CONTEMPORARY AND LONG-RUN CORRELATIONS: A COVARIANCE COMPONENT MODEL AND STUDIES ON THE S&P 500 CASH AND FUTURES MARKETS

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In this article, a multivariate component model for conditional asset return covariance is developed as an extension to the univariate volatility component model of Engle & Lee (1999). The conditional covariance now is decomposed into a long-run (trend) component and a short-run (transitory) component. Through the decomposition, relationships like the long-run correlation and volatility copersistence can be studied solely upon examining the long-run trend of the conditional covariance. The decomposition also has important implications in studying portfolio hedging problems such as the multi-period minimum-variance hedging for long-term portfolio management. The empirical study in this article focuses on estimating the covariance component structure between the S&P 500 cash and futures markets and their contemporary and long-run correlation relationship and the volatility copersistence relationship. © John Wiley & Sons, Inc. Jrl Fut Mark 19: 877–894, 1999

I have benefited greatly from the discussions with Clive Granger, Bruce Lehmann, Alex Kane, Ben Lum, and David Reiner. Special thanks to Robert F. Engle for his insightful comments during the model-developing process. Of course, all remaining errors are solely my responsibility.

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The Journal of Futures Markets, Vol. 19, No. 8, 877–894 (1999)

CCC 0270-7314/99/080877-18

INTRODUCTION

The fundamental linkage between the S&P 500 cash and futures markets has always interested academics as well as practitioners—ever since the introduction of the stock index futures trading in 1982. The apparent divergence between the two markets during the "October '87 Crash" has raised tremendous concerns with respect to the economic function of the futures market as a price discovery and risk management tool. In recent literature, issues of a wide range about the S&P 500 cash and futures market relationship have been addressed, including the price and volatility lead–lag relationship (for example, see Herbst et al., 1987; Kawaller et al., 1987, 1990; Stoll & Whaley, 1990; Cheung & Ng, 1991; Chan et al., 1991; Chan, 1992; Koutmos, & Tucker, 1996), the price cointegration (for example, see Wahab & Leshagri, 1993; Wang & Yau, 1994), the impact of the "October '87 Crash" on price and volatility movements (for example, see Harris, 1989; Schwert, 1990; Kleidon & Whaley, 1992; Arshanapalli & Doukas, 1994), and the futures hedging issues related to the price integration and conditional covariance heteroskedasticity (for example, Myers, 1991; Ghosh, 1993; Lien & Luo, 1994).

To understand the long-run and short-run behaviors of individual cash and futures market movements and their interactions is crucial to studies that address the aforementioned issues. For example, the cointegration test originated by Engle and Granger (1987) is now used widely in studying the price comovement between the S&P cash and futures markets with growing understandings that their close relationship is of a long-run nature, and that this long-run relationship can be frequently disturbed by short-run deviations between the two markets. Thus the error correction model (ERM), the key concept addressing the long-run and short-run interactions, needs to be applied.

We would pose the question as to whether similar long-run and short-run interactions also exist in the volatility movements between the S&P 500 cash and futures markets. The time-varying stock market volatility behavior has been widely documented in the literature (for example, see French, Schwert, & Stambaugh, 1987; Chou, 1988; Nelson, 1989, 1990; Pagan & Schwert, 1990; and Poterba & Summers, 1986, for empirical evidence of historical stock volatility as well as Black-Scholes-type option-implied volatility). A recent study by Engle and Lee (1999) continued the investigation by decomposing the volatility movement into a long-run component and a short-run component. Engle and Lee's empirical studies showed that the stock market data including the S&P 500 index as well as individual stocks strongly support the volatility component structure. The main purpose of this article is to extend the component model of Engle and Lee (1999) into a multivariate framework and to pursue specific implications of the conditional covariance component structure between the S&P 500 cash and futures markets.

In the covariance component framework, two aspects seem to reflect the long-run cross-market relationship: one is the volatility "copersistence" relationship originally addressed by Bollerslev and Engle (1993); the other is the return correlation implied by the long-run covariance component or the "long-run return correlation"—in contrast to the "contemporary return correlation" implied by the time-varying covariance. We would like to seek in this article both the theoretical interpretation and the empirical evidence of these relationships in terms of the covariance component structure in the S&P 500 cash and futures markets.

One important application of the proposed covariance component structure is in studying the hedging issues between the cash and futures markets. Given the time-varying nature of the covariance movement in many financial markets, the classical assumption of the time-invariant optimal hedge ratio is apparently inappropriate. The time-varying covariance estimation method like the bivariate GARCH model has been used to estimate optimal futures hedge ratios (see Cecchetti, Cumby, & Figlewski, 1988; Baillie & Myers, 1991; Myers, 1991; Kroner & Sultan, 1993; Lien & Luo, 1994; and Park & Switzer, 1995, among others). Although these studies were successful in capturing the time-varying covariance/correlation features, many of them focused mainly on the one-period hedging problems. For those studies on the multi-period problems, the implications of the typical dynamics of the conditional covariance movement revealed by the models have not been fully exploited. With the covariance component model, the dynamics of the decomposed covariance movements can facilitate typically the studies on the multi-period hedging problems. In particular, the "long-run correlation" concept introduced may shed a new angle for studies on the long-term optimal hedge ratio problem.

SPECIFICATIONS OF THE CONDITIONAL COVARIANCE COMPONENT MODEL

Let $\{\mathbf{r}_t\}$ denote an $N \times 1$ vector of returns on the assets of interest, F_{t-1} as the historical asset return information set available at time $t - 1$, the conditional mean, $\mathbf{m}_t$, and the conditional covariance, $\mathbf{H}_t$, are defined by

$$\mathbf{m}_t \equiv E(\mathbf{r}_t | F_{t-1}) \qquad (2.1)$$

$$\mathbf{H}_t \equiv Var(\mathbf{r}_t | F_{t-1}) \equiv E_{t-1}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') \qquad (2.1)$$

where $\mathbf{m}_t$ is an $N \times 1$ vector, $\mathbf{H}_t$ is an $N \times N$ matrix which is almost surely positive definite. $\boldsymbol{\varepsilon}_t = (\mathbf{r}_t - \mathbf{m}_t)$ is an $N \times 1$ return residual vector. Because $\mathbf{H}_t$ and $\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'$ are symmetric matrices, only the upper (lower) triangle elements of the matrices are of interest. We adopt the vector-half operator, $\mathbf{vech}(\cdot)$, used in Baba, Engle, Kraft, and Kroner (1989), which stacks the upper (lower) triangle elements of a symmetric $N \times N$ matrix into an $(N + 1)N/2 \times 1$ vector. The conditional covariance thus can also be defined in vector form by

$$\mathbf{vech}(\mathbf{H}_t) = \mathbf{vech}(E_{t-1}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')) \tag{2.3}$$

Following the specification of Engle and Lee (1999) for the univariate conditional variance component model, the multivariate vector form of the component model for the conditional covariance is defined as

$$(\mathbf{vech}(\mathbf{H}_t) - \mathbf{vech}(\mathbf{Q}_t)) = (\alpha + \boldsymbol{\beta})(\mathbf{vech}(\mathbf{H}_{t-1}) - \mathbf{vech}(\mathbf{Q}_{t-1}))$$

$$+ \alpha\,(\mathbf{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') - \mathbf{vech}(\mathbf{H}_{t-1})) \tag{2.4}$$

$$\mathbf{vech}(\mathbf{Q}_t) = \boldsymbol{\omega} + \boldsymbol{\rho}\mathbf{vech}(\mathbf{Q}_{t-1}) + \phi(\mathbf{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')$$

$$- \mathbf{vech}(\mathbf{H}_{t-1})) \tag{2.5}$$

where $\mathbf{Q}_t$ is a symmetric positive-definite $N \times N$ matrix. The model in eqs. (2.4) (2.5) decomposes the covariance into two components: $\mathbf{Q}_t$ represents the long-run (trend) component of $\mathbf{H}_t$; $(\mathbf{H}_t - \mathbf{Q}_t)$ is the deviation of the covariance from its long-run component representing the short-run (transitory) component of the covariance movement; both are subject to market changes, $\mathbf{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')$, over time. The dynamics of the long-run and short-run components are specified by two mean-reverting processes, which are characterized by $\boldsymbol{\rho}$ and $(\alpha + \boldsymbol{\beta})$, respectively, with ϕ and α defining the sensitivities of the long-run and short-run components to market changes, respectively.

PROPERTIES OF THE CONDITIONAL COVARIANCE COMPONENT MODEL

Decomposition to the Conditional Covariance

The properties of the decomposition in eqs. (2.4) (2.5) to the conditional covariance can be best illustrated by the forecast profile of the model. Define the k-step conditional covariance expectation as

$$\mathbf{vech}(\mathbf{H}_{t+k}) \equiv E_{t-1}(\mathbf{vech}(\boldsymbol{\varepsilon}_{t+k}\boldsymbol{\varepsilon}'_{t+k})) \tag{3.1}$$

and $\mathbf{Q}_{t+k}$ as the trend of $\mathbf{H}_{t+k}$. The multi-step expectation of the short-run component is

$$(\mathbf{vech}(\mathbf{H}_{t+k}) - \mathbf{vech}(\mathbf{Q}_{t+k})) = (\boldsymbol{\alpha} + \boldsymbol{\beta})(\mathbf{vech}(\mathbf{H}_{t+k-1}) - \mathbf{vech}(\mathbf{Q}_{t+k-1})) = (\boldsymbol{\alpha} + \boldsymbol{\beta})^k (\mathbf{vech}(\mathbf{H}_t) - \mathbf{vech}(\mathbf{Q}_t)) \tag{3.2}$$

using eqs. (2.4) (3.1). Assuming that $(\boldsymbol{\alpha} + \boldsymbol{\beta})$ is non-singular with distinct eigenvalues, we have by the Jordan decomposition that

$$(\mathbf{vech}(\mathbf{H}_{t+k}) - \mathbf{vech}(\mathbf{Q}_{t+k})) = \mathbf{P}_{(\alpha+\beta)} \boldsymbol{\Lambda}^k_{(\alpha+\beta)} \mathbf{P}^{-1}_{(\alpha+\beta)} (\mathbf{vech}(\mathbf{H}_t) - \mathbf{vech}(\mathbf{Q}_t)) \tag{3.3}$$

where $\boldsymbol{\Lambda}_{(\alpha+\beta)}$ is an $((N + 1)N/2) \times ((N + 1)N/2)$ diagonal matrix with the diagonal elements being the eigenvalues of $(\boldsymbol{\alpha} + \boldsymbol{\beta})$, $\mathbf{P}_{(\alpha+\beta)}$ is an $((N + 1)N/2) \times ((N + 1)N/2)$ non-singular matrix with the columns being the corresponding eigenvectors. If some eigenvalues of $(\boldsymbol{\alpha} + \boldsymbol{\beta})$ coincide, $\boldsymbol{\Lambda}_{(\alpha+\beta)}$ will be upper triangular with the eigenvalues along the diagonal. For ease of exposure, we discuss here the case of distinct eigenvalues.

Now let us turn to the long-run component in eq. (2.5). Following the same recursive rule used above, the multi-step forecast of $\mathbf{Q}_t$ becomes

$$\mathbf{vech}(\mathbf{Q}_{t+k}) = \boldsymbol{\omega} + \boldsymbol{\rho}\mathbf{vech}(\mathbf{Q}_{t+k-1}) = (\mathbf{I} + \boldsymbol{\rho} + \boldsymbol{\rho}^2 + \cdots + \boldsymbol{\rho}^{k-1})\, \boldsymbol{\omega} + \boldsymbol{\rho}^k\mathbf{vech}(\mathbf{Q}_t) \tag{3.4}$$

Again, assuming that $\boldsymbol{\rho}$ is non-singular with distinct eigenvalues, eq. (3.4) can be written as,

$$\mathbf{vech}(\mathbf{Q}_{t+k}) = (\mathbf{I} + \mathbf{P}_\rho\boldsymbol{\Lambda}_\rho\mathbf{P}_\rho^{-1} + \cdots + \mathbf{P}_\rho\boldsymbol{\Lambda}_\rho^{k-1}\mathbf{P}_\rho^{-1})\, \boldsymbol{\omega} + \mathbf{P}_\rho\boldsymbol{\Lambda}_\rho^k\mathbf{P}_\rho^{-1}\mathbf{vech}(\mathbf{Q}_t) \tag{3.5}$$

where $\boldsymbol{\Lambda}^\rho$ is the diagonal matrix of the eigenvalues of $\boldsymbol{\rho}$, $\mathbf{P}^\rho$ is the matrix of the corresponding eigenvectors.

Assume that both components are mean-reverting, which requires that $|\lambda_{\rho,\max}| < 1$ and $|\lambda_{(\alpha+\beta),\max}| < 1$, the mean-reversion of the conditional covariance components can be summarized by

$$(\mathbf{vech}(\mathbf{H}_{t+k}) - \mathbf{vech}(\mathbf{Q}_{t+k})) \Rightarrow \mathbf{0} \tag{3.6}$$

$$\mathbf{vech}(\mathbf{Q}_{t+k}) \Rightarrow (\mathbf{I} - \boldsymbol{\rho})^{-1}\boldsymbol{\omega} \tag{3.7}$$

Furthermore, we would define that the largest eigenvalue of the long-run component is larger than that of the short-run component (that is, $|\lambda_{(\alpha+\beta),\max}| < |\lambda_{\rho,\max}| < 1$), the mean-reversion of the conditional covariance thus becomes

$$\mathbf{vech}(\mathbf{H}_{t+k}) \Rightarrow \mathbf{vech}(\mathbf{Q}_{t+k}) \Rightarrow (\mathbf{I} - \boldsymbol{\rho})^{-1}\boldsymbol{\omega} \tag{3.8}$$

Eq. (3.8) states that any deviations of $\mathbf{H}_t$ from $\mathbf{Q}_t$ will die out first, as the forecast horizon extends, and the conditional covariance will converge to its long-run trend, which itself mean-reverts slowly to a long-run average level of $(\mathbf{I} - \boldsymbol{\rho})^{-1}\boldsymbol{\omega}$. Exactly in this sense, we interpret $\mathbf{Q}_t$ as the trend of the covariance movement and $(\mathbf{H}_t - \mathbf{Q}_t)$ as the transitory component. In the appendix of this article, we show the rigorous derivation of the stationary conditions of the conditional covariance component of (2.4) (2.5), which are consistent with the mean-reversion conditions discussed in this section.

Copersistence Relationship

Many empirical studies on stock market volatility, either the historical volatility or the Black-Scholes-type option implied volatility, found that stock market volatility is highly persistent over time (for example, see French, Schwert, & Stambaugh, 1987; Chou, 1988; Nelson, 1989, 1990b; Pagan & Schwert, 1990; Poterba & Summers, 1986; Bollerslev & Engle, 1987; and Engle & Lee, 1999). Empirically, the volatility persistence pattern shows up in the close-to-unit mean-reverting rates of the volatility dynamics—for example, the estimations of the mean-reverting rate of the long-run volatility component in Engle and Lee (1999) are shown to be above 0.98 for the S&P 500 index, the CRSP value-weighted index, the NIKKEI index and most of the individual stock data. It is not necessary to restrict the persistence feature only to the integrated process.[1] With these close-to-unit mean-reverting rates, the market shock effect on conditional variance expectation decays extremely slowly, so a high persistence exists in the market volatility movement.

The copersistence problem, first addressed in Bollerslev and Engle (1993), now turns to find a linear relationship of the return series of interest, which can dramatically reduce the volatility persistence level

[1]The cases with unit-roots are much more complicated for the conditional variance/covariance process. For example, Nelson (1990) proved that the second moment does not exist for the IGARCH(1,1) process but the mean process may still be strictly stationary and ergodic (see Engle & Lee, 1999, for more detailed discussions). From the application point of view, the stationary or mean-reverting case is discussed in this article.

shown in individual series. This relationship exists only when the two return series share a common long-run volatility trend.

Let an $N \times 1$ vector, $\boldsymbol{\gamma}$, define the linear combination of asset returns, the conditional variance forecast of the linear combination $\{\boldsymbol{\gamma}'\mathbf{r}_t\}$, under the assumption of a slower mean-reverting long-run covariance component, becomes

$$\begin{aligned}
\gamma'\mathbf{H}_{t+k}\gamma &= \mathbf{vec2}(\gamma)'\mathbf{vech}(\mathbf{H}_{t+k}) \\
&\Rightarrow \mathbf{vec2}(\gamma)'\mathbf{vech}(\mathbf{Q}_{t+k}) \\
&= \mathbf{vec2}(\gamma)'(\mathbf{I} + \rho + \cdots + \rho^{k-1})\,\omega \\
&\quad + \mathbf{vec2}(\gamma)'\rho^{k}\mathbf{vech}(\mathbf{Q})_t) \\
&= [\mathbf{vec2}(\gamma)'\mathbf{P}_\rho][(\mathbf{P}_\rho^{-1} + \Lambda_\rho\mathbf{P}_\rho^{-1} + \cdots + \Lambda_\rho^{k-1}\mathbf{P}_\rho^{-1})\omega \\
&\quad + \Lambda_\rho^{k}\mathbf{P}_\rho^{-1}\mathbf{vech}(\mathbf{Q})_t)]
\end{aligned} \tag{3.9}$$

using a vector quadratic operator, $\mathbf{vec2}(\cdot)$.[2] Persistence is a long-run characteristic of volatility movements and a common persistence implies common long-run volatility trends. So it is natural to see in eq. (3.9) that the examination of the copersistence relationship is solely targeted towards the long-run covariance trend.

The copersistence relationship should be chosen such that the variance of $\{\boldsymbol{\gamma}'\mathbf{r}_t\}$ has a mean-reverting rate less than the maximum rate of the trend, $\lambda_{,max}$. It is obvious that a relationship should hold that

$$(\mathbf{vec2}(\boldsymbol{\gamma})'\mathbf{p}_{\rho,max}) = 0 \tag{3.10}$$

where $\mathbf{p}_{\rho,max}$ is the eigenvector corresponding to $\lambda_{\rho,max}$. So the volatility persistence of $\mathbf{Q}_t$ due to $\lambda_{\rho,max}$ will not be carried over to the linear combination. In the case that there are several close-to-unit eigenvalues, the derivation can be easily extended by requiring $(\mathbf{vec2}(\boldsymbol{\gamma})'\mathbf{p}_{\rho,i}) = 0$ for each eigenvalue targeted. Note that the copersistence vector is not guaranteed but depends on whether there exist solutions in the linear system (3.10).

Long-Run Correlation

If the asset return covariance is time varying, the correlation between the returns may be time varying as well. The covariance component model

[2]The vector quadratic operator, $\mathbf{vec2}(\ )$ for an $N \times 1$ vector $\boldsymbol{\gamma}$ is defined as $\mathbf{vech}(2\boldsymbol{\gamma}\boldsymbol{\gamma}' - diag(\boldsymbol{\gamma})diag(\boldsymbol{\gamma}))$. The case with $N = 2$ is that $\mathbf{vec2}((\gamma_{1},\gamma_2)') = (\gamma_1{}^2, 2\gamma_1\gamma_2, \gamma_2{}^2)$.

provides a way to examine the time-varying features of the correlation movement. The contemporary correlation can be calculated from the conditional covariance estimation

$$Corr_{ij,t} = h_{ij,t}/(h_{ii,t}^{1/2}h_{jj,t}^{1/2}) \tag{3.11}$$

Of more importance, the decomposition to the covariance in the model provides a way to examine the long-run features of the correlation movement. A new measure of correlation, called the "long-run correlation," can be calculated from the long-run component of the conditional covariance,

$$LR\text{-}Corr_{ij,t} = q_{ij,t}/(q_{ii,t}^{1/2}q_{jj,t}^{1/2}) \tag{3.12}$$

It should be noted that the long-run correlation is just a new measure of the correlation relationship. It is not necessarily to imply the copersistence or cointegration relationship. With strong cointegration or copersistence between two markets, however, high and stable long-run correlations between the two markets should be evident.

HEDGING ISSUES USING THE CONDITIONAL COVARIANCE COMPONENT MODEL

Consider a multi-asset investment problem of T period. The portfolio consists of assets with the returns denoted by $\mathbf{r}_t$, the conditional return covariance by $\mathbf{H}_t$, and the portfolio weights by $\mathbf{w}$. The terminal wealth of the portfolio, W_T, can be expressed as

$$W_T = W_{t-1} + (\mathbf{w}'\mathbf{r}_t + \mathbf{w}'\mathbf{r}_{t+1} + \cdots + \mathbf{w}'\mathbf{r}_T) \tag{4.1}$$

The conditional variance of the terminal portfolio wealth changes thus can be expressed as

$$\begin{aligned} Var[(W_T - W_{t-1})|F_{t-1}] &= \mathbf{w}'\mathbf{H}_t\mathbf{w} + \mathbf{w}'\mathbf{H}_{t+1}\mathbf{w} \\ &+ \cdots + \mathbf{w}'\mathbf{H}_T\,\mathbf{w} = \mathbf{vec2}(\mathbf{w})'(\mathbf{vech}(\mathbf{H}_t) \\ &+ \mathbf{vech}(\mathbf{H}_{t+1}) + \cdots + \mathbf{vech}(\mathbf{H}_T)) \end{aligned} \tag{4.2}$$

under the assumption of no significant serial correlation in $\{\mathbf{r}_t\}$.

Multi-Period Minimum Variance Hedging

The minimum variance hedging is the most popular portfolio risk control strategy used. The optimal hedge ratio is the solution to the following optimization problem,

$$\underset{w}{\text{Min}}\ \text{Var}[(W_T - W_{t-1})|F_{t-1}] \quad (4.3)$$

with respect to the portfolio weight vector, **w**.

When a two-asset problem is of interest, the solution to (4.3) becomes

$$w_t^* = h_{12,t}^*/h_{22,t}^* \quad ((4.4)$$

with $\mathbf{w}_t = (1, w_t^*)$ now is the weight vector and $h_{ij,t}^*$ is the element of the resulting matrix of $(\mathbf{vech}(\mathbf{H}_t) + \mathbf{vech}(\mathbf{H}_{t+1}) + \cdots + \mathbf{vech}(\mathbf{H}_T))$. It is easy to see that, for a one-period two-asset problem, (4.4) becomes the classical minimum variance hedging ratio

$$w_t^* = h_{12,t}/h_{22,t} = corr_{12,t}h_{11,t}^{1/2}/h_{22,t}^{1/2} \quad (4.5)$$

Apparently, the one-period optimal hedge ratio (4.5), would no longer be optimal when a multi-period horizon is of the concern. In the multi-period case, the dynamic structure of the covariance/correlation movements will explicitly affect the solution to the optimization problem.

Long-Term Hedging Using the Long-Run Covariance Component Estimation

If the short-run covariance movement is merely reflecting recent market fluctuations and short-lived, the hedging strategy that incorporates this short-run information, like that of eq. (4.4), can be highly unstable—and thus becomes costly and ineffective when a long-term investment horizon is under consideration. On the other hand, the long-term hedging strategy should really focus on the covariance trend if the fundamental relationships among different assets are reflected by the covariance trend movement.

The idea can be illustrated by the properties of the covariance component model proposed. With the assumption that the long-run covariance component is much more persistent than the short-run component, the variance of the terminal wealth changes of the portfolio in (4.2) can be well approximated by

$$Var[(W_T - W_{t-1})|F_{t-1} \approx \mathbf{vec2}(\mathbf{w})'(\mathbf{vech}(\mathbf{Q}_t) + \mathbf{vech}(\mathbf{Q}_{t+1}) + \cdots + \mathbf{vech}(\mathbf{Q}_T)) \quad (4.6)$$

when T is long enough relative to the half-life decay time of the short-run covariance component.

TABLE I

The Descriptive Statistics of Daily Returns on the S&P 500 Index and the Index Futures 1982.1–1990.12[a]

	Mean	*St. Dev.*	*Skew*	*Kurtosis*	*Cross Corr.*
Cash	0.1726e-5	0.0110	−4.33	92.55	0.9381
Futures	0.1971e-5	0.0138	−5.914	174.5	

EMPIRICAL RESULTS FOR THE S&P 500 CASH AND FUTURES MARKETS

The empirical studies in this article focus on describing the long-run and short-run behaviors of the covariance movement between the S&P 500 cash index and the S&P 500 index futures, including the contemporary and long-run correlations and the copersistence relationship.

Data

The data used is the daily closing S&P 500 cash index and the S&P 500 index futures data for the period 1982.1–1990.12. For the futures, only the data from the nearby contracts is used because the trading is the most active. The daily returns are constructed by the log difference of the daily closing prices on two consecutive business days, adjusted for the cost of carry using the U.S. three-month treasury bill rate. To concentrate on the covariance estimation, the return process is assumed to have a constant expected return.

Table I shows the descriptive statistics of the returns on the S&P 500 stock index and the index futures. The two markets are highly correlated with the (unconditional) correlation of 0.9381; the futures market has higher volatility than the cash market; the distributions of both markets have skewness and excess kurtosis—a typical pattern in many financial markets.

Estimation Results of the Conditional Covariance Component Model

The full specification of the bivariate covariance component model of (2.4) (2.5) requires estimations of total 39 parameters, of which some may not be statistically significant. After some preliminary experiments, the Quasi Maximum Likelihood Estimation (QMLE) results are as follows[3]

$$
\begin{aligned}
\mathbf{Vech(H_t)} &= \mathbf{Vech(Q_t)} \\
&+ \begin{bmatrix} 0.158 & -0.052 & 0.000 \\ (0.022) & (0.036) & (—) \\ 0.000 & 0.292 & -0.152 \\ (—) & (0.070) & (0.049) \\ 0.000 & 0.337 & -0.136 \\ (—) & (0.120) & (0.089) \end{bmatrix}_{\alpha} (\mathbf{Vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) - \mathbf{Vech(Q_{t-1})}) \\
&+ \begin{bmatrix} 0.578 & -0.000 & 0.000 \\ (0.033) & (—) & (—) \\ 0.000 & 0.581 & 0.000 \\ (—) & (0.031) & (—) \\ 0.000 & 0.000 & 0.544 \\ (—) & (—) & (0.049) \end{bmatrix}_{\beta} (\mathbf{Vech(H_{t-1})} - \mathbf{Vech(Q_{t-1})})
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{Vech(Q_t)} &= \begin{bmatrix} 0.173e-5 \\ (0.076e-5 \\ 0.154e-5 \\ (0.114e-5 \\ 0.197e-5 \\ (0.161e-5) \end{bmatrix}_{\omega} \\
&+ \begin{bmatrix} 1.289 & -0.393 & 0.085 \\ (0.146) & (0.183) & (0.049) \\ 0.552 & 0.252 & 0.174 \\ (0.115) & (0.131) & (0.048) \\ 0.790 & -1.065 & 1.248 \\ (0.151) & (0.160) & (0.063) \end{bmatrix}_{\rho} (\mathbf{Vech(Q_{t-1})}) \\
&+ \begin{bmatrix} -0.041 & 0.000 & 0.039 \\ (0.024) & (—) & (0.018) \\ -0.009 & -0.102 & -0.103 \\ (0.034) & (0.077) & (0.064) \\ 0.000 & -0.166 & 0.154 \\ (—) & (0.137) & (0.101) \end{bmatrix}_{\phi} (\mathbf{Vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) - \mathbf{Vech(H_{t-1})})
\end{aligned}
$$

The lead–lag relationship in the volatility movements between the S&P 500 cash and futures markets has been examined by several studies in the literature (for example, see Cheung & Ng, 1991; Chan, Chan, & Karolyi, 1991). In the component model, the analysis of this lead–lag relationship lies on examining the structure of α and ϕ. If α and ϕ are upper-triangular (lower-triangular) matrices, then there is a lead of the volatility information flow from futures (cash) market to the cash (futures) market. The empirical results here indicate that such a lead or lag

[3]The estimation method is the QMLE method suggested by Engle (1992) and Bollerslev (1986). In the parenthesis are the standard errors of the estimates.

of information flow between the two markets is not supported by the data. The two markets actually interact with each other by showing some statistically significantly off-diagonal elements of α and ϕ. Similar results are found in Chan, Chan, and Karolyi (1991), studying the intraday volatility using the multivariate GARCH model.

Now we turn to discuss the persistence feature of the covariance components. The eigenvalues of the autoregressive parameter matrices of the components, $\boldsymbol{\rho}$ and $(\alpha + \boldsymbol{\beta})$, are as the follows,

$$\lambda_{(\alpha+\beta)} = (0.7364,\ 0.6928,\ 0.5876)'$$

$$\lambda_{\rho} = (0.9968,\ 0.9712,\ 0.8217)'$$

The largest eigenvalue of the long-run component is very close to one, 0.9968, indicating that the covariance/variances of the S&P 500 cash and futures markets are highly persistent, jointly. The largest eigenvalue of the short-run component, 0.7364, is much smaller, even smaller than the smallest eigenvalue of the trend, 0.8217. This implies a big gap between the long-run component and the short-run component in terms of the speed at which the components mean-revert.

Empirical Results of the Copersistence Vector

Now the question becomes whether there exists a copersistence relationship between the S&P 500 cash and futures markets to represent the common long-run volatility trend. With the eigenvector, (1.0, 0.7583, 0.0718)′, for the largest eigenvalue of $\boldsymbol{\rho}$, solving eq. (3.10) delivers two solutions,

$$\gamma = (\gamma_1, \gamma_2) = (1.00,\ -0.68)' \text{ or } (1.00,\ -20.0)'$$

Both solutions will eliminate the persistence in the corresponding linear combination of the returns, but only the first one seems to make sense. From the economic point of view, the copersistence vector should be around (−1.0, 1.0) if, in an ideal situation, the cash market and the futures market are perfectly correlated and have the same volatility level. In reality, as shown in Table I, the futures market is more volatile and the idiosyncratic noises in both the markets cause the correlation to be less than perfect.

To give a final examination, we estimate the univariate component model again using the linear combination of the returns based on the copersistence vector estimated above. It is not surprising the component model is no longer a good-fitting model because the volatility of the linear combination is now more of a temporary nature. The estimation of the simple GARCH(1,1) model of Bollerslev (1986) shows the mean-reverting rate, $(\alpha + \beta)$, of only 0.7934, indicating that the volatility persistence level has been largely reduced through the linear combination.

Long-Run Correlation Between the S&P 500 Cash and Futures Markets

In Figure 1, we plot the series of the contemporary correlations and the long-run correlations derived from the estimations of the covariance components over the entire sample period. There are several interesting points worthy of discussion. First, the long-run correlation between the two markets varies over time but remains very high, over 0.93 on average, close to the unconditional correlation. This indicates that the cash market and the futures market of the S&P 500 index are found not to be fundamentally different. Secondly, the temporary correlation between the two markets varies over time and it can go as low as around 20%. This means the two markets can deviate from each other considerably over certain occasions, even if they move in the same direction over the long run. If we look into the pattern of these deviations, we also found that the departure or divergence is only temporary as suggested by the strong mean-reverting feature of the transitory covariance component.

The arbitrage theory argues that arbitrage activities by market participants lead to a close alignment between of the two markets. Therefore, theoretically, it is easy to claim that the cash market and the futures market should be perfectly correlated. However, the differences between the two markets do exist, ranging from micro market mechanisms like market liquidity constraints to market participant status. Such factors are often put forth, as the reasons cash and futures markets sometimes diverge resulting low correlations like the ones shown in the graph. However, the temporary behavior of the divergence and the close-to-perfect long-run correlation is consistent with the claim that there is no fundamental difference between the S&P 500 stock index and the S&P 500 stock index futures markets.

Another issue that needs to be noted is the occasion when serious divergence of the two markets occur. It can be seen from the graph that most of the divergence occurs when the markets experience severe shocks; the worst case, where the correlation is around only 20%, occurred during the "October '87 Crash." The breakdown of the fundamental linkage between the S&P 500 cash and futures markets during

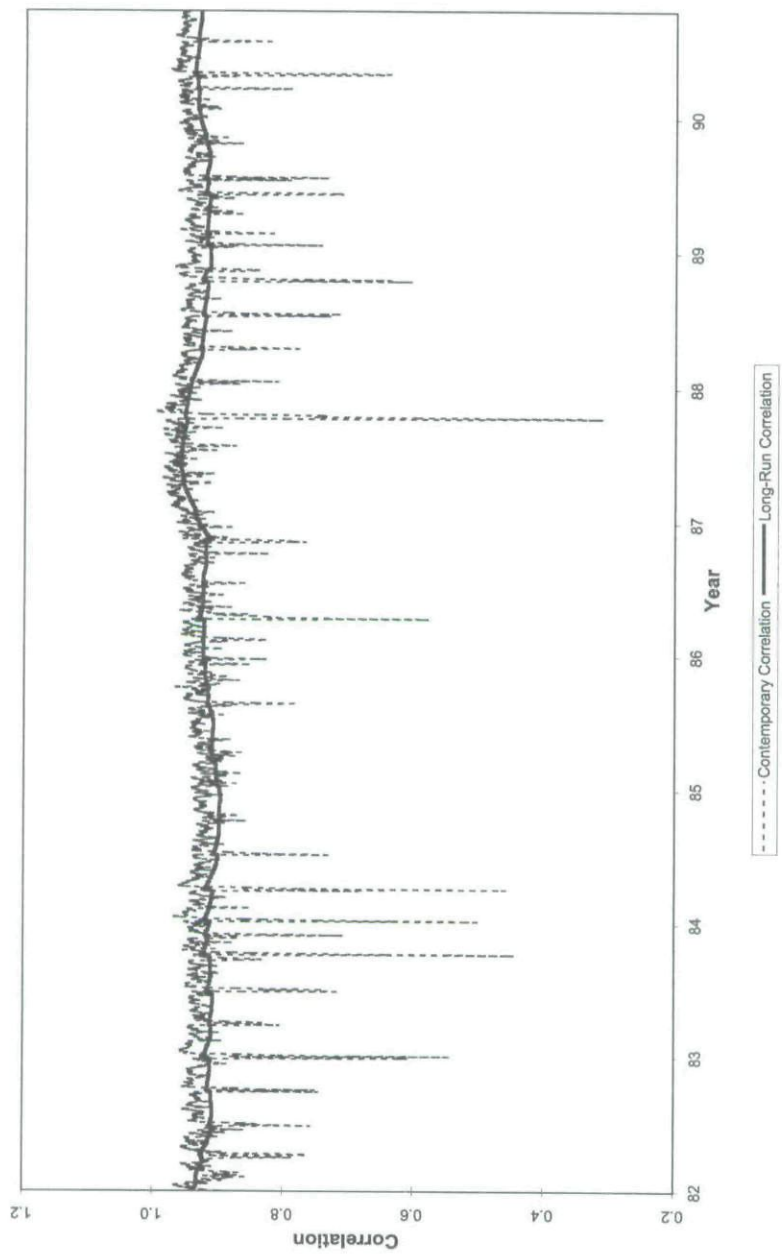

FIGURE 1
Contemporary and Long-Run Correlations between S&P 500 Cash and Futures Markets.

the "October '87 Crash" has brought serious concerns to market participants, including the regulators, about the fundamental functionality of the futures market (see the Brady Commission Report, 1988; the NYSE Report on Market Volatility and Investor Confidence, 1990; and the research in Kleidon & Whaley, 1992, among many others). Many researchers have suggested the microstructure difference between the cash and futures markets contributes much to the breakdown of the normal relation when facing serious shocks. The empirical estimation of the conditional covariance component here is capable of providing a quantitative measure of the divergence through estimating the contemporary correlation between the two markets.

CONCLUSIONS

In this article, we extend the component model of Engle and Lee (1999) into a multivariate frame, in which the conditional covariance matrix is also decomposed into the transitory and the permanent component. The decomposition can benefit many studies of the long-run relationship of asset return volatility by focusing on the long-run trend component. The empirical studies on the S&P 500 stock index and the index futures reveal that not only do the two markets share a common long-run trend in volatility, but they also share a common long-run trend in returns through the long-run correlation analysis derived from the permanent component of the covariance between the two markets.

APPENDIX

The Stationarity Conditions of the Conditional Covariance Component Model

The reduced form of the conditional covariance process defined in the component model (2.4) (2.5) can be shown as

$$\begin{aligned}\mathbf{vech}(\mathbf{H}_t) = \omega^* + \alpha_1^*\mathbf{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \mathbf{vech}(\varepsilon_{t-2}\varepsilon'_{t-2}) \\ + \beta_1^*\mathbf{vech}(\mathbf{H}_{t-1}) + \mathbf{vech}_2^*\mathbf{vech}(\mathbf{H}_{t-2}) \qquad (A.1)\end{aligned}$$

which is a multivariate GARCH(2,2) process with the parameters

$$\alpha_1^* = \alpha + \phi \qquad (A.2)$$

$$\begin{aligned}\alpha_2^* = -(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})\,\alpha - (\boldsymbol{\rho} - \alpha - \boldsymbol{\beta}) \\ (\alpha + \boldsymbol{\beta})(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})^{-1}(\alpha + \phi) \qquad (A.3)\end{aligned}$$

$$\beta_1^* = \boldsymbol{\rho} - \alpha - \phi + (\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})(\alpha + \boldsymbol{\beta})(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})^{-1} \quad (A.4)$$

$$\beta_2^* = -(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})\,\boldsymbol{\beta} - (\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})$$

$$(\alpha + \boldsymbol{\beta})(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})^{-1}(\alpha + \phi) \quad (A.5)$$

Applying Theorem 1 in Bollerslev and Engle (1993) to eq. (3.9), the stationarity conditions for $\{\mathbf{r}_t\}$ to be covariance stationary in the conditional covariance component model is that the roots of the characteristic polynomial

$$det[\lambda^2\, \mathbf{I} - \lambda(\alpha_1^* + \beta_1^*) - (\alpha_2^* + \beta_2^*)] = 0 \quad (A.5)$$

lie inside the unit circle. (A.5) implies that

$$det[\lambda \mathbf{I} - (\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})(\alpha + \boldsymbol{\beta})(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})^{-1}] = 0 \quad (A.7)$$

$$det[\lambda \mathbf{I} - \boldsymbol{\rho}] = 0 \quad (A.7)$$

It is easy to see that the solutions to eqs. (A.6) (A.7) actually are the eigenvalues of $(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})(\alpha + \boldsymbol{\beta})(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})^{-1}$ and $\boldsymbol{\rho}$. Note that the eigenvalues of $(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})(\alpha + \boldsymbol{\beta})(\boldsymbol{\rho} - \alpha - \boldsymbol{\beta})^{-1}$ are the same as that of $(\alpha + \boldsymbol{\beta})$. Therefore, the stationarity conditions of the conditional covariance component model are that all the eigenvalues of $(\alpha + \boldsymbol{\beta})$ and $\boldsymbol{\rho}$ lying inside the unit circle.

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