VOLATILITY AND MATURITY EFFECTS IN THE NIKKEI INDEX FUTURES

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Many financial data series are found to exhibit stochastic volatility. Some of these time series are constructed from contracts with time-varying maturities. In this paper, we focus on index futures, an important subclass of such time series. We propose a bivariate GARCH model with the maturity effect to describe the joint dynamics of the spot index and the futures-spot basis. The setup makes it possible to examine the Samuelson effect as well as to compare the hedge ratios under scenarios with and without the maturity effect. The Nikkei-225 index and its futures are used in our empirical analysis. Contrary to the Samuelson effect, we find that the volatility of the futures price decreases when the contract is closer to its maturity. We also apply our model to futures hedging, and find that both the optimal hedge ratio and the hedging effectiveness critically depend on both the maturity and GARCH effects. © 1999 John Wiley & Sons, Inc. Jrl Fut Mark 19: 895–909, 1999

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INTRODUCTION

The Samuelson effect is a well-known theory in futures literature. It refers to an increasing volatility of futures prices as the contract approaches expiration. The theory (Samuelson, 1965) is based on the premise that more information on futures prices is released when the contract approaches its maturity. The empirical evidence on the Samuelson effect is mixed, however. Rutledge (1976) argued that increasing volatility with a shorter time to maturity is not a general result. His empirical studies rejected the Samuelson hypothesis for the futures contracts on wheat and soybean oil, but could not reject the same hypothesis for the futures contracts on silver and cocoa. Milonas (1986) examined eleven commodities (five agricultural, three financial, and three metal) from 1972–1983. Milonas' results supported the Samuelson effect in 10 out of 11 futures markets, which for the first time established evidence for the Samuelson effect in financial futures and metals. In addition to the evidence on American commodity futures, Khoury and Yourougou (1993) found strong support for the Samuelson effect using six Canadian agricultural commodities.

In this paper, we take a different approach to the Samuelson effect. We construct an econometric model focusing on the basis between futures and spot prices. The rationale for doing so is somewhat apparent. Because the basis must converge to zero as the contract expires, it is natural to expect that the volatility of the basis also approaches zero as the maturity is shortened. This simple observation allows us to specify an econometric model with a priori knowledge about the sign of the parameter pertaining to the contract maturity. We then use this specification to formulate a prediction about the Samuelson effect. Specifically, we model the basis and spot price jointly using a bivariate GARCH model. Our specific model incorporates a maturity variable so that the remaining maturity of the contract is allowed to play a role in determining the behavior of the basis. Our adoption of the GARCH model is motivated by the empirical success of this class of models. Many financial time series are known to exhibit both heteroscedasticity and leptokurtosis. The autoregressive conditional heteroscedastic (ARCH) process of Engle (1982) and its generalized version, the GARCH process of Bollerslev (1986), have become a standard modeling tool for dealing with these empirical features. The justification for our modification of the standard GARCH model with a maturity variable comes from the fact that the futures contract, thus the basis, has a time-varying maturity.

The presence of the maturity and GARCH effects in asset returns could have many important implications for financial economics. In hedging applications, for example, a hedge is typically lifted prior to the contract's expiration, and the hedge is therefore subject to "basis risk." Properly modeling the basis volatility is of considerable importance for practitioners. Castelino and Francis (1982) showed that the volatility of changes in the basis declines as maturity is shortened. This implies that hedging with a nearby contract involves less basis risk than with a more distant one. The fact that basis risk declines when maturity is shortened is clearly due to an arbitrage condition. Because the futures contract becomes a spot contract at maturity, basis must approach zero as the maturity is shortened. Naturally, a sensible specification for the conditional mean and variance of the basis series must depend upon the maturity of the futures contract. Stochastic volatility affecting the optimal hedge ratio is not surprising either. Because the optimal hedge ratio is essentially the regression coefficient of the change in spot prices on the change in futures prices, time-varying volatilities naturally lead to an optimal hedge ratio that is time-varying. The article by Kroner and Sultan (1994) provides such an example.

In this article, we use the Nikkei-225 index spot and futures to examine the Samuelson effect and study the hedging implications under both stochastic volatility and time-varying futures maturities. Our results are against the Samuelson effect. We also find that the optimal hedge ratios are very different under scenarios with and without stochastic volatility. The maturity of the futures contract is also found to have a pronounced effect on the optimal hedge ratio and the hedging effectiveness.

MODELING FUTURES VIA SPOT AND BASIS

In this section, we model the joint dynamics of spot and futures prices. Instead of directly modeling futures price, we have chosen to model the futures-spot basis in order to take advantage of an important property possessed by the basis. By arbitrage, the basis equals zero at maturity, and so does its volatility. This boundary condition guides us in coming up with a more sensible model specification. We use a simple bivariate nonlinear asymmetric GARCH(1,1) model for the joint dynamics. The nonlinear asymmetric version of the model comes from Engle and Ng (1993). The bivariate specification follows that of Bollerslev (1988) which imposes a constant conditional correlation structure. Our choice of the GARCH dynamic is motivated by the empirical evidence in support of this class of models (see, for example, Bollerslev, Chou, & Kroner (1992)).

Let S_t and $F_t(m_t)$ be the spot index and the futures price with maturity m_t at time t, respectively. Define the basis at time t as $B_t(m_t) \equiv$

$F_t(m_t) - S_t$. Because the future-spot basis is expected to approach zero as m_t goes to zero, it is convenient to make the volatility of the basis a power function of maturity. First, we specify a process for the stock index return:

$$\frac{\Delta S_t}{S_{t-1}} = a_0 + \lambda\sqrt{h_t} + \sqrt{h_t}\varepsilon_t \tag{1}$$

$$h_t = \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 h_{t-1}(\varepsilon_{t-1} - \theta)^2 \tag{2}$$

$$\varepsilon_t | \mathcal{F}_{t-1} \sim D_\varepsilon(0,1)$$

where $\alpha_0 \tau 0$, $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, and $\alpha_1 + \alpha_2(1 + \theta^2) < 1$. The information set available at time t is denoted by $\mathcal{F}_t$. $D_\varepsilon(0,1)$ is some distribution with mean 0 and variance 1. It turns out that the exact form of the distribution is not critical in parameter estimation as long as its mean equals 0 and variance equals 1. A further elaboration on this issue is provided in later in this article. We then model the basis (normalized by the index level) by

$$\frac{\Delta B_t(m_t)}{S_{t-1}} = b_0 m_t^\eta + b_1 \frac{B_{t-1}(m_{t-1})}{S_{t-1}} + m_t^\gamma \sqrt{q_t}\xi_t \tag{3}$$

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 q_{t-1}(\xi_{t-1} - \delta)^2 \tag{4}$$

$$\xi_t | \mathcal{F}_{t-1} \sim D_\xi(0,1)$$

where $\beta_0 \tau 0$, $\beta_1 \geq 0$, $\beta_2 \geq 0$, and $\beta_1 + \beta_2(1 + \delta^2) < 1$. The distribution $D_\xi(0,1)$ again denotes some distribution with mean 0 and variance 1. The conditional correlation of two innovations is assumed to be constant ρ, i.e., $Cov_{t-1}(\varepsilon_t, \xi_t) = \rho$. The parameter restrictions: $\alpha_1 + \alpha_2(1 + \theta^2) < 1$ and $\beta_1 + \beta_2(1 + \delta^2) < 1$ ensure that h_t and q_t are stationary processes (see Duan, 1997). The other parameter restrictions are used to guarantee positive conditional variances.

The GARCH feature of our model is reflected in eq. (2) where the conditional variance is a function of the previous conditional variance discounted by a factor α_1. The conditional variance is also subject to the shock arising from the previous unanticipated return innovation. The difference between the standard linear GARCH model and the nonlinear asymmetric GARCH model lies in the term θ. If $\theta = 0$, the standard linear GARCH(1,1) model emerges. Parameter θ in the nonlinear asymmetric GARCH model is used to capture a potential asymmetric volatility response to a positive and negative innovation. This asymmetry parameter is useful in reflecting an important empirical regularity, known as the leverage effect, in equity returns. The term "leverage effect" refers to the negative correlation between return and volatility innovations in equity time series, and was first analyzed by Black (1976). A positive θ implies a negative correlation between return and volatility innovations, and hence leverage effect. Leverage effect for equity returns has been previously documented by, for example, Nelson (1991), Engle and Ng (1993), Glosten et al. (1993) and Duan (1997) using several different GARCH specifications with the non-linear asymmetric GARCH model being one of them. The return dynamic in eq. (1) has a term λ to reflect risk premium. Parameter λ measures the index return's risk premium per unit of the conditional standard deviation.

The dynamic for the normalized basis is assumed to also follow the nonlinear asymmetric GARCH(1,1) process except that the maturity effect has been built into the model. The effect of the futures' maturity is reflected by a power function of maturity. According to our earlier discussion on the boundary condition for the basis, parameter γ should be positive, meaning that the shorter the maturity the smaller is the basis volatility. Because the boundary condition directly imposes a restriction on the level of the basis, we also take it into account by having another power function with parameter η for the conditional mean of the basis dynamic. Except for the complication related to maturity, the volatility dynamic of the basis remains in the same form as that of the spot index return. Unlike the dynamic for the spot index return, however, the asymmetry parameter δ for the basis dynamic does not have the natural interpretation as a leverage parameter. We simply use it here to capture any potential asymmetric volatility response to the innovation in the basis.

DATA AND ESTIMATION RESULTS

The data series is daily Nikkei-225 index spot and futures series from 24 November 1988 to 6 June 1996. The futures contracts are the ones traded in the Osaka exchange. We examine the nearby three-month futures contract which is switched over to the next maturing contract five days before the expiration date of the nearby contract. We have used both zero and ten days before maturity to construct the data sample in an attempt to examine the robustness of our conclusions. Since the results remain essentially the same, we will concentrate our exposition only on the series constructed by switching over five days before maturity.

Our bivariate system can be estimated using the maximum likelihood method. We assume conditional normality in our empirical analysis. It is well known in the time series literature that even if the conditional normality assumption is violated, the parameter estimates obtained under such an assumption continue to be consistent. The price is the loss of efficiency for such parameter estimates. This conclusion is based on the quasi-maximum likelihood estimation principle which only requires a correct specification for the conditional mean and variance of a dynamic system (see Bollerslev and Wooldridge, 1992).

The estimation results of the bivariate system of spot price and basis are presented in Table I. For comparison purposes, we have also estimated a restricted version of the bivariate system. This restricted version sets $\gamma = \eta = 0$ so that the maturity effect is disabled. Our results clearly indicate the presence of the GARCH effect for both the spot index return and basis. The parameter estimates for the spot index return are typical. Parameter value for α_1 is much larger than that for α_2, and $\alpha_1 + \alpha_2(1 + \theta^2) < 1$. The fact that $\alpha_1 + \alpha_2(1 + \theta^2) < 1$ implies that the spot index return is stationary. The estimated value for $\alpha_1 + \alpha_2(1 + \theta^2)$ is very close to one, implying that the volatility is highly persistent. Parameter θ has a significantly positive value, which suggests that the leverage effect is present in the Nikkei-225 index return. Our results also suggest that the specification with or without the maturity effect does not give rise to much different parameter estimates if one is concerned only with the spot index return.

Interestingly, the parameter estimates for the basis are qualitatively similar to those for the spot index return except for the asymmetry parameter δ. The estimate for δ suggests that innovations in the basis and its volatility are positively correlated. It should be noted, however, that the magnitude of δ, rather than its sign, is meaningful, because changing the definition of the basis from the futures price minus the spot price to the other way round immediately flips the sign of δ. The results pertaining to the maturity parameters suggest that maturity effect is highly significant. It is clear from the individual t values for parameters η and γ, or the likelihood ratio test statistic computed as two times the difference of the two log-likelihood functional values reported in Table I. (The numbers suggest that the chi-square test statistic with 2 degrees of freedom equals 30.1318 which is highly significant.) As argued earlier, only positive values for the maturity parameters can be consistent with the basis behavior implied by the arbitrage boundary condition. Our findings are therefore consistent with the theoretical prediction. Failing to consider the maturity effect can affect the parameter estimates for the basis dynamic; for example, the estimate for δ has changed from being significantly negative to insignificantly positive. The estimated correlation coefficient between

TABLE I

Parameter Estimates and Standard Errors for Nikkei-225 Index Spot and Futures Series from 24 November 1988 to 6 June 1996

$$\frac{\Delta S_t}{S_{S-1}} = a_0 + \lambda\sqrt{h_t} + \sqrt{h_t}\varepsilon_t$$

$$h_t = \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 h_{t-1}(\varepsilon_{t-1} - \theta)^2$$

$$\varepsilon_t|F_{t-1} \sim N(0,1)$$

$$\frac{\Delta B_t(m)_t}{S_{t-1}} = b_0 m_t^{\eta} + b_1 \frac{B_{t-1}(m_{t-1})}{S_{t-1}} + m_t^{\gamma}\sqrt{q_i}\,\xi_i$$

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 q_{t-1}(\xi_{t-1} - \delta)^2$$

$$\xi_t|F_{t-1} \sim N(0,1)$$

	Bivariate GARCH Model (with maturity effect)		*Bivariate GARCH Model (without maturity effect)*	
Spot price dynamic	*Parameter value*	*Standard error*	*Parameter value*	*Standard error*
α_0	0.0932	0.0501	0.0931	0.0502
λ	−0.0860	0.0505	−0.0859	0.0518
α_0	0.0170	0.0047	0.0171	0.0038
α_1	0.8581	0.0155	0.8576	0.0100
α_2	0.0719	0.0114	0.0721	0.0079
θ	0.9653	0.1267	0.9652	0.0869
Basis dynamic				
η	0.8650	0.2021		
γ	0.0627	0.0257		
b_0	0.0035	0.0030	0.0798	0.0117
b_1	−0.2184	0.0178	−0.1880	0.0138
β_0	0.0034	0.0009	0.0051	0.0008
β_1	0.8321	0.0193	0.8431	0.0058
β_2	0.1588	0.0191	0.1499	0.0073
δ	−0.1237	0.0687	0.0115	0.0571
Correlation				
ρ	−0.1948	0.0225	−0.1949	0.0194
Log-likelihood	−2313.0358		−2328.1017	
Sample size	1809		1809	

two conditional innovations is significantly negative with a value approximately equal to -0.19. The correlation estimate is practically unaffected by removing the maturity effect from the bivariate system. The correlation coefficient ρ and the maturity parameter γ turn out to be the key parameters in deciding whether there is the Samuelson effect, an issue addressed in the next section.

SAMUELSON EFFECT

Recall that the Samuelson effect refers to the theory developed by Samuelson (1965). It predicts that the volatility of futures prices will increase as the contract approaches expiration. In our context, the Samuelson effect can be analyzed using the bivariate GARCH system in eqs. (1)–(4). For this purpose, we need to first derive from the bivariate system an expression for the conditional variance of the change in futures prices:

$$\begin{aligned}
\Phi(m_t) &\equiv Var_{t-1}[\Delta F_t(m_t)] \\
&= Var_{t-1}[\Delta S_t + \Delta B_t(m_t)] \\
&= S_{t-1}^2 Var_{t-1}\left[\frac{\Delta S_t}{S_{t-1}} + \frac{\Delta B_t(m_t)}{S_{t-1}}\right] \\
&= S_{t-1}^2 \left(Var_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}\right] + Var_{t-1}\left[\frac{\Delta B_t(m_t)}{S_{t-1}}\right]\right. \\
&\qquad \left. + 2Cov_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}, \frac{\Delta B_t(m_t)}{S_{t-1}}\right]\right) \\
&= S_{t-1}^2 \left(h_t + q_t m_t^{2\gamma} + 2\rho\sqrt{h_t q_t}\, m_t^{\gamma}\right) \qquad (5)
\end{aligned}$$

This allows us to take a derivative of $\Phi(m_t)$ with respect to m_t to examine the Samuelson effect:

$$\begin{aligned}
\Phi'(m_t) &= S_{t-1}^2\left(2\gamma q_t m_t^{2\gamma-1} + 2\rho\gamma\sqrt{h_t q_t}\, m_t^{\gamma-1}\right) \qquad (6) \\
&= 2S_{t-1}^2\, \gamma\sqrt{h_t q_t}\, m_t^{\gamma-1}\left(m_t^{\gamma}\sqrt{\frac{q_t}{h_t}} + \rho\right) \qquad (7)
\end{aligned}$$

If $\Phi'(m_t) < 0$, then the futures volatility increases when its maturity becomes shorter, and the Samuelson effect is supported. On the other hand, the Samuelson effect is rejected if $\Phi'(m_t)$ τ 0. Because $S_{t-1}^2\gamma\sqrt{h_t q_t}\, m_t^{\gamma-1}$ is positive, the sign of $\Phi'(m_t)$ entirely depends on $m_t^{\gamma}\sqrt{q_t/h_t} + \rho$. Figure 1 plots the time series of $m_t^{\gamma}\sqrt{q_t/h_t} + \rho$ to see how this quantity varies over the time period covered by our sample. It is clear from Figure 1 that the values are mostly positive, a finding contrary to the Samuelson effect.

Although the volatility of the price change in futures is functionally related to its maturity as in eq. (5), the relationship is complicated by three stochastic variables in the system. In order to gain a better under-

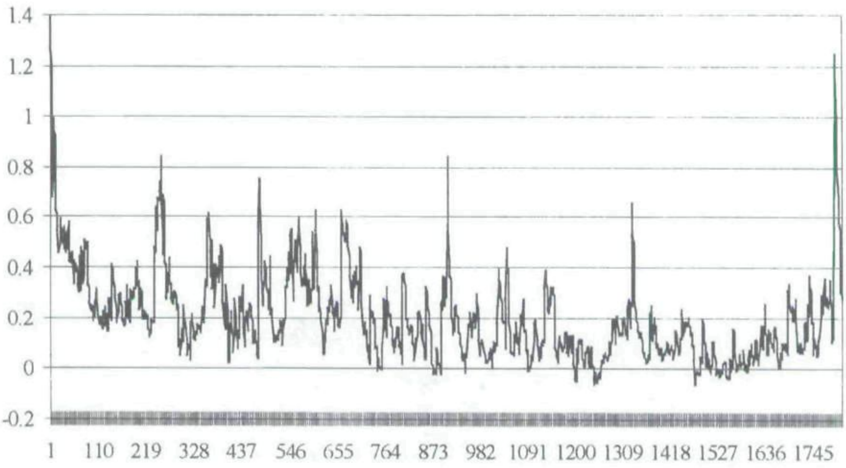

FIGURE 1
The derivative of the futures price volatility with respect to maturity over the sample period. A negative (positive) value is supportive of (against) the Samuelson effect.

standing, we could plot the historical relationship between the conditional volatility of the percentage change in the futures price, $Var_{t-1}[\Delta F_t(m_t)/F_{t-1}(m_{t-1})]$, as a function of maturity, m_t. Because $Var_{t-1}[\Delta F_t(m_t)/F_{t-1}(m_{t-1})] = \Phi(m_t)/[F_{t-1}(m_t - 1)]^2$, plotting this quantity against maturity is less likely subject to the level effect arising from the spot index value. There are two other stochastic variables, h_t and q_t, in the system. Because h_t is the conditional volatility of the spot index return which is independent of futures contracts, it will be better to remove its impact so that the maturity effect can be isolated. We use a linear regression to filter out the effect of h_t. In other words, we end up dealing with the regression residual rather than $Var_{t-1}[\Delta F_t(m_t)/F_{t-1}(m_{t-1})]$ directly. The relationship between the residual and the maturity is plotted in Figure 2. A trend emerges in Figure 2; that is, the volatility of the percentage change in the futures price, after applying a linear filter, generally increases with maturity. This result thus collaborates with our previous evidence against the Samuelson effect.

OPTIMAL HEDGE RATIO AND HEDGING EFFECTIVENESS

The interest in futures contracts among financial professionals has much to do with the role of futures in risk management. Stock index futures are often used for portfolio risk management; for example, it can be used to temporarily alter the systematic risk of a portfolio without having to

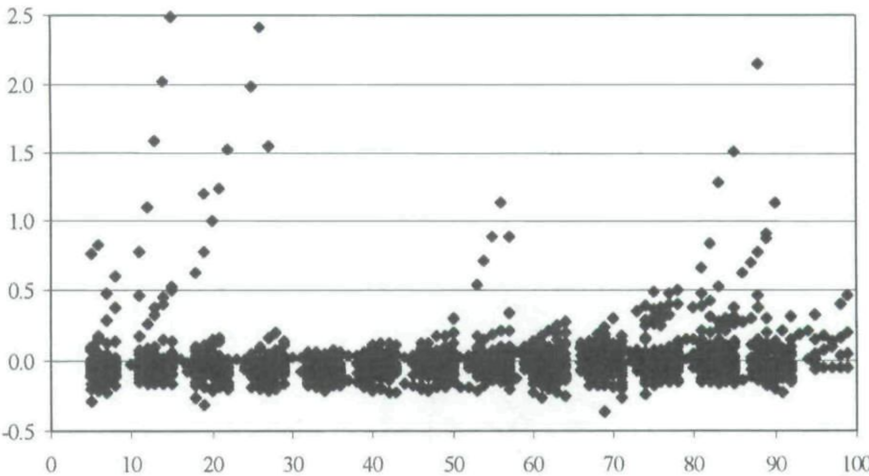

FIGURE 2
The conditional variance of the percentage change in futures prices in relation to the maturity (in days) of the futures. The conditional variance has first been filtered through a linear model to remove the effect of the spot index volatility.

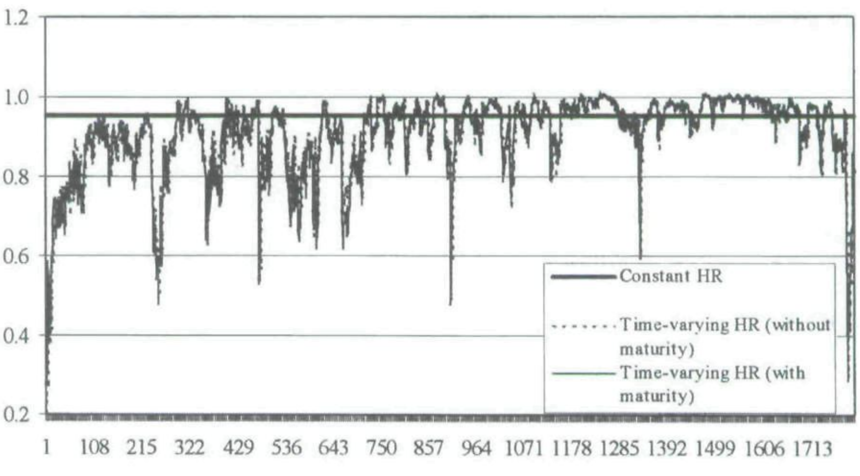

FIGURE 3
The optimal hedge ratios over the sample period under three modeling assumptions: constant volatility, time-varying volatility without the maturity effect and time-varying volatility with the maturity effect.

buy or sell its constituent stocks. Index futures are sometimes used in the dynamic trading program to achieve the portfolio insurance or arbitrage purpose. Here, we will examine the hedging implications of our bivariate specification. Although our bivariate system can be used for general risk management objectives, we only concentrate on the issue of minimizing portfolio risk in this paper. An optimal hedge ratio for our purpose is the relative position in index futures that attains a lowest hedged portfolio variance.

By the standard hedging result, the optimal hedge ratio (conditional on the available information) must be

$$\varphi_t = \frac{Cov_{t-1}[\Delta F_t(m_t), \Delta S_t]}{Var_{t-1}[\Delta F_t(m_t)]}. \tag{8}$$

Using the bivariate system in (1)–(4) the optimal hedge ratio can be derived as follows.

$$\begin{aligned}
\varphi_t &= \frac{Cov_{t-1}[\Delta B_t(m_t) + \Delta S_t, \Delta S_t]}{Var_{t-1}[\Delta B_t(m_t) + \Delta S_t]} \\
&= \frac{Cov_{t-1}\left[\frac{\Delta B_t(m_t)}{S_{t-1}} + \frac{\Delta S_t}{S_{t-1}}, \frac{\Delta S_t}{S_{t-1}}\right]}{Var_{t-1}\left[\frac{\Delta B_t(m_t)}{S_{t-1}} + \frac{\Delta S_t}{S_{t-1}}\right]} \\
&= \frac{Cov_{t-1}\left[\frac{\Delta B_t(m_t)}{S_{t-1}}, \frac{\Delta S_t}{S_{t-1}}\right] + Var_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}\right]}{Var_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}\right] + Var_{t-1}\left[\frac{\Delta B_t(m_t)}{S_{t-1}}\right] + 2Cov_{t-1}\left[\frac{\Delta B_t(m_t)}{S_{t-1}}, \frac{\Delta S_t}{S_{t-1}}\right]} \\
&= \frac{\rho\sqrt{h_t q_t} m_t^{\gamma} + h_t}{h_t + q_t m_t^{2\gamma} + 2\rho\sqrt{h_t q_t} m_t^{\gamma}}
\end{aligned} \tag{9}$$

The above formula indicates that the optimal hedge ratio is a function of the maturity effect parameter γ. To study the implications of stochastic volatility and time-varying maturity, we first compute the optimal hedge ratio under the assumption that the volatility is constant and the maturity effect is absent. This case leads to the classical hedge ratio which is directly obtainable from regressing the change in spot prices on the change in futures prices. Using our data set, this hedge ratio equals 0.951084. For two scenarios under stochastic volatility, we use the parameter values reported in Table 1 to compute the optimal hedge ratios over the entire sample. The results are plotted in Figure 3. The horizontal line corresponds to the case of constant volatility and no maturity effect. The dashed line is for the optimal hedge ratio without the maturity effect. The optimal hedge ratio corresponding to the general model is depicted by the solid line. Figure 3 shows that the two optimal hedge ratios under stochastic volatility differ from the traditional constant hedge ratio. Moreover, the maturity of the futures contract affects the optimal hedge ratio under the scenario of stochastic volatility. In general, the constant volatility assumption leads to over-hedging.

In order to see how the optimal hedge ratio is related to maturity, we present Figure 4, in which the optimal hedge ratio based on our GARCH-maturity model is plotted against maturity for the entire data sample. In this figure, the lowest possible maturity is five days because our particular choice of the futures data series. The data selection procedure was described previously in this article. The result in Figure 4 indicates that the optimal hedge ratio is inversely related to maturity. This is not surprising because the index futures and spot prices become less correlated as maturity increases. Although this relationship is complicated by two stochastic variables, h_t and q_t, the effect of maturity is evident.

We now turn to the question of hedging effectiveness. A standard measure of hedging effectiveness is the proportion of variance in the spot price return that is eliminated by hedging. Because our setup considers stochastic volatility, the hedging effectiveness is best measured in terms of the conditioning information. Using the optimal hedge ratio formula given in eq. (9), we can arrive at the following hedging effectiveness formula:

$$e_t = \frac{Cov_{t-1}[\Delta F_t(m_t),\ \Delta S_t]^2}{Var_{t-1}[\Delta F_t(m_t)]Var_{t-1}[\Delta S_t]} = \frac{(\rho\sqrt{h_t q_t}m_t^{\gamma} + h_t)^2}{h_t(h_t + q_t m_t^{2\gamma} + 2\rho\sqrt{h_t q_t}m_t^{\gamma})} \tag{10}$$

The hedging effectiveness measure is determined by maturity and two conditional variances. The maturity effect cannot be isolated unless we artificially fix the values for h_t and q_t. We plot the hedging effectiveness measure against maturity over our sample as an attempt to uncover their overall relationship. The result is presented in Figure 5. Although h_t and q_t complicate the relationship, Figure 5 still shows a clear inverse relationship between hedging effectiveness and maturity. Our result suggests that futures hedging becomes less effective if the futures contract employed has a longer maturity. This result is inherent to dynamic hedging, and it is true even if one improves the overall hedging performance by employing a model with the maturity effect.

CONCLUSION

In this article, a bivariate GARCH time series model with the maturity effect is employed to model the joint dynamics of the spot index and the

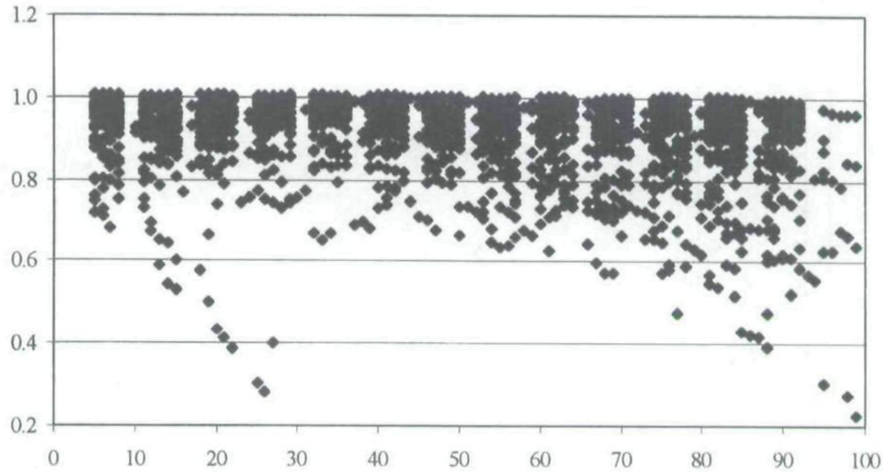

FIGURE 4
The optimal hedge ratio in relation to maturity (in days). The optimal hedge ratio is computed using the time-varying volatility model with the maturity effect.

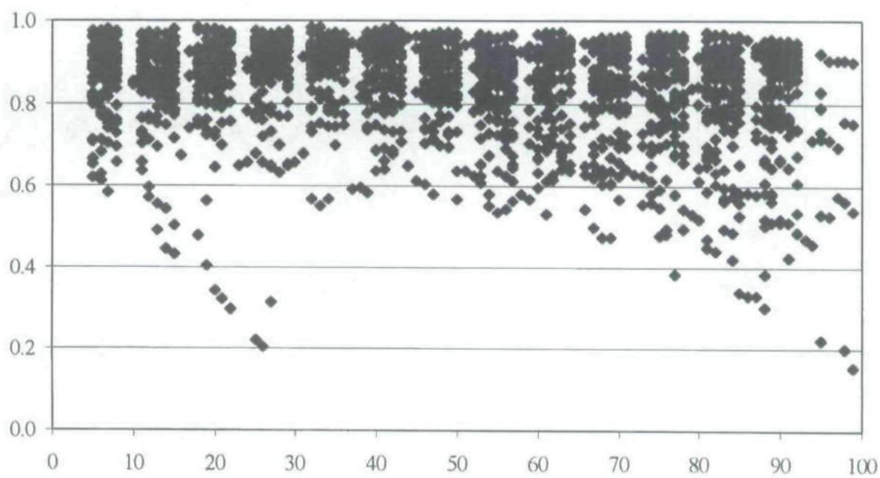

FIGURE 5
Hedging effectiveness in relation to maturity (in days). Hedging effectiveness is for the time-varying volatility model with the maturity effect.

futures-spot basis. Our results show that both the maturity and GARCH effects are simultaneously present. Moreover, our results show that the conditional variance of the futures price decreases if its maturity is shortened. Our finding is thus against the Samuelson effect.

We also examine the hedge implications of our model. Our results show that a significant difference exists between the traditional constant hedge ratio and the one allowing for time-varying volatilities. When the maturity of the futures contract in a futures-spot hedge is taken into account, the optimal hedge ratio decreases when the maturity increases. The hedging performance is also found to be inversely related to the maturity of the futures. Our results have an important implication for futures hedging in general. Because futures hedging always involves the futures contract with changing maturities, the effect of maturity should not be overlooked. Failing to consider the maturity effect could, as demonstrated in this article, lead to overhedging and induce unnecessary risk exposures.

BIBLIOGRAPHY

Black, F. (1976). Studies in stock price volatility changes. Proceedings of the 1976 Meeting of the Business and Economics Statistics Section, American Statistical Association, 177–181.

Bollerslev, T. (1986). General autoregressive conditional heteroskedasticity. Journal of Econometrics, 31, 307–327.

Bollerslev, T. (1988). On the correlation structure on the generalized autoregressive conditional heteroskedastic process. Journal of Time Series Analysis 9, 121–131.

Bollerslev, T., Chou, R. & Kroner, K. (1992). ARCH modeling in finance: A review of the theory and empirical evidence. Journal of Econometrics, 52, 5–59.

Bollerslev, T. & Wooldridge, J. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Review 11, 143–172.

Castelino, M. & Francis, J. (1982). Basis speculation in commodity futures: The maturity effect. The Journal of Futures Markets, 2, 195–207.

Duan, J.-C. (1997). Augmented GARCH(p,q) process and its diffusion limit. Journal of Econometrics 79, 97–127.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation. Econometrica 50, 987–1008.

Engle, R., & Ng, V. (1993). Measuring and testing the impact of news on volatility. Journal of Finance 48, 1749–1778.

Glosten, L., Jagannathan, R. & Runkle, D. (1993). Relationship between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48, 1779–1801.

Khoury, N., & Yourougou, P. (1993). Determinants of agricultural futures prices volatilities: Evidence from Winnipeg Commodity Exchange. The Journal of Futures Markets, 13, 345–356.

Kroner K., & Sultan, J. (1993). Time varying distributions and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis, 28, 535–551.

Milonas, N. (1986). Price variability and the maturity effect in futures markets. The Journal of Futures Markets, 6, 443–460.

Nelson, D. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59, 347–370.

Park T. & Switzer, L. (1995). Bivariate GARCH estimation of the optimal hedge ratios for stock index futures: A note. The Journal of Futures Markets, 15, 61–67.

Rutledge, D. (1976). A note on the variability of futures prices. Review of Economics and Statistics, 58, 118–120.

Samuelson, P. (1965). Proof that properly anticipated prices fluctuate randomly. Industrial Management Review, 6, 41–49.