Valuation of Futures and Commodity Options with Information Costs

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In this article, futures and commodity options are analyzed in the context of Merton's (1987) model of capital market equilibrium with incomplete information. First, following Dusak (1973) and Black (1976), the conditions under which Merton's model can be applied to the valuation of forward and futures contracts are proposed. Then an application to futures markets is given. We provide a partial differential equation and the formulas for European commodity options, futures contracts, and American options in the same context. The models are simulated and compared to standard models with no information costs. We find that model prices are not significantly different from standard model prices. However, our models correct for some pricing biases in standard models. In particular, they reduce the overvaluation bias for European and American commodity options. It seems that the costs of gathering and processing information regarding the option and its underlying asset play a role in explaining the biases observed in standard models. This work can be applied to other futures markets. © 1999 John Wiley & Sons, Inc. Jrl Fut Mark 19: 645–664, 1999

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INTRODUCTION

The first specific option pricing model for the valuation of futures options is introduced by Black (1976). Black (1976) derives the formula for futures and forward contracts and options under the assumption that investors create riskless hedges between options and the futures or forward contracts. The formula relies implicitly on the CAPM.[1] Since then, biases are reported by several authors. Black's model is used in Barone-Adesi and Whaley (1987) for the valuation of American commodity options. This model is referred to as the BAW model.[2]

Information plays a key role in the determination of asset prices in capital markets. Merton (1987) develops a model of capital market equilibrium with incomplete information and studies the equilibrium structure of asset prices and its connection with empirical anomalies in financial markets in the same context. By introducing information costs, this article examines the effects of these costs on the risk premium in futures contracts and options values. This work may be justified by the reported anomalous evidence, which has stimulated research to make explicit the limitations of the basic finance models with complete information. The valuation of futures contracts is important for market participants who implement different strategies in these markets where information costs are expected to be important. The derivation of an option pricing model with information costs might help to understand why the Black (1976) and BAW's (1987) type models, lead to theoretical prices that are systematically biased.

This article develops a valuation framework for forward and futures contracts as well as options on these contracts in the presence of information costs. The models might offer explanations for the pricing biases in Black's and BAW's models, and especially for the relative pricing of in and out-of-the money options.[3] Because these models have been tested in several empirical studies, they are used as a benchmark for our simulations.

[1]Futures markets are not different in principle from the market for any other asset. The returns on any risky asset are governed by the asset contribution to the risk of a well-diversified portfolio. The classic CAPM is applied by Dusak (1973) in the analysis of the risk premium and the valuation of futures contracts.

[2]The Barone-Adesi and Whaley (1987) model has been used recently in an empirical work done by Melick and Thomas (1997) to recover an asset's implied PDF from option prices. Some biases are reported in that study using BAW's model for crude oil prices.

[3]We thank the reviewer for this precision.

APPLICATION OF MERTON'S MODEL TO FORWARD AND FUTURES CONTRACTS

This section reviews Merton's model, defines the exact nature of information costs and examines the conditions under which the CAPMI can be applied to the valuation of forward and futures contracts.

Merton's Model

Merton's model is a two period model of capital market equilibrium in an economy where each investor has information about only a subset of the available securities. The main assumption in the Mertons model is that an investor includes a security S in his portfolio only if he has some information about the first and second moment of the distribution of its returns. Information costs have two components: the costs of gathering and processing data for the analysis and the valuation of financial assets, and the costs of information transmission. Merton's model may be stated as follows:

$$\tilde{R}_S - r = \beta_S[\tilde{R}_m - r] + \lambda_S - \beta_S\lambda_m \qquad (1)$$

where:

- $\tilde{R}_S$: the equilibrium expected return on security S,
- $\tilde{R}_m$: the equilibrium expected return on the market portfolio,
- r: one plus the riskless rate of interest,
- $\beta_S = \text{cov}(\tilde{R}_S/\tilde{R}_m)/\text{var}(\tilde{R}_m)$: the beta of security S,
- λ_S: the equilibrium aggregate "shadow cost" for the security S,
- λ_m: the weighted average shadow cost of incomplete information over all securities in the market place.

The CAPMI is an extension of the CAPM to a context of incomplete information. Note that when $\lambda_m = \lambda_S = 0$, this model reduces to the standard CAPM.

The Role of Information Costs

Consider an investor who intends to buy or sell a futures contract or an option on that contract. The investor needs to buy the necessary information for the analysis and the valuation of these financial instruments. This includes a data base, the elaboration of the models, the costs of gathering and treating this information, the costs of implementing the valuation models and the resulting appropriate strategies, and so forth. All these costs might have an effect on the distribution of returns of financial assets.

From Merton's model (1987), it appears that taking into account the effect of information costs on the equilibrium price of an asset is similar to applying an additional discount rate to this asset's future cash flows. These costs enter the portfolios of individual and institutional investors, who spend huge amounts of money in research and development, financial engineering and reingineering, information analysis, the management of financial risks, and the like. An investor dealing with these contracts would bear at least some costs, which must be accounted for when constructing a hedge or adjusting a portfolio. The information costs regarding the decision of trading, the allocation and adjustments of the portfolios in forward and futures markets, are justified on the grounds of the tradeoff between risk and return. These costs enable the financial institution to better understand and hedge its financial risk, which implies a higher required expected return from the investment in these markets. The importance of information costs varies from one market to another according to the degree of sophistication of the financial contract.[4]

In all cases, an investor who does not know about the existence of a financial asset or a trading opportunity in the marketplace will not include it in his portfolio strategies. The choice of a financial asset, the valuation models, the use of an asset in portfolio strategies and risk management, the microstructure of the market place, and so forth imply the existence of information costs. When these costs are accounted for in financial models, the resulting models may have the potential to explain some anomalies reported when using standard costless models.

An Application of the Model

The major difficulty arising from the application of the CAPMI to the risk return relationship on a capital asset comes from the definition of the appropriate rate of return. More specifically, Merton's model cannot be directly applied to the futures contract because the initial value of the contract is zero. However, it is possible to take the margin as the capital investment and treat the ratio of the net profit at closeout to the initial

[4]Contrary to spot assets, some financial contracts are difficult to analyze and to price. This is the case for the Treasury bond futures contract with its embedded delivery options: quality options, wildcard options, timing options, and so forth. The analysis, valuation, and trading of such contracts imply the existence of information costs. These costs exist also in options markets for the same reasons. For more details, see, for example, Bellalah (1990), Briys, Bellalah, et al. (1998).

margin as the rate of return on investment. Although this procedure is appealing, the margin in the sense of Merton's model can not be considered as a portfolio asset. As the initial investment in the futures contract is zero, it is not possible to talk about the percentage or fractional change on the investor's position in the futures market.

Following the work of Dusak (1973), when applying the classic CAPM, the percentage change in the futures price is used as a candidate because it can always be computed. However, the percentage change in the future price cannot be interpreted as a rate of return comparable to the $\tilde{R}_S$, because the holder of the position does not invest current resources in the contract. It can rather be seen as a risk premium $(\tilde{R}_S - r)$ on the underlying spot assets. In fact, the buyer of the futures contract takes the risk and has no capital on his own invested. Hence he earns no interest on the capital.

Merton's equilibrium conditions can then be restated by saying that the expected return on any asset S, is written as:

$$E(\tilde{R}_S) = r(1 - \beta_S) + \lambda_S + \beta_S(E(\tilde{R}_m) - \lambda_m) \tag{2}$$

where $E(\tilde{R}_S)$ can be represented in terms of periods 0 and 1 prices for the asset S as: $E(S_0 - S_0)/S_0$. The equilibrium risk-return relationship on asset S can then be expressed as:

$$S_0 = E(S_1) - \beta_S(E(\tilde{R}_m) - r)S_0 - \lambda_S S_0 + \lambda_m \beta_S S_0 \tag{3}$$

where S_0 is the price of asset S in the start of period and S_1 is the asset's price at the end of the period.

In present value form, the above equation is equivalent to:

$$S_0 = \frac{E(S_1) - \beta_S(E(\tilde{R}_m) - r)S_0 \div \lambda_S S_0 + \lambda_m \beta_S S_0}{(1 + r)} \tag{4}$$

On the other hand, the current price for asset S under an agreement to buy the asset at time 0 but with payment deferred to time 1 is $S_0(1 + r)$. This can be seen as the current futures price for payment of the spot price in a period. The term $E(S_1)$ is interpreted as the spot price expected to prevail at time 1. Multiplying the above equation by $(1 + r)$ gives:

$$(1 + r)S_0 = E(S_1) - \beta_S(E(\tilde{R}_m) - r)S_0 - \lambda_S S_0 + \lambda_m \beta_S S_0 \tag{5}$$

Setting $F_0 = (1 + r + \lambda_S)S_0$ and rearranging terms gives:

$$\frac{E(S_1) - F_0}{S_0} = \beta_S(E(\tilde{R}_m) - r - \lambda_m) \tag{6}$$

This equation can be interpreted as expressing the risk premium on the underlying spot asset as the change in the futures price divided by the period zero spot price in the context of information costs.

An analysis similar to the one adopted in Black (1976) can be implemented in this context. In fact, let's rewrite the CAPMI in the following form:

$$E(\tilde{R}_S) - r - \lambda_S = \beta_S(E(\tilde{R}_m) - r - \lambda_m).$$

Writing S_0 and S_1 as the start and end-of-period prices of asset i and using the definition of β_i, the CAPMI can be written as:

$$E\left(\frac{(S_1 - S_0)}{S_0}\right) - r - \lambda_S$$

$$= \left(cov\left(\frac{(S_1 - S_0)}{S_0}, \tilde{R}_m\right) \Big/ var(\tilde{R}_m)\right)(E(\tilde{R}_m) - r - \lambda_m) \tag{7}$$

If we multiply by S_0, we get the expected dollar return on asset i:

$$E(S_1 - S_0) - (r + \lambda_S)S_0$$

$$= \frac{cov((S_1 - S_0), \tilde{R}_m)}{var(\tilde{R}_m)}(E(\tilde{R}_m) - r - \lambda_m) \tag{8}$$

The price S_0 can be set equal to zero since the futures contact's value at that time is zero. Ignoring daily limits, we set S_1 equal to ΔS. In fact, if we apply this equation to a futures contract, the change in the futures price over the period is:

$$E(\Delta S) = \frac{cov(\Delta S, \tilde{R}_m)}{var(\tilde{R}_m)}(E(\tilde{R}_m) - r - \lambda_m) \tag{9}$$

Eq. (9) can be written for the futures prices as:

$$E(\Delta S) = \beta_*(E(\tilde{R}_m) - r - \lambda_m) \tag{10}$$

Eq. (10) shows that the expected change in the futures price is proportional to the dollar "beta" of the futures price. The expected change in the futures price may be zero, positive or negative.[5]

[5]Also, the expected change in the futures price is zero when the covariance of the change in the futures price with the market portfolio is zero, i.e., $E(\Delta S) = 0$ when $cov(\Delta S, \tilde{R}_m) = 0$

COMMODITY OPTION VALUATION WITH INFORMATION COSTS

Black (1976) shows that in the absence of interest rate uncertainty, a European commodity option on a futures (or a forward) contract can be priced using a minor modification of the Black and Scholes (1973) option pricing formula. In deriving expressions for the behavior of the futures price, he assumes that both taxes and transaction costs are zero and that the CAPM applies at each instant of time.

Following Black (1976), we assume that the fractional change in the futures price is distributed lognormally, with a known constant variance rate, σ. We also assume that all the parameters of the CAPMI are constant through time. Under these assumptions, the value of the futures commodity option, $C(S, t)$, can be written as a function of the underlying futures price and time. In this context, it is possible to create a riskless hedge by taking a long position in the option and a short position in the futures contract with the same transaction date. Black (1976) assumed that a continuously rebalanced self-financing portfolio of the underlying futures contracts and the riskless asset can be constructed to duplicate the payoff of the futures option. The spot price follows the following equation:

$$dS/S = \mu dt + \sigma dz \tag{11}$$

where μ and σ refer to the instantaneous rate of return and the standard deviation of the underlying asset, and dz is a Brownian motion.

The absence of costless arbitrage opportunities implies the following relationship between the futures or the forward price and its underlying asset:

$$F = Se^{(b+\lambda_S)T}$$

where F is the current forward price, T is the option's maturity date, b is the constant proportional cost of carrying the commodity and λ_S is the information cost on the spot asset.

If this relationship holds, the dynamics of the forward prices are given by:

$$dF/F = (\mu - b - \lambda_S)dt + \sigma dz \tag{12}$$

The relationship between a commodity option's beta and its underlying security's beta is given by:

$$\beta_C = S\left(\frac{C_S}{C}\right)\beta_S \tag{13}$$

where β_c and β_S refer respectively to the betas of the commodity option and its underlying commodity contract.

The expected return on a security in the context of Merton's model is:

$$\tilde{R}_S - r = \beta_S[\tilde{R}_m - r] + \lambda_S - \beta_S\lambda_m$$

This equation can be written for the expected return on a commodity contract in the presence of a carrying cost over a small interval of time as:

$$E\left(\frac{\Delta S}{S}\right) = [b + \beta_S(\tilde{R}_m - r) + \lambda_S - \beta_S\lambda_m]\Delta t \tag{14}$$

where b is the cost of carrying the commodity.[6]

Using Merton's model, the expected return on a commodity call option must be:

$$E\left(\frac{\Delta C}{C}\right) = [r + \beta_C(\tilde{R}_m - r) + \lambda_C - \beta_C\lambda_m]\Delta t \tag{15}$$

where an information cost λ_c enters the option's expected return as explained above. This information cost is accounted for in the option market as a discounting factor required by investors for holding options.

When the expected returns on the commodity option and its underlying contract are multiplied by C and S, this gives:

$$E(\Delta S) = [bS + S\beta_S(\tilde{R}_m - r) + \lambda_S S - S\beta_S\lambda_m]\Delta t \tag{16}$$

$$E(\Delta C) = [rC + C\beta_C(\tilde{R}_m - r) + \lambda_C C - C\beta_C\lambda_m]\Delta t \tag{17}$$

Substituting for the option's elasticity from equation (13), equation (17) can be written as:

$$E(\Delta C) = [rC + SC_S\beta_S(\tilde{R}_m - r) + \lambda_C C - SC_S\beta_S\lambda_m]\Delta t \tag{18}$$

When a hedged position is constructed and "continuously" rebalanced,

[6]As suggested by the reviewer, the approach adopted in what follows [eqs. (15) to (21)] is similar to that used in Bellalah and Jacquillat (1995). However, this derivation accounts besides for the cost of carry and the specificities of futures contracts within the context of costly information. It is given for illustrative purposes.

using limiting arguments as in Omberg (1991), dC or ΔC can be written as:

$$dC = \frac{1}{2} C_{SS} dS^2 + C_S dS + C_t dt \quad (19)$$

or:

$$\Delta C = \frac{1}{2} C_{SS}(S)^2\sigma^2 + C_S \Delta S + C_t \Delta t \quad (20)$$

When the expectation is applied to (19) or (20) and dS is replaced by its value, this gives:

$$E(dC) = \frac{1}{2} \sigma^2 S^2 C_{SS} dt + C_S[bS + S\beta_S(\tilde{R}_m - r) + \lambda_S S - S\beta_S\lambda_m]dt + C_t dt \quad (21)$$

Combining (21) and (18) and rearranging yields:

$$\frac{1}{2} \sigma^2 S^2 C_{SS} + (b + \lambda_S)SC_S - (r + \lambda_C)C + C_t = 0 \quad (22)$$

When λ_S and λ_C are set equal to zero, this equation collapses to that in Barone-Adesi and Whaley (1987).

Let T be the maturity date of the call and K be its strike price. Equation (22) must be solved under the following boundary condition at maturity:

$$C(S, T) = S - K \text{ if } S \geq K$$
$$C(S, T) = 0 \text{ if } S < K \quad (23)$$

The value of a European commodity call is:

$$C(S, T) = Se^{((b-r-(\lambda_C-\lambda_S))T)} N(d_1) - Ke^{-(r+\lambda_C)T}N(d_2) \quad (24)$$

with:

$$d_1 = \left[\ln\left(\frac{S}{K}\right) + \left(b + \frac{1}{2}\sigma^2 + \lambda_S\right)T\right] \bigg/ \sigma\sqrt{T}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

and where $N(\cdot)$ is the cumulative normal density function.

When λ_S and λ_C are equal to zero and $b = r$, this formula is the same as that in Black and Scholes.[7] If, besides the cost of carrying, the commodity is zero, this equation is equivalent to that in Black. Using the futures price $F = Se^{(b+\lambda_S)T}$ instead of the spot price, equation (24) is rewritten for a European futures call as:

$$C(F, T) = e^{-(r+\lambda_C)T} (FN(d_1) - KN(d_2)) \tag{25}$$

with:

$$d_1 = \left[\ln\left(\frac{F}{K}\right) + \frac{1}{2}\sigma^2 T\right] \Big/ \sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The solution for a European futures put option in the same context is:[8]

$$P(F, T) = e^{-(r+\lambda_C)T} (-FN(-d_1) + KN(-d_2)) \tag{26}$$

The term $FN(d_1) - KN(d_2)$ shows that the expected value of the futures

[7]In fact, to obtain this equation, the CAPMI can be written as:

$$\tilde{R}_S - r - \lambda_S = \beta_S[\tilde{R}_m - r - \lambda_m], \text{ or:}$$

$$\tilde{R}_S - r - \lambda_S = a\beta_S \text{ with } a = [\tilde{R}_m - r - \lambda_m].$$

This equation can be written for the expected return on the spot asset and the option as:

$$E\left(\frac{\Delta S}{S}\right) = (r + \lambda_S)\Delta t + a\beta_S\Delta t$$

$$E\left(\frac{\Delta C}{C}\right) = (r + \lambda_C)\Delta t + a\beta_C\Delta t$$

Multiplying this last equation by C and substituting for β_C gives:

$$E(\Delta C) = (r + \lambda_C)C\Delta t + aS\beta_S C_S\Delta t$$

Taking the expected value of (20) and replacing $E(\Delta S)$ gives:

$$E(\Delta C) = (r + \lambda_S)SC_S\Delta t + aS\beta_S C_S\Delta t + C_t\Delta t + \frac{1}{2}C_{SS}S^2\sigma^2\Delta t$$

Making the equality between this equation and

$$E(\Delta C) = (r + \lambda_C)C\Delta t + aS\beta_S C_S\Delta t, \text{ and simplifying gives:}$$

$$\frac{1}{2}C_{SS}S^2\sigma^2 + (r + \lambda_S)SC_S - (r + \lambda_C)C + C_t = 0.$$

If $\lambda_S = \lambda_C = 0$, this equation is the B-S equation.

[8]It is possible to use directly the put-call parity to write down the put formula. Recall that the put-call parity relationship between puts and call with identical strike prices is an arbitrage-based relationship that holds regardless of the distribution of financial asset prices. More general results about calls and puts with different strike prices can be written down for symmetric and asymmetric processes using the propositions in Bates (1997).

call at expiration, is the expected difference between the futures price and the strike price conditional upon the option being in-the-money times the probability that it will be in-the-money. The term $e^{-(r+\lambda_C)T}$ is the appropriate discount factor within a framework of incomplete information by which the expected expiration value is brought to the present.

The following equation (with information costs) is the analogous of that in Black (1976):[9]

$$\frac{1}{2} C_{SS} S^2 \sigma^2 - (r + \lambda_C)C + C_t = 0.$$

This equation can be applied to the valuation of forward contracts. If we denote by $w(F(t, t^*), t, K, t^*)$, the value of a long forward contract as a function of the current futures price, $F(t, t^*)$, the price of a commodity K at which the commodity will be bought, t the current time, t^* the time of the transaction, then the futures price is the price at which the current value of the contract is zero. When the time comes for the transaction, the value of a forward contract is given the difference between the futures price and the spot price:

$$w(F, t^*, K, t^*) = F - K$$

This is the boundary condition which must be imposed on the following equation:

$$\frac{1}{2} w_{SS} F^2 \sigma^2 - (r + \lambda_w)w + w_t = 0 \qquad (27)$$

On the maturity date, this equation must be solved under the condition:

[9]It is possible to use the previous footnote to see this result. In fact, since the value of a futures contract is zero, the equity in the position is just the value of the option. In this context, the system:

$$E(\Delta C) = (r + \lambda_S)SC_S\Delta t + aS\beta_S C_S \Delta t + C_t\Delta t + \frac{1}{2} C_{SS}S^2\sigma^2\Delta t$$

$$E(\Delta C) = (r + \lambda_C)C\Delta t + aS\beta_S C_S\Delta t,$$

becomes:

$$E(\Delta C) = C_t\Delta t + \frac{1}{2} C_{SS}S^2\sigma^2\Delta t$$

$$E(\Delta C) = (r + \lambda_C)C\Delta t,$$

which gives:

$$\frac{1}{2} C_{SS}S^2\sigma^2 - (r + \lambda_C)C + C_t = 0.$$

If $\lambda_C = 0$, this equation is the Black equation with no information costs.

$$w(F, t^*) = F - K \tag{28}$$

The solution to this system gives the value of the forward contract:

$$w(F, t) = (F - K)e^{(r+\lambda_w)(t-t*)} \tag{29}$$

This value is the difference between the futures price and the forward contract price. This difference is discounted to the present at the appropriate interest rate by accounting for information costs. This eq. (22) applies to forward contracts and to European and American options. A direct application of the approach in BAW (1987), allows to write down immediately the formulas for American commodity options with information costs. In this context, the American commodity option value $C_A(S, T)$ is given by:

$$C_A(S, T) = C(S, T) + A_2(S/S^*)^{q2} \text{ when } S < S^*$$

$$C_A(S, T) = S - K \text{ when } S \geq S^*$$

with:

$$A_2 = \frac{S^*}{q2}(1 - e^{(b+\lambda_S-r-\lambda_C)T} N(d_1(S^*))) \tag{30}$$

$$q_2 = \frac{1}{2}\left(-(N - 1) + \sqrt{(N - 1)^2 + 4\frac{M}{k}}\right)$$

$$N = \frac{2(r + \lambda_C)}{\sigma^2},$$

$$M = \frac{2(b + \lambda_S)}{\sigma^2} \quad k = 1 - e^{-(r+\lambda_C)T}.$$

The critical underlying commodity price is given by an iterative procedure from the following equation:

$$S^* - K = C(S^*, T) + \frac{S^*(1 - e^{(b+\lambda_S-r-\lambda_C)T} N(d_1(S^*))}{q2} \tag{31}$$

In the same context, the American commodity option value $P_A(S, T)$ is given by:

$$P_A(S, T) = P(S, T) + A_1(S/S^*)^{q1} \text{ when } S > S^{**}$$

$$P_A(S, T) = K - S \text{ when } S \leq S^{**}$$

with:

$$A_1 = \frac{-S^{**}}{q1}\left(1 - e^{(b+\lambda_S-r-\lambda_C)T}\, N(-d_1(S^{**}))\right) \tag{32}$$

$$q_1 = \frac{1}{2}\left(-(N-1) - \sqrt{(N-1)^2 + 4\frac{M}{k}}\right),$$

$$N = \frac{2(r+\lambda_C)}{\sigma^2},$$

$$M = \frac{2(b+\lambda_S)}{\sigma^2},$$

$$k = 1 - e^{-(r+\lambda_C)T}.$$

The critical underlying commodity price is given by an iterative procedure from the following equation:[10]

$$K - S^* = P(S^*, T) - \frac{S^*(1 - e^{(b+\lambda_S-r-\lambda_C)T}\, N(-d_1(S^*)))}{q1} \tag{33}$$

A similar algorithm as the one developed in BAW (1987) or Bellalah (1998) can be used to determine the critical underlying price.[11]

SIMULATIONS

The model presented above is simulated with reference to the Black (1976) and BAW (1987) model. The pricing model requires the use of the following parameters: the underlying asset price, the strike price, the time to maturity, the interest rate, the cost of carrying, the volatility and information costs.

Table I presents a sensitivity analysis of the theoretical European futures call and put values as given by Black's model and the model derived in the previous section. Eqs. (25) and (26) are used in the simulations. The calculations are based on two alternative assumptions regarding information costs, 1% and 5%.

When $F = K = 100$, $T = 0.25$, $\sigma = 0.2$, $r = 0.08$ and $b = 0$, results show that an increase in information costs from 1% to 5% reduces call values respectively from 3.8990 to 3.8602.

The simulations show that model prices are less than Black's prices for out-of-the money and in-the-money calls and are nearly equal to Black's prices for at-the-money calls.

[10]The above solutions are written as in BAW (1987). They can be rewritten as a function of the futures price as in Whaley (1986) using the cost-of-carry model: the relationship between the futures price and the spot price.

[11]It is also possible to use the new bounds and approximations in Broadie and Detemple (1996) for the valuation of American options with information costs.

TABLE I

Simulations of Futures Call (Put) Option Values [Eqs. (25) and (26)]

Panel A, F	C_{Black}	$C_{M,\lambda_C=0.01}$	$C_{M,\lambda_C=0.05}$	P_{Black}	$P_{M,\lambda_C=0.01}$	$P_{M,\lambda_C=0.05}$
80	0	0	0	19.6611	19.6203	19.4549
90	0.6963	0.6353	0.6310	10.4365	10.3128	10.3112
100	3.9087	3.8990	3.8602	3.9087	3.8990	3.8602
110	10.7427	10.7358	10.6290	0.9487	0.9483	0.9482
120	19.7554	19.7260	19.5297	0.1414	0.1370	0.1363
Panel B	$T = 0.25$	$\sigma = 0.2$	$r = 0.12$			
80	0	0	0	19.4461	19.4249	19.2912
90	0.6820	0.6810	0.6767	10.4356	10.4112	10.3106
100	3.8698	3.8602	3.8217	3.8698	3.8602	3.8217
110	10.6356	10.6290	10.5237	0.9412	0.9408	0.9393
120	19.5487	19.5297	19.3354	0.1497	0.1493	0.1476
Panel C	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$			
80	0	0	0	18.1417	18.0566	17.6991
90	1.6913	1.6833	1.6520	11.3091	11.2433	11.1226
100	5.4161	5.3890	5.2823	5.4161	5.3870	5.2823
110	11.7325	11.6939	11.4235	2.1246	2.1139	2.0916
120	19.9157	19.8562	19.4630	0.6999	0.6961	0.6916
Panel D	$T = 0.5$	$\sigma = 0.4$	$r = 0.08$			
80	2.6940	2.6806	2.6275	21.9098	21.8005	21.3688
90	6.1331	6.1025	5.9817	15.7410	15.6625	15.3524
100	10.8051	10.7512	10.5383	10.8151	10.7512	10.5383
110	16.7917	16.7080	16.6123	7.1838	7.1480	7.0165
120	19.9557	19.8562	19.4630	4.6800	4.6567	4.5645

Table uses Black's model, C_{Black}, (P_{Black}) and the proposed model, C_M, (P_M) for the following parameters $K = 100$, $T = 0.25$, $\sigma = 0.2$, $r = 0.08$, $b = 0$.

This result[12] shows that our model explains at least the relative mispricing of out and at-the-money calls, which is an advantage with respect to Black's model. It is important to note that the differences in option

[12]In an extensive empirical work, Rubinstein (1985, 1994), Melick and Thomas (1997), and Bellalah (1990) among others document the main following bias when testing the B-S and Black type models for European options:

- When pricing out of the money calls, (puts), B-S type models tend to overvalue (undervalue) these options with respect to market values.
- When pricing in the money calls, (puts) B-S type models tend to undervalue (overvalue) these options with respect to market values.
- The degree of mispricing is a function of the option's moneyness. Similar bias are reported in the French stock option market. This latter is regarded as a forward market. For more details, see the work of Bellalah (1990) and Briys, Bellalah, et al. (1998).

values increase with the interaction between information costs and the other option valuation parameters. In general, call prices are less than those given by Black's model. This reduces the overvaluation bias for these options and particularly for in the money calls. The degree of reduction of this bias depends on the values given to information costs and the interaction between these costs and the other parameters.

When the information costs vary from 1% to 5%, a decrease is observed in put values. The simulations performed indicate that model prices are nearly equivalent to Black's prices for at and out-of-the-money puts. However, model prices are less than those reported in Black's model for in-the-money puts. Because our model eliminates the overvaluation bias of these puts, this is an advantage with respect to Black's model. It is of interest to note that the interaction between information costs and the interest rates as well as time to maturity shows less impact on option values than the combined effect between information costs and the volatility parameter. One final comment is that the interaction between incomplete information and interest rates diminishes the gap between the two models.

In general, put prices are less than those given by Black's model. This reduces the overvaluation bias for in the money puts. The degree of reduction of this bias depends on information costs and their interaction with the other parameters.

The above results can be improved in empirical tests since because model has an additional parameter with respect to Black's model (one degree of freedom) and can be more easily calibrated to market data.[13] The theory of futures option valuation suggests that the early exercise privilege of American futures options contributes meaningfully to the futures option value. To demonstrate the plausible magnitudes of the early exercise premium on American futures options, the European and American model prices are compared for a range of option pricing parameters within the context of information costs. Results are reported in Table II

[13]It is possible to use the put-call parity as suggested by the reviewer to link the information costs. In this context, we will have two degrees of freedom to calibrate the model prices to market prices. This is useful in empirical tests since it allows the use of a single volatility parameter to get close to market prices. In this spirit, we can use an implicit estimation of the information costs with respect to market data in the same way as we extract an implicit volatility. A similar procedure is used in Bellalah and Jacquillat (1995). This approach allows the calibration of model prices to market prices and can reduce the smile effect. In fact, using two more parameters than standard models, and a single figure of volatility, it is possible to reduce the difference between market prices and model prices. This shows that our model can account for the skewness premium as defined in Bates (1997). Because we are simulating the model (rather than estimating its parameters), we consider the futures contract price as given and do not care about the estimation of the underlying asset information costs when using the formulas (25) and (26).

TABLE II

Comparisons of European and American Futures Call Option Values

Panel, A, F	$C_{European}$	$C_{American}$	$F_{Critical}$	C_{BAW}
90	0.6950	0.7407	121.770	0.70
100	3.8000	3.9380	121.758	3.93
110	10.7427	10.8158	121.778	10.81
120	19.7500	20.0270	121.770	20.02
Panel B	$T = 0.25$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.02$
90	0.6340	0.6419	121.629	0.70
100	3.8800	3.9201	121.612	3.93
110	10.7000	10.8186	121.629	10.81
120	19.6700	20.0230	121.624	20.02
Panel C	$T = 0.25$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.05$
90	0.6310	0.6478	121.3529	0.70
100	3.8602	3.910	121.3110	3.93
110	10.6290	10.8056	121.3244	10.81
120	19.5297	20.0183	121.3240	20.02
Panel D	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.01$
90	1.5830	1.61087	128.240	1.72
100	5.3800	5.46840	128.290	5.48
110	11.6930	11.9060	128.136	11.90
120	19.8560	20.3413	121.139	20.34
Panel E	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.02$
90	1.5750	1.61220	127.920	1.72
100	5.3600	5.4639	127.929	5.48
110	11.6300	11.8896	127.935	11.90
120	19.7500	20.3423	127.922	20.34
Panel F	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.05$
90	1.6520	1.6265	127.5502	1.72
100	5.2823	5.4647	127.5533	5.48
110	11.4235	11.8724	127.5430	11.90
120	19.4630	20.3218	127.5470	20.34

Table uses the proposed models, $C_{European}$, $C_{American}$, C_{BAW} for the following parameters $K = 100$, $T = 0.25$, $\sigma = 0.2$, $r = 0.08$, $b = 0$, $\lambda_C = 0.01$.

for different parameters. The BAW price (their table 3, page 314) is given as a benchmark for comparisons.

The calculations performed are based on three alternative assumptions regarding information costs for European and American call options when the futures price varies from 90 to 120. Table II shows that our model prices are most of the time less or equal to BAW's model prices for call options. When testing BAW's model, Whaley (1986) finds that American futures option valuation equations generate mispricing errors that are systematically related to the option moneyness and time to expiration.[14] If we compare the results to those in Whaley (1986), we find that our model reduces the overvaluation bias.

It is interesting to see that out-of-the-money futures options have negligible early exercise premiums. When the option is considerably in-the-money, the early exercise premium account for a significant proportion of the option's price. It is important to note that when we fix the futures price and vary the information costs from 1% to 5%, we observe a decrease in the futures price path.

Table III provides the theoretical values for European and American futures put options. Again, the BAW prices are used for comparisons. The difference between European and American option values reflect the early exercise premium. The table shows that our model prices are most of the time greater than or equal to BAW's prices for put options. When testing BAW's model, Whaley (1986, p 139) finds that out-of-the-money puts are overpriced by BAW's model and in-the-money puts are underpriced. If we compare the results to those in Whaley (1986), we see that our model reduces the undervaluation bias since model prices are greater than BAW prices. Besides, the early exercise premium seems to be important for put options. This result is in accordance with the predictions of the theory of futures option valuation which suggests that the early exercise privilege of American options accounts for a significant percentage of the option price.[15]

It is important to note that the interaction between information costs and the volatility parameter, seems to have the same effect on futures option values as the interaction between information costs and time to maturity.

[14]He finds (p 139) that out-of-the money calls are overpriced by BAW's model and in-the-money calls are underpriced. When examining the maturity bias, he shows that the model prices are higher than the observed prices for short-term options.

[15]The simulations done by Whaley (1986) show that this is true for typical parameters of the SP 500 futures contracts, but he reports this affect only for in-the-money options.

TABLE III

Comparisons of European and American Futures Put Option Values

Panel, A, F	$P_{European}$	$P_{American}$	$P_{Critical}$	P_{BAW}
90	10.3128	10.6440	86.780	10.58
100	3.8990	4.0130	86.132	3.93
110	0.9584	0.9850	86.770	0.94
120	0.1710	0.1800	86.710	0.15
Panel B	$T = 0.25$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.02$
90	10.1870	10.6310	86.821	10.58
100	3.8890	4.00600	86.190	3.93
110	0.9450	0.98420	86.817	0.94
120	0.1370	0.17950	86.756	0.15
Panel C	$T = 0.25$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.05$
90	10.3112	10.6050	86.9420	10.58
100	3.8602	3.99600	86.3589	3.93
110	0.9482	0.98000	86.940	0.94
120	0.1363	0.17900	86.889	0.15
Panel D	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.01$
90	11.2433	11.7240	82.251	11.48
100	5.3890	5.5846	82.144	5.48
110	2.1139	2.21400	82.190	2.15
120	0.6961	0.7757	82.228	0.70
Panel E	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.02$
90	11.0870	11.69200	82.3570	11.48
100	5.3620	5.5656	82.2556	5.48
110	2.1232	2.20680	82.299	2.15
120	0.7325	0.77000	82.336	0.70
Panel F	$T = 0.5$	$\sigma = 0.2$	$r = 0.08$	$\lambda_C = 0.05$
90	11.1226	11.6400	82.6640	11.48
100	5.2823	5.5209	82.5700	5.48
110	2.0916	2.1843	82.614	2.15
120	0.6916	0.77100	82.640	0.70

Table uses the proposed models, $P_{European}$, $P_{American}$, P_{BAW} for the following parameters $K = 100$, $T = 0.25$, $\sigma = 0.2$, $r = 0.08$, $b = 0$, $\lambda_C = 0.01$.

CONCLUSION

Using Merton's model of capital market equilibrium with incomplete information, we develop a method for its application in the valuation of forward and futures contracts. The approach parallels that applied by Dusak (1973) and Black (1976) in the case of the standard CAPM.

Following the approach in Black (1976) and BAW (1987), the model of capital market equilibrium with incomplete information is applied to the valuation of European and American commodity options. When information costs are ignored, the model collapses to the ones in Black (1976) and BAW (1987). Because the model presents an additional parameter with respect to these models, it may be calibrated to market prices. We have simulated option values and compared our results to those generated by Black's model. It is important to note that the two models do not differ significantly for the pricing of at and out-of-the-money options. This result may be explained by the interaction between information costs and the volatility parameter. The simulations of the model show that it reduces the amplitude of the biases when compared to the other models prices. For example, it over-estimates prices of out of the money calls but in a proportion less than B-S type models.

Using a similar approach as that in BAW (1987), we provide simple analytic approximations for pricing American options written on futures contracts. We have simulated the plausible magnitude of the early exercise premium on American futures options and the critical futures price levels. Changes in the value of information costs affect the other option valuation parameters and hence have an impact on option values. Two points are noteworthy. First, the difference in values between European and American futures options are larger, the larger is the difference between information costs. Second, the impact of information costs is most noticeable for deep-in-the-money options and may affect the probability of early exercise.

Our results, although useful for valuing European options-on-futures within the context of incomplete information, may have some implications regarding the optimal exercise of American options. They can also be used to explain some shapes of the volatility smile in the marketplace.

BIBLIOGRAPHY

Barone-Adesi, G. & Whaley, R. E. (1987) Efficient analytic approximation of American option values. Journal of Finance, 42, 301–320.

Bates, D. (1997) The skewness premium: Option pricing under asymmetric processes. Advances in Futures and Options Research, 9, 51–82.
Bellalah, M. (1990), Quatre essais pour l'évaluation des options sur indices et sur contrats à terme d'indice. Doctorat de l'Université de Paris-Dauphine.
Bellalah, M. (1999, forthcoming) Options et dividendes. Canadian Revue, FINECO, 2.
Bellalah, M. & Jacquillat, B. (1995). Option valuation with information costs: theory and tests. The Financial Review, 30(3), 617–635.
Briys, E., Bellalah, M., et al. (1998). Options, futures and other exotics. John Wiley & Sons.
Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 81, 637–659.
Black, F. (1976) The pricing of commodity contracts. Journal of Financial Economics, 79(3), 167–179.
Broadie, M. & Detemple, J. (1996). American option valuation: New bounds, approximations, and a comparison of existing methods. The Review of Financial Studies, 9(4), 1211–1250.
Dusak, K. (1973). Futures trading and investor returns: An investigation of commodity market risk premiums. Journal of Political Economy, 1387–1404.
Merton, R. C. (1987). A simple model of capital market equilibrium with incomplete information. Journal of Finance, 3, 483–510.
Melick, W., & Thomas, C. P. (1997). Recovering an asset's implied PDF from option prices: An application to crude oil during the Gulf Crisis. Journal of Financial and Quantitative Analysis, 32, 91–115.
Omberg, (1991). On the theory of perfect hedging. Advances in Futures and Options Research, 5.
Rubinstein, M. (1983). Nonparametric tests of alternative pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978. Journal of Finance, 11(2), 455–480.
Rubinstein, M. (1994). Implied binomial trees. Journal of Finance, 3, 771–818.
Whaley, R. E. (1986). Valuation of American futures options: Theory and empirical tests. Journal of Finance, 41, 127–150.