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A Theory of Negative Prices for Storage

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A conspicuous feature of commodity markets is the substantial quantities held in store even when a commodity's price for future delivery is below the price for immediate delivery. Such storage in the face of backwardations, also known as inverse carrying charges, has been rationalized in various ways. The most popular interpretation is that stock holders gain a convenience yield from having inventories close at hand (Kaldor (1939)).

Another explanation is that the backwardation reflects a risk premium that drives the futures price below the spot price expected for a later date. The expected spot price nonetheless encompasses full carrying charges. Theories that appeal to a risk premium do not claim that it accounts for those instances when the backwardation is much larger than "normal." Such theories, especially the subset focusing on stockholding dealers, Dow (1940) and Stein (1979 and 1986, Chapter 4) concede convenience yield as a premise, and represent it as a unique, stable function of stocks.

Convenience yield as the explanation of storage at inverse carrying charges has not been analyzed beyond demonstration of its plausibility with brief examples related to the behavior of individual processors. Many of these emphasize the role of stocks in lowering costs. For example, Telser (1958, p. 235) has argued, "First, the availability of stocks permits a processor or producer to maintain a given *level* of output at a lower cost than would be required without stocks. . . . Second, holding stocks permits the rate of production or sales to be *varied* at lower cost than would be incurred if the firm attempted to purchase stocks as they were needed" (emphasis in original). Earlier, Working (1948, 1949a, 1949b) made much the same point by observing that storage is an adjunct to firms' main processing or merchandising business and may save them enough there to justify the loss from backwardation. Others have emphasized the revenue effects. Brennan (1958), Cootner (1967), and Weymar (1974) have suggested that wholesalers hold inventories because they increase the chances of a sale or of repeat business.

This paper defines the apparent negative price for storage as a positive difference between the full carrying cost (interest, insurance, warehousing fees, and spoilage) and the

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expected rate of change of the price for immediate delivery. It could alternatively be measured by the extent to which the spread between the price for immediate delivery and the nearest conceivable futures price (say for delivery the next day) falls below full (one-day) carrying charges. The related question of backwardation between futures prices for nearby and distant delivery dates is not addressed here.[1]

Observation of storage under backwardation is an aggregation phenomenon. As disaggregation proceeds, the expected rate of change of the price of each subaggregate with positive stocks tends towards full carrying charges. On close inspection the price differentials attributable to convenience yield vanish.

There are two necessary conditions for storage at an apparent opportunity cost not explicitly identified in the literature thus far. The first is the existence of economically distinct subaggregates of the commodity. The second is intertemporal variation in the marginal costs of transformation of one subaggregate into another by transportation, processing, or marketing operations. Inferences of convenience yield occur when the subaggregate commodities are sufficiently substitutable to be lumped together in market reports but are separated by a time-varying cost of transforming one subaggregate into its close relative.

The reasons for storage under backwardation become apparent in a simple model of the markets for two commodities such as: corn in Iowa and corn in Louisiana; or certified and uncertified stocks of coffee. These commodities are economically distinct but so closely related as to be commonly aggregated for reporting prices and quantities in store. Although the cost-reducing or revenue-raising services of stocks emphasized by Telser help to explain why individual processors hold stocks when the price is falling, they do not sufficiently explain storage at spreads below full carry as a marketwide phenomenon. This is true even if the services are nonlinearly related to the size of the stocks. Cootner (1967, p. 69) concludes that risk aversion is not necessary for storage at an apparent opportunity çost. But the phenomenon can arise under conditions of perfect foresight, contrary to his position that uncertainty is necessary.

The model developed here duplicates the empirically observed relationship between the extent of backwardation and the amount of inventories stored, the "supply-of-storage" curve identified by Working (1948), without resort to convenience yield as a rationale. The model reveals an additional argument of the supply-of-storage curve omitted from previous econometric work: The dispersion of supplies across subaggregates importantly affects the measured curve. One testable hypothesis generated by this model is that as the definition of stocks and their prices become more precise, observations of significant stockholding in times of backwardation diminish. The example of coffee, considered recently in this *Journal* by Thompson (1986), confirms the hypothesis.

I. A SIMPLE MODEL OF CONVENIENCE YIELD

The two commodities in the model, a and b, are closely related substitutes linked by a production technology defined broadly to include costs of transportation, cleaning, and other transformations necessary to convert a into b. The two products could be, for example, the same grade of wheat in Kansas City and in New Orleans or dirty and clean

[1]The opportunity cost to storage is usually measured as the extent the spread between the spot price and the nearest traded futures contract falls below full carrying charges. Our definition is meant to exclude the possibility that the spread between the spot price and a futures contract for even nearer delivery, were it traded, would be at full carrying charges while the spread between the two futures prices would be below full carrying charges due to the possibility of a stock-out over the intervening period. The issue is a seeming loss on the current holding of stocks.

corn at a given elevator. The transformation may well run in only one direction (wheat cannot be reconstituted from flour), but it might also be reversible (at some expense).

Each period t begins with some of each commodity already available (from the previous period's storage, if positive, or from supplies newly arriving). The economic problem is the allocation of the endowments of a and b given that additional quantities are anticipated. How much of each commodity should be consumed immediately, stored for the next period, or converted into the other commodity? Within that broader problem, the purpose is to determine the circumstances under which there will be storage of either a or b even when the price for one of the commodities in period t is above the price expected for period $t + 1$.

The following notation will be used:

A_t^a, A_t^b: amounts available period t

X_t^a, X_t^b: random new supplies in period t with time-invariant means $\mu^a \equiv E_{t-1}[X_t^a]$ and $\mu^b \equiv E_{t-1}[X_t^b]$

T_t: the amount of commodity a transformed into b in period t

$C_t[T_t]$: total cost of transforming amount T from a into b, $C_t[0] = 0$, $C_t' \equiv \partial C_t/\partial T > 0$ and $C_t'' \equiv \partial^2 C_t/\partial T^2 \geq 0$ if $T > 0$

S_t^a, S_t^b: amounts stored from period t to period $t + 1$

P_t^a, P_t^b: prices in period t

q_t^a, q_t^b: amounts consumed in period t

$P^a[q^a], P^b[q^b]$: inverse demand curves for a and b; the same functions apply for each period

r: the rate of interest between periods

Note that there are no costs to physical storage. It was decided to avoid cluttering the model with other determinants of storage behavior apart from the transformation technology and uncertainty. Randomness is located in supply here; but random demand would generate similar results. The source of the randomness is not important; but how it is reflected in the excess supply for a particular commodity is important. Consider the case where T must be positive. T can, in principle, have a negative sign. But if T is negative, C' and C'' would have signs opposite from those with T positive. A linear cost of transformation over the full range of T is generally unrealistic.[2] (Money is paid to move wheat from Kansas City to New Orleans; more money must be paid to move it back to Kansas City.)

The model is an application of the partial equilibrium theory of investment under uncertainty (Scheinkman and Schechtman (1983)). Without loss of generality, one can treat the separate activities of storage of a, storage of b, and transformation of a into b as performed together in a single firm. One can also simplify notation in what follows by defining the risk-neutral firm as the industry while maintaining the assumption of competitive price-taking with respect to outputs and factors.[3]

The competitive firm has rational expectations, and solves in period 1

$$\text{Max } E_1\left[\sum_{t=1}^{\infty}(1 + r)^{-t}\{P_t^a(X_t^a + S_{t-1}^a - S_t^a - T_t) + P_t^b(X_t^b + S_{t-1}^b - S_t^b + T_t) - C_t[T_t]\}\right] \tag{1}$$

[2]Moreover, for positive T the costs concern highly nonlinear variable costs, the idea being that investments in processing plants or transportation networks are much longer lived than the crop year.

[3]A corollary to this assumption of risk neutrality is that futures prices equal expected spot prices.

subject to

$$X_1^a + S_0^a = A_1^a$$

$$X_1^b + S_0^b = A_1^b,$$

$$S_t^a, S_t^b \geq 0.$$

for the variables it controls, S^a, S^b, and T. The precise conditions for maximization with respect to these variables provide the main insights of the formal model. These first-order conditions for period 1 (similar first-order conditions apply as each period in turn becomes the present) are the complementary inequalities

$$\begin{aligned} -P_1^a + (1+r)^{-1}E_1[P_2^a] &= 0, \qquad S_1^a > 0 \\ &\leq 0, \qquad \quad = 0, \end{aligned} \tag{2}$$

$$\begin{aligned} -P_1^b + (1+r)^{-1}E_1[P_2^b] &= 0 \qquad S_1^b > 0 \\ &\leq 0 \qquad \quad = 0, \end{aligned} \tag{3}$$

$$-P_1^a + P_1^b - C_1'[T_1] = 0 \qquad T_1 > 0. \tag{4}$$

Clearly, a backwardation in the price of a or b or, more generally, a spread below full carrying charges can emerge only when there is no storage of that commodity. Otherwise $P_t = (1 + r)^{-1}[P_{t+1}]$, the standard of full carrying charges in this model. Profit-maximizing storage takes place only at full carrying charges, properly calculated.

However, the first-order conditions for industrywide equilibrium for two related commodities neither ensure nor require that both (2) and (3) hold with equality at the same time. Therefore, storage of commodity a could appear against the backdrop of a backwardation in the set of prices for b. If the prices of b are attributed to the aggregated commodity $(a + b)$, the phenomenon of storage at an apparent opportunity cost is observed.

On the other hand, if neither commodity is stored there is no such appearance. Likewise, no backwardation occurs and, hence, no supposition of convenience yield arises if there is simultaneous storage of a and b. Thus, the storage at an apparent negative price occurs when there is storage of one of the commodities but not the other.

An algebraic manipulation of the first-order conditions reveals the circumstances under which storage at an apparent opportunity cost occurs. If a strict backwardation in b exists in period 1 along with storage of a

$$-P_1^a + (1+r)^{-1}E_1[P_2^a] = 0, \tag{5}$$

$$-P_1^b + (1+r)^{-1}E_1[P_2^b] < 0. \tag{6}$$

Using (4), given $T_1 > 0$ and $T_2 > 0$, one can substitute for P_1^b and $E_1[P_2^b]$ in (6) to obtain

$$-P_1^a - C_1'[T_1] + (1+r)^{-1}\{E_1[P_2^a] + E_1[C_2'\{T_2]]\} < 0, \tag{7}$$

that is,

$$C_1'[T_1] > (1+r)^{-1}E_1[C_2'[T_2]]. \tag{8}$$

This is the fundamental necessary condition for backwardation in b with storage of a: The current marginal cost for transforming a into b must be above that expected for the next period.

This condition concerning marginal transformation costs in one period compared to those in the next may seem a long way from the accepted intuitive understanding of storage at spreads below full carrying charges. But the intuitive understanding relies on loose analysis, on two counts. First, it considers individual firms and not the whole market. For individual firms, who take the extent spreads are below full carrying charges as given, it is possible to show (as in Williams (1986)) that they will store under many circumstances. For example, a firm with soybeans to crush now or next period will not crush them all immediately in the face of a backwardation in their products' prices, because of capacity constraints. But the phenomenon of storage at an apparent opportunity cost concerns the whole industry, at which level prices are endogenous.[4] Although soybean crushers may not be able to produce all they want for prompt delivery, that they so strive may collectively provide enough products immediately, and for the same reason less for the later period, to reduce, if not eliminate, the premium for immediate delivery. In brief, many of the plausible examples of convenience yield at the level of individual firms disappear if investigated rigorously at the level of the industry.

Second, many intuitive examples of convenience yield, although purporting to represent a single commodity, implicitly imagine several related commodities. This is most obvious in the example of a flour mill said to keep wheat in its own premises to ensure the smooth operation of its mill. Clearly, wheat in Chicago and wheat in a distant country mill are not identical. But several related commodities being implicit is also true of the examples involving dealers said to keep supplies on hand to facilitate sales. Clearly, an outsider would pay more for the dealership if most of its stock was sold except for some closing paperwork than if all its stock was in the warehouse for possible sales. The difference between "cocoa with sale pending" and "cocoa waiting for a buyer," although not large, suffices to make them distinct commodities. In this and the other cases, the proper analytical approach is surely to make the different commodities explicit.

II. THE ROLE OF TRANSFORMATION COSTS

It has been shown that storage in the face of spreads below full carrying charges can arise if the marginal cost of transformation in the first period is different from the marginal cost of transformation expected for the second period.[5] Because the transformation links the contemporaneous prices for *a* and *b*, for one set of prices to be in backwardation and the other to be at full carrying charges, the expected marginal cost of transformation, by algebraic necessity, must be different in the two periods.

Several sets of circumstances can lead to a marginal cost of transformation in the first period that is above the marginal cost expected in the second period. Three of these are worth considering in more detail.

(i) Transformation costs are different if the technology is different in various periods (and the change foreseeable). If it is known that the cost of transforming commodity *a* into *b* at all levels of output will be lower in the next period (as would be the case if a current freeze should force all current barge traffic to go by more expensive routes), then storage of *a* will occur simultaneously with backwardation in the prices of *b*. Similarly, if it is known that the cost of transformation will increase soon, the equilibrium will be transformation of much *a* into *b* immediately and the storage of *b* and, possibly, backwardation in *a*.

[4]Blau (1944) confuses the behavior of individual firms with that of the industry in arguing against the existence of convenience yield. She claims that because any individual firm in an emergency can easily arrange for shipment of such standardized commodities as are traded in futures markets, it need not hold stocks against such an eventuality when there is a backwardation. She forgets that the intertemporal price structure reflects the anticipation that many firms might have such short-term needs, in which case such needs would be expensive to satisfy collectively.

[5]All prices mentioned in this section refer to present values.

Very often it is cheaper to transform a particular amount if the transformation is done more slowly. An obvious example is shipping by water instead of by air freight. Another example is the lower transactions cost associated with a seller's less hurried search for a buyer. Such lower costs to slower transformation can lead to the appearance of continuous storage whenever the subaggregate of goods in process is included in the stocks of the aggregate commodity, as when goods in transit are included in visible supply.

(ii) A second category covers circumstances under which a technology that is the same in all periods will give rise to expected marginal costs for the next period that are different from the current marginal costs. This occurs when the marginal costs are increasing in T over the relevant range and the amounts available in the two locations are variable.

Imagine circumstances in the current period such that commodity a is abundant compared to the expected supply of it in the next period, whereas b is scarce compared to the expected supply of b in the next period. The equilibrium will call for a substantial amount of a to be converted into b immediately and some of a to be stored. There is no point in storing b, however, because soon it is expected to be relatively abundant, and have a lower price.

More precisely, if the inverse demand functions are stable and weakly convex in consumption q, then in this case $S_1^b = 0$ implies, from (7), substituting for $q_1^b = A_1^b - S_1^b + T_1$, and $A_2^b = X_2^b + S_1^b$,

$$0 > A_1^b + T_1 - (\mu^b - E_1[S_2^b - T_2]) . \tag{9}$$

With $T_1 \geq E_1[T_2]$ already being a condition for backwardation for $C'' \geq 0$, (9) amounts to another necessary condition:

$$A_1^b < \mu^b - E_1[S_2^b] . \tag{10}$$

In this case of stable $C[\cdot]$, available supply of b must be less than expected production; supplies of commodity b must be scarce temporarily.

Obviously, condition (10) cannot hold if μ^b equals zero, since S_2^b must be nonnegative. If there were no expectation of supply of b from sources other than a in the second period, the equilibrium would reflect the certain need to transform commodity a into b during both periods. Moreover, it would reflect the allocation of that transformation process so as to minimize the total transformation costs. This is achieved by equating the marginal transformation cost in the first period to the expected marginal cost in the second period. That equality precludes storage in the face of a backwardation; condition (8) cannot hold.

In short, nonlinear transformation costs alone do not suffice to give storage and backwardation simultaneously, contrary to the message of Telser's examples. There must also be an expected supply of b from other sources; commodity b cannot be solely a transformed version of a. This independent supply of b allows the market as a whole to avoid the cost of transforming a into b in the second period, so that the expected marginal transformation costs in the second period can be below those in the first period. By extension, the set of prices for b can be in backwardation even as the set for a is at full carrying charges.

Another important lesson from cases (i) and (ii) is that uncertainty is not necessary for circumstances ascribed to convenience yield, contrary to the implication of Cootner. The phenomenon can arise in a model of perfect foresight. Anticipated variability is essential, but that is a quite distinct condition from uncertainty. Indeed, if there were no variability (that is, if A_1^a equaled μ^a and A_1^b equaled μ^b), there would be no appearance of convenience yield no matter how great the uncertainty about future supplies.

(iii) Uncertainty can give rise to storage at an apparent opportunity cost in a variant of case (ii) which is of practical importance. In certain transformation processes approximate constancy of $C'[\cdot]$ is very plausible for $T > 0$. But except in unusual cases (e.g., perfectly efficient pumped storage and power generation in a hydroelectric scheme) $C'[\cdot]$ has a discontinuity at zero; a round trip between a and b typically costs twice the one-way charge.

Modify case (ii) to allow $T \leq 0$, and specify $C'[-T] = C'[T]$ = a constant, which makes $C[\cdot]$ a *V*-shaped function (see Figure 1). Then backwardation can occur in b, with storage of a, if a is initially abundant and b is scarce, and supplies of both in the next period are uncertain. The current differential between the prices of a and b, with $T_1 > 0$, will be above that expected for the second period, because it is possible T_2 will need to be either zero or less than zero, that is, it is possible that there will be no transformation or that the transformation will be from b to a.

The economic forces in operation here can be perceived more easily in an example from the demand for currency. Why is currency held in a wallet even though interest would be earned if the trouble were taken to deposit the money in a bank? If it were known that the money had to be deposited sometime and a trip to the bank were no more inconvenient today than tomorrow, the deposit would surely be made today in order to earn the interest. On the other hand, if currency might be needed again tomorrow, having to make a trip back to the bank would be regretted. To avoid the chance of having to pay to undo what had just been done, some currency is retained. Measured against the foregone interest for money in the bank, the decision will appear to be costly, but not if measured against the shadow price of currency in a wallet, that currency being in effect a different commodity.

Likewise, because of the chance of having to ship copper back to Arizona whence it had come, some amount is not shipped even though there is a premium for immediate delivery compared to the price for later delivery in New York. Although there may be strong demand for grain on track, not all grain is removed from warehouses; some is left

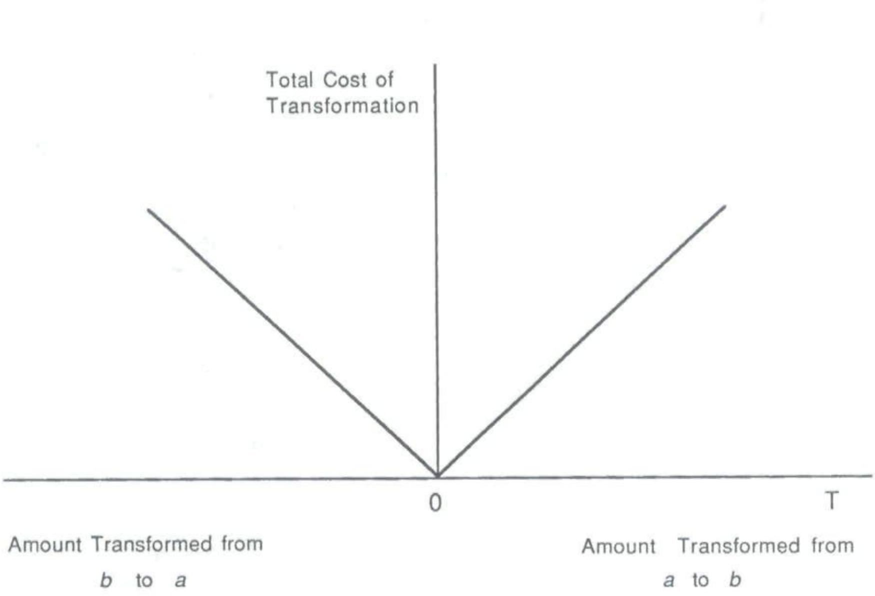

Figure 1
Transformation costs when one-way marginal costs are constant and positive.

houses; some is left there because of the possibility of having to pay the charge for elevating it again. An importer with cocoa certified for delivery on futures contracts may not go to the expense of selling it despite a premium for such stocks, because he anticipates having to incur the expense of buying similar cocoa in just a few days. Measured against the prices for copper in New York, for grain on track, and for certified cocoa, these stockpiles will appear to be held in the face of spreads below full carrying charges. But in terms of prices in Arizona, in warehouses, and in the cocoa importer's accounts, the storage will take place at full carrying charges.

In sum, these three broad categories share a common feature: The production technology is nonlinear over the relevant range.[6] In the case of time-varying technology, this nonlinearity is a feature of the time dimension. In the case of constant marginal costs, the technology is nonlinear because it is costly in both directions; that is, a plot of the total transformation costs is *V*-shaped rather than linear.

But more than nonlinearity is needed. The nonlinearity must be in the relevant range. That is why uncertainty is needed in the case of the *V*-shaped curve. The uncertainty allows both sides of the *V* to come into play. That is why variability is needed in the case of a general nonlinear curve. Without the anticipated variability in excess supply, current marginal cost would be set equal to expected marginal cost, and both commodities would display full carrying charges if either did.

Of course, to admit that the total costs of such transformations are complex, nonlinear functions is to admit that the commodities are more than trivially distinct. Once again, storage at an apparent opportunity cost has more to do with differences among related commodities than with their similarities.

III. SUPPLY-OF STORAGE CURVES

The two-commodity model developed here has more to offer than a demonstration that storage of one commodity is consistent with a backwardation in the price of the other, the aggregated commodity displaying storage at spreads below full carrying charges. Even such a simple model produces a supply-of-storage curve, to use Working's (1948) terminology for the empirically observed relationship between the amount stored and the extent the nearest-term spread is below full carrying charges. Indeed, a supply-of-storage curve emerges even from a two-period version of the model.

In the two-period version of the model, let the demand curves for commodities a and b have the simple linear form $P^a = \alpha^a + \beta^a q^a$ and $P^b = \alpha^b + \beta^b q^b$ in each period. Let the interest rate r be zero. Let T be either sign and $C[T]$ have the form cT^2. Under these circumstances, storage of a when there is a backwardation in the prices for b has the precise form

$$S_1^a \mid (E_1[P_2^b] - P_1^b < 0) = \frac{A_1^a - \mu^a}{2} - \frac{\beta^b(A_1^b - \mu^b)}{2(2c - \beta^b)}. \tag{11}$$

The backwardation associated with a particular amount in store is

$$E_1[P_2^b] - P_1^b = \beta^b(\mu^b - A_1^b) + \frac{\beta^b[\beta^b(\mu^b - A_1^b) - \beta^a(\mu^a - A_1^a + 2S_1^a)]}{(2c - \beta^a - \beta^b)}. \tag{12}$$

[6]A fourth category could be added, one in which there is a large fixed cost to the transformation. Such fixed costs are probably especially important in explaining why small holdings are not sold or shipped despite backwardations in prices; it does not pay to incur the expense of searching for a buyer or of chartering a whole ship. A model with such fixed costs is much more complex. Yet it would share the importance of a nonlinearity, which, after all, is the effect of a fixed cost.

Together, expressions (11) and (12) determine the extent of the measured negative price of storage for any pair of initial availabilities, A_1^a and A_1^b. Let $A_1^a = \overline{A}_1 + \varepsilon$ and $A_1^b = \overline{A}_1 - \varepsilon$. Upon varying this average initial availability, $\overline{A}_1$, a synthetic supply-of-storage curve is traced out.

In Figure 2, two synthetic supply-of-storage curves are illustrated for particular parameters. Specifically, commodities a and b are set to be symmetric with $\alpha^a = \alpha^b = 300$; $\beta^a = \beta^b = -2$; and $\mu^a = \mu^b = 100$. Without randomness in harvests, P^a and P^b would each equal 100. The coefficient for transformation costs, c, is set at 1.0. Two different dispersions in initial availabilities are shown, one with $\varepsilon = 20$ and the other with $\varepsilon = 30$.

These synthetic supply-of-storage curves look remarkably like empirically observed curves, all of which strongly display the pattern of the negative component to the price of storage falling as the amount in store increases. Yet in the disaggregated synthetic data, no commodity is stored at less than full carrying charges, measured in terms of the true own price of that commodity.

The supply-of-storage curves deduced from the model expose a specification problem in the econometric estimation of supply-of-storage curves that renders any generalization about empirically observed curves unreliable. In essence, empirical studies all presume

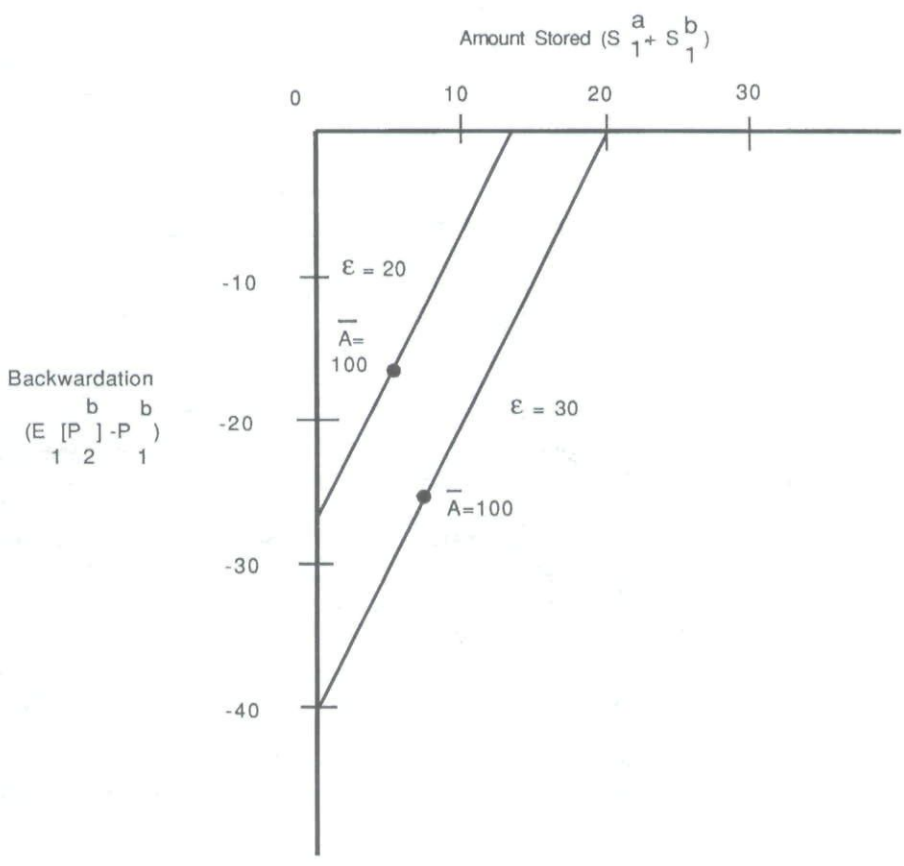

Figure 2
Synthetic supply-of-storage curves.

that the curve remains stable except for variations in $\overline{A}$, the extreme variations in the average harvest providing a wide range of data points. This assumption is defensible for parameters such as β^a and μ^b, which move only secularly and, even then, not much. However, the assumption is indefensible in the case of ε, the dispersion in availability. Because dispersion is mainly the result of the weather and similar random factors, it varies from year to year just as $\overline{A}$ does. Two years can have much the same nationwide harvest but very different regional results. In those years when regions have much the same production, collective storage will be much lower than when some regions have had bumper crops while others have suffered droughts. Likewise, those years when the quality is highly variable will have more storage than when the quality is more nearly uniform.

In short, empirical estimation of supply-of-storage curves should account for the location and grades of stocks, not just the total stocks. (The importance of locations and grades is emphasized in the insightful study of the wheat market by Gray and Peck (1981).) Were this done, the plots of carryover and spreads would show even tighter relationships. This criticism is not mere carping as the dispersion in locations and grades is a central reason for the relationship in the first place. For the same reasons, the facile approach in theoretical work of treating the relationship between observed stocks and backwardation as a stable function is inappropriate.

IV. EMPIRICAL EVIDENCE

According to this model, in a competitive market positive storage of a commodity precisely defined and measured will always be at full carry measured by the current rate of change of that commodity's own price. In other words, the "supply of storage," thus measured, is horizontal for positive stocks. An empirically testable extension of this hypothesis is that as the definition in terms of grade and location of a commodity and its prices is made more precise the supply-of-storage relationship should become more like the letter *L* rotated ninety degrees clockwise. A test of this hypothesis is, of course, a joint test of the underlying theory, quality of data, and market competition.

The case of coffee examined in a recent issue of this *Journal* (Thompson (1986)) can be extended to illustrate this type of test. Total U.S. stocks and the Coffee, Sugar, & Cocoa Exchange's prices should show a typical supply-of-storage relationship, nearly vertical at the left, horizontal at the right, and with a region of intermediate slope in between corresponding to backwardation along with low to medium stocks. But stocks certified for delivery on the futures contracts should show little evidence of being held in the face of a backwardation.

The squares in Figure 3 show U.S. quarterly stocks 1973–1982 for March, September, and December plotted against the March–May, September–December, and December–March New York spreads, respectively, each calculated on the last trading day of the near futures contract.[7] Spreads are cents per pound per month after subtraction of interest expenses, using the 3-month eurodollar rate as the cost of funds. The scatter of squares shows a supply-of-storage relationship typical of those observed for other agricultural commodities.

In Figure 4 the plot of the same spreads, extended to 1986, against coffee stocks deliverable against the coffee "C" contract, measured in the week of expiration of the near

[7]Extension of the series to the present is not possible due to lack of data. U.S. stocks, as reported in *Fruit and Tropical Products* (Commonwealth Secretariat), became negative in 1984. Publication ceased, pending construction of a revised series.

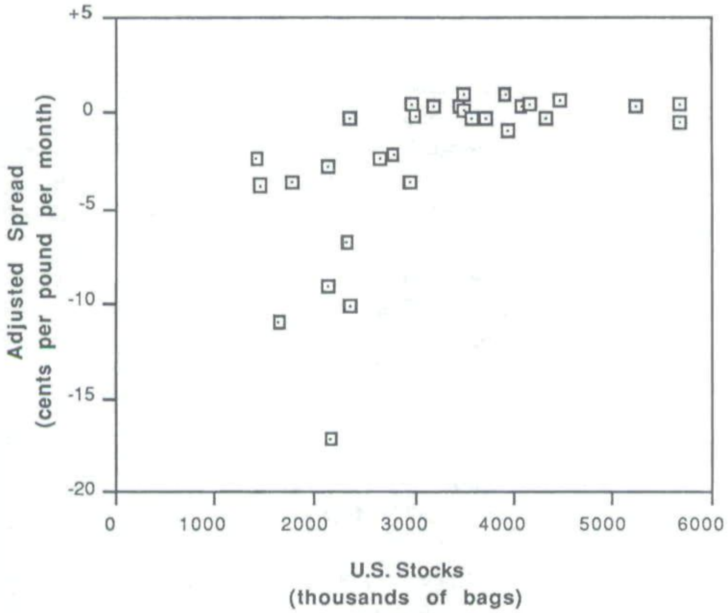

Figure 3
Supply-of-storage scatter for U.S. coffee stocks. (1973–1982)

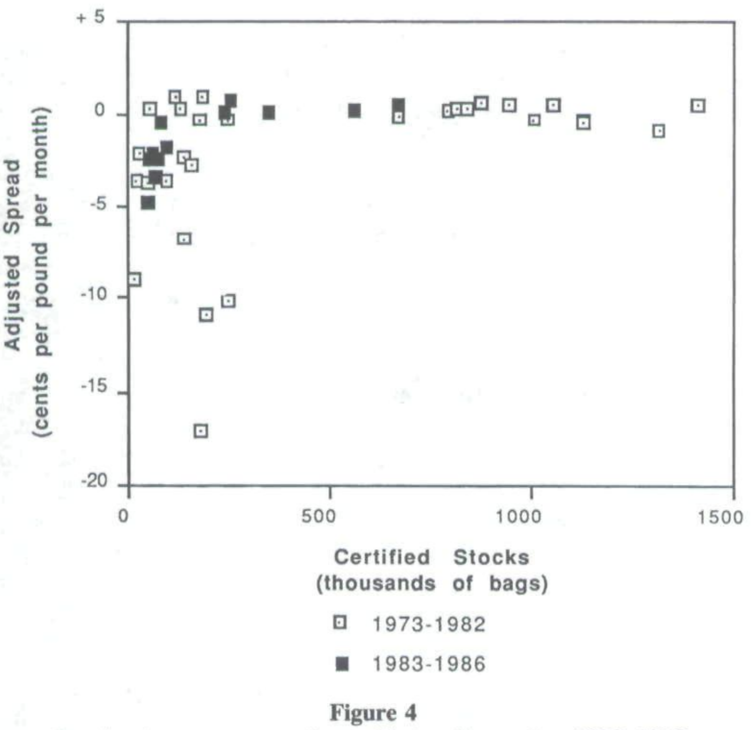

Figure 4
Supply-of-storage scatter for certified coffee stocks. (1973–1986)

contract, is much closer to the rotated L. Consistent with our hypothesis, no substantial amount of deliverable stocks is carried in backwardation.

The observed relationship is not a perfect rotated L, but that is not to be expected with these data. Deliverable stocks include stocks pending certification, which cannot be delivered immediately. They also cover a number of grades. If the exchange-determined grade differentials are inconsistent with market evaluations, the reported spreads are inappropriate for some grades.[8] In short, even "deliverable stocks" are not the precise commodity corresponding to the prices used to construct the spreads. Yet they correspond more closely than the more highly aggregated U.S. stocks. The comparison of the two plots is eminently consistent with the extended hypothesis that increasing disaggregation diminishes the observation of positive storage in the presence of backwardation.

IV. CONCLUSION

At least three lessons come to light with a formal model of spreads and storage. The first is the importance of nonlinearities or intertemporal shifts in technology whether that technology concerns transporting, processing, or merchandising commodities. The phenomenon of storage at an apparent opportunity cost can arise because these activities are more inflexible in the short run than in the long run, or are unusually costly in the short run. For such cases "total transformation cost minimization" is a more descriptive, although more grandiloquent, rationale than "convenience yield." This emphasis on costs is more than a restatement of the standard arguments, which were in terms of benefits. Stocks are not held in the face of spreads below full carrying charges because of the convenience of having them in that location or in that form but because of the inconvenience of transforming them into the commodity for which there is a premium for immediate delivery.

The second lesson is that storage at an apparent opportunity cost can arise because commodities normally lumped together are actually different and are held apart by the cost and time of transforming them into one another. This is true even in a world with perfect foresight; uncertainty, let alone risk aversion, is not necessary for observing aggregate storage at a seeming opportunity cost. It follows, therefore, that empirical investigations of storage at inverse carrying charges should focus on the differences among apparently related commodities. As a preliminary effort in this direction, we have used evidence from the coffee market to show how the observations of significant storage in times of backwardation can be sharply diminished by greater precision in the definitions of relevant prices and stocks.

A third lesson is that distortions in commodity markets, such as export subsidies or the strategic petroleum reserve, change the relationship between the amount stored and the backwardation. It may be a convenient approximation to assume, as have Pliska (1973), Peck (1978), and Wright and Williams (1982 and 1984), that the marginal cost of storage can be represented as a stable function of aggregate available supplies. But if one does, an important aspect of the distortion is lost. Public stockpiles distort not only the total quantity of private storage but also the location and grades of private stocks. These effects can be investigated only if such differences are included in the formal model as they have been in the simple two-commodity model presented here.

[8] Also, the points with the extreme backwardations are from 1978 and 1980 when it is believed the producers' cartel held deliverable stocks (Edmunds (1982) and Shaviro (1987)). As part of their operations to raise the price, they were willing to hold stocks at spreads below full carrying charges.

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