FRACTIONAL COINTEGRATION AND FUTURES HEDGING

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This article examines the performance of various hedge ratios estimated from different econometric models: The FIEC model is introduced as a new model for estimating the hedge ratio. Utilized in this study are NSA futures data, along with the ARFIMA-GARCH approach, the EC model, and the VAR model. Our analysis identifies the prevalence of a fractional cointegration relationship. The effects of incorporating such a relationship into futures hedging are investigated, as is the relative performance of various models with respect to different hedge horizons.

Findings include: (i) Incorporation of conditional heteroskedasticity improves hedging performance; (ii) the hedge ratio of the EC model is consistently larger than that of the FIEC model, with the EC providing better post-sample hedging performance in the return–risk context; (iii) the EC hedging strategy (for longer hedge horizons of ten days or more) incorporating conditional heteroskedasticty is the dominant strategy; (iv) incorporating the fractional cointegration relationship does not improve the hedging performance over the EC model; (v) the conventional regression method provides the worst

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hedging outcomes for hedge horizons of five days or more. Whether these results (based on the NSA index) can be generalized to other cases is proposed as a topic for further research. Jrl Fut Mark 19: 457–474, 1999

INTRODUCTION

Futures contracts are popular instruments for risk management. If we adopt the convention that equates risk to variance, the best futures hedging strategy is as follows. The ratio of the futures position to the spot position (that is, the hedge ratio) should equal to the ratio of the conditional covariance between spot and futures returns to the conditional variance of the futures return.

In empirical applications, the hedge ratio varies according to the conditioning information adopted. The vector autoregression (VAR) method incorporates the history of both spot and futures prices as the conditional information, whereas the error correction (EC) model adds the previous basis (defined as the difference between the futures and spot prices) as a conditioning variable. Statistical properties (such as the cointegration between the spot and futures prices) have been documented to support the information value of the lagged basis and prices.[1]

In comparing the VAR and EC hedge ratios Ghosh (1993) and Chou, Denis, and Lee (1996) found that the latter leads to better performance. Lien (1996) provided some analytical support for these empirical findings. An interesting property of the EC model is that only the most recent basis is incorporated as a conditioning variable. All other previous bases are irrelevant. When the basis has long-range dependence, this assumption appears to be invalid.

A class of time series that allows for long-range dependence is the fractionally integrated time series. Cheung (1993) established fractional integration in four major spot exchange rates. Similar conclusions were found in Fang, Lai, and Lai (1994) for currency futures prices. Barkoulas, Labys, and Onochie (1997) found fractional orders in the spot prices of a number of commodities. Corazza, Malliaris, and Nardelli (1997) identified fractal structures in the futures prices of six major agricultural commodities.

When the basis is characterized by a fractionally integrated time series, spot and futures prices are "fractionally cointegrated." In general,

[1]More generally, the cointegration relationship between the spot and futures prices is characterized by a linear combination. The basis restricts the coefficients of the linear combination. Empirical studies, however, often found the restriction to be valid; for an example, see Pesaran and Shin (1996). For ease of economic interpretation, we use the basis as the cointegration relationship in this article.

two economic variables are fractionally cointegrated if both are of the same fractional order, whereas a linear combination of them has a smaller fractional order. Evidence in support of fractional cointegration in economic and financial time series can be found in Baillie and Bollerslev (1994), Martin (1997), and Masih and Masih (1995). Granger (1986) established a model, referred to as the fractionally integrated error correction (FIEC) model, when fractional cointegration prevails.

In this article we compare the effectiveness of the hedge ratios estimated from the regression, VAR, EC, and FIEC models using the Nikkei Stock Average (NSA) futures. We examine the performance of these hedge ratios with respect to different hedge horizons. To better characterize the price processes, we allow for conditional heteroskedasticity. Thus generalized autoregressive conditional heteroskedasticity (GARCH) equations are adopted to describe the conditional second moments of the spot and futures prices. We found, through post-sample comparisons, that the hedging performance improves after the GARCH effects are incorporated. Overall, the EC-GARCH model provides the best hedging performance. The FIEC model is unable to provide better hedging performance than the EC model. On the other hand, the conventional regression method almost always produces the worst outcome.

STATISTICAL MODELS FOR THE ESTIMATION OF HEDGE RATIOS

We consider a *bona fide* hedger within a one-period framework. The period begins at time $t - \eta$ and ends at time t. At time $t - \eta$, the hedger has a non-tradable spot position Q. Let $p_{t-\eta}$ and p_t denote the spot price (in logarithms) at $t - \eta$ and t, respectively. To reduce the risk exposure, the hedger assumes short positions in a futures contract. Let $f_{t-\eta}$ and f_t denote the prices of the futures contract (in logarithms) at $t - \eta$ and t, respectively. Suppose the hedger sells X futures contracts. Let $h = X/Q$ denote the hedge ratio. The portfolio return at the end of the period is $r_m = r_p - hr_f$, where $r_p = \Delta_\eta p_t = p_t - p_{t-\eta}$ and $r_f = \Delta_\eta f_t = f_t - f_{t-\eta}$. The risk of the hedger is measured by the conditional variance of r_m based on the information available at the beginning of the period: $\text{var}(r_m \mid \Phi_{t-\eta})$, where $\Phi_{t-\eta}$ denotes the information set.

The hedger chooses the optimal hedge ratio h^* to minimize the risk given by $\text{var}(r_m \mid \Phi_{t-\eta})$. The solution is as follows:

$$h^* = \text{cov}(\Delta_\eta p_t,\ \Delta_\eta f_t|\Phi_{t-\eta})/\text{var}(\Delta_\eta f_t|\Phi_{t-\eta}), \tag{1}$$

where cov(.) is the covariance operator.

A conventional approach in estimating h* relies upon the simple linear regression method. Hereby we apply the daily data to estimate the relationship between the spot and futures prices irrespective of the hedge horizon η. Let $\Delta p_t = p_t - p_{t-1}$ be the dependent variable and let $\Delta f_t = f_t - f_{t-1}$ be the independent variable. We have the following regression model:

$$\Delta p_t = \alpha + \beta \, \Delta f_t + \varepsilon_t. \tag{2}$$

The estimated coefficient associated with Δf_t is the estimated optimal one-day hedge ratio. For longer hedge horizons, we derive from eq. (2) the following model:

$$\Delta_\eta p_t = \eta\alpha + \beta \, \Delta_\eta f_t + (\varepsilon_t + \varepsilon_{t-1} + \ldots + \varepsilon_{t-\eta+1}). \tag{3}$$

Again, the optimal hedge ratio is the slope of the regression. That is, regardless of the hedge horizon, the optimal hedge ratio remains the same.

The above approach ignores many information variables that may be incorporated into the model. Notably, previous price movements in the spot and futures markets may affect the current price movements. The following VAR model incorporates these considerations[2]

$$\Delta p_t = \alpha_p + \sum_{i=1}^{m} \beta_{pi}\Delta p_{t-i} + \sum_{j=1}^{m} \gamma_{pj}\Delta f_{t-j} + \varepsilon_{pt}, \tag{4}$$

$$\Delta f_t = \alpha_f + \sum_{i=1}^{m} \beta_{fi}\Delta p_{t-i} + \sum_{j=1}^{m} \gamma_{ff}\Delta f_{t-j} + \varepsilon_{ft}, \tag{5}$$

where $(\varepsilon_{pt}, \varepsilon_{ft})$ are independently identically distributed (IID) random vectors. Let $\mathrm{var}(\varepsilon_{pt}) = \sigma_{pp}$, $\mathrm{var}(\varepsilon_{ft}) = \sigma_{ff}$, and $\mathrm{cov}(\varepsilon_{pt}, \varepsilon_{ft}) = \sigma_{pf}$. The optimal one-day hedge ratio is σ_{pf}/σ_{ff}, which will be called the VAR hedge ratio. When the hedge horizon is η, the optimal hedge ratio is $\mathrm{cov}(\varepsilon_{pt} + \ldots + \varepsilon_{p,t-\eta+1}, \varepsilon_{ft} + \ldots + \varepsilon_{f,t-\eta+1})/\mathrm{var}(\varepsilon_{ft} + \ldots + \varepsilon_{f,t-\eta+1})$, which, under the IID assumption, becomes σ_{pf}/σ_{ff}. Thus the optimal hedge ratio is invariant with respect to the hedge horizon.

Due to arbitrage-free relationship it is often argued that the basis may help determine the price movements. Consequently, the basis should be incorporated into the model, resulting in the following equations:

[2]Vector autoregression model was first proposed by Sims (1980). It was applied extensively in the literature to investigate lead–lag relationships. For an example, see Kawaller, Koch, and Koch (1987).

$$\Delta p_t = \alpha_p + \sum_{i=1}^{m} \beta_{pi}\Delta p_{t-i} + \sum_{j=1}^{n} \gamma_{pj}\Delta f_{t-j} + \theta_p z_{t-1} + \varepsilon_{pt}, \tag{6}$$

$$\Delta f_t = \alpha_f + \sum_{i=1}^{m'} \beta_{fi}\Delta p_{t-i} + \sum_{j=1}^{n'} \gamma_{fj}\Delta f_{t-j} + \theta_f z_{t-1} + \varepsilon_{ft}, \tag{7}$$

where $z_t = f_t - p_t$ is the basis. Indeed, when the spot and futures prices are found to be cointegrated, the results in Engle and Granger (1987) showed that an EC model as described by eqs. (6) and (7) is appropriate.

The presence of the basis as a conditioning variable distinguishes an EC model from a VAR model. When the two models contain the same lag orders, the former encompasses the latter. Similar to the VAR case, if we assume the variance–covariance matrix of ε_{pt} and ε_{ft} to be constant over time, we have the same hedge ratio irrespective of the hedge horizon.

The above approach relies heavily on the assumption that the basis is integrated of order zero. Recent evidence has often indicated that the autocorrelation coefficient of the basis decays geometrically instead of exponentially. Thus a fractionally integrated process may describe the basis more appropriately. In other words, there exists a "fractional cointegration" relationship between the spot and futures prices.

Let B denote the backward shift operator such that $Bz_t = z_{t-1}$, then z_t is integrated of order d if it can be described by the following process:

$$\theta(B)\,(1 - B)^d z_t = \phi(B)\varepsilon_t, \tag{8}$$

where $\theta(B)$ and $\phi(B)$ are finite-order polynomials, and ε_t is a white noise. We assume that all roots of $\theta(B)$ and $\phi(B)$ are outside the unit circle, and $-0.5 < d < 0.5$. Under these conditions z_t is stationary. In particular, when $0 < d < 0.5$, z_t exhibits long-memory characteristics. Note that

$$(1 - B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k - d)B^k}{\Gamma(-d)\Gamma(k + 1)}, \tag{9}$$

where $\Gamma(.)$ is the gamma function. Following Granger (1986), a FIEC model representation can be written as follows:

$$\Delta p_t = \omega_p + \sum_{i=1}^{m} \omega_{pi}\Delta p_{t-i} + \sum_{j=1}^{n} \tau_{pj}\Delta f_{t-j} + \delta_p$$

$$[(1 - B)^d - (1 - B)]z_t + \varepsilon_{pt}, \tag{10}$$

$$\Delta f_t = \omega_f + \sum_{i=1}^{m'} \omega_{fi}\Delta p_{t-i} + \sum_{j=1}^{n'} \tau_{fj}\Delta f_{t-j} + \delta_f$$

$$[(1 - B)^d - (1 - B)]z_t + \varepsilon_{ft}. \tag{11}$$

Note that, eqs. (10) and (11) reduce to (6) and (7), respectively, when d = 0. Thus the EC model is encompassed in the FIEC model.

By expanding the operator $(1 - B)^d$, we find that price movements are affected not only by z_{t-1} (as predicted by the EC model), but also by all z_{t-j}, $j > 1$. That is, whereas the EC model indicates that only the most recent basis is relevant information variable, the FIEC model considers the whole history of the basis. The characteristic of long-memory dependence of the FIEC model suggests that if there is any superiority in hedging performance for the model, it may be exhibited only for long hedge horizons. Later in this article we consider hedge horizons of up to forty days.

In the above discussions, we have assumed that the residuals have constant variances and covariances. Recent empirical investigations, however, found dissatisfaction with the assumption of constant second moments. To accommodate time-varying second moments, we consider the following bivariate GARCH model:

$$\sigma_{pp,t} = \delta_p + \sum_{i=1}^{r} a_{pi}\sigma_{pp,t-i} + \sum_{j=1}^{s} b_{pj}\varepsilon_{p,t-j}^2, \tag{12}$$

$$\sigma_{ff,t} = \delta_f + \sum_{i=1}^{r'} a_{fi}\sigma_{ff,t-i} + \sum_{j=1}^{s'} b_{fj}\varepsilon_{f,t-j}^2, \tag{13}$$

$$\sigma_{pf,t} = \rho\sigma_{pp,t}^{1/2}\sigma_{ff,t}^{1/2}, \tag{14}$$

where ρ is the correlation coefficient, assumed to be constant over time. The bivariate GARCH process may be incorporated into the VAR, EC, or FIEC models. Consequently, we obtain the VAR-GARCH, EC-GARCH, or FIEC-GARCH hedge ratios. In each case, the one-period hedge ratio is $\sigma_{pf,t}/\sigma_{ff,t}$, which is time varying. To calculate the hedge ratio over multiple days, we rely upon iterations of eqs. (12) through (14). Due to the nonlinearity caused by the GARCH effects, the optimal hedge ratio changes as the hedge horizon changes.

Intuitively, the EC model would generate a hedge ratio larger than that of the VAR model, because it takes into account the tighter long-run equilibrium relationship between the spot and futures prices.[3] In the Appendix, we provide an analytical comparison of the hedge ratios of various models, assuming that the true data generating process follows a simple

[3]A tighter relationship between the spot and futures prices leads to a larger R^2. On the other hand, a larger hedge ratio is indicated by a larger slope coefficient [that is, β in eq. (2)]. Note that $R^2 = [\mathrm{cov}(\Delta p, \Delta f)]^2/(\mathrm{var}(\Delta p)\mathrm{var}(\Delta f))$ and $\beta = \mathrm{cov}(\Delta p, \Delta f)/\mathrm{var}(\Delta f)$. When $\mathrm{var}(\Delta p)$ is close to $\mathrm{var}(\Delta f)$, R^2 and β^2 are approximately equal.

FIEC model. We find that in the simple model the FIEC hedge ratio is not less than the EC hedge ratio, which in turn is not less than the VAR hedge ratio. However, when the futures return is not affected by the lagged basis (which we find to be the case in the empirical results reported below), the hedge ratios are equal. In empirical applications, the estimated hedge ratios may differ due to estimation errors, and the relative performance of the hedge ratios becomes an empirical question.

DATA DESCRIPTION AND ESTIMATION RESULTS

Our data set consists of 2,106 daily observations of the spot index and the futures price of the Nikkei Stock Average 225 (NSA), covering the period from January 1989 through August 1997. The futures contract is traded on the Singapore International Monetary Exchange (SIMEX). In terms of trading in futures, SIMEX ranked 14 among global futures exchanges for 1996 and 1997 (Futures Industry, 1998, p.12). NSA futures account for nearly 20% of the total trading volume in SIMEX, next only to the Eurodollar and Euroyen futures. Transactions in the NSA futures are denominated in Yen.[4] The trading features and price behavior of the NSA futures have been well documented in the literature. Readers may refer to Brenner, Subrahmanyam, and Uno (1989a, 1989b, 1990) for further details.

Daily closing values of the spot index and settlement price of the futures contracts are used. For the futures price series, we use the nearest regular contracts and roll over to the next contract around the tenth of the contract month. The regular contract months are March, June, September, and December. Each contract expires on the third Wednesday of the contract month.

To examine the post-sample hedging performance of various strategies, we use the data from January 1989 through August 1996 (a total of 1861 observations) for the in-sample analysis and estimation, and the data from September 1996 through August 1997 (a total of 245 observations) for post-sample comparisons. The mean and variance of each hedged portfolio are used to determine the desirability of each strategy.

The regression model was estimated using ordinary least squares (OLS), whereas the VAR and EC models (with and without conditional

[4]Note that the NSA futures contract traded in the Chicago Mercantile Exchange (CME) is denominated in Dollar. As pointed out by Hull (1998, p.63), the cost-of-carry pricing arguments and the hedging measures break down when the spot and futures prices are in different currencies. This problem, however, does not exist for the SIMEX contract. We are indebted to a referee for pointing out this difference in the CME contract.

TABLE I

Estimation Results of the ARFIMA (1,d,1)-GARCH (1,1) Models

Parameter	*Futures Return*	*Spot Return*	*Basis*	*Basis*
μ	−0.0067	0.0141	0.1405[b]	0.1395
	(0.0329)	(0.0318)	(0.0513)	(0.1078)
θ	−0.5397[a]	−0.5283	−0.1332[b]	−0.1198[b]
	(0.2344)	(0.6767)	(0.0513)	(0.0405)
ϕ	−0.4784	−0.5152	−0.0147	
	(0.2497)	(0.6866)	(0.0578)	
d	0.0010	−0.0011	0.3763[b]	0.3774[b]
	(0.0278)	(0.0295)	(0.0262)	(0.0308)
ω	0.0509[b]	0.0573[b]	0.0021[a]	0.0021[a]
	(0.0165)	(0.0201)	(0.0009)	(0.0009)
α	0.0661[b]	0.0841[b]	0.1032[b]	0.1032[b]
	(0.0139)	(0.0197)	(0.0192)	(0.0216)
β	0.9095[b]	0.8873[b]	0.8852[b]	0.8852[b]
	(0.0191)	(0.0265)	(0.0207)	(0.0234)

Notes: The numbers in the parentheses are the corresponding standard errors of the estimates.
[a]Statistical significance at the 5% level.
[b]Statistical significance at the 1% level.

heteroskedasticity) were estimated using the maximum likelihood (ML) method. For the FIEC model we employed the conditional sum of squares (CSS) method as in Baillie, Chung, and Tieslau (1995). Ling and Li (1997) proved the consistency and asymptotic normality of the CSS estimators in the presence of conditional heteroskedasticity. The method, however, depends upon the (theoretically) infinite past history of the series. In empirical estimation, the pre-sample observations are truncated at a finite lag value. Unlike Ling and Li (1997), who imposed the pre-sample observations to be zero, we use the actual data as pre-sample values. This method reduces the effective sample size of the estimation. As long-memory processes are characterized by hyperbolically declining expansions, long lags are required. After some experimentation, we truncate the pre-sample observations at lag 500. Thus, the effective sample size of the estimated models is 1,361.

The above approach is applied to determine possible fractional integration orders for the spot return, the futures return and the basis via eq. (8). We consider the ARFIMA(1,d,1)-GARCH(1,1) model:

$$(1 - \theta B)(1 - B)^d (x_t - \mu) = (1 - \phi B)\varepsilon_t, \tag{15}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{16}$$

where θ, d, μ, ϕ, ω, α and β are parameters, and x_t stands for the spot/

futures return or the basis. The results are presented in Table I. We estimated two equations for the basis. The last column of Table I presents the equation when the insignificant parameter ϕ is deleted. Thus the hypotheses that the spot and futures prices are I(1) cannot be rejected. We further conducted the Dickey-Fuller (DF) test to ascertain this conclusion. The t-ratios of the DF regressions (with intercept but without time trend) of the spot and futures prices were found to be -1.536 and -1.568, respectively, which are higher than the approximate critical value (-2.865) at the 5% level. Thus nonstationarity of the spot and futures prices cannot be rejected.

Based on the last column of Table I, the basis was found to be I(0.3774). Thus the fractional cointegration variable (see eqs. (10) and (11)) can be defined as $z_t^* = [(1 - B)^{0.3774} - (1 - B)]z_t$. The AIC was applied to select the lag orders of the models.

The estimation results of various models are summarized in Table II. The lag orders of the VAR and the EC/FIEC models were identified as two and one, respectively. Thus the VAR and EC models are nonnested. The mean equations of the VAR and VAR-GARCH models indicate that the spot and futures prices exhibit mean reversion by responding negatively to their own lagged values. For the EC and EC-GARCH models, the futures price is affected only by its history. The spot index adjusts positively to the basis. That is, a positive basis increases the futures price, and a negative basis reduces the futures price. Similar results were observed for the FIEC and FIEC-GARCH models. In all cases, we conclude that the Nikkei spot index unidirectionally follows the Nikkei futures prices. This result is consistent with Tse (1995).

The estimation results for the variance equations support the existence of conditional heteroskedasticity for both the spot index and the futures price. There is strong persistence in the variance movement. The coefficients for the lagged variance terms are around 0.90. The futures price exhibits a stronger variance persistence than the spot index. As expected, the two series are highly correlated.

For the models without the GARCH effect, the hedge ratios can be calculated from the results from Table IIb. The EC hedge ratio is found to be 0.9231, which, as expected, is larger than the VAR hedge ratio of 0.9199. On the other hand, it is also larger than the FIEC hedge ratio of 0.9212. However, as the futures price is not affected by the lagged basis, the latter result is perhaps not unexpected (see the discussions in the Appendix). Finally, we note that the regression hedge ratio of 0.9170 is the smallest among all models.

TABLE II

Estimation Results of Various Models of Hedge Ratios

(A) Mean Equations

Model	*Dependent Variable*	*Constant*	Δp_{t-1}	Δp_{t-2}	Δf_{t-1}	Δf_{t-l}	z_{t-1}	z_t^*
Regression	Δp_t	0.0122			0.9170			
(R^2 = 0.911)		(0.0113)			(0.0078)			
VAR	Δp_t	0.0018	−0.6008	−0.2698	0.5878	0.2738		
		(0.0246)	(0.0428)	(0.0330)	(0.0406)	(0.0326)		
(LLF = −802.1)	Δf_t	−0.0261	−0.0129	0.0142	−0.0322	−0.0140		
		(0.0257)	(0.0458)	(0.0326)	(0.0434)	(0.0322)		
EC	Δp_t	−0.1269					0.2108	
		(0.0262)					(0.0113)	
(LLF = −1084.8)	Δf_t	−0.0263			−0.0666			
		(0.0265)			(0.0053)			
FIEC	Δp_t	−0.0271						0.8323
		(0.0238)						(0.0273)
(LLF = −848.1)	Δf_t	−0.0256				−0.0402		
		(0.0251)				(0.0050)		
VAR-GARCH	Δp_t	0.0324	−0.5523	−0.2143	0.5179	0.2606		
		(0.0514)	(0.0265)	(0.0282)	(0.0250)	(0.0293)		
(LLF = −552.3)	Δf_t	0.0074	0.0440	0.0763	−0.1103	−0.0301		
		(0.0557)	(0.0282)	(0.0325)	(0.0272)	(0.0348)		
EC-GARCH	Δp_t	−0.0611					0.1930	
		(0.0253)					(0.0112)	
(LLF = −851.1)	Δf_t	0.0278			−0.0727			
		(0.0263)			(0.0060)			
FIEC-GARCH	Δp_t	0.0154						0.8268
		(0.0186)						(0.0281)
(LLF = −610.5)	Δf_t	0.0223			−0.0414			
		(0.0196)			(0.0056)			

POST-SAMPLE HEDGING PERFORMANCE

Using the above estimation results, we calculate the post-sample hedge ratios and compare the hedging performance of each strategy. As the performance of the models may vary according to the hedge horizon, we consider hedge horizons of 1, 5, 10, 20, 30 and 40 day(s). The results are summarized in Tables III through VIII.

When conditional heteroskedasticity is not taken into account, the optimal hedge ratio is a constant regardless of the length of the hedge period. First, we focus on the hedging strategies with constant hedge ratios. In the one-day hedge case, a tradeoff between risk and return occurs. Whereas the regression method generates the smallest return, it also incurs the smallest risk. On the other hand, the EC hedge ratio has both the highest return and the highest risk. Extending to longer hedge horizons, some regularities seem to emerge. The EC model generates the

TABLE II (Continued)

Estimation Results of Various Models of Hedge Ratios

(B) Variance and Covariance Equations

Model	*Dependent Variable*	*Constant*	$\sigma_{pp,t-1}$	$\sigma_{ff,t-1}$	$\varepsilon^2_{p,t-1}$	$\varepsilon^2_{f,t-1}$
VAR	$\sigma_{pp,t}$	1.8950				
		(0.0384)				
	$\sigma_{ff,t}$	2.1023				
		(0.0424)				
	ρ	0.9689				
		(0.0010)				
EC	$\sigma_{pp,t}$	1.9343				
		(0.0525)				
	$\sigma_{ff,t}$	2.1021				
		(0.0570)				
	ρ	0.9623				
		(0.0014)				
FIEC	$\sigma_{pp,t}$	1.9034				
		(0.0518)				
	$\sigma_{ff,t}$	2.1011				
		(0.0571)				
	ρ	0.9679				
		(0.0012)				
VAR-GARCH	$\sigma_{pp,t}$	0.0966	0.8758		0.0684	
		(0.0150)	(0.0142)		(0.0078)	
	$\sigma_{ff,t}$	0.0876		0.8951		0.0591
		(0.0119)		(0.0101)		(0.0060)
	ρ	0.9662				
		(0.0013)				
EC-GARCH	$\sigma_{pp,t}$	0.0924	0.8788		0.0690	
		(0.0163)	(0.0155)		(0.0085)	
	$\sigma_{ff,t}$	0.0788		0.9033		0.0557
		(0.0119)		(0.0101)		(0.0059)
	ρ	0.9584				
		(0.0016)				
FIEC-GARCH	$\sigma_{pp,t}$	0.0677	0.9043		0.0563	
		(0.0120)	(0.0119)		(0.0068)	
	$\sigma_{ff,t}$	0.0639		0.9167		0.0497
		(0.0098)		(0.0086)		(0.0052)
	ρ	0.9647				
		(0.0013)				

Notes: The numbers within the parentheses are the corresponding standard errors. LLF denotes the log-likelihood values of estimated models.

highest return and the lowest risk, whereas the regression method produces the worst performance, giving rise to the lowest return and the highest risk. Overall, it appears that the EC model provides the best strategy, followed by the FIEC and VAR models.

TABLE III

Post-Sample One-Day Hedging Performance

Model	*Mean of the Hedge Ratio*	*Variance of the Hedge Ratio*	*Mean of the Return of the Hedged Portfolio*	*Variance of the Return of the Hedged Portfolio*
Regression	0.9170		−0.0005	0.1382
VAR	0.9199		−0.0004	0.1383
VAR-GARCH	0.9279	0.0012	0.0047	0.1401
EC	0.9231		−0.0002	0.1385
EC-GARCH	0.9248	0.0010	0.0005	0.1349
FIEC	0.9212		−0.0003	0.1384
FIEC-GARCH	0.9200	0.0010	0.0033	0.1384

Notes: The hedge ratios of the models with GARCH effects are time varying, while other hedge ratios are constant. The hedged portfolio returns are measured in percentage.

TABLE IV

Post-Sample Five-Day Hedging Performance

Model	*Mean of the Hedge Ratio*	*Variance of the Hedge Ratio*	*Mean of the Return of the Hedged Portfolio*	*Variance of the Return of the Hedged Portfolio*
Regression	0.9170		−0.0076	0.1618
VAR	0.9199		−0.0070	0.1595
VAR-GARCH	0.9273	0.0009	0.0004	0.1604
EC	0.9231		−0.0064	0.1571
EC-GARCH	0.9244	0.0011	0.0005	0.1622
FIEC	0.9212		−0.0068	0.1585
FIEC-GARCH	0.9238	0.0008	0.0006	0.1599

Notes: The hedge ratios of the models with GARCH effects are time varying, while other hedge ratios are constant. The hedged portfolio returns are measured in percentage.

When the GARCH effects are considered, we find, on average, that the EC-GARCH hedge ratio remains larger than the FIEC-GARCH hedge ratio. In contrast, the relative size between these two ratios and the VAR hedge ratio varies with the hedge horizon. For the post-sample hedging performance, the EC-GARCH model produces the best results when the hedge horizon is ten days or more.

Incorporating the GARCH effects generally improves the hedging performance. For example, for the twenty-day hedge, the risk of the VAR, FIEC, and EC strategies are 0.3016, 0.2874, and 0.2958, respectively. The corresponding risk levels when the GARCH effects are considered are 0.2727, 0.2723, and 0.2754, respectively. Furthermore, the latter

TABLE V

Post-Sample Ten-Day Hedging Performance

Model	*Mean of the Hedge Ratio*	*Variance of the Hedge Ratio*	*Mean of the Return of the Hedged Portfolio*	*Variance of the Return of the Hedged Portfolio*
Regression	0.9170		−0.0129	0.2081
VAR	0.9199		−0.0119	0.2027
VAR-GARCH	0.9274	0.0007	0.0085	0.1916
EC	0.9231		−0.0108	0.1971
EC-GARCH	0.9263	0.0009	0.0062	0.1865
FIEC	0.9212		−0.0115	0.2004
FIEC-GARCH	0.9245	0.0007	0.0062	0.1891

Notes: The hedge ratios of the models with GARCH effects are time varying, while other hedge ratios are constant. The hedged portfolio returns are measured in percentage.

TABLE VI

Post-Sample Twenty-Day Hedging Performance

Model	*Mean of the Hedge Ratio*	*Variance of the Hedge Ratio*	*Mean of the Return of the Hedged Portfolio*	*Variance of the Return of the Hedged Portfolio*
Regression	0.9170		−0.0559	0.3151
VAR	0.9199		−0.536	0.3016
VAR-GARCH	0.9266	0.0005	−0.0023	0.2727
EC	0.9231		−0.0511	0.2874
EC-GARCH	0.9275	0.0006	−0.0003	0.2723
FIEC	0.9212		−0.0526	0.2958
FIEC-GARCH	0.9250	0.0005	−0.0055	0.2754

Notes: The hedge ratios of the models with GARCH effects are time varying, while other hedge ratios are constant. The hedged portfolio returns are measured in percentage.

strategies also generate higher returns. In fact, the EC-GARCH model is the dominant strategy when the hedge horizon exceeds five days, followed by the VAR-GARCH model. In the cases of one-day and five-day hedges, the GARCH strategies always generate higher returns—but sometimes at the cost of higher risk. Overall, in the one-day case, the best choice is among the three GARCH strategies: EC-GARCH has the lowest risk, whereas VAR-GARCH has the highest return.

In sum, although we have established the presence of fractional cointegration, the post-sample performance of the resulting FIEC hedge ratio does not compare favorably against the EC hedge ratio. Several factors may be responsible for the outcome. First, as shown in the analysis, frac-

TABLE VII

Post-Sample Thirty-Day Hedging Performance

Model	*Mean of the Hedge Ratio*	*Variance of the Hedge Ratio*	*Mean of the Return of the Hedged Portfolio*	*Variance of the Return of the Hedged Portfolio*
Regression	0.9170		−0.0742	0.4174
VAR	0.9199		−0.0711	0.3970
VAR-GARCH	0.9256	0.0004	−0.0073	0.3578
EC	0.9231		−0.0678	0.3755
EC-GARCH	0.9267	0.0005	−0.0056	0.3377
FIEC	0.9212		−0.0698	0.3881
FIEC-GARCH	0.9246	0.0004	−0.0122	0.3591

Notes: The hedge ratios of the models with GARCH effects are time varying, while other hedge ratios are constant. The hedged portfolio returns are measured in percentage.

TABLE VIII

Post-Sample Forty-Day Hedging Performance

Model	*Mean of the Hedge Ratio*	*Variance of the Hedge Ratio*	*Mean of the Return of the Hedged Portfolio*	*Variance of the Return of the Hedged Portfolio*
Regression	0.9170		−0.0975	0.4689
VAR	0.9199		−0.0936	0.4429
VAR-GARCH	0.9243	0.0003	−0.0185	0.4019
EC	0.9231		−0.0892	0.4155
EC-GARCH	0.9255	0.0004	−0.0095	0.3749
FIEC	0.9212		−0.0918	0.4316
FIEC-GARCH	0.9238	0.0003	−0.0216	0.4098

Notes: The hedge ratios of the models with GARCH effects are time varying, while other hedge ratios are constant. The hedged portfolio returns are measured in percentage.

tional cointegration has little impact on the hedge ratio when the futures price is independent of the fractional cointegration variable (which is exactly the case for the NSA spot and futures prices). Secondly, to derive the FIEC hedge ratio, we need to estimate the fractional integration order. The estimate may be sensitive to the sample data. In our case, during the later part of the estimation period, there were events such as the Kobe earthquake and the Nick Leeson incident.[5] Finally, as in the usual case of post-sample comparisons, structural change may occur in the post-sample.

[5]Hunt and Heinrich (1996) gave detailed accounts of both events.

CONCLUSIONS

In this article we examine the performance of various hedge ratios estimated from different econometric models. The FIEC model is introduced as a new model for estimating the hedge ratio. We use the NSA futures for illustration. Adopting the ARFIMA-GARCH approach, we identify the prevalence of a fractional cointegration relationship. We examine the effects of incorporating such a relationship into futures hedging and investigate the relative performance of various models with respect to different hedge horizons.

Optimal post-sample hedge ratios based on the fractional cointegration relationship are estimated. This strategy is compared against other competitive alternatives, including the regression method, the VAR model, and the EC model—with and without conditional heteroskedasticity. Our findings are as follows. First, incorporating conditional heteroskedasticity improves the hedging performance. Second, the hedge ratio of the EC model is consistently larger than that of the FIEC model. The former also provides better post-sample hedging performance in the return–risk context. Third, for longer hedge horizons of ten days or more, the hedging strategy based on the EC model incorporating conditional heteroskedasticty is the dominant strategy. Fourth, incorporating the fractional cointegration relationship does not improve the hedging performance over the EC model. Fifth, the conventional regression method provides the worst hedging outcomes for hedge horizons of five days or more.

Finally, we should add that our findings are based on our experience with the NSA index. Whether the results (especially the favorable performance of the EC model against the FIEC model) can be generalized to other cases is a topic for further research.

APPENDIX

To analyze the effects of fractional cointegration on the hedge ratio, we adopt the approach in Lien (1996) and consider the following FIEC model:

$$\Delta p_t = \alpha[(1 - B)^d - (1 - B)]\,(f_t - p_t) + \varepsilon_{pt}, \tag{A.1}$$

$$\Delta f_t = -\gamma[(1 - B)^d - (1 - B)]\,(f_t - p_t) + \varepsilon_{ft}, \tag{A.2}$$

where α, γ, ≥ 0. We note that the model considered here is a simple one. It does not allow for lagged spot and futures prices (apart from in the basis implicitly) in the right-hand-side of either equation. Although acknowledging this formulation does not fully describe most of the real-world data, we hope to generate some qualitative conclusions within a simple framework. Furthermore, for analytical tractability, we impose the condition that $\alpha + \gamma = 1$ (see also Garbade & Silber, 1993). Upon subtracting eq. (A.1) from eq. (A.2), we obtain

$$(1 - B)^d (f_t - p_t) = \varepsilon_{ft} - \varepsilon_{pt}. \tag{A.3}$$

Now let $u_t = [1 - (1 - B)^{1-d}] (\varepsilon_{ft} - \varepsilon_{pt})$. Eq. (A.3) implies $\Delta_{pt} = \alpha u_t + \varepsilon_{pt}$ and $\Delta_{ft} = -\gamma_{ut} + \varepsilon_{ft}$. Consequently, the VAR ratio is

$$\frac{\operatorname{cov}(\Delta p_t, \Delta f_t)}{\operatorname{var}(\Delta f_t)} = \frac{-\alpha\gamma \operatorname{var}(u_t) + \sigma_{pf}}{\gamma^2 \operatorname{var}(u_t) + \sigma_{ff}} \leq \frac{\sigma_{pf}}{\sigma_{ff}},$$

assuming ε_{ft} and ε_{pt} are both white noises. Noting that σ_{pf}/σ_{ff} is the FIEC hedge ratio, we conclude that the VAR hedge ratio is smaller than the FIEC hedge ratio.

To derive the EC hedge ratio, from eq. (A.3) we have

$$f_{t-1} - p_{t-1} = (1 - B)^d (\varepsilon_{f,t-1} - \varepsilon_{p,t-1}).$$

By projection, $[(1 - B)^d - (1 - B)] (f_t - p_t) = [1 - (1 - B)^{1-d}] (\varepsilon_{ft} - \varepsilon_{pt}) = \theta (1 - B)^{-d} (\varepsilon_{f,t-1} - \varepsilon_{p,t-1}) + w_t$, for some constant θ. Thus the EC model is as follows:

$$\Delta p_t = \alpha\theta(f_{t-1} - p_{t-1}) + \alpha w_t + \varepsilon_{pt},$$

$$\Delta f_t = -\gamma\theta(f_{t-1} - p_{t-1}) - \gamma w_t + \varepsilon_{ft}.$$

The EC ratio is, therefore,

$$\frac{\operatorname{cov}(\Delta p_t, \Delta f_t)}{\operatorname{var}(\Delta f_t)} = \frac{-\alpha\gamma \operatorname{var}(w_t) + \sigma_{pf}}{\gamma^2(\operatorname{var}(w_t) + \sigma_{ff}} \leq \frac{\sigma_{pf}}{\sigma_{ff}}.$$

Moreover, because $\operatorname{var}(u_t) \geq \operatorname{var}(w_t)$ we conclude that FIEC hedge ratio $\geq$ EC hedge ratio $\geq$ VAR hedge ratio. The differences in the three hedge ratios depend upon α, γ, $\operatorname{var}(u_t)$, and $\operatorname{var}(w_t)$.

When $\gamma = 0$ (that is, the futures price is not affected by the history of the basis), all three hedge ratios are equal. If $\alpha = 0$ (that is, the spot price is not affected by the history of the basis), the VAR hedge ratio is strictly smaller than the EC hedge ratio, which in turn is strictly smaller than the FIEC hedge ratio. In general, we expect $\operatorname{var}(w_t)$ to be small and, therefore, the EC hedge ratio is close to the FIEC hedge ratio. Thus, in practice, we expect the relative sizes of the estimated hedge ratios of the EC and FIEC models to be indeterminate.

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