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Cash Settlement of Futures Contracts: An Economic Analysis

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Futures contracts have traditionally provided for delivery of a physical commodity as the means of ultimate settlement. This provision is important because delivery promotes convergence of cash and futures prices and thereby enhances the risk-transfer and price-discovery functions of futures markets.

Despite the apparent importance of final delivery to the operation of futures markets, fewer than 1% of all contracts end with physical settlement. Instead, longs and shorts usually eliminate their obligations to take and make delivery by offsetting their contracts before maturity. Their liquidating transactions produce a simple financial settlement of gains and losses and, more importantly, avoid the costs and uncertainties of physical delivery.

The costs associated with physical delivery, and the convenience of financial settlement, have led to several proposals to replace physical delivery with cash settlement in certain types of contracts (Bakken, 1966; Paul, Kahl, and Tomek 1981). A cash settlement futures contract establishes an obligation for shorts to deliver cash to longs in an amount equal to a "settlement index" that reflects the spot market value of a particular commodity (or group of related commodities).[1] Such a provision leaves most current practices unchanged, but has the advantage that market participants are never forced to make or take physical delivery. The eurodollar time deposit contract traded on the Chicago Mercantile Exchange and the stock index contracts traded on the Chicago Mercantile Exchange, the New York Futures Exchange, and the Kansas City Board of Trade are examples of contracts which use cash settlement to save on delivery costs. Cash settlement also makes feasible futures contracts on commodities for which physical delivery would

[1]In practice, a short would pay the algebraic difference between the settlement index and his sale price, e.g., $10 if he agreed to deliver at $90 and the index is $100, and a long would receive that difference. (If the difference is negative, a short would receive funds and a long would pay funds.) This is accomplished by marking all outstanding contracts to the index after the close of trading in a contract, and then making a final variation margin payment. See Garbade (1982, pp. 317–320) for a description of variation margin payments.

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be virtually impossible, such as a contract on the consumer price index (see Bakken, 1966, p. 20).

This article examines several issues raised by cash settlement of futures contracts. Section I reviews how some existing cash settlement practices emerged. Section II explores the specific factors that make cash settlement more or less desirable, and discusses whether cash settlement should be optional or mandatory. Section III examines how a cash settlement index should be constructed. Finally, Section IV treats the special case of cash settlement of futures contracts on heterogeneous grades of the same commodity. That section reports results which suggest that, even ignoring delivery costs, a cash settlement contract can be a superior hedging vehicle compared with conventional contracts.

This last point is of special importance because the Commodity Futures Trading Commission approved cash settlement on stock indices only because of excessive costs of physical delivery. Our results suggest that cash settlement can enhance contract design in other circumstances as well.

I. HISTORICAL EVOLUTION OF PARTIAL CASH SETTLEMENT

All futures contracts presently contain at least some of the following partial cash settlement provisions:

(a) settlement by offset,
(b) settlement by an exchange of futures for physicals (with a cash side payment if needed),
(c) settlement by nonpar physical delivery with a cash adjustment,
(d) cash settlement in the event of a corner on the deliverable commodity.

As described below, the first two provisions are voluntary between buyers and sellers and stem from the costs and uncertainties of making or taking physical delivery. The last two are at the sole option of sellers and are attributable to the squeezes and corners sometimes associated with physical delivery.

A. Settlement by Offset

Working emphasized (1977, p. 277) that the most successful futures contracts are those that attract commercial hedgers. He also noted that merchandising use of a contract by commercial hedgers is important in the early stages of contract development. (Merchandising use refers to members of the trade using futures contracts to make or take delivery of the physical commodity.) The usefulness of a futures contract as a merchandising device is established when contract specifications of commodity grade and delivery location match trade practices.[2]

Once the merchandising potential of a futures contract is established, cash prices and futures prices will move together. Given this comovement, however, most participants will try to avoid physical delivery.[3] The standardized nature of

[2]Compare, for example, the close parallel between the standard cash market contracts in soybean meal and soybean oil as specified in the trading rules of the National Soybean Processors Association and the futures contracts on those commodities specified by the Chicago Board of Trade. See also the discussions of the terms of the frozen pork bellies contract in Powers (1967) and the plywood contract in Sandor (1973).

[3]See the discussion in Working (1977, pp. 280–284) on the transition between actual merchandising use of futures and using futures as a *temporary* substitute for a merchandising contract.

futures contracts make them less economical for commercial users than receiving or delivering the precise grade of a commodity in the location that best fits the business requirements of the moment. In addition, *non*commercial speculators always try to avoid dealing in the relatively unfamiliar cash markets. These preferences are the main reasons why most futures contracts are liquidated by offsetting transactions, with cash payments made or received depending upon the difference between a contract's purchase and sale prices.

B. Settlement by Exchange of Futures for Physicals

The second practice involving partial cash settlement is known as an "exchange of futures for physicals." In these transactions a stock of the commodity which is not deliverable under the terms of a futures contract (such as grain graded as substandard) is used to liquidate a short futures position. Because the commodity is not contractually deliverable, the long must agree voluntarily to accept delivery, including (if necessary to complete the exchange) a *privately negotiated* cash adjustment to reflect the difference between the price of the standardized commodity and the price of what was delivered. The main objective of an exchange of futures for physicals is to reduce delivery costs by allowing shorts to arrange for delivery of available stocks of the commodity, regardless of their quality and location.

C. Settlement by Nonpar Delivery

Even physical delivery of a commodity as provided in a futures contract can involve aspects of partial cash settlement. Hoffman (1932, Chap. 15) notes that in the early years of futures trading, futures contracts required delivery of a particular grade of a commodity in a given location, e.g., No. 2 Spring wheat in Chicago. This specificity promoted accurate pricing, but also led to "artificial" increases in the local price of the deliverable grade when stocks were in short supply.

To avoid such squeezes and, more generally, price increases induced solely by precautionary accumulations of deliverable stocks, most futures contracts now provide options for sellers to deliver nonstandard grades at nonstandard locations. *Contractually specified* cash premia and discounts on the alternative grades and locations compensate buyers for receiving nonstandard delivery.[4]

D. Cash Settlement as the Result of a Corner

The possibility of buyers squeezing sellers takes on a more sinister character when a purposeful effort is made to control, or "corner," the deliverable supply of a commodity. To discourage intentional price distortions, futures contracts provide for liquidating cash settlements when complaint of a corner by a member of a futures exchange is upheld by an arbitration committee. Such emergency provisions eliminate the need for physical delivery, but they are *ex post* and discretionary measures. Working (1977, p. 279) emphasizes that an *ex ante* provision for cash settlement is necessary to eliminate entirely the problems of corners.

[4]Garbade and Silber (1983a) discuss the basis for, and structure of, premia and discounts on nonstandard grades and locations. See also Kilcollin (forthcoming) and Commodity Futures Trading Commission (1981, Part Three, pp. 98–117).

II. ALTERNATIVE FORMS OF CASH SETTLEMENT

The costs and uncertainties associated with physical delivery, plus the ease of settling differences in cash, illustrate the practical benefits of cash settlement. This section examines when cash settlement as the rule rather than the exception might be preferred to physical delivery. We also consider the basis for selecting alternative forms of cash settlement, including mandatory cash settlement, optional cash settlement with a penalty, and optional cash settlement without a penalty. The analysis shows that the choice depends on three factors: (1) the need to promote convergence of cash and futures prices because delivery costs are large, (2) the need to reduce the incidence of squeezes and corners, and (3) whether the settlement index is inaccurate and/or subject to manipulation.

In presenting the analysis, it is important to examine first the problems caused by physical settlement when delivery costs are high and squeezes likely. The discussion then considers the advantages of mandatory cash settlement with an accurate settlement index. Finally, the analysis shows that optional cash settlement with a penalty is preferred when there is measurement error in (or potential manipulation of) the settlement index.

A. Limits on Price Convergence with Physical Settlement

There are two primary problems with physical delivery. First, high delivery costs impair the convergence of cash market prices and futures prices as a contract approaches its final settlement date. To illustrate this point, suppose a contract is near settlement and the cash market price of the commodity is less than the futures price by a substantial amount. Ignoring transaction costs, an arbitrageur would acquire the physical commodity, sell a futures contract, and liquidate the contract by delivering the commodity. These transactions would push up cash prices and depress futures prices, thereby promoting convergence. However, the incentive to undertake such transactions declines as a function of delivery costs. If those costs are high, even relatively large cash/futures price differences may not generate arbitrage orders, so that price convergence will not necessarily occur.

The second problem with physical delivery is that precautionary demands by shorts preparing to deliver the commodity can push up the local cash market price of deliverable grades. The resulting squeeze increases the cash market price at the delivery point above the "true" commercial value of the commodity. Similarly, if longs expect delivery of large unwanted stocks of the commodity ("dumping" by the shorts), their anticipatory cash market sales may push down the cash market price, thereby causing it to decrease below the true commercial value.

Figure 1 illustrates the convergence problems just described. Ideally, the cash price should equal the commercial value of the commodity (represented by the point at which line bb crosses the vertical axis). In addition, the futures price should equal the cash market price on the final settlement date (satisfied by any point on the 45° line aa). Thus, the cash/futures price combination should converge to point X in Figure 1, so that both cash and futures prices equal the commercial value of the commodity. However, the presence of high delivery costs will impair cash/futures arbitrage. In particular, the futures price can be above or below the cash price by the size of delivery costs (c_1 and c_2 in Fig. 1). The combination of cash and futures prices, therefore, can be anywhere in region A. Moreover,

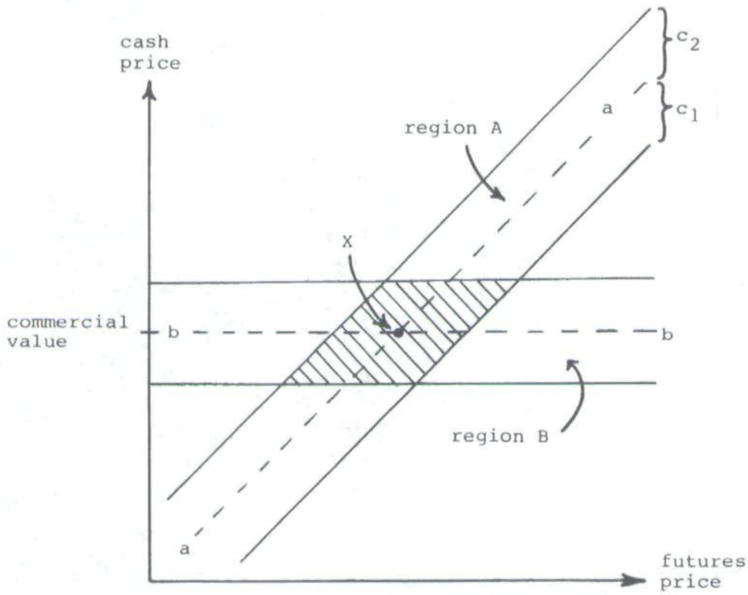

Figure 1

Price convergence with physical settlement. c_1 is transaction cost of buying in the cash market and delivering on a futures contract. c_2 is transaction cost of selling in the cash market and taking delivery on a futures contract.

squeezes and dumping may push the cash price above or below the commercial value of the commodity, so that line bb expands to region B. The net effect is that the cash/futures price combination on the settlement date may be anywhere within the shaded region, i.e., the intersection of regions A and B, in Figure 1.

B. Price Convergence with Mandatory Cash Settlement

If there exists a reliable settlement index that accurately reflects cash market prices and is free of manipulative influences, mandatory cash settlement will eliminate the problems illustrated in Figure 1. The two benefits of mandatory cash settlement are presented in Figure 2. First, because the futures contract neither allows nor requires physical delivery, squeezes and dumping vanish and local cash market prices of deliverable grades will not diverge from the commercial value of the commodity. Thus, region B in Figure 1 collapses to line bb in Figure 2, indicating that the cash price always equals the true commercial value.

The second gain from mandatory cash settlement is that riskless transactions in the futures market force the futures price into equality with the index on the settlement date.[5] Therefore, if the index measures accurately the commercial value of the commodity, the futures price will also converge to the true commercial value. In Figure 2 this implies that the futures price will lie on line cc. Thus, cash settle-

[5]If the futures price is above the index, arbitrageurs will sell the futures contract to earn the futures price minus the settlement index. This puts downward pressure on the futures price. Conversely, if the futures price is below the index, arbitrageurs will buy the futures contract and put upward pressure on its price.

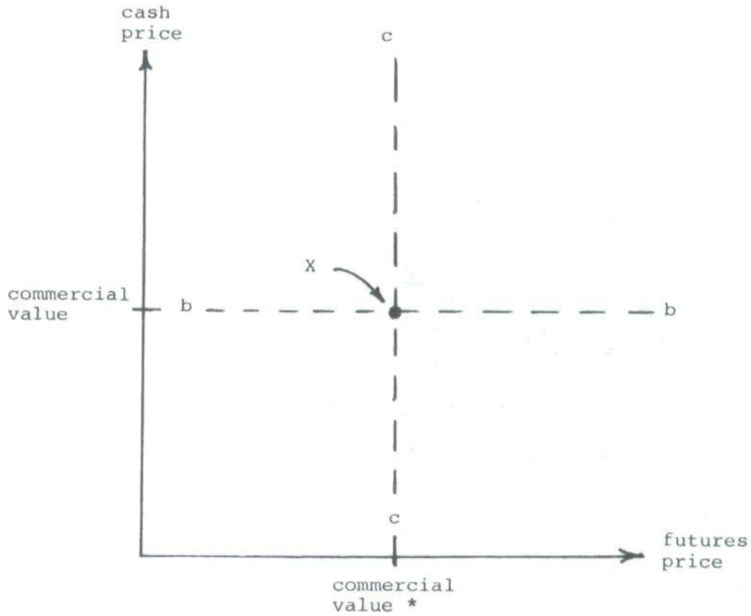

Figure 2

Price convergence with mandatory cash settlement based on an accurate settlement index. (*)The futures price converges to the commercial value of the commodity because (a) it converges to the settlement index, which (b) is equal to the commercial value.

ment based on an accurate settlement index produces convergence of both cash prices and futures prices to the commercial value of the underlying commodity, as indicated by point X in the figure.

Consequences of an Inaccurate Settlement Index

The desirability of *mandatory* cash settlement depends on the accuracy with which the settlement index measures the commercial value of the commodity. Figure 3 illustrates the problems created by measurement error in the index. Suppose the index can take any value in band dd around the commercial value, where the width of the band reflects likely measurement errors. Since the futures price must converge to the index, the combination of cash and futures prices must lie within region D. The fact that cash prices never diverge from true commercial value (because squeezes and dumping do not exist) collapses the feasible cash/futures price combinations to any point on line ee.

Choosing between physical settlement and mandatory cash settlement with an imperfect index is essentially a matter of choosing between (a) convergence to the shaded region in Figure 1 and (b) convergence to line ee in Figure 3. The extreme cases are as follows. If line ee in Figure 3 is short (that is, if the settlement index is quite accurate) and if regions A and/or B in Figure 1 are wide (that is, if delivery costs are high and/or squeezes likely), then cash settlement is preferable. Conversely, cash settlement is not desirable if the settlement index is inaccurate (so

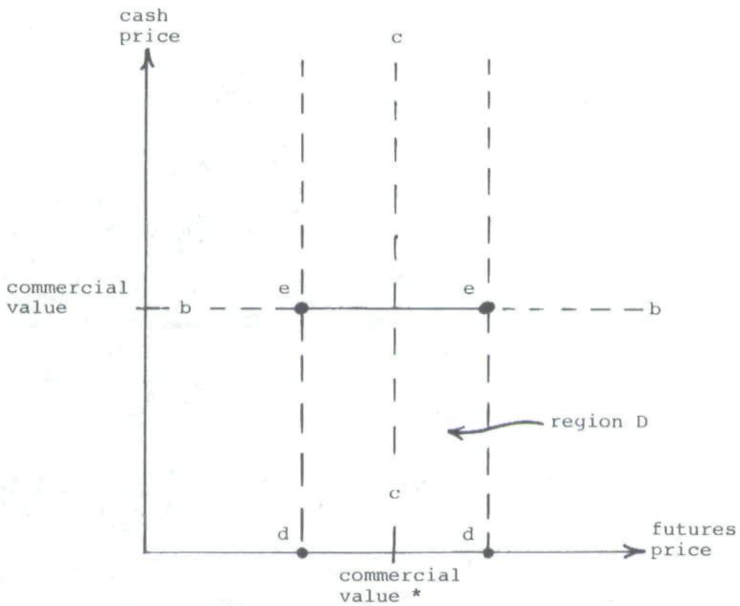

Figure 3
Price convergence with mandatory cash settlement based on a possibly inaccurate settlement index. (*)The settlement index can differ from the commercial value of the commodity, but must lie within the band dd. The futures price must converge to the settlement index regardless of the value of the index. Thus, the futures price can lie within band dd.

that the line ee is long) and if delivery costs are low and price distortions due to squeezes and dumping are unlikely (so that the shaded region in Fig. 1 is small).

The choice between cash settlement and physical settlement is ambiguous when the settlement index is not accurate *and* where delivery costs are high and/or squeezes and dumping are likely. The inaccurate index suggests physical settlement is better, while high delivery costs and squeeze potential indicate a role for cash settlement. As we will see in the next section, it may be desirable to allow an option for both cash and physical settlement under such conditions.

C. Combining Physical Settlement with Cash Settlement Based on an Inaccurate Index

In analyzing futures contracts with provisions for both cash settlement and physical delivery, we begin with a contract that provides for either form of settlement without penalty. Examining such a contract shows the need for a rule of precedence to resolve imbalances between those preferring physical settlement and those wanting cash settlement. Our results show that the cash settlement option must take precedence, but that a penalty should be imposed on market participants who prefer cash settlement.

First, let's see what happens when both shorts and longs can choose either cash settlement or physical delivery and there are no penalties. We know that riskless transactions within the futures market force the futures price to the settlement index on the settlement date. This point is indicated by the intersection of line ff with the horizontal axis in Figure 4. Since (thus far) there is no reason for distortion in the cash price, the initial cash/futures price combination is represented by point Y in that figure.

If, as shown in Figure 4, we assume that the error in the index (the distance between lines cc and ff) exceeds the cost of buying and delivering the commodity (shown as c_1), the cash/futures price combination cannot remain at point Y. In particular, since the futures price exceeds the cash market price by more than the costs of delivery at that point, arbitrageurs will buy the commodity in the cash market and sell a futures contract in order to make physical delivery. These transactions will push up cash market prices and push down futures prices. However, the effect on futures prices will be offset by futures market arbitrageurs who, anticipating cash settlement, buy futures contracts as soon as the futures price falls below the settlement index. Thus, the only effect of the arbitrage with the cash market is to raise the cash price until it is equal to the futures price less delivery costs. In the end, therefore, the combination of cash and futures prices is at point Z in Figure 4.

The most important implication of our analysis is that the case of optional cash or physical settlement plus an inaccurate index creates an inconsistency in the settlement intentions of longs and shorts. In our example, shorts want to deliver the commodity while longs want to settle for cash. Thus, one form of settlement must be given priority.

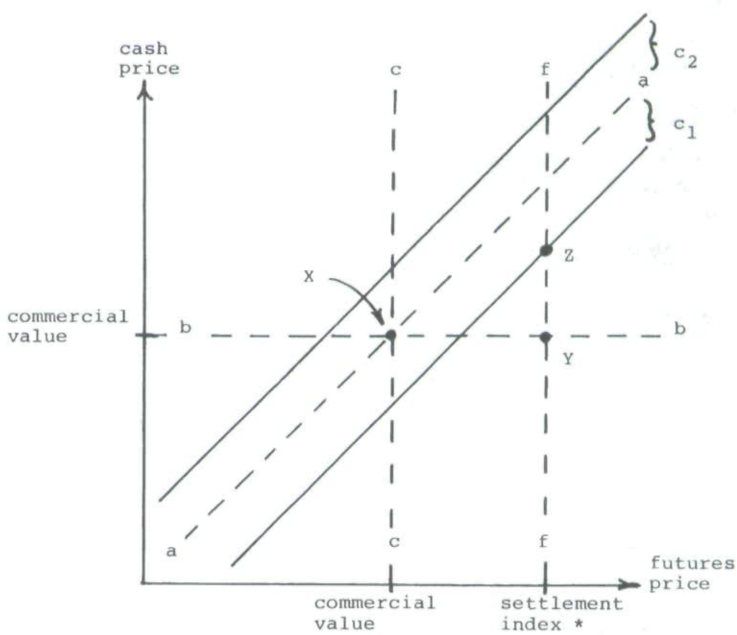

Figure 4

Price convergence with combined physical settlement and cash settlement with an inaccurate settlement index. (*)Measurement error causes the settlement index to exceed the commercial value of the commodity. The futures price must converge to the settlement index regardless of the value of the index. c_1 and c_2 are defined in Figure 1.

Allowing those who want physical delivery to impose that settlement option on others would remove all of the benefits of cash settlement. In particular, a participant in the futures market could never be certain of avoiding the costs of making or taking physical delivery. Even if the futures price far exceeded the settlement index, an arbitrageur would be reluctant to short the contract if longs were likely to demand physical settlement.

Giving precedence to cash settlement insures that no one must receive or deliver the physical commodity involuntarily. To allow for inaccuracies in the settlement index, however, a penalty should be imposed on those choosing cash settlement. The penalty makes arbitrage *within* the futures market unprofitable as long as the difference between the futures price and the settlement index is less than the penalty. This allows the futures price to diverge from the settlement index, and avoids forcing price convergence to an inaccurate index. Moreover, this slippage permits arbitrage with the cash market to influence the futures price within the band created by the penalty.

The size of the penalty should be related directly to the likely inaccuracy in the index. If the index is prone to error, a large penalty imposed on those choosing cash settlement will encourage physical delivery, thereby loosening the linkage between futures prices and the inaccurate index. If the index is quite accurate, a small penalty should be imposed so that futures prices converge more closely to the index.

IV. CONSTRUCTION OF A SETTLEMENT INDEX

The results of the preceding section indicate that construction of an accurate settlement index is crucial in designing a cash settlement futures contract. If the index reflects accurately the commercial value of the underlying commodity, mandatory cash settlement improves price convergence substantially. On the other hand, if the index is inaccurate or subject to manipulation, a preference (but not a requirement) for cash settlement should be the rule. Thus, the design of the settlement index influences the cash settlement formula. In constructing a cash settlement index, two issues must be addressed. First, what types of prices should be used? Second, how should prices be aggregated? This section examines each of these issues.

A. Prices for Index Construction

There are three sources of price data for constructing a settlement index: (1) transaction prices, (2) bid and offer quotations, and (3) price indications. These data differ substantially in their reliability and availability. Our analysis suggests that transaction and quotation prices are superior to price indications in constructing an index. Transaction prices are preferred if the frequency of trading is high and if the terms of different transactions are reasonably homogeneous. On the other hand, quotations are better if trading is thin, or if transaction prices vary significantly because of different terms and conditions of each trade. However, special attention must be given to excluding from the index unrepresentative and intentionally distorted quotations.

Transaction Prices

Transaction prices are the best estimates of the commercial value of a commodity because they are generated by adversary bargaining, i.e., they reflect the efforts of buyers and sellers to transact at the best possible prices. There are, however, two significant disadvantages to constructing a settlement index using transaction prices. First, each price reflects the peculiar elements of an individual transaction. For example, even in a market as competitive and well integrated as the New York Stock Exchange, transaction prices vary as a result of selling at bid prices versus buying at higher offer prices and as a result of the size of a trade (Kraus and Stoll, 1972; Garbade and Lieber, 1977). The idiosyncratic components of transaction prices are much larger for a commodity that is less homogeneous and less frequently traded compared with an exchange-listed equity. The second disadvantage to using transaction prices to construct a settlement index occurs when trading is infrequent. Sparse transactions reduce the ability to eliminate idiosyncratic effects by averaging over a large number of approximately synchronous transactions.

Bid and Offer Quotations

Bid and offer quotations are the second possible source of data for a settlement index. The main advantage of dealer quotations is that they can be obtained for a standardized transaction. This specificity eliminates much of the idiosyncratic variation associated with transaction prices. In addition, quotations can usually be obtained in quantities sufficient for the construction of averages.

There are two major disadvantages to bid and offer quotations. First, quotations vary as a function of a market participant's desire to buy or sell. For example, a dealer's bid and offer prices will be high relative to other bids and offers if he is more interested in buying than selling (Garbade, Pomrenze, and Silber, 1979). When such purchase or sale interests are especially pronounced, the quotations may be unrepresentative of prevailing market conditions.

The second disadvantage to bid and offer quotations is that a market participant may be able to manipulate the value of an index by altering his quotations. This is particularly likely if quotations are obtained outside of normal trading, e.g., by inquiry from a futures exchange or government agency (because the dealer would not have to trade at his quoted bid or offer).

On the other hand, it is possible to identify distorted or unrepresentative quotations, which can then be excluded from the settlement index. For example, given a sample of bids and offers, a bid that is above the lowest offering price (or an offer that is below the highest bid price) implies unexploited but mutually compatible trading interests, suggesting that the quotation is suspect. Similarly, quotations with an unusually wide bid–ask spread are likely to have at least one unrepresentative component, suggesting that one of the quotes be eliminated.

Price Indications

The third class of data for constructing a settlement index is price indications obtained from market participants. Price indications are neither transaction prices nor bid or offer quotations; they are merely "expert opinions" of the prevailing cash market price. Price indications are least suitable for use in a cash settlement futures contract. In addition to being distorted more easily than either transaction prices or quotations, they are likely to reflect relatively more ignorance of current market conditions. More importantly, unlike the case where paired bid and offer quotations are sampled, there are no simple detection devices to indicate whether a price indication is unrepresentative.

B. Aggregation of Observations

Individual price observations should be aggregated when constructing a settlement index to dilute the impact of an error in any single observation. The simplest aggregation procedure is to average several synchronous price observations. In some cases, however, a sufficient number of price observations cannot be collected over a short time interval. Under such conditions, the settlement index should be computed by aggregating observations over time as well as cross sectionally; but the observations need not be given equal weight. For example, suppose the index seeks to measure the value of the commodity at noon on a particular day. Price observations obtained at 11:30 a.m. or 12:30 p.m. that day will clearly be more informative than price observations obtained at 10:00 a.m. or 3:00 p.m. on the same day. Thus, a price observation should receive less weight the further away it is from the time at which the settlement index is applicable.

The proper pattern of weights as a function of time depends on the volatility of the commercial value of the commodity. If the value is relatively stable, observations over several days may be quite informative and could receive nearly equal weight. If the commercial value of the commodity fluctuates rapidly (as with stock prices or gold), time differences of even a few hours may render an observation essentially useless, suggesting rapidly decaying weights over time.

We conclude that the reliability of a cash settlement index varies both with the source of the price data and the aggregation procedures. Each index must be evaluated on an individual basis. For example, the stock index contracts traded on the Chicago Mercantile Exchange and the New York Futures Exchange use transaction prices for their respective settlement indexes. In contrast, the eurodollar time deposit contract traded on the Chicago Mercantile Exchange uses price indications obtained from a sample of 12 banks. The CME drops the two highest and two lowest observations in order to eliminate potentially unreliable prices, and then averages the remaining eight prices. Thus, both transaction prices and price indications, with different calculation procedures, have received the approval of the Commodity Futures Trading Commission. The success of these contracts will depend, in part, on the degree of public acceptance of the alternative approaches.

CASH SETTLEMENT OF A "MARKET—BASKET" CONTRACT

In order to maximize the attractiveness of a futures contract for commercial hedgers, delivery specifications usually correspond with trade practices. For example, the wheat contract on the Chicago Board of Trade calls for the delivery of 5000 bushels of No. 2 Soft Red Winter wheat in an approved warehouse in Chicago. Other varieties of wheat are deliverable at the seller's option (such as No. 1 Dark

Northern Spring wheat), but only at a specified premium to or discount from the contract price. (These are examples of nonpar delivery as described in Sec. II C above.) The premiums and discounts are set so that the wheat contract reflects the price of the No. 2 Soft Red Winter standard grade most of the time, although it can reflect a nonstandard grade some of the time.

Since most commodities exist in multiple varieties, such as different types of corn, various coupons and maturities of Treasury bonds, and so on (Garbade and Silber, 1983a), it might make sense to design a futures contract to reflect the *average* value of different varieties of a commodity. In order to avoid the excessive cost of delivering small (noncommercial) amounts of several different grades, cash settlement based on a multiple-variety index would be desirable. Because the pricing and hedging characteristics of such contracts will differ from conventional contracts, it is useful to examine them in some detail.

In the simulation results reported below, we find that if two varieties of a commodity are of comparable importance in commercial trade, and if the *difference* in their prices is highly volatile, then a market-basket cash settlement contract provides a better hedge for the market as a whole than a seller's option physical delivery contract. This implies that, under such conditions, cash settlement can lead to important social benefits through greater flexibility in the design of futures contracts.

The intuition behind these results is as follows. A futures contract clearly cannot provide a perfect hedge for two grades of the same commodity if the two prices vary somewhat independently of each other. There will always be some unhedgeable residual risk. The contract specifications determine how the residual risk is allocated between those hedging the standard commodity and those hedging the nonstandard variety. For example, a contract for delivery of *only* the standard variety imposes all the residual risk on those hedging the nonstandard grade. On the other hand, a contract with a seller's option for nonpar delivery sometimes imposes most of the residual risk on those hedging the nonstandard variety (this happens when the standard variety is clearly cheapest) and sometimes imposes most of the residual risk on those hedging the standard grade (this happens when the nonstandard variety is clearly cheapest). Finally, a market-basket contract *always* imposes *some* (smaller) residual risk on *both* types of hedgers.

Our results show that where two product varieties are of comparable importance (in terms of market share) and when the differences in their prices are quite volatile, it is better for all hedgers to bear some residual risk all of the time instead of forcing some hedgers to bear most of the residual risk some of the time. More generally, market-basket contracts with cash settlement provide a device for dividing large residual risks among participants on a cross-sectional rather than temporal basis.

Since these results depend crucially on the magnitude of price volatility and market share, it is necessary to detail carefully the analytical foundations. We proceed in three steps. First, we present a simple model of the behavior of cash market prices for two varieties of the same commodity. Next, we consider the relationship between cash market prices and futures prices for the three different types of futures contracts noted above. Finally, we simulate the model to compare the advantages of the different contracts for hedging cash market positions in each of the two varieties.

A. A Model of Cash Market Prices

Our model of cash market prices is designed to capture the behavior of prices on two varieties of a commodity that are close substitutes in the long run, but which may be subject to different short-run changes in supply and/or demand.[6]

Suppose there exist two types of a particular commodity: a premium grade with price P_A and a standard grade with price P_B. The normal commercial premium on A is denoted Q. We assume that, over the long run, the average value of the price ratio P_A/P_B is $1 + Q$. (Nothing in our analysis would change if we let A be an inferior grade and $Q < 0$.) To model the dynamic evolution of P_A and P_B, we assume the logarithm of each price (net of the normal commercial premium on A) is composed of a common element called E, and an "abnormal" difference component, labeled D:

$$\ln[P_A/(1 + Q)] = E + \alpha D, \tag{1a}$$

$$\ln(P_B) = E - \beta D, \tag{1b}$$

where $\alpha + \beta = 1$. Changes in E through time reflect broad changes in supply and/or demand for the commodity as whole. Changes in D reflect differential changes in supply and/or demand between the two grades.

Our model incorporates two basic ideas. First, in the long run, the abnormal difference component (D) is expected to be zero and the prices of the grades are determined by the common component (E) and the normal commerical premium (Q) on variety A. Second, in the short run, the ratio of P_A to P_B can differ from $1 + Q$ as a result of transient deviations of the difference component (D) from zero. Thus, the two varieties are close substitutes in the long run (because the abnormal difference element tends toward zero in the long run), but not in the short run.

From Eq. (1) the abnormal difference component is

$$D = \ln[P_A/(1 + Q)] - \ln(P_B). \tag{2}$$

To model this abnormal difference as a transitory process with a mean of zero, we assume it varies through time as a stable first-order process around zero:

$$dD = -\lambda D\, dt + \omega\, dz_D, \tag{3}$$

where dz_D is a Gauss–Weiner process and ω^2 is the variance of the random change in the difference component per unit time. λ is the rate at which the abnormal difference component is expected to decay toward zero.

From Eq. (1) the common price element (E) is a weighted average of the two prices:

$$E = \beta \ln[P_A/(1 + Q)] + \alpha \ln(P_B), \tag{4}$$

where $\alpha + \beta + 1$. Note that the price of the standard variety is "more important" than the price of the nonstandard variety when $\alpha > \beta$. It is in this sense that grade B would be the standard or dominant type. We assume that changes in the com-

[6]Garbade and Silber (1983a) report the results of using the model to describe the price behavior of different varieties of wheat, corn, and soybeans.

mon component are permanent and persist indefinitely through time. More specifically, we assume that E varies as a random walk with zero drift:

$$dE = \sigma \, dz_E, \tag{5}$$

where dz_E is a Guass–Weiner process which is not correlated with dz_D, and σ^2 is the variance per unit time of the change in the common component.

To summarize our model, there are two price components which are important in the long run: (1) the common component (E) whose changes are permanent and which affect both P_A and P_B on an equal basis, and (2) the normal commercial premium (Q) on variety A. In the short run, a third factor, the abnormal difference component (D), is also important. This element has an expected value of zero, but can have nonzero realizations which are expected to disappear over time.

B. Alternative Futures Contract Specifications

We are interested in the advantages of different types of futures contracts for hedging the premium and standard grades of the commodity just described. We consider three contracts:

(1) a *single-delivery* contract on which only the standard variety (grade B) is deliverable;

(2) a seller's option *multiple-delivery* contract on which grade B is deliverable at the contract price and grade A is deliverable at the contract price times a contract premium of $1 + Q_c$. The contract premium can vary from zero to the normal commercial premium. If $Q_c = Q$ then grade A is deliverable at the contract price times the normal commercial premium on the variety. If $Q_c < Q$ then grade A is deliverable at a penalty relative to the normal commercial premium on that variety (most futures contracts, in fact, specify $Q_c < Q$ to encourage delivery of the standard grade most of the time);

(3) a *market-basket* cash settlement contract with an index defined as a weighted average of the prices of the two grades, where the weights are taken from Eq. (3). More specifically, the settlement index is

$$I = \beta[P_A/(1 + Q)] + \alpha P_B. \tag{6}$$

The price behavior, and hence hedging characteristics, of each of these contracts differs considerably. In particular, as shown in the Appendix, the settlement price at time t on the single-delivery contract settling at $t^* > t$ will be

$$F_{SD} = P_B \exp[R(t^* - t)], \tag{7}$$

where R is the continuously compounded rate of interest per unit time. Thus, the price of the single-delivery contract varies only with the value of the standard variety.

We also show in the Appendix that the settlement price at time t on the seller's option multiple-delivery contract settling at time $t^* > t$ will be

$$F_{MD} = \{P_B[1 - N(d_1)] + P_A(1 + Q_c)^{-1}N(d_2)\} \exp[R(t^* - t)], \tag{8a}$$

where

$$d_1 = \frac{\ln[P_B(1 + Q_c)/P_A] + \frac{1}{2}\omega^2(t^* - t)}{\omega(t^* - t)^{1/2}}, \tag{8b}$$

$$d_2 = d_1 - \omega(t^* - t)^{1/2}, \tag{8c}$$

$$N(x) = (2\pi)^{-1/2} \int_{-\infty}^{x} \exp(-\tfrac{1}{2}u^2)\, du. \tag{8d}$$

This expression for F_{MD} reflects the fact that a seller has an option to deliver either grade on his contract. (See the Appendix for more details.) Equation (8a) shows that, in two extreme cases, the price of the multiple-delivery contract will vary with the cash market price of the net cheapest deliverable grade. In the first case, if $P_{\mathrm{A}}/(1 + Q_c)$ is much greater than P_{B} [so that d_1 and d_2 are near zero in Eqs. (8b) and (8c)], then F_{MD} will be nearly equal to $P_{\mathrm{B}} \exp[R(t^* - t)]$. This corresponds to the case where grade A is at such a large premium in the cash market that sellers are unlikely to deliver it on their multiple-delivery futures contracts, so those contracts are essentially single-delivery contracts on B. On the other hand, if $P_{\mathrm{A}}/(1 + Q_c)$ is much less than P_{B}, then F_{MD} will approximately equal $[P_{\mathrm{A}}/(1 + Q_c)] \exp[R(t^* - t)]$, i.e., the settlement price on a contract which permits delivery of variety A only. This case corresponds to the situation where the price of A is sufficiently low that sellers are unlikely to deliver variety B on their multiple-delivery futures contracts, so those contracts behave as if they were single-delivery contracts on A. In intermediate cases, F_{MD} will vary with both P_{A} and P_{B}. Thus, the price of the multiple-delivery contract offers a poorer hedge for the standard variety than the single-delivery contract.

The market-basket contract provides for cash settlement at an index price which is, to a first approximation, equal to the common element in the two prices. As shown in the Appendix, the settlement price on a market-basket contract will be

$$F_{\mathrm{MB}} = \{\beta[P_{\mathrm{A}}/(1 + Q)] + \alpha P_{\mathrm{B}}\} \exp[R(t^* - t)]. \tag{9}$$

This contract will provide a hedge against changes in the common price component of the two varieties, but not against changes in the abnormal difference component.

C. Implications for Hedging

To evaluate the implications of each of the contracts for hedging, we performed the following Monte Carlo experiment. We assume an investor has a long position that he wants to hedge for 30 days. One way to hedge is to sell out the position and invest in short-term debt securities yielding R per unit time. The investor then bears no risk as a result of commodity price fluctuations. Another way to hedge is to short a futures contract, hold the short position for 30 days, and then liquidate the position. We measure the "residual risk" on the futures hedge as the standard deviation of the ratio of (a) the net value of the hedged position at the end of 30 days to (b) the value of the debt securities at the end of 30 days. For example, a residual risk of 5% implies approximately a 66% probability that the net value of the hedged position at the end of 30 days would fall within 5% of the value which the debt securities would have reached.

The experiment just described was undertaken for long cash market positions in both grades A and B for each of the three future contracts listed above.[7] The result,

[7]All simulations assumed $\sigma^2 = (0.05)^2$/day, $Q = 0.01$, $\lambda = 0.075$/day, and $R = 0.10$/year. The value of E at the start of the hedging interval was set to ln[100.], and the value of D at the start of the hedging interval was chosen randomly from a normal distribution with mean 0 and variance $\tfrac{1}{2}\omega^2/\lambda$. As noted in footnote 8, this is the asymptotic distribution of D. All estimates of residual risk are based on 100 replications.

for any given contract, is a pair of numbers characterizing the residual risks on hedged positions in each of the varieties.

Figures 5 and 6 show the simulated residual risks on hedged positions in grades A and B for the different types of futures contracts. Figure 5 shows the results for relatively low intergrade price volatility, and Figure 6 shows the simulation for high intergrade price volatility.[8] The vertical axis in each figure measures the residual risk for grade A. The horizontal axis measures the residual risk for variety B.

In both figures, the residual risks on the single-delivery contract (denoted point a) are located on the vertical axis because, by construction, the single-delivery contract provides a perfect hedge for variety B (the only grade deliverable under that contract).

For the multiple-delivery contract, points b and c show the risk combinations for the cases where the contract premium (Q_c) is zero and equal to the normal commer-

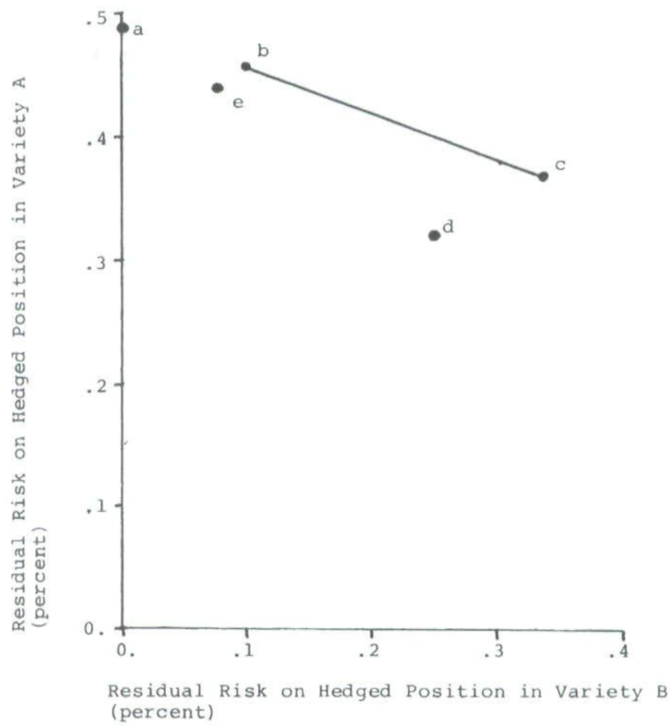

Figure 5

Residual risks with low intergrade price volatility (abnormal difference component with an asymptotic standard deviation of 0.26%): (a) single-delivery contract, (b) multiple-delivery contract ($Q_c = 0$), (c) multiple-delivery contract ($Q_c = Q$), (d) market-basket contract ($\alpha = 0.5$, $\beta = 0.5$), (e) market-basket contract ($\alpha = 0.8$, $\beta = 0.2$).

[8]The intergrade price volatility (measured by the asymptotic variance of the abnormal difference component D) is $\frac{1}{2}\omega^2/\lambda$. This follows because Eq. (3) implies that the value of D at time $t_0 + t$ is related to the value of D at time t_0 by the stochastic equation: $D(t_0 + t) = D(t_0) \cdot \exp[-\lambda t] + d$, where d has a mean of zero and a variance of $\frac{1}{2}\omega^2[1 - \exp(-2\lambda t)]/\lambda$. This implies that D is asymptotically normally distributed with zero mean and variance $\frac{1}{2}\omega^2/\lambda$. Thus, $\frac{1}{2}\omega^2/\lambda = (0.0026)^2$ in Fig. 5, and $\frac{1}{2}\omega^2/\lambda = (0.026)^2$ in Fig. 6.

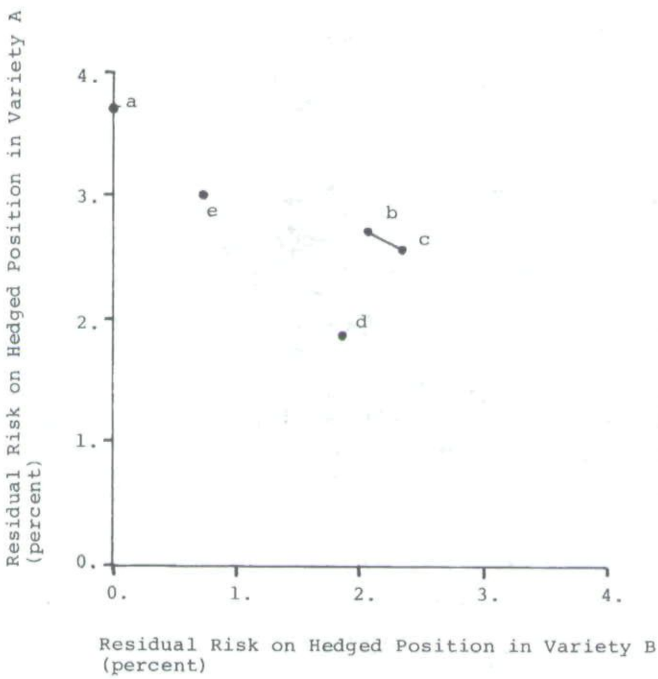

Figure 6

Residual risk with high intergrade price volatility (abnormal difference component with an asymptotic standard deviation of 2.6%): (a) single-delivery contract, (b) multiple-delivery contract ($Q_c = 0$), (c) multiple-delivery contract ($Q_c = Q$), (d) market-basket contract ($\alpha = 0.5$, $\beta = 0.5$), (e) market-basket contract ($\alpha = 0.8$, $\beta = 0.2$).

cial premium (Q), respectively. The arc connecting points b and c shows risk exposure for intermediate values of the contract premium ($Q < Q_c < Q$).

Both figures illustrate that for variety B, any multiple-delivery contract is a poorer hedge than the single-delivery contract, i.e., line segment bc lies to the right of point a. This result occurs because the multiple-delivery contract allows delivery of grade A and hence is influenced by fluctuations in P_A that may not occur in P_B. The opposite is true for the value of the single-delivery and multiple-delivery contracts for hedging a position in A. Our results (not shown) also indicate that the residual risks on hedged positions in the two varieties are largely independent of the values of α and β in Eq. (1).

We turn now to the market-basket cash settlement contract. Points d and e in Figures 5 and 6 show the residual risks on positions in the two grades hedged with a market-basket cash settlement contract. Point d is the case where the varieties are of comparable importance (with $\alpha = \beta = 0.5$), and point e is the case where grade B has a larger market (with $\alpha = 0.8$ and $\beta = 0.2$). In this case, as a result of the weighting scheme used to construct the settlement index [see eq. (6)], the relative values of α and β affect the residual risks significantly. Higher values of α move the combination of residual risks up and to the left, i.e., from point d to point e. This is reasonable because as α increases the price of the standard variety (B) gets a bigger weight in the index [see eq. (6)], so that the market-basket contract becomes a better hedge for that grade and a poorer hedge for grade A.

Comparing Figures 5 and 6 suggests when a market-basket futures contract may be especially valuable. If the volatility of the abnormal difference in grade prices is high (as in Fig. 6), and if the varieties are of comparable importance (so that $\alpha = \beta$), then a market-basket contract will generate smaller residual risks on a hedged position for both A and B than *any* multiple-delivery contract (no matter what value Q_c takes on). That is, point d lies below and to the left of arc bc in Figure 6. Note that a market-basket contract does not dominate every multiple-delivery contract when the volatility of the difference in prices is low (as in Fig. 5 where point b is to the left of point d), or when the standard variety is far more important than the premium variety (as at point e in both figures).

These results reflect the fact that sometimes it is better to distribute large risks among all market participants rather than forcing some participants to bear most of the risk some of the time. Since this is true only for certain values of price volatility and market share, the desirability of market-basket contracts must be evaluated on a case by case basis.

VI. CONCLUDING REMARKS

Cash settlement can improve the contribution of futures contracts to economic welfare in several ways. First, by promoting closer convergence of cash and futures prices, the hedging and risk transfer functions of futures contracts are enhanced. Second, there is a direct cost saving from eliminating expensive physical delivery on the settlement date. Third, new types of futures contracts become feasible, such as when physical delivery is nearly impossible. Fourth, cash settlement adds flexibility to contract design, such as permitting a market-basket contract when multiple product varieties are important.

While cash settlement has many potential benefits, and while the evolution of futures contracts has moved in the direction of cash settlement, there is one key requirement that must be satisfied at all times: The price index used for settlement must be a reliable indicator of the true commercial value of the commodity. When the cash index is inaccurate or subject to manipulation, cash settlement may be inferior to physical delivery. The necessity of a reliable cash index makes the subject of index construction extremely important.

Appendix

DERIVATION OF EQS. (7), (8), AND (9)

The settlement prices F_{SD}, F_{MD}, and F_{MB} specified in Eqs. (7), (8), and (9) for the three types of futures contracts can be derived in a four-step process:

(1) derive diffusion equations for P_A and P_B from the diffusion equations for E and D,
(2) derive a differential equation for dE_k (k = SD, MD, or MB) as a function of dP_A, dP_B, and dt.
(3) derive a partical differential equation for the equilibrium settlement price F_k (k = SD, MD, or MB) based on riskless arbitrage with the cash market,

(4) solve the partial differential equation, subject to the respective boundary conditions implied by the three contracts, for the behavior of the equilibrium settlement price.

The methodology used here is similar to that of Black and Scholes (1973) and Black (1976). In particular, we assume a flat and stationary term structure of interest rates, no costs of storage (other then financing costs), no limitation on short sales or margin financing, and an infinitely elastic supply of arbitrage services. [See Garbade and Silber (1983) for an analysis of the behavior of futures prices when arbitrage services are not infinitely elastic.]

Diffusion Equations for P_A and P_A

The prices of the two varieties (A and B) in terms of the common and difference components E and D, respectively, are given by Eq. (1) in the text:

$$P_{\mathrm{A}} = \exp(E + \alpha D)(1 + Q), \tag{A1a}$$

$$P_{\mathrm{B}} = \exp(E - \beta D). \tag{A1b}$$

From Ito's Lemma we have

$$dP_i = (\partial P_i/\partial E)\, dE + (\partial P_i/\partial D)\, dD + \tfrac{1}{2}(\partial^2 P_i/\partial E^2)\,(dE)^2 + \tfrac{1}{2}(\partial^2 P_i/\partial D^2)(dD)^2 + [\partial^2 P_i/(\partial E\, \partial D)]\, dE\, dD, \qquad i = \text{A or B}. \tag{A2}$$

The diffusion processes for the common and difference components are given by Eqs. (3) and (5) in the text:

$$dE = \sigma\, dz_E, \tag{A3a}$$

$$dD = -\lambda D\, dt + \omega\, dz_D, \tag{A3b}$$

where dz_E and dz_D are uncorrelated Gauss–Weiner processes.

Substituting (A3) into (A2) for i = A and i = B, and using the multiplication rules of stochastic calculus [$(dt)^2 = 0$; $(dt)(dz_j) = 0$ for $f{=}E$ or D; $(dz_j)^2 = dt$ for $j{=}E$ or D; and $(dz_E)(dz_D) = 0$] gives

$$dP_{\mathrm{A}} = P_{\mathrm{A}}\sigma\, dz_E + P_{\mathrm{A}}\alpha\omega\, dz_D + P_{\mathrm{A}}(-\alpha\lambda D + \tfrac{1}{2}\sigma^2 + \tfrac{1}{2}\alpha^2\omega^2)/dt, \tag{A4a}$$

$$dP_{\mathrm{B}} = P_{\mathrm{B}}\sigma\, dz_E - P_{\mathrm{B}}\beta\omega\, dz_D + P_{\mathrm{B}}(\beta\lambda D + \tfrac{1}{2}\sigma^2 + \tfrac{1}{2}\beta^2\omega^2)\, dt \tag{A4b}$$

Differential Equations for dF_k, k = SD, MD, and MB

In general, the settlement price on a futures contract can be written as a function of time and the contemporaneous prices of the underlying varieties:

$$F_k = F_k(P_{\mathrm{A}}, P_{\mathrm{B}}, t), \qquad k = \text{SD, MD, or MB}. \tag{A5}$$

From Ito's lemma we have

$$dF_k = (\partial F_k/\partial P_{\mathrm{A}})\, dP_{\mathrm{A}} + (\partial F_k/\partial P_{\mathrm{B}})\, dP_{\mathrm{B}} + (\partial F_k/\partial t)\, dt + \tfrac{1}{2}(\partial^2 F_k/\partial P_{\mathrm{A}}^2)\,(dP_{\mathrm{A}})^2 + \tfrac{1}{2}(\partial^2 F_k/\partial P_{\mathrm{B}}^2)(dP_{\mathrm{B}})^2 + [\partial^2 F_k/(\partial P_{\mathrm{A}}\, \partial P_{\mathrm{B}})]\, dP_{\mathrm{A}}\, dP_{\mathrm{B}}, \qquad k = \text{SD, MD, or MB}. \tag{A6}$$

Computing $(dP_A)^2$ and $(dP_B)^2$ from (A4), subject to the multiplication rules noted above, gives

$$dF_k = (\partial F_k/\partial P_A)\, dP_A + (\partial F_k/\partial P_B)\, dP_B + [(\partial F_k/\partial t) + \tfrac{1}{2}(\partial^2 F_k/\partial P_A^2)\, P_A^2\, (\sigma^2 + \alpha^2\omega^2) + \tfrac{1}{2}(\partial^2 F_k/\partial P_B^2)\, P_B^2\, (\sigma^2 + \beta^2\omega^2) + [\partial^2 F_k/(\partial P_A\, \partial P_B)]\, P_A P_B\, (\sigma^2 - \alpha\beta\omega^2)]\, dt,$$

$$k = \text{SD, MD, or MB.} \tag{A7}$$

Partial Differential Equation for an Equilibrium Futures Price

Suppose a market participant establishes a long position in $h_A = \partial F_k/\partial P_A$ units of variety A, a long position in $h_B = \partial F_k/\partial P_B$ units of variety B, and a short position in 1 unit of futures contract k, where k = SD, MD, or MB. Since the futures contract has no value (Black, 1976), his net worth is

$$W = h_A P_A + h_B P_B. \tag{A8}$$

With continuous mark-to-market settlement of the futures contract (Black, 1976), the differential of his net worth is

$$dW = h_A dP_A + h_B dP_B - dF_k. \tag{A9}$$

In view of Eq. (A7) and the definitions of h_A and $_B$, this becomes

$$dW = -(\partial F_k/\partial t)dt - \tfrac{1}{2}(\partial F_k/\partial P_A^2)\, P_A^2(\sigma^2 + \alpha^2\omega^2)\, dt - \tfrac{1}{2}(\partial^2 F_k/\partial P_B^2) \times P_B^2(\sigma^2 + \beta^3\omega^2)\, dt - [\partial^2 F_k/(\partial P_A \cdot P_B)]\, P_A P_B(\sigma^2 - \alpha\beta\omega^2)\, dt \tag{A10}$$

Since the differential of net worth in Eq. (A10) depends only on the passage of time, and not on changes in either cash or futures prices, the position is fully hedged. The rate of change of net worth must, therefore, be equal to the rate of interest on risk-free debt (which we assume is equal to R for debt of all maturities):

$$dW/W = R\, dt. \tag{A11}$$

Using Eqs. (A8) and (A10), this implies that

$$\partial F_k/\partial t = -RP_A(\partial F_k/\partial P_A) - RP_B(\partial F_k/\partial P_B) - \tfrac{1}{2}(\partial^2 F_k/\partial P_A^2\, P_A^2\, (\sigma^2 + \alpha^2\omega^2) - \tfrac{1}{2}(\partial^2 F_k/\partial P_B^2\, P_B^2\, (\sigma^2 + \beta^2\omega^2) - [\partial^2 F_k/(\partial P_A \partial P_B)] P_A P_B\, (\sigma^2 - \alpha\beta\omega^2),$$

$$k = \text{SD, MD, or MB.} \tag{A12}$$

Equation (A12) is the partial differential equation for the equilibrium settlement price which must be satisfied by each of the futures contracts.

Solution to the Partial Differential Equation

The equilibrium prices on the three futures contracts specified in Section V B can be found by solving Eq. (A12) subject to appropriate boundary conditions.

The single-delivery contract allows delivery only of variety B. On the final settlement date t^* we must, therefore, have convergence of F_{SD} and P_B, so that

$$F_{SD}(P_A, P_B, t^*) = P_B. \tag{A13}$$

Equations (A12) and (A13) together imply that for $t \leq t^*$:

$$F_{SD}(P_A, P_B, t) = \exp[R(t^* - t)]P_B. \tag{A14}$$

The multiple-delivery contract allows delivery, at the seller's option, of either variety B at the settlement price or variety A at the settlement price times a contract premium of $1 + Q_c$. On the final settlement date t^* we must have convergence of F_{MD} and the price of the net cheapest deliverable variety, so that

$$F_{\mathrm{MD}}(P_{\mathrm{A}}, P_{\mathrm{B}}, t^*) = \min[P_{\mathrm{B}}, P_{\mathrm{A}}(1 + Q_c)^{-1}]. \tag{A15}$$

Note that if $P_{\mathrm{B}} < P_{\mathrm{A}}(1 + Q_c)^{-1}$ at t^*, the settlement price will converge to P_{B} and only variety B will be delivered (at the contract price). If $P_{\mathrm{B}} > P_{\mathrm{A}}(1 + Q_c)^{-1}$ at t^*, the settlement price will converge to $P_{\mathrm{A}}(1 + Q_c)^{-1}$ and only variety A will be delivered (at the contract price times the premium $1 + Q_c$, or at P_{A}). Equations (A12) and (A15) together imply that for $t \leq t^*$,

$$F_{\mathrm{MD}}(P_{\mathrm{A}}, P_{\mathrm{B}}, t) = \exp[R(t^* - t)\{P_{\mathrm{B}}[1 - N(d_1)] + P_{\mathrm{A}}(1 + Q_c)^{-1}N(d_2)\}, \tag{A16}$$

where

$$d_1 = \frac{\ln[P_{\mathrm{B}}(1 + Q_c)/P_{\mathrm{A}}] + \tfrac{1}{2}\omega^2(t^* - t)}{\omega(t^* - t)^{1/2}}, \tag{A17a}$$

$$d_2 = d_1 - \omega(t^* - t)^{1/2}, \tag{A17b}$$

$$N(x) = (2\pi)^{-1/2}\int_x^{-\infty} \exp(-\tfrac{1}{2}u^2)\, du. \tag{A17c}$$

The market-basket contract provides for cash settlement at the settlement index $I = \beta[P_{\mathrm{A}}/(1 + Q)] + \alpha P_{\mathrm{B}}$. On the final settlement date t^* we must have convergence of F_{MB} and the index, so that

$$F_{\mathrm{MB}}(P_{\mathrm{A}}, P_{\mathrm{B}}, t^*) = \beta[P_{\mathrm{A}}/(1 + Q)] + \alpha P_{\mathrm{B}}. \tag{A18}$$

Equations (A12) and (A18) together imply that, for $t \leq t^*$,

$$F_{\mathrm{MB}}(P_{\mathrm{A}}, P_{\mathrm{B}}, t) = \{\beta \cdot [P_{\mathrm{A}}/(1 + Q)] + \alpha P_{\mathrm{B}}\} \exp[R(t^* - t)]. \tag{A19}$$

It should be noted that eqs. (A14), (A16), and (A19) can be written in the form:

$$F_k = \exp[R(t^* - t)]\, S_k, \qquad k = \mathrm{SD, MD, or MB}, \tag{A20}$$

where

$$S_{\mathrm{SD}} = P_{\mathrm{B}}, \tag{A21a}$$

$$S_{\mathrm{MD}} = P_{\mathrm{B}} - C, \tag{A21b}$$

$$C = P_{\mathrm{B}}N(d_1) - P_{\mathrm{A}}(1 + Q_c)^{-1}N(d_2), \tag{A21c}$$

$$S_{\mathrm{MB}} = \beta[P_{\mathrm{A}}/(1 + Q)] + \alpha P_{\mathrm{B}}. \tag{A21d}$$

Equation (A20) says that the futures price is equal to some spot market price S_k, compounded over the time interval remaining to settlement at the interest rate R. In the case of the single-delivery contract the relevant spot price is simply the price of the deliverable commodity (P_{B}). For the market-basket contract the relevant spot price is the value of the settlement index $\{\beta[P_{\mathrm{A}}/(1 + Q)] + \alpha P_{\mathrm{B}}\}$. For the multiple-delivery contract, however, the spot price is the price of the standard variety (P_{B}), *less* the value C of a call option on one unit of that variety where the cost of exercising the option is $P_{\mathrm{A}}(1 + Q_c)^{-1}$. This shows that the existence of an option to deliver more than one variety of a commodity on a futures contract will generally act to depress the price of the contract below the price of a single-delivery contract. This result is reasonable, because the choice of which variety to deliver rests with the seller. He pays for the right to choose by contracting to sell at a lower price.

It should also be noted that the prices of all three futures contracts follow the same partial differential equation, Eq. (A12). What distinguishes the contracts is the boundary conditions of Eq. (A13) compared with those of Eqs. (A15) and (A18). These boundary conditions are defined by the delivery specifications of the contracts. This shows, quite directly, that delivery specifications are crucial to the behavior of futures prices.

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