MODELING NONLINEAR DYNAMICS OF DAILY FUTURES PRICE CHANGES

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The purpose of this article is to characterize linear and nonlinear serial dependence in daily futures price changes. The daily prices of four futures are included in this study: (i) S&P 500; (ii) Japanese yen; (iii) Deutsche mark; and (iv) Eurodollar. Our major empirical findings are: (i) Based on the results of nonlinearity tests (that is, the BDS, the Q^2, and the TAR-F tests), we found all futures price changes contain nonlinearity in the series; (ii) a GARCH model can explain the source of nonlinearity for three out of four series; (iii) a threshold autoregressive model and autoregressive volatility model can adequately represent nonlinear dynamics of S&P 500 series; and (iv) deterministic chaos is not evident in the scaled residuals from the nonlinear time series models. Hence we favor a statistical time series approach to represent the data-generating mechanism of futures price changes. © 1999 John Wiley & Sons, Inc. Jrl Fut Mark 19: 325–351, 1999

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INTRODUCTION

The primary purpose of this article is to investigate the existence and sources of nonlinear dependence in daily futures price changes. The secondary purpose is to reexamine the market anomalies such as the days-of-the-week and the holiday effects in a more general model framework that adjusts for linear and nonlinear dependence.

Understanding the dynamic behavior of daily futures prices is of critical importance for empirical financial researchers. For example, the validity of testing an alternative financial hypothesis depends on the adequacy of the empirical model upon which the hypothesis is performed. To calculate the market risk of a financial instrument from a Value-at-Risk (VaR) model, users need to understand the dynamic behavior of the daily price changes in order to select appropriate assumptions for the construction of confidence intervals for market risk. Previous research on futures price behavior generally follows two approaches. The first approach documents the phenomenon that the distribution of futures price changes is not normal but leptokurtic. In particular, the empirical distribution of daily futures price changes has more observations around the mean and in the extreme tails than does a normal distribution. Stable paretian, Student's t, and mixture of normal distributions have often been suggested as possible candidates to describe the empirical distribution of futures price changes. Past studies along this approach include Cornew et al. (1984), Hudson et al. (1987), Hall et al. (1988)—as well as others. This approach implicitly assumes that futures price changes follow a random walk hypothesis.

The second approach is to employ a time series model to characterize the linear and nonlinear serial dependence in daily price changes. Then the distribution properties of residuals from the selected models are examined. In general, linear or nonlinear dependence has been found in futures price changes. It is often found that the distribution properties of the residuals deviate less from a normal distribution in comparison with an empirical distribution of daily futures price changes. A GARCH(1,1) model is often selected as an underlying data-generating process model for futures price changes. The literature that follows this approach includes Taylor (1986), Akgiray (1989), Yang and Brorsen (1993, 1994), and Hsieh (1991, 1993)—in addition to many others.

This article extends the previous research by using more recent data and additional futures contracts to provide additional empirical evidence on the existence of nonlinear dependence in daily futures price changes. The appropriate models for daily futures price changes considered in this article are selected within a larger class of nonlinear time series models.

For example, we found that a GARCH(1,1) model can only explain the sources of nonlinearity for three out of four series. A threshold autoregressive model and autoregressive volatility model can adequately represent nonlinear dynamics of the S&P 500 index futures. Within a general model framework, the market anomalies in the conditional mean as well as in the conditional variance are also reexamined. The BDS statistic is used to distinguish the hypothesis that daily futures price changes are generated by a deterministic chaos mechanism from the hypothesis that daily futures price changes are generated by a nonlinear statistical data-generating process.

METHODOLOGY

In the first subsection, statistical tests for detecting nonlinear serial dependence in time series are presented. In the second subsection, the structure of three nonlinear time series models and their associated model building procedures are reviewed.

Statistical Tests for Nonlinearity in Time Series

The BDS Statistic

The BDS statistic proposed by Brock, Dechert, and Scheinkman (1987, 1996) is derived from the correlation integral. The statistic is used to test whether a time series is independent and identically distributed (IID). Brock (1987) proved that the BDS statistic can be applied to the residuals of a linear fitted model for detecting the chaos and nonlinearity in the time series. The BDS statistic can also be used to test the adequacy of the forecasting model by checking if there is any structure in the forecasting errors.

For the time series $\{x_t\}$, at embedding dimension of M, its M-histories $x(M)_t$ is defined by:

$$x(M)_t = (x_t, x_{t+1}, \ldots, x_{t+M-1}), \tag{1}$$

and its correlation integral $C_M(L)$ at embedding dimension of M and correlation length L (in the unit of sample standard deviation) is:

$$C_{M,T}(L) = \frac{2}{(T - M + 1)(T - M)} \sum_{1 \le i < j \le T-M+1} I_L\,(x(M)_i,\ x(M)_j) \tag{2}$$

where T is the sample size, and the indicator function $I_L(.)$ is defined by:

$$I_L(x(M)_i, x(M)_j = 1 \text{ if sup norm } |x(M)_i - x(M)_j)| < L,$$

$$I_L(x(M)_i, x(M)_j) = 0 \text{ elsewhere.} \quad (3)$$

The BDS statistic of the time series at embedding dimension M, correlation length L, and sample size T is:

$$BDS(M, L, T) = T^{1/2}(C_{M,T}(L) - C_{1,T}(L)^M)/\sigma, \quad (4)$$

where σ is the estimate of the asymptotic standard deviation of $T^{1/2}(C_{M,T}(L)\text{-}C_{1,T}(L)^M)$ when time series is IID.

Brock, Dechert, and Scheinkman (1987,1996) showed that if a time series is IID, then its BDS statistic is asymptotically standard normal distributed. Given a time series, after calculating its BDS statistic, the percentile values of the standard normal distribution can be used to decide the acceptance or rejection of the IID null hypothesis.

The finite sample properties of the BDS statistic have been studied by Brock, Dechert, and Scheinkman (1987, 1996), Hsieh and LeBaron (1988), Brock, Hsieh, and LeBaron (1991), and Lee, White, and Granger (1993). More recently, Gao, Hazilla, and Wang (1993) did a more comprehensive study on the finite sample properties of the BDS statistic. They found that the BDS statistic has power to detect varieties of non-IID time series. In a study of the BDS statistic, Gao (1994) also suggested using $M = 3$ and $L = 1.5$ to calculate the BDS statistic.

The TAR-F Statistic

The TAR-F statistic developed by Tsay (1996) is intended to detect the threshold autoregressive(TAR) type of nonlinearity in the time series. That is, the TAR-F statistic can be used to detect whether the time series follows different linear process when a threshold variable falls into different threshold regions.

To calculate the TAR-F statistic of the time series $\{x_t\}$, a regression with an AR order p and a threshold lag d is performed. The regressor of the regression is $\{(1, x_{t-1}, \ldots, x_{t-p}) \mid t = p + 1, \ldots, T\}$. The observation x_t and the regressor $(1, x_{t-1}, \ldots, x_{t-p})$ are arranged by the ascending order of the threshold variable x_{t-d}. After the arrangement, the n-th observation is $x_{\pi n}$ and its regressor is $(1, x_{\pi n-1}, \ldots, x_{\pi n-p})$. Now the threshold variable is $x_{\pi n-d}$, and $x_{\pi(n-1)-d} \leq x_{\pi n-d}$. For the arranged regression, a least square estimation is run with first b observations, then a recursive regression is employed to the remaining observations. Thus we can obtain the standardized predictive residuals of the arranged recursive regression

$e_{\pi i}$. We perform a least square regression with $e_{\pi i}$ on the regressor $(1, x_{\pi n-1}, \ldots, x_{\pi n-p})$, obtain the residuals of the regression, $\varepsilon_{\pi i}$, and compute the associated F-statistic, the TAR-F statistic:

$$F(p, d) = \frac{(\Sigma\, e_t^2 - \Sigma\, \varepsilon_t^2)/(p + 1)}{\Sigma\, e_t^2/(T - d - b - p - h)} \tag{5}$$

where T is the sample size, $h = \max\{1, p + 1 - d\}$. Tsay (1989) proved that if $\{x_t\}$ is a linear stationary AR process of order p, then for large T, F(p,d) follows approximately an F-distribution with $p + 1$ and T-d-b-p-h degrees of freedom. Furthermore, $(p + 1)F(p,d)$ is asymptotically a chi-squared random variable with $p + 1$ degrees of freedom.

The finite sample properties of the TAR-F statistic have been studied by Tsay (1989) and Gao(1994). Gao(1994) found that the TAR-F statistic is effective in detecting nonlinear time series such as nonlinear AR, nonlinear MA, and TAR processes.

The Q^2 Test

McLeod and Li (1983) developed the Q^2-statistic, following the suggestion of Granger and Andersen (1978) that the autocorrelation function of the squares of a time series can be useful for identifying the bilinear type nonlinear time series. McLeod and Li also suggested the Q^2-statistic to be used for identifying other types of nonlinear time series such as ARCH-type time series.

For the time series $\{x_t\}$, its Q^2-statistic is given by:

$$Q^2(p) = T(T + 2)\, \Sigma_{k=1,p}\, r^2(k)/(T - k),$$

$$r^2(k) = \sum_{t=1,T-k} (x_t^2 - u^2)(x_{t+k}^2 - u^2) \Big/ \sum_{t=1,T} (x_t^2 - u^2)^2,$$

$$u^2 = \sum_{t=1,T} x_t^2/T. \tag{6}$$

Under the null hypothesis of there is no autocorrelation in the squared values of the time series, the asymptotic distribution of the Q^2-statistic will have a chi-squared distribution with p degrees of freedom. Gao (1994) studied the finite sample properties of the Q^2 statistic and found the Q^2 test can detect certain types of serial dependence, especially when the serial dependence is in the conditional variance of the time series.

Nonlinear Time Series Models

The GARCH Model

The generalized autoregressive conditional heteroskedastic (GARCH) model developed by Bollerslev (1986) is an extension of the ARCH model of Engle (1982). In general, the ARCH-type process is applied to the error terms of a regression model. For example, if e_t is the error term of an regression model, then the GARCH process in the error term will be specified as:

$$e_t \mid \Psi_{t-1} \sim N(0, h_t),$$
$$h_t = \alpha_0 + \sum_{i=1,q} \alpha_j e_{t-i}^2 + \Sigma_{j=1,p} \beta_j h_{t-j}, \tag{7}$$

where $p \geq 0$, $q \geq 0$, $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i = 1, \ldots, q$, $j = 1, \ldots, p$. The GARCH process allows the lagged conditional variances as well as the lagged squares of error terms to determine the current conditional variance. It has the property that large (small) changes to be followed by large (small) changes in the time series generally seen in financial markets. Bollerslev, Chou, and Kroner (1992) present an excellent review of the application of the GARCH family for modeling volatility of financial markets.

The GARCH type nonlinear process can be detected by the Q^2 test. The estimation of GARCH type model requires maximum likelihood method. The Berndt, Hall, Hall, and Hausman (1974) algorithm is generally used to obtain the maximum likelihood estimates of this kind of model.

The Threshold Autoregressive Model

The threshold autoregressive (TAR) model was first developed by Tong (1978). A time series $\{x_t\}$ is a threshold autoregressive process if it follows the model below:

$$x_t = \alpha_0^{(j)} + \sum_{i=1,p} \alpha_i^{(j)} x_{t-1} + \varepsilon_t^{(j)}, \; r_{j-1} \leq x_{t-d} < r_j, \tag{8}$$

where $j = 1, \ldots, k + 1$, k is the number of thresholds, p is the AR order, d is the threshold lag, and x_{t-d} is the threshold variable. The threshold values of the TAR model are $-\infty = r_0 < r_1 < \ldots < r_k < r_{k+1} = \infty$. For each j, $\{\varepsilon_t^{(j)}\}$ are I.I.D. normal random variables with zero mean and variance σ_j^2 such that $e^{(i)}$ and $e^{(j)}$ are independent if $i \neq j$. The TAR model partitions the one-dimensional Euclidean space of x_{t-d} into k + 1 threshold regions and follows a linear AR process in each region. The overall model for x_t is not linear when there are at least two regions with two different linear processes. The model (8) is also called a self-exciting TAR model because the model uses its own lagged value of x_{t-d} as the threshold variable. The TAR model can be interpreted as one member of the switching linear regression models. The switching mechanism is controlled by threshold variable x_{t-d}, not by the time index t.

Tong (1978, 1983), Tong and Lim (1980) proposed this nonlinear time series model as an alternative model for describing periodic time series. The TAR model has certain features—such as limit cycles, amplitude dependent frequencies, and jump phenomena—that cannot be captured by linear time series models. For instance, Tong and Lim (1980) showed that the TAR model is capable of producing the asymmetric, periodic behavior exhibited in the annual Wolf's sunspot data and the Canadian LYNX data. The TAR models have also been applied in financial and economic time series. For example, Pope and Yadav (1990) employed a TAR model to characterize the mispricing behavior of FTSE 100 index futures. Tiao and Tsay (1994), using a TAR Model, studied the cyclical properties of real U. S. GNP quarterly series.

The TAR model uses threshold values to partition a nonlinear time series into piecewise linear autoregressive processes within each threshold region. This mechanism makes TAR a useful model to represent the empirical dynamic patterns of financial return time series evolving with past return series. This dynamic return pattern often refers to large return follows large returns and small return follows small returns. Furthermore, the TAR model permits to estimate varying autoregressive coefficients within different threshold regions. This feature has the capacity to capture the characteristics of time-irreversibility, asymmetric limit cycle behavior, and jump phenomena in time series of futures price changes if they exist.[1]

In this article, the modeling procure for the TAR model proposed by Tsay (1989) is used. This modeling procedure consists of the following steps:

1. To determine the threshold lag d, TAR-F test should be run for different threshold lags with an AR order p that is not too small. The threshold lag that has the largest TAR-F statistic should be selected.
2. To identify threshold r_i values, the arranged recursive autoregression (described earlier in this article) with the threshold lag determined in

[1]Further discussion of the properties of the TAR model and on comparison of the TAR with other linear and nonlinear time series models is referred to Tong (1990).

step (1) should be run. The scatter-plot of t-ratios of the AR coefficients at each recursive stage should be obtained. If the arranged recursive autoregression passes a threshold value by even a small amount, then the pattern of gradual convergence of one or more t-ratios will be destroyed, and these t-ratios will be greatly changed. The values of x_{t-d} where t-ratios show dramatic changes are the locations of the threshold values.

3. Once the threshold values are determined, the AR order in each threshold regime should be selected by using Akaike information criteria (AIC).

The Autoregressive Volatility Model

Hsieh (1991, 1993) has applied the autoregressive volatility model to capture nonlinear dependence in daily S&P 500 index and daily foreign exchange rate changes. The general specification of this model is as follows:

$$X_t + u_t = \sigma_t Z_t \tag{9}$$

X_t denotes log price changes, $Z_t \sim$ IID N (0,1) and σ_t is treated as a random variable and it evolves according to:

$$\text{Log}(\sigma_t) = b_0 + \Sigma_{i=1,l}\, b_i \log(\sigma_{t-i}) + v_t \tag{9a}$$

where v_t is IID and distributed independent of Z_t, and u_t, the conditional mean of X_t, is often treated as zero for daily log price changes or u_t is filtered out by least squares estimates from the X_t. The motivation of autoregression in $\log(\sigma_t)$ is to capture the common feature in financial time series - large (small) return follows large (small) returns. In GARCH type model, the conditional variance h_t is deterministic and is only influenced by lagged conditional variance and lagged squared residuals up to $t - 1$ time period. However, in the autoregressive volatility model, σ_t is treated as a random variable and is influenced not only by lagged σ_{t-i}, but also by the current innovation term v_t. This is the major difference between these two models.[2]

The estimation of parameters of model (9) can be complicated because the σ_t is not directly observable.[3] The estimation procedure sug-

[2]Taylor (1994) presents an excellent review of the differences between a class of GARCH models and a class of stochastic volatility models. It should be mentioned that the autoregressive volatility model suggested by Hsieh (1991, 1993) is a special member of stochastic volatility models.

[3]See Taylor (1994) for a discussion on the estimation problems of general class of stochastic volatility models.

TABLE I

Futures and Sample Period

Futures	*Sample Period*	*Contract Months*	*Sample Size*
S&P 500	1/84–9/93	Mar May Sep Dec	2444
J. yen	1/84–6/93	Mar May Sep Dec	2401
D. mark	1/84–6/93	Mar May Sep Dec	2401
Eurodollar	1/84–6/93	Mar May Sep Dec	2401

gested by Hsieh (1991, 1993) is adopted in this study. The estimation and diagnostic checking of the adequacy of model (9) is described as follows: (i) an ex-post estimate of σ_t is used to replace the unobserved σ_t in eq. (9a); (ii) the estimated σ_t, generated by OLS estimates from eq. (9a), is used as the estimate of conditional variance; (iii) Q, and BDS statistics are used to check the adequacy of volatility model in (9a); and (iv) apply the same set of diagnostic checking statistics to examine whether the standardized residual $(X_t/(\sigma_t))$ has captured all nonlinear dependence in the time series X_t.

THE DATA AND EXPLORATORY DATA ANALYSIS

Daily prices of four actively traded futures contracts are selected in this study. They are: (i) S&P 500 Index; (ii) Japanese yen; (iii) Deutsche mark; and (iv) Eurodollar. The sample period is from the beginning of 1984 to June of 1993. The details of the data set are described in Table I.

The daily prices are the daily settlement prices for each contract. The data is from the CFTC market data bank. A continuous series of nearby daily futures prices is constructed. During the maturity months the nearby futures prices are rolled over by the daily prices of first deferred contract. The date of rollover is occurred when the trading volume of the first deferred contract is greater than the trading volume of the nearby contract. The date of rollover is often occurred in the middle or later parts of maturity month. Following this way, each data set has more than 2300 observations. The large data set for each futures series provides enough degrees of freedom for the justification in using large sample results for our statistical inference.

For exploratory data analysis, we are interested in testing an unit root against the alternative of a stationary process, but we are willing to entertain the possibility of two unit roots. A sequential test suggested by

TABLE II

Test of Unit Root in Log-prices and Log-Price Changes

	$(\alpha$-$1)$ τ statistic Log-price Changes		$(\alpha$-$1)$ τ statistic Log-prices	
S&P 500	−1.23	−15.64[a]	−0.0085	−2.81
J. yen	−0.89	−13.57[a]	−0.0016	−1.57
D. mark	−0.94	−14.10[a]	−0.0020	−1.44
Eurodollar	−0.89	−14.02[a]	−0.0021	−1.75

[a]Denotes rejection of the null hypothesis of an unit root at 1% level. The critical value of t statistic for 1% level is −3.96 (see Fuller, 1976) p. 373.

TABLE III

Summary Statistics of Daily Futures Log-Price Changes

Futures	*Maximum*	*Minimum*	*Mean*	*Var.*	*Skew.*	*Kurt.*	λ
S&P 500	17.75	−33.70	0.0277	1.654	−6.690[a]	210.8[a]	4540000[a]
J. yen	5.33	−4.13	0.0251	0.453	0.405[a]	4.12[a]	1763[a]
D. mark	4.83	−3.31	0.0162	0.609	0.156[a]	1.90[a]	371[a]
Eurodollar	0.297	−0.106	0.0015	0.00053	1.128[a]	13.76[a]	19400[a]

[a]Denotes rejection of the null hypothesis of a normal distribution at 1% level.
λ is the Bera and Jarque normality test and λ = (sample size)*((skew)²/6 + (kurt)²/24)) follows $\chi^2_{(2)}$ distribution.
The critical value of Chi-square with 2 degree of freedom at 1% level is 9.21.

Dickey and Pantula (1987) is employed. We begin with the greatest number of unit roots one is willing to consider. In our case, we begin by testing a second unit root under the maintain hypothesis of a single unit root. If y_t is the log-price of the futures, $x_t = y_t - y_{t-1}$, $x_t - x_{t-1} = \Delta x_t$, then the model for argumented Dickey and Fuller (ADF) test (Fuller, 1976; Dickey & Fuller, 1979) is specified as:

$$\Delta x_t = \beta_0 + \beta_1 t + (\alpha - 1)x_{t-1} + \sum_{i=1,10} a_i \Delta x_{t-i} + v_t. \tag{10}$$

The empirical results (reported in Table II) reject the hypothesis of a second unit root. We now test a single unit root for the log price of original futures price. Let y_t be the log-price of the futures and $y_t - y_{t-1} = \Delta y_t$, then the model for ADF test is specified as:

$$\Delta y_t = \beta_0 + \beta_1 t + (\alpha - 1)y_{t-1} + \sum_{i=1,10} a_i\, \Delta y_{t-i} + u_t. \tag{11}$$

Empirical results in Table II indicate that we can not reject $(\alpha - 1) =$

TABLE IV

Test of Serial Dependence in Futures Log-Price Changes

Futures	*Q(24)*	*$Q^2(24)$*	*BDS*	*TAR-F*
S&P 500	109[a]	207[a]	8.73[a]	32.92[a]
J. Yen	29.2	90.9[a]	6.97[a]	0.84
D. Mark	24.3	156[a]	3.09[a]	2.45[a]
Eurodollar	44.6[a]	105[a]	6.12[a]	3.94[a]

[a]Denotes rejection of the null hypothesis at 1% level.
The BDS statistic is calculated at M = 3 and L = 1.5, and the TAR-F statistic is calculated at d = 3 and p = 10.

0, this confirms that there is a single unit root in log of futures prices. Thus, we conclude that the log-price changes are stationary time series.

Table III provides summary statistics of the log-price changes of the four futures. All the means are not statistically different from zero. The estimated variances of the daily log-price changes are very different from futures to futures. All of the futures daily log-price changes are skewed. The skewness is negative for S&P 500 futures, but is positive for Japanese yen futures, for Deutsche mark futures, and for Eurodollar futures. The kurtosis of all four futures is significantly different from that of the normal distribution. These results suggest that the distributions of log-price changes are deviated from a normal distribution.

EMPIRICAL RESULTS

In this section, linear and nonlinear time series models are fitted to characterize serial dependence in daily futures log-price changes. In the real world, the true model that generates observed time series are unknown. A time series model is considered as adequate if the residuals are distributed as IID series. For this reason, the empirical results of diagnostic tests are used as the guide in the selection of an adequate empirical time series model. In other words, a fitted time series model is considered as adequate if its residuals pass all the diagnostic tests.

The diagnostic tests used here are: (i) Ljung-Box test for linear serial dependence; (ii) the Q^2 test for nonlinear serial dependence in conditional variance; (iii) the TAR-F test for nonlinear serial dependence in conditional mean; and (iv) the BDS statistic test for non-IID time series. If a time series is not IID, it may contain linear or nonlinear serial dependence or it may be generated by a deterministic chaos mechanism.

The empirical results of these tests on log-price changes of the four futures contracts are presented in Table IV. The Ljung-Box Q statistic shows that there is linear dependence in log-price changes of S&P 500 futures and of Eurodollar futures. The Q^2 statistic shows that there is serial dependence in the square of the log-price changes of all four futures. The large Q^2 statistic can be the results of linear dependence or the results of nonlinear dependence such as the GARCH process. The BDS statistic rejects the null hypothesis of IID for log-price changes of all four futures. The TAR-F statistic rejects the linear time series model for log-price changes of S&P 500 futures, of Deutsche mark futures, and of Eurodollar futures. The large TAR-F statistic can be the results of nonlinearity in the conditional mean, such as TAR process.[4]

Linear Time Series Models

Because the BDS statistic rejects the IID null hypothesis for log-price changes of all four futures, we first investigate whether the rejection of IID is due to the results of days-of-the-week effect in the conditional mean and of the linear AR process. Thus a linear regression model of log-price changes is fitted for each of the four futures:

$$x_t = C + \sum_{i=1,4} \beta_i D_i + \beta_H DH + \sum_{j=1,p} b_j x_{t-j} + e_t, \qquad (12)$$

where e_t follows a stationary process.

In eq. (12), the D_is are the dummy variables for days-of-the-week, D_1 is for Monday, D_2 is for Tuesday, D_3 is for Wednesday, and D_4 is for Thursday, the Friday is the base case. The DH is the dummy variable for holidays (that is, the day following the nontrading holiday). The lag length p of the AR process is determined by the Akaike information criterion. The estimated parameters of the models and the test results of serial dependence in the residuals are presented in Table V.

The major empirical results of Table V are summarized as follows:

1. The days-of-the-week effect in the conditional mean is not significant for all four futures contracts except for Eurodollar futures. We found that the mean log price changes is highest on Friday and lowest on Monday for Eurodollar futures.
2. We found negative coefficients of the holiday effect in conditional mean for all four contracts. However, the coefficient of the holiday effect is significant at 5% level only for Eurodollar futures. The neg-

[4]The bispectral nonlinearity test proposed by Hinich (1982) was also applied to the log price changes of the four futures contracts. The bisspectral test rejects linear time series process for all four futures in this study.

TABLE V

Estimated Linear Models and Test of Serial Dependence in Residuals

	S&P 500	*J. Yen*	*D. Mark*	*Eurodollar*
C	−0.044	0.0015	−0.026	0.00202[b]
	(0.058)	(0.031)	(0.036)	(0.00105)
D_1	0.082	0.027	0.058	−0.0023
	(0.082)	(0.043)	(0.051)	(0.0015)
D_2	0.115	0.0079	0.049	0.0021
	(0.081)	(0.043)	(0.050)	(0.015)
D_3	0.132	0.023	0.059	−0.0010
	(0.081)	(0.043)	(0.050)	(0.0015)
D_4	0.058	0.068	0.056	−0.0010
	(0.081)	(0.043)	(0.050)	(0.0015)
D_H	−0.070	−0.054	−0.058	−0.0057[b]
	(0.149)	(0.080)	(0.092)	(0.0027)
x_{t-1}	−0.016			−0.084[b]
	(0.020)			(0.020)
x_{t-2}	0.159[b]			
	(0.020)			
x_{t-3}	−0.019			
	(0.020)			
x_{t-4}	−0.062[b]			
	(0.020)			
x_{t-5}	0.072[b]			
	(0.020)			
Test of Serial Dependence in Residuals				
Q(24)	33.3	29.1	24.9	29.6
Q^2 (24)	67.0[a]	90.1[a]	157[a]	114[a]
BDS (M = 3, L = 1.5)	8.18[a]	7.14[a]	3.11[a]	6.21[a]
Test of Normality in Residuals				
Skew	−8.04[a]	0.41[a]	0.17[a]	1.07[a]
Kurt	224[a]	4.09[a]	1.89[a]	13.0[a]
λ	573000[a]	1740[a]	369[a]	17400[a]

[a]Denotes rejection of the null hypothesis at 1% level.
[b]Significant at 5% level.
Standard errors of the estimated parameters are in parentheses.
λ is the Bera and Jarque normality test. It is defined in Table III.

ative coefficient of the holiday effect suggested that the mean of log price changes for the day after holiday is lower than the mean of log price changes on Friday.

3. There are linear serial dependence in S&P 500 and Eurodollar futures. The residuals of estimated linear time series models for all futures considered in this study pass the Ljung-Box test. These results suggest that our fitted linear time series models are adequate to represent linear serial dependence in the series.

4. The residual of estimated linear time series models for all four futures passed neither the Q^2 test and the BDS test. These results confirm there are nonlinear dependence in the series. For this reason, alternative nonlinear time series model are fitted to the data in order to characterize the sources of nonlinear dynamics of these series.

Nonlinear Time Series Models

The strategy in fitting nonlinear time series models is as follows: first, the GARCH(1,1) model is fitted to all four series. Other nonlinear time series will be fitted to the series if the GARCH(1,1) model fails to represent nonlinear dynamics of the series.

The GARCH Model

The regression model with GARCH(1,1) error terms is examined first to determine whether it is adequate for modeling the log-price changes of these four futures.

The GARCH(1,1) model used in this article is specified as:

$$x_t = C + \sum_{i=1,4} \beta_i D_i + \beta_H DH + \sum_{j=1,p} b_j x_{t-j} + e_t, \; e_t \sim N(0,h_t)$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 h_{t-1} + \sum_{i=1,4} a_i D_i + a_H DH. \tag{13}$$

The D_is are the dummies for the days-of-the-week effect and the DH is the dummy for the day after holiday effect. In this model, the conditional mean of the log-price changes has three components: AR process, the days-of-the-week effect, and the holiday effect. The lag of the AR process in the conditional mean is determined by the Akaike information criterion in the last section. The conditional variance of log-price changes is an autoregressive process of past sample variance and past conditional variance. The model also has the days-of-the-week effect and the holiday effect in the conditional variance. The parameters of the GARCH model is estimated by the maximum likelihood method. The results of the estimation of the GARCH model are presented in Table VI.

The findings based on the GARCH model are as follows. For the conditional mean of log-price changes in the model, the results of the days-of-the-week and the holiday effect show little change from the results of the linear model estimated in the last section. However, fewer coefficients of the AR process remain significant for S&P 500 futures.

TABLE VI

Estimated GARCH(1,1) Models and Test of Serial Dependence in Residuals

	S&P 500	*J. Yen*	*D. Mark*	*Eurodollar*
C	0.017	−0.013	−0.015	0.0022[b]
	(0.045)[b]	(0.031)	(0.035)	(0.0010)
D_1	0.269[b]	0.032	0.039	−0.0033[b]
	(0.063)	(0.046)	(0.050)	(0.0012)
D_2	0.098	0.034	0.033	−0.00018
	(0.061)	(0.043)	(0.049)	(0.00122)
D_3	0.075	0.028	0.051	−0.00072
	(0.056)	(0.039)	(0.045)	(0.00122)
D_4	−0.017	0.055	0.050	−0.00053
	(0.065)	(0.042)	(0.048)	(0.01134)
D_H	−0.241[b]	−0.125	−0.072	−0.0037
	(0.108)	(0.071)	(0.095)	(0.0024)
x_{t-1}	−0.0017			−0.078[b]
	(0.019)			(0.024)
x_{t-2}	−0.0068			
	(0.018)			
x_{t-3}	−0.0019			
	(0.020)			
x_{t-4}	0.0088			
	(0.019)			
x_{t-5}	0.017			
	(0.019)			
Variance:				
α_0	0.083[b]	0.118[b]	0.142[b]	0.00027[b]
	(0.026)	(0.019)	(0.036)	(0.00002)
e^2_{t-1}	0.0196[b]	0.090[b]	0.0604[b]	0.114[b]
	(0.0011)	(0.011)	(0.0087)	(0.006)
h_{t-1}	0.864[b]	0.820[b]	0.887[b]	0.837[b]
	(0.014)	(0.020)	(0.017)	(0.0092)
D_1		−0.054	−0.097	−0.000531[b]
		(0.036)	(0.062)	(0.000036)
D_2	−0.054	−0.126[b]	−0.215[b]	−0.00026[b]
	(0.068)	(0.029)	(0.056)	(0.000026)
D_3	−0.178[b]	−0.171[b]	−0.215[b]	−0.00028[b]
	(0.058)	(0.026)	(0.048)	(0.000024)
D_4	0.290[b]	−0.037	−0.053	−0.00017[b]
	(0.061)	(0.030)	(0.052)	(0.000028)
D_H	0.280[b]	0.058	0.220[b]	0.00021[b]
	(0.068)	(0.036)	(0.049)	(0.000023)
Test of Serial Dependence in Standardized Residuals				
Q (24)	22.0	31.1	22.5	28.4
Q^2 (24)	7.68	14.7	20.3	18.1
BDS (M = 3, L = 1.5)	1.78[b]	0.76	−1.55	−2.85
Test of Normality in Standardized Residuals				
Skew	−5.04[a]	0.49[a]	0.072[a]	0.44[a]
Kurtosis	106[a]	3.56[a]	1.12[a]	4.24[a]
λ	1150000[a]	1363[a]	127[a]	1909[a]

[a]Denotes rejection of the null hypothesis at 1% level.
[b]Significant at 5% level.
Standard errors of the estimated parameters are in parentheses.
At 1% significant level, the critical value of the BDS test on the standardized residuals of GARCH model is 1.59 (see Brock et al., 1991, p. 279).
λ is defined in Table III.

TABLE VII

TAR-F Statistic for Log-price Changes

Threshold Lag	*S&P 500*
1	39.12[a]
2	33.83[a]
3	35.82[a]
4	35.52[a]
5	24.56[a]

[a]Denotes the rejection of null hypothesis at 1% level.
The AR order used for calculating TAR-F statistic is 5. Its selection is based on Akaike Information criterion.

This implies that, if we omit the effects of a GARCH process, we may incorrectly make inferences about the existence of the AR process in the conditional mean of the log-price changes.

For the conditional variance of the log-price changes in the model, the effects of sample variance and conditional variance in the last period are significant. But the effects of the past sample variance is less than the effect of past conditional variance. The persistence measure of volatility ranges from 0.884 (S&P 500 index futures) to 0.51 (Eurodollars). These measures are consistent with the persistent measures for S&P 500 index and foreign exchanges obtained by Hsieh (1991, 1993). These persistent measures confirm that there are strong cluster effects in conditional variances and conditional variances have a mean-revision property (that is, conditional variances are stationary). All futures have significant days-of-the-week and holiday effects on conditional variance.[5] For all the futures except Eurodollar futures, the variance in midweek is smaller than the variance on Mondays and Fridays. For all four futures, the variance over the holidays is larger than other period and statistically significant, supporting the calendar time hypothesis rather than the trading time hypothesis. These results are consistent with previous findings by Hsieh (1988) and Baillie and Bollerslev (1989) on the positive impact of holiday effects on conditional variance of spot exchange rates and with the study done by Yang and Brorsen (1993, 1994) on log futures price changes.

[5]The TSP program used for estimation of GARCH model reported that the day-of- the week effect on Monday cannot be estimated for the S&P 500 futures due to singularity of the data. This is probably caused by the large decline of futures prices on Black Monday of 1987. For this reason, a GARCH model for the S&P 500 futures is reestimated with the omission of the 1987 crash period (that is, from 16 October 1987 to 28 October 1987), and the empirical results are reported in Table A1. It is interesting to observe that the GARCH model reported in Table A1 passes all the diagnostic tests. Hence it is an adequate model for this sample period only when the sample data of the crash period is omitted.

The test results for serial dependence in the standardized residual of the GARCH model are also presented in Table VI. The standardized residuals of Japanese yen, Deutsche mark, and Eurodollar futures pass the Ljung-Box test, Q^2 test, and BDS test. These results imply that the standardized residuals of the GARCH model are IID. Therefore, the GARCH process can adequately model the log-price changes of these three futures. The standardized residuals of S&P 500 futures pass the Ljung-Box test and the Q^2 test, but they do not pass the BDS test. Hence, GARCH(1,1) model cannot adequately represent S&P 500 futures log price changes in this sample period. For this reason, the TAR model and the autoregressive volatility model are examined to decide whether they can adequately represent the dynamic behaviors of the log-price changes of S&P 500 futures in this sample period.[6]

The TAR Model

The TAR-F statistics calculated from the log price changes of S&P 500 futures are statistically significant at the 5% level (see Table IV). These findings suggested that we should consider a TAR model as a possible candidate to represent the nonlinear dynamics of S&P 500 futures price changes. The model identification and estimation procedure for a TAR process proposed by Tsay (1989) is used in this study.

The first step in modeling the TAR process is to determine the threshold lag. The TAR-F statistic of the log-price changes at threshold lags of 1 to 5 are calculated. The AR order used in the calculation is 5 for S&P 500 futures, where the AR orders are determined (as described previously in this article) by using Akaike information criterion. The results of the TAR-F statistic for S&P 500 futures are presented in Table VII.

Table VII suggests that the TAR-F statistic is very large at threshold lags of 1 to 5 for log-price changes of S&P 500 futures. The TAR-F statistic at threshold lag of 1 is the highest. Thus the threshold lag of 1 for the TAR model of log-price changes of S&P 500 futures is selected.

For S&P 500 futures, the arranged recursive regression of the log-price changes is performed with the threshold lag of 1 and the AR order of 5. The scatter-plot of the t-values of the AR coefficients is presented in Figure 1. The scatter-plot of the t-values of the AR coefficients is used

[6]Yang and Barson (1993) found similar testing results for the standardized residuals of their GARCH(1,1) model. They suggested the use of a higher order of GARCH model may be required. Brock, Hsieh, and Lebaron (1991) also found that GARCH model cannot capture all the nonlinarities in log price changes of some foreign currencies that they studied.

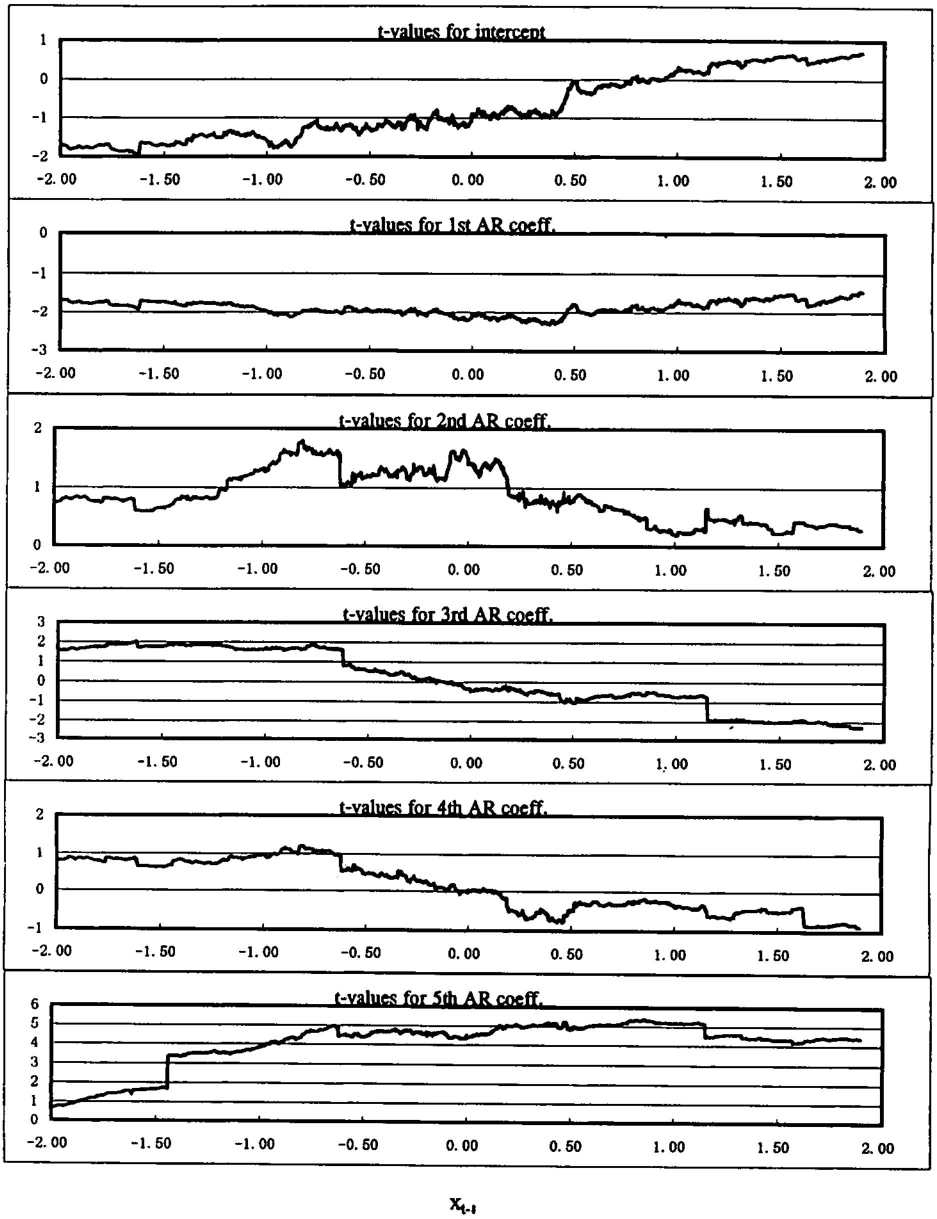

FIGURE 1
Scatter-plot of t-values for recursive autoregression of S&P500 futures daily returns

for determining the threshold values of the model. The places where the t-values have big changes are the location of the threshold values. Figure 1 shows that the largest changes in t-values of the coefficient have occurred at the values of -0.62 and 1.15. Thus, these two values are identified as the threshold values of the model, and the following TAR model is specified:

$$
\begin{aligned}
x_t &= C^1 + \sum_{i=1,4} \beta_i^1 D_i + \beta_H^1 DH + \sum_{j=1,5} b_j^1 x_{t-j} + e_t^1, \\
&\quad x_{t-1} < -0.62, \\
&= C^2 + \sum_{i=1,4} \beta_i^2 D_i + \beta_H^2 DH + \sum_{j=1,5} b_j^2 x_{t-j} + e_t^2, \\
&\quad -0.62 \leq x_{t-1} < 1.15, \\
&= C^3 + \sum_{i=1,4} \beta_i^3 D_i + \beta_H^3 DH + \sum_{j=1,5} b_j^3 x_{t-j} + e_t^3, \\
&\quad x_{t-1} \geq 1.15
\end{aligned} \tag{14}
$$

Where D_is are the dummy variables for days-of-the-week, DH is the dummy variable for holidays, and e_t^i's are error terms with a zero mean and a variance of σ^{i2}. The results of the estimated parameters of the TAR model for the log-price changes of S&P 500 futures [eq. (14)] are presented in Table VIII.

The empirical results of TAR model (in Table VIII) partition daily S&P 500 log future price changes into three threshold regions. In each region, daily price changes follow different linear processes.

Daily log price changes of the S&P 500 fall into threshold region 1 if the last period log price change is less than −0.62%. There are 469 observations (about 19.3% of the total sample) in this region. The day-of-the-week effects and holiday effect in the conditional mean is not significant. Two of the AR coefficients however are significant: −0.10 at lag one and 0.28 at lag five. The linear process in threshold II characterizes daily price changes when the last period price change between −0.62% and 1.15%. None of the AR coefficients are significant, but the day-of-the-week effect in the conditional mean are significant for Mondays and Wednesdays. The holiday effect is not significant. This region represents the majority of daily log price changes of the S&P 500 in this sample period because there are 1728 observations (about 70.8% of the total sample) in this region. The threshold region III characterizes time series properties of daily price changes when the last period price changes are greater than 1.15%. There are 242 (about 9% of the total sample) observations in this region. None of the day-of-the-week or holiday effect are significant. The AR coefficients at lags 2,4 and 5 are significant. Excepting lag one, the other four AR coefficients have negative signs.

TABLE VIII

Estimated TAR Models and Test of Serial Dependence in Residuals

	Region I ($x_{t-1} < -0.62$)	Region II ($-0.62 \leq x_{t-1} < 1.15$)	Region III ($x_{t-1} \geq 1.15$)
C	−0.237	−0.053	0.360
	(0.212)	(0.050)	(0.286)
D_1	−0.195	0.142[b]	0.590
	(0.275)	(0.071)	(0.313)
D_2	0.396	0.078	0.047
	(0.287)	(0.070)	(0.287)
D_3	0.143	0.134[b]	−0.194
	(0.278)	(0.071)	(0.282)
D_4	0.182	0.045	−0.032
	(0.286)	(0.069)	(0.300)
D_H	0.099	−0.134	−0.162
	(0.526)	(0.125)	(0.642)
x_{t-1}	−0.100[b]	−0.011	−0.192[b]
	(0.051)	(0.051)	(0.102)
x_{t-2}	0.137	−0.039	−0.395[b]
	(0.080)	(0.024)	(0.035)
x_{t-3}	0.132	−0.004	−0.021
	(0.085)	(0.023)	(0.051)
x_{t-4}	0.050	−0.018	−0.348[b]
	(0.050)	(0.024)	(0.076)
x_{t-5}	0.276[b]	−0.024	−0.194[b]
	(0.059)	(0.024)	(0.050)
σ^2	3.65	0.85	1.93
n	469	1728	242
Test of Serial Dependence in Standardized Residuals			
Q		17.9	
Q^2		24.4	
BDS (M = 3, L = 1.5)		−1.00	
Test of Normality in Standardized Residuals			
Skew		−2.8[a]	
Kurt		43.2[a]	
λ		193000[a]	

[a]Denotes rejection of the null hypothesis at 1% level.
[b]Significant at 5% level.
Standard errors of the estimated parameters are in parentheses.
n denotes the number of observations in each threshold region.
λ is defined in Table 3.

In comparing with the properties of linear process in these regions, three interesting empirical results are observed: (i) There are no autocorrelations in majority of daily price changes in region II, however, autocorrelations exist in daily price changes when the last period price changes are less than −0.62% or greater than 1.15%. (ii) In terms of the magnitude of the standard deviation of residuals of linear processes, the standard deviation in region I (3.65) is the largest, the next largest one is 1.93 in region III, and the least standard deviation is 0.85 in region II. When the standard deviation in region I is greater than the standard deviation in region III, these results support the phenomenon of asymmetric volatility. (iii) The day-of-week effect exists in region II, but does not exist in either region I or in region III. This suggests that the day-of-the-week effect most likely occurs in normal daily price changes in region II.

The test results for serial dependence in the standardized residuals of the TAR model for S&P 500 futures are also presented in Table VIII. The standardized residuals pass the Ljung-Box Q test, the Q^2 test and the BDS test. These results confirm that the standardized residuals are IID and the log-price changes of S&P 500 futures can be modeled quite well by a TAR process in this sample period.

The Autoregressive Volatility Model

The estimation procedure for autoregressive volatility model suggested by Hsieh (1993) is used in this article. Daily measure of volatility is the Parksion's (1980) range estimator of standard deviation, $\sigma_t = (0.361)^{1/2}\text{Log}(\text{Hight}_t/\text{Low}_t)$, where the Hight_t and Low_t are, respectively, the maximum and the minimum transaction prices during each trading day. The ordinary least squares estimator is used to estimate the model, and the Schwarze Beyse information criterion is used to obtain the optimal lag length of six.[7] The empirical results for S&P 500 futures are as follows:

$$
\begin{aligned}
\text{Log}(\sigma_t) =\; & \underset{(0.12)}{-1.11} - \underset{(0.027)}{0.21\,D1} + \underset{(0.027)}{0.036\,D2} + \underset{(0.027)}{0.0006\,D3} - \underset{(0.027)}{0.021\,D4} + \underset{(0.049)}{0.172\,DH} \\
& + \underset{(0.02)}{0.170\log(\sigma_{t-1})} + \underset{(0.02)}{0.170\log(\sigma_{t-2})} + \underset{(0.02)}{0.106\log(\sigma_{t-3})} + \underset{(0.02)}{0.142\log(\sigma_{t-4})} \\
& + \underset{(0.02)}{0.107\log(\sigma_{t-5})} + \underset{(0.02)}{0.082\log(\sigma_{t-6})}
\end{aligned}
$$

$$\text{Adjusted } R^2 = 0.3027 \tag{15}$$

The standard errors of the estimated coefficients are in the parentheses.

[7] In eq. (15), the initial lag length was specified with 24 lags. Schwartz Bayes information criterion guides the selection of the optimal lag length of six. All of the coefficients of the higher lags are either statistical insignificant or very small in magnitude.

TABLE IX

Diagnostics for an Autoregressive Volatility for S&P500 Futures Log-Price Changes

	Part A v_t^*	*Part B* e_t^*
Q	33.6	15.2
Q^2	—	12.4
BDS (M = 3, L = 1.5)	0.77	−1.16
Skew	0.50[a]	−2.8[a]
Kurt	1.43[a]	42.2[a]
λ	311[a]	184000[a]

[a]Denotes rejection of the null hypothesis at 1% level.
v_t is the OLS residuals from:

$$\text{Log}(\sigma_t) = b_0 + \Sigma_{i=1,6}\, b_i \log(\sigma_{t-1}) + v_t \text{ (see equation(15))}$$

e_t is the residuals from:

$$\underset{(0.068)}{z_t = -0.127} \; \underset{(0.097)}{+0.244\, D_1} + \underset{(0.096)}{0.238\, D_2} + \underset{(0.096)}{0.208\, D_3} + \underset{(0.096)}{0.136\, D_4} - \underset{(0.176)}{0.208\, D_H} + e_t$$

where the standard errors of the estimated parameters are in parentheses.

Di, i = 1,2,3,4 and DH are dummy variables for the days-of-the-week effect and the holiday effect. The days-of-the-week effect is not significant, but the coefficient of holiday effect, DH, is positive and significant. This result is consistent with results obtained from GARCH models reported in Table VI. The persistence of volatility is measured by the sum of the coefficients of lagged $\log(\sigma_{t-i})$, $i = 1, \ldots, 6$, which is equal to 0.777 with the standard error (0.0243). The one-sided test on the null hypothesis that the sum of coefficients is equal to one versus the sum of the coefficients is less than one is performed by both t test and F(1,2444) test with 1 and 2444 degrees of freedoms. The calculated values of t and F statistics are −9.17 and 84.07 respectively. These results reject the null hypothesis at 1% level and confirm that there is a mean revision property in volatility of S&P 500 index futures. Using the autoregressive volatility model for the S&P 500 returns, Hsieh (1991) found that the value of persistence measure is equal to 0.7713 and is statistically less than one. In comparing the magnitude of persistence measure for the daily log S&P 500 futures price changes, it is found that the measure obtained by Autoregressive Volatility model is smaller than the measure estimated from GARCH(1,1) model (reported in Table VI).

Table IX presents diagnostic for the fitted autoregressive volatility model in eq. (15) for the S&P 500 futures log price changes. Part A contains the diagnostics for the residuals from eq. (15). It is found that the residuals from eq. (15), v_t, pass Q and the BDS tests. Part B in Table IX also confirms that the residuals $\hat{e}_t$ calculated from the regression of standardized S&P 500 log price changes (that is, $Z_t/(\hat{\sigma}_t)$ on days-of-the-week dummies and holiday effects, also pass Q, Q^2, and the BDS tests at the 1% level. This result suggests that there are some day-of-the-week effects left in the conditional mean of the standardized S&P 500 log futures price changes. These diagnostic test results suggest that the autoregressive volatility model can adequately represent the nonlinear serial dependence of S&P 500 futures log price changes. In addition, it is worth mentioning that the kurtosis of S&P 500 futures is 210.8 in Table III, the kurtosis of the residual of the GARCH(1,1) model for the S&P 500 futures is 106, and the kurtosis of the residual of the standardized S&P 500 futures log price changes in the Table IX is 42.18. These results demonstrate that most of the sources of leptokurtosis are due to the change in variance. These empirical results are consistent with the empirical results in Hsieh (1991) for the autoregressive volatility model for daily S&P 500 returns. The only minor difference is that Hsieh (1991) found there is first order autoregression and no days-of-the-week effect left in the standardized S&P 500 daily returns, while this study shows no autoregressions but some days-of-the-week effects left in the standardized daily S&P 500 futures log price changes.

SUMMARY AND CONCLUSION

This study finds that the testing results of the BDS and Q^2 statistics confirm that all of the daily log futures price changes considered in this study exhibit nonlinear serial dependence. The daily futures log price changes of four futures contracts are linearly uncorrelated when the errors are modeled with GARCH processes. The estimates of skewness and kurtosis suggest that daily futures log price changes deviate significantly from normal distribution.

The regression model with GARCH(1,1) error process is an adequate model for representing daily log price changes of Japanese yen, Deutsche mark, and Eurodollar contracts during the sample period of this study. However, it is an imperfect model for representing the behavior of daily futures log price changes of the S&P 500. On the other hand, the threshold autoregressive model or autoregressive volatility model are adequate models for representing daily futures log price changes of S&P 500 in the sample period of this study. Within the context of appropriate models, our empirical results indicate that most of the series do have the days-of-the week and holiday effect in their conditional variance; only one series

TABLE A1

Estimated GARCH(1,1) Models and Test of Serial Dependence in Residuals
S&P500 Futures Log-price Changes with 1987 Crash Period Omitted

C	−0.0891[b]
	(0.0438)
D_1	0.212[b]
	(0.064)
D_2	0.150[b]
	(0.059)
D_3	0.134[b]
	(0.057)
D_4	0.130[b]
	(0.059)
D_H	−0.0178)
	(0.0152)
Variance:	
α_0	0.142[b]
	(0.024)
e^2_{t-1}	0.0229[b]
	(0.0027)
h_{t-1}	0.969[b]
	(0.0037)
D_1	−0.174[b]
	(0.041)
D_2	−0.198[b]
	(0.051)
D_3	−0.267[b]
	(0.040)
D_4	−0.0412
	(0.0475)
D_H	0.0597[b]
	(0.0314)
Test of Serial Dependence in Standardized Residuals	
Q(24)	14.7
Q^2 (24)	8.9
BDS (M = 3, L = 1.5)	−0.69
Test of Normality in Standardized Residuals	
Skew	−0.706[a]
Kurtosis	6.98[a]
λ	5152[a]

[a]Denotes rejection of the null hypothesis at 1% level.
[b]Significant at 5% level.
Standard errors of the estimated parameters are in parentheses.
At 1% significant level, the critical value of the BDS test on the standardized residuals of GARCH model is 1.59 (see Brock et al., 1991, p. 279).
λ is defined in Table 3.

exhibits the days-of-the-week effect in its conditional mean. The strong holiday effects in volatility provide empirical evidence to support the hypothesis that trading and nontrading periods contribute to volatility in futures markets.

These empirical results have several important implications for researchers in financial markets. First, autocorrelations and Ljung-Box tests fail to detect nonlinear serial dependence in the series. Hence, researchers should employ nonlinearity tests to check the adequacy of their empirical time series models. Second, empirical results suggest that the GARCH model cannot represent all daily futures log price changes considered in this study. Thus researchers should consider a larger class of nonlinear models from which a possible candidate can be selected to represent the behavior of the futures price changes of their interests. For example, a TAR model or autoregressive volatility model should also be considered. Third, the results of the BDS test do not lend support for a conclusion of deterministic chaos. Thus a nonlinear statistical model is preferred as a data-generating process for daily futures price changes. Finally, hypotheses on the intervention effects of policy changes or the existence of market anomalies should be tested within the context of more appropriate general models in order to increase the reliability of our empirical results.

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