}^2$
OEX calls	86,561	0.17 (.0001)	3.34 (.0001)	22.53 (.0001)	0.67
OEX puts	97,004	0.18 (.0001)	3.40 (.0001)	22.38 (.0001)	0.58
SPX calls	15,823	0.58 (.0001)	3.35 (.0001)	−5.97 (.0001)	0.57
SPX puts	30,097	0.51 (.0001)	2.72 (.0001)	5.07 (.0001)	0.61

Note: This table presents ordinary least squares results for the relationship between bid–ask spread and moneyness for OEX and SPX call and put options. The bid–ask spread measure is the actual dollar size of the spread. *p* values are in parentheses.

On the basis of previous research by Day and Lewis (1988), Jameson and Wilhelm (1992), and George and Longstaff (1993), significant regression coefficients were anticipated. Furthermore, the results of previous research suggest that the smallest bid–ask spreads will occur when the moneyness measure, M, is near zero. It is possible for the estimated coefficients to be significantly different from zero, yet the true relationship could be of a higher order in M. Furthermore, even if the estimated coefficients are significantly different from zero, the estimated coefficients may or may not be consistent with a minimum bid–ask spread value occurring when M is near zero.

Table IV summarizes the results for the quadratic ordinary least squares regressions. All parameter estimates were significant at the .0001 level for all four option types, and the adjusted R^2s were all greater than .5.[5]

Quadratic Regression Results for OEX Options

For the OEX options, the results suggest that a strong quadratic relationship exists between the continuous moneyness measure, M, and the dollar bid–ask spread. With the estimated coefficients, the predicted minimums for both the OEX calls and OEX puts were at a strike price approximately 7% above or below the at-the-money strike price, respectively. This percentage difference between the index value and the strike price suggests that the minimum bid–ask spread for OEX options did not occur at the

[5]Of course, the significant parameters also could result from the large sample size. To test for this, the sample was divided into 14 subsets based on days to expiration, and then the regressions were reestimated. The results were qualitatively similar. Thus, it is likely that the statistical significance was not driven by a large sample.

money. For example, if the OEX index value was \$300, the put strike price with the lowest bid-ask spread was predicted to be about \$280.[6]

If market makers use put–call parity (Stoll, 1969) to hedge their inventory, one would expect to observe approximately equal liquidity for the call and put strikes, even though the moneyness levels of the call and put are different. To examine this proposition, a test was conducted for the equality between sets of regression coefficients with a two-sample Hotelling's T^2 statistic (Morrison, 1976). With this test statistic, the null hypothesis that the quadratic coefficients are equal for OEX calls and OEX puts was not rejected ($p = .4300$).

Based on the two-sample Hotelling's T^2 test and the results shown in Table IV, a quadratic function describes the relationship between moneyness and liquidity (as measured by the dollar bid–ask spread) for OEX calls and puts.

Quadratic Regression Results for SPX Options

For the SPX options, the results suggest that a quadratic relationship does not describe the relationship between the continuous moneyness measure, M, and the dollar bid–ask spread.

For SPX calls, the coefficients in the quadratic regression predicted a maximum bid–ask spread occurring at the money. Although the parameter estimates were significantly different from zero, the resulting economic predictions from the model suggest that a quadratic regression was misspecified for the SPX options. Furthermore, from these parameter estimates the SPX puts had a predicted minimum at a strike price approximately 27% below the at-the-money strike price. For example, if the SPX index value was \$300, the put strike price with the lowest bid–ask spread was predicted to be about \$220.

With the two-sample Hotelling's T^2 statistic, the null hypothesis that the quadratic coefficient estimates are equal for SPX calls and SPX puts was rejected ($p < .0001$). Thus, on the basis of these results, a quadratic function did not describe the relationship between moneyness and liquidity for the SPX options. Therefore, the investigation turned to examining a higher order relationship between moneyness and liquidity for the SPX options.

[6]With the size of the sample and the standard error of the estimate provided, the 99% confidence interval around the predicted strike price with the minimum bid–ask spread was 279.7 to 280.3.

Tests for a Higher Order Relationship for the SPX Options

Regression Model for the SPX

In this section, we investigate the relationship between moneyness, represented by M, and the dollar bid–ask spread measure for SPX options. A cubic OLS regression equation was estimated:

$$BASPREAD = \alpha + \beta_1 M + \beta_2 M^2 + \beta_3 M^3 + \varepsilon \tag{9}$$

A cubic relationship between the bid–ask spread and moneyness has two possible functional forms. For calls, if the local minimum is to the right of the local maximum, the bid–ask spread decreases as the option moves from out of the money to in the money. For puts, if the local minimum is to the right of the local maximum, the bid–ask spread increases as the option moves from out of the money to in the money.

Cubic and Higher Order Regression Results for the SPX

From the cubic regressions, the local minimum bid–ask spread was predicted to occur at a moneyness level of about −15% for both SPX call and put options. The local maximum bid–ask spreads were predicted to occur at moneyness levels of 22% for the SPX calls and 18% for the SPX puts. As the options continued to move deep in the money beyond these moneyness levels, the model predicted that the bid–ask spread would decrease.

Fourth-order and fifth-order polynomial regression equations were also estimated. The coefficients on the fourth-order and fifth-order terms were not significant either individually or as a pair, even when the sample was stratified into subsets based on time to expiration. The results of these tests were consistent with the proposition that the statistical significance of the higher order terms was a result of sample size.[7]

On the basis of the higher order results and the results shown in Table V, it appears that a cubic function approximately describes the relationship between moneyness and liquidity for the SPX calls and puts. This proposition was tested with a two-sample Hotelling's T^2 statistic. Although the null hypothesis that the cubic coefficients are equal for SPX

[7]In some regressions involving the subsets, there were as many as 3,600 observations, but the coefficients on the higher order terms were not always significant. However, with a sample size of approximately 30,000, the coefficients were significant.

TABLE V

Tests of a Cubic and Fifth-Order Relationship for Standard & Poor's 500 Index (SPX) Options

Panel A. $BASPREAD = \alpha + \beta_1 M + \beta_2 M^2 + \beta_3 M^3 + \varepsilon$

Option	*Observations*	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\bar{R}^2$
SPX calls	51,823	0.54 (.0001)	3.34 (.0001)	4.93 (.0001)	36.56 (.0001)	0.61
SPX puts	30,097	0.52 (.0001)	3.25 (.0001)	2.11 (.0001)	−26.93 (.0001)	0.64

Panel B. $BASPREAD = \alpha + \beta_1 M + \beta_2 M^2 + \beta_3 M^3 + \beta_4 M^4 + \beta_5 M^5 + \varepsilon$

Option	*Observations*	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\bar{R}^2$
SPX calls	51,823	0.51 (.0001)	4.06 (.0001)	11.12 (.0001)	−105.47 (.0001)	−52.15 (.0001)	605.8 (.0001)	0.64
SPX puts	30,097	0.47 (.0001)	4.09 (.0001)	12.73 (.0001)	−56.95 (.0001)	−212.0 (.0001)	−15.8 (.4833)	0.68

Note: This table presents ordinary least squares regression results for the relationship between bid–ask spread and moneyness for SPX call and put options. The bid–ask spread measure is the actual dollar size of the spread. *p* values are in parentheses.

calls and SPX puts was rejected ($P < 0.0001$), this rejection could have stemmed from small standard errors generated by the large sample size.

SUMMARY AND CONCLUDING REMARKS

Previous research has implicitly assumed, or even suggested, that the relationship between option moneyness and liquidity is quadratic, maximized for at-the-money options. The analysis presented in this study rejects this simple relationship. The results of this study suggest that the relationship between moneyness and liquidity is complex.

With either volume or the time between quotes as a liquidity proxy, this study found that liquidity was maximized near the money but did not decrease symmetrically as option strikes moved deeper in the money or deeper out of the money.

With dollar bid–ask spreads as a liquidity proxy, this study found the greatest liquidity occurred for away-from-the-money options. In terms of the functional form that best describes the relationship between moneyness and bid–ask spread, a quadratic function fits this relationship for OEX calls and puts. For SPX call and put options, the relationship between moneyness and bid–ask spread is best described by a cubic function.

In this study, we found the relationship between moneyness and liquidity depends, in part, on the proxy chosen for liquidity. The results suggest that option market participants should decide which liquidity proxy is most important for their trading pattern and then trade on the basis of their assessment of that liquidity proxy.

These results also suggest that it is important to consider both the distance an option is from at the money and whether the option is in the money or out of the money when option liquidity is measured.

Finally, the results of this study show that liquidity risk can vary across moneyness classes in a predictable way. Therefore, empirical tests of violations of theoretical no-arbitrage relationships, such as put–call parity, could easily incorporate varying liquidity risk.

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