Pricing Eurodollar Futures Options with the Ho and Lee and Black, Derman, and Toy Models: An Empirical Comparison

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This article compares empirically the Ho and Lee (1986) and Black, Derman, and Toy (1990) discrete-time debt option pricing models in the pricing of Eurodollar futures options over the period from March 1997 through February 1998 using daily data. The results indicate that both models performed well. The average absolute pricing errors over the sample period were less than one tick (0.01) in every case. The Black, Derman, and Toy model slightly outperformed the Ho and Lee model in the pricing of in-the-money call options and out-of-the-money put options over the period studied. © 1999 John Wiley & Sons, Inc. Jrl Fut Mark 19: 291–306, 1999

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INTRODUCTION

The Ho and Lee (1986) model and the Black, Derman, and Toy (1990) model are two competing one-factor binomial models that are useful in the pricing of interest rate contingent claims. These models specify differently how interest rates may evolve over time on a binomial lattice and can give different values to various contingent claims. In this article, the models are used to specify empirically the values of Eurodollar futures options and an analysis of variance is used to compare the performance of the two models in valuing these options.

Eurodollar futures options began trading in March 1985 on the International Money Market of the Chicago Mercantile Exchange (CME). These contracts grant the holder the right to purchase or sell a Eurodollar futures contract at the exercise price prior to the option's expiration date. A Eurodollar futures contract, if held until maturity, calls for the delivery of a 90-day Eurodollar time deposit with a face value of one million dollars.

Eurodollar futures options have several features that make them attractive for use in this study. The Eurodollar futures options market is the most active option market for short-term interest rate–sensitive securities. The underlying deliverable asset, a 90-day Eurodollar time deposit, is essentially a pure discount bond and is not complicated by coupon effects. Eurodollar futures contracts are actually settled in cash; thus there is no need to determine the cheapest-to-deliver security. Moreover, Eurodollar futures option contracts and Eurodollar futures contracts expire concurrently.

REVIEW OF THE MODELS

The Ho and Lee and Black, Derman, and Toy models both describe interest rate movements on a binomial lattice. In this article, a movement from time n and state i on the lattice to time $n + 1$ and state $i + 1$ represents an upward movement in the term structure; and a movement to time $n + 1$ and state i represents a downward movement in the term structure. At each date n on the lattice there are $n + 1$ states that can occur.

In each of the models, restrictions are placed on the term structure movements to ensure there are no arbitrage opportunities over time or across states. A key feature of these models is that the security prices generated by the models are consistent with the natural price boundaries possessed by debt securities. For example, the price volatility of default free debt securities eventually declines as the maturity date approaches—

even though the shape and position of the term structure becomes increasingly uncertain. The two models differ primarily with respect to the stochastic process of the term structure movements and the corresponding measures of volatility along the binomial lattice.[1]

The Ho and Lee Model

In the Ho and Lee model, the initial term structure is taken as given, and it is assumed to shift over time so that profitable arbitrage opportunities fail to occur. The price at time n and state i of a pure discount bond that pays $1.00 at the end of T periods is denoted as $P_{n,i}(T)$. The price, $P_{n,i}(T)$, as a function of T is a discount function that can be used at time n in state i to find the value of any cash flow to occur at time $n + T$.[2] The prices are a function of their maturities and of the term structure appearing at time n in state i. Initially, when n and i are both equal to zero, the price of a pure discount bond maturing at the end of T periods is P(T). The discount functions are subject to the following constraints:

$$P_{n,i}(0) = 1 \text{ for all } n, i \text{ and} \tag{1}$$

$$\lim_{T \to \infty} P_{n,i}(T) = 0 \text{ for all } n, i. \tag{2}$$

Beginning[3] at any time n and state i on the lattice, there are two possible states that can occur one period later in the Ho and Lee binomial model. The discount function can move to an upstate, designated as state $i + 1$ at time $n + 1$, and can be written as

$$P_{n+1,i+1}(T) = \frac{P_{n,i}(T + 1)}{P_{n,i}(1)} h^*(T) \tag{3}$$

or it can move to a downstate, designated as state i at time $n + 1$, and is denoted as

[1]The Kalotay, Williams, and Fabozzi (1993) model for valuing bonds and embedded options shares the same underlying interest rate process as the Black, Derman, and Toy model. As such, the analysis of the Black, Derman, and Toy model presented in this article will apply to the Kalotay, Williams, and Fabozzi model as well.

[2]In the Ho and Lee (1986) article an upstate is said to occur when the discount function increases. However, in the presentation given here, this represents the downstate. This makes the notation used here consistent with that used by the Black, Derman, and Toy model, in which an increase in the short rate is defined as an upward move.

[3]The short rate at time n and state i, $R_{n,i}$, can be determined from the one-period discount function at time n and state i as follows:

$$R_{n,i}(1) = \frac{1}{P_{n,i}(1)} - 1.$$

$$P_{n+1,i}(T) = \frac{P_{n,i}(T+1)}{P_{n,i}(1)} h(T). \tag{4}$$

Here, $P_{n,i}(T + 1)/P_{n,i}(1)$ is the forward price of the bond as determined in the term structure at time n, state i. The perturbation functions, $h^*(T)$ and $h(T)$, are functions only of T, and they specify the deviations of the discount functions from the implied forward functions. The perturbation functions must be positive for all T with

$$h^*(0) = h(0) = 1 \text{ and} \tag{5}$$

$$h^*(T) < 1 < h(T).$$

In effect, the price at time n + 1 is either smaller or larger than the forward price in the term structure at time n, state i.

Eqs. (3) and (4) can be solved recursively so that a discount function at any point along the lattice can be expressed as a function of the perturbation functions and the initial discount functions, which are given by the current term structure.[4]

In order to ensure that arbitrage opportunities cannot be created by forming portfolios of discount bonds, Ho and Lee show that the following restriction on the perturbation functions hold at each point on the lattice:

$$\pi h^*(T) + (1 - \pi)\, h(T) = 1 \tag{6}$$

where π is a constant that is independent of T. The constant π is known as the implied binomial probability or martingale probability of an upward move. Under the probability π, the expected price to rule next period is the current forward price and the expected one period return is the risk-

[4]For example, using eq. (4) we have

$$P_{2,0}(T) = \frac{P_{1,0}(T+1)}{P_{1,0}(1)} h(T),$$

$$P_{1,0}(T+1) = \frac{P(T+2)}{P(1)} h(T+1),$$

and

$$P_{1,0}(1) = \frac{P(2)}{P(1)} h(1).$$

Hence,

$$P_{2,0}(T) = \frac{P(T+2)h(T+1)h(T)}{P(2)h(1)}$$

where, $P(T + 2)$ and $P(2)$ are given by the current term structure.

free return. Individuals for whom π represents the probability of an upward movement of the term structure are risk neutral.

Ho and Lee also assume that the term structure movements along the binomial lattice are path independent. Path Independence requires that the discount function at any point on the lattice be determined solely on the basis of the number of upward movements that must occur in order to reach that point on the lattice. Therefore, knowledge of the sequence in which the movements occurred is not needed. Given the definition of the discount function in eqs. (3) and (4), it is clear that i represents the number of upstate movements to have occurred since $n = 0$. Ho and Lee show that path independence and the absence of profitable risk-free arbitrage opportunities imply that the perturbation functions can be expressed as

$$h^*(T) = \frac{\delta^T}{\pi\delta^T + (1 - \pi)} \text{ for } T \geq 0 \text{ and} \tag{7}$$

$$h(T) = \frac{1}{\pi\delta^T + (1 - \pi)} \text{ for } T \geq 0 \tag{8}$$

where, $0 \leqslant \delta \leqslant 1$. Clearly, $\delta^T = h^*(T)/h(T)$ so that δ is a measure of the spread between the T-period discount functions in states i and i+1 at any time n on the lattice. Lattices having different 'spreads' are easily constructed by simply specifying the ratio $h^*(1)/h(1)$ differently.

Eqs. (3), (4), (7), and (8) can be used to determine the discount functions at each point on the binomial lattice. Thus, given the constants π and δ, the above constraints, and the initial discount functions, which are given by the initial term structure, the Ho and Lee model is completely specified and the term structure movements determined by the model can be used to price interest rate path independent contingent claims that can be represented as cash flows to occur at the time-state nodes, (n,i).

The Black, Derman, and Toy Model

Black, Derman, and Toy (1990) developed a discrete-time model of interest rate movements in which the prices of interest rate contingent claims depend upon the stochastic movement of short rates. The model utilizes the yields and the estimated yield volatilities of pure discount bonds as inputs.

Define $R_{n,i}(T)$ as the yield to maturity, compounded once per period, at time n and state i for a pure discount bond that matures in T-periods.

Hence, $R_{n,i}(1)$ equals the short rate at time n and state i , which is also denoted simply as $R_{n,i}$. A pure discount bond, as defined, returns \$1 at maturity. Hence the price, $P_{n,i}(T)$, of such a bond is

$$P_{n,i}(T) = \frac{1}{(1 + R_{n,i}(T))^T} \tag{9}$$

At any time n and state i, the probability of an upstate is assumed to equal the probability of a downstate, that is, $q = 0.5$[5]. The expected return per dollar over one period is assumed to be the same for every interest rate–sensitive security. It follows that

$$P_{n,i}(T) = \frac{0.5[P_{n+1,i+1}(T - 1) + P_{n+1,i}(T - 1)]}{1 + R_{n,i}}, \text{ for all } T > 1. \tag{10}$$

Eq. (10) allows the pure discount bond prices that may occur at time period n to be expressed as a function of the prices at time period n + 1 for every maturity T. In addition, Black, Derman, and Toy specify the yield volatility for a pure discount bond with maturity T as

$$\sigma_y(T) = \frac{\ln\left(\frac{R_{1,1}(T - 1)}{R_{1,0}(T - 1)}\right)}{2} \tag{11}$$

and the volatility of the short rate at time n as

$$\sigma_{s,n} = \frac{\ln\left(\frac{R_{n,i+1}}{R_{n,i}}\right)}{2} \text{ for } i = 0 \text{ to } n. \tag{12}$$

Given the yield, $R_{0,0}(T)$, and yield volatility, $\sigma_y(T)$, from the current term structure for all T, eqs. (9), (10), (11), and (12) can be used to determine the short rates at all nodes on the binomial lattice.

DATA AND METHODOLOGY

This study utilizes the Eurodollar futures prices, futures options prices, and interest rates reported in the CME's *Daily Information Bulletin* over the period from March 1997 through February 1998 to analyze the pricing performance of the Ho and Lee and Black, Derman, and Toy models. At any given time, Eurodollar futures options with three or more different

[5]This assumption is not critical.

contract maturities may trade. However, this study focuses on the contracts that have between 42 and 132 days remaining until maturity. These contracts are highly liquid and have a substantial open interest. A one-day binomial period was employed and the Eurodollar interest rates were adjusted to reflect daily compounding.[6]

To create a full spectrum of maturities for the Eurodollar rates, cubic spline interpolation was employed. In this manner, zero coupon bond prices could be obtained for each time date on the binomial lattice. These rates/zero coupon bond prices, along with the short rate volatility and the binomial probability, are the inputs required for the parameterization of the term structure models.

Pricing Eurodollar Futures Options

In this section, the application of the Ho and Lee and Black, Derman, and Toy models to the pricing of Eurodollar futures options is formulated. It is important to note that once the models have been parameterized—that is, the lattice of short rates has been specified, the pricing procedure is the same regardless of the model being used.

Because Eurodollar futures options are American-style options, which can be exercised prior to maturity, the price of the underlying security (a Eurodollar futures contract) must be determined at each node on the binomial lattice. Eurodollar futures contracts, although actually settled in cash, have a 90-day Eurodollar time deposit as their underlying security. Thus, the value of a Eurodollar futures contract at maturity in each state on the binomial lattice is equal to the value of a zero coupon bond at that state which matures 90 days after the maturity of the futures contract. In order for the models to price Eurodollar futures option contracts, they must first be applied to value the underlying Eurodollar time deposit and the Eurodollar futures contract.

The value of a default free zero coupon bond (that is, a Eurodollar time deposit) at any time n and state i prior to its maturity can be determined by discounting the expected value of the bond at time $n+1$ back one period using the short rate at time n and state i on the lattice. Thus, if the binomial probability of an upstate is 0.5, the value of the zero coupon bond at time n and state i, $Z_{n,i}$, is given as

[6]The binomial lattices will, thus, consist of a minimum of 42 periods and as many as 132 periods. This should be sufficient to ensure that the empirical results are not influenced by the size of the binomial lattice (see Tian 1993).

$$Z_{n,i} = \frac{0.5Z_{n+1,i+1} + (1 - 0.5)Z_{n+1,i}}{1 + R_{n,i}} \tag{13}$$

where $R_{n,i}$ is the short rate. Because the value of a default free zero coupon bond at maturity is equal to its face value in all states, the value of the zero coupon bond at any period prior to maturity can be determined via the use of eq. (13) beginning one period prior to maturity and working backward to the desired time and state. The value of the time deposit 90 days prior to maturity then becomes the maturity value of the Eurodollar futures contract.

Once the value of the Eurodollar futures contract in each state at maturity has been determined using the above technique, the value of the Eurodollar futures contract at each time and state prior to maturity can be determined as the expected value of the contract one period into the future. Thus, $F_{n,i}$, the value of the Eurodollar Futures contract at any time n and state i prior to maturity, is given as

$$F_{n,i} = 0.5F_{n+1,i+1} + (1 - 0.5)F_{n+1,i}. \tag{14}$$

An entire lattice of Eurodollar futures prices can be constructed using eq. (14) beginning one period prior to the maturity of the futures contract and working backward.

The remaining step is to calculate the Eurodollar futures option contract prices. Here, the procedure for determining the value of a call option is illustrated.

At maturity, the value of a Eurodollar futures call option will equal either the underlying Eurodollar futures contract price minus the option's strike price or zero, whichever is greater. The value of a futures call option at maturity is then

$$FO_{m,i} = \text{Max}[F_{m,i} - S,0] \text{ for } i = 0 \text{ to } m \tag{15}$$

where m = the maturity of the Eurodollar futures option contract and S = the exercise price of the Eurodollar futures option.

Because Eurodollar futures options can be exercised prior to maturity, the value of a Eurodollar futures call option at any node on the binomial lattice prior to maturity is equal to the greatest of the value of the option if exercised, the value of the option if held, or zero. The value of the option if exercised at time n and state i, $E_{n,i}$, is given by

$$E_{n,i} = F_{n,i} - S. \tag{16}$$

The value of the option if held at time n and state i, $H_{n,i}$ is given by

$$H_{n,i} = \frac{0.5FO_{n+1,i+1} + (1 - 0.5)FO_{n+1,i}}{1 + R_{n,i}}. \tag{17}$$

Thus, the value of the Eurodollar futures call option at any time n and state i prior to maturity is given by[7]

$$FO_{n,i} = \text{Max}[E_{n,i}, H_{n,i}, 0]. \tag{18}$$

To capture properly the value of the early exercise feature of Eurodollar futures options, the model Eurodollar futures price must equal the observed Eurodollar futures price. In practice the two prices may be slightly different. To correct for this possibility, an adjusted Eurodollar futures price is calculated at each node on the binomial lattice using the following procedure. First, the difference between the observed futures price and the model futures price at time zero is calculated. Then, the difference is added to the model Eurodollar futures price at each node on the lattice to calculate an adjusted Eurodollar futures price at each node on the lattice. These adjusted futures prices are used to calculate the price of the Eurodollar futures option using the procedure described above.[8]

The Empirical Procedure

In order to parameterize the term structure models, the binomial probability and the short rate volatility at time zero must be specified. In this study, a binomial probability of 0.5 was used, implying that, at each node on the lattice, an upstate or downstate was equally likely to occur. The short rate volatility was varied until the model Eurodollar futures option price was equal to the actual Eurodollar futures option price for the at-the-money call option.

The empirical procedure compares the performance of the models in the pricing of in-the-money call options, out-of-the-money call options, in-the-money put options, and out-of-the-money put options. Because of potential empirical problems in the pricing of both deep in-the-money and deep out-of-the-money options, the analysis focuses only on the in-the-money and out-of-the-money options whose exercise prices were

[7]For Eurodollar futures put options, the value of the option if exercised at time n and state i, En,i, will equal the strike price minus the futures price at time n and state i.

[8]The Eurodollar futures prices calculated by the models were not sensitive to the parameters of the models. Thus, it is not generally possible to force the Eurodollar futures contract prices determined by the models to equal exactly the actual prices simply by varying the parameters. The use of the adjustment procedure did not alter the conclusions of the article; however, the performance of both models improved with the use of the adjustment procedure.

nearest to the at-the-money strike price.[9] The pricing error, Ym,n, that results from using model m to price a Eurodollar futures option on day n is given by the following:

$$Y_{m,n} = OP_n - MP_{m,n}. \tag{19}$$

where OP_n = the observed futures option price on day n, and $MP_{m,n}$ = the futures option price determined by model m on day n.

The mean pricing error for model m, μ_m, is calculated as follows.

$$\mu_m = \sum_{n=1}^{N} \frac{Y_{m,n}}{N} \tag{20}$$

where N = the number of observations.

To test for differences in the mean pricing errors between the two models the following analysis of variance (ANOVA) model was employed.

$$Y_{m,n} = \mu_m + \varepsilon_{m,n} \tag{21}$$

where $\varepsilon_{m,n}$ = the residuals that are assumed to be independent and identically distributed.

Because the pricing error on any given day can be either positive or negative, the mean pricing error could be close to zero even if the pricing error on most days was far from zero. To correct for this problem, the average absolute pricing error, ν_m, is also used to evaluate the performance of the Black, Derman, and Toy and Ho and Lee models. The average absolute pricing error for model m is calculated as follows.

$$\nu_m = \sum_{n=1}^{N} \frac{|Y_{m,n}|}{N} \tag{22}$$

EMPIRICAL RESULTS

Tables I and II present the average absolute errors in the pricing of call options and put options respectively. Tables III and IV provide the summary statistics for the corresponding options. The average absolute errors and summary statistics are calculated for the entire sample period, options with less than 90 days remaining to maturity (42 to 89 days to

[9]For deep in-the-money options the early exercise feature may dominate. Similarly, deep out-of-the-money options may have little or no value. In both cases, the option prices may not be sensitive to the model parameters.

TABLE I

Average Absolute Errors Call Options

	In-the-Money			Out-of-the-Money		
Sample	*Ho and Lee*	*BDT*	*Difference*	*Ho and Lee*	*BDT*	*Difference*
All	0.008362	0.007575	0.000787	0.007449	0.007341	0.000107
Days to Maturity						
<90 Days to Maturity	0.008017	0.007697	0.000320	0.006030	0.006149	−0.000119
≥90 Days to Maturity	0.008734	0.007443	0.001290	0.008975	0.008624	0.000351
Month						
March 1997	0.005912	0.004427	0.001485	0.004804	0.003404	0.001400
April 1997	0.003744	0.003418	0.000326	0.001998	0.002174	−0.000176
May 1997	0.010010	0.008334	0.001677	0.004811	0.003878	0.000933
June 1997	0.011018	0.010047	0.000971	0.008807	0.008176	0.000631
July 1997	0.009521	0.009327	0.000194	0.004290	0.004385	−0.000095
August 1997	0.009628	0.008295	0.001333	0.009848	0.008610	0.001238
September 1997	0.012068	0.011270	0.000798	0.010467	0.009726	0.000741
October 1997	0.008735	0.008281	0.000453	0.007351	0.007335	0.000016
November 1997	0.011771	0.009560	0.002211	0.014261	0.015789	−0.001528
December 1997	0.010468	0.009057	0.001411	0.009499	0.009774	−0.000275
January 1998	0.004612	0.005105	−0.000493	0.009809	0.010559	−0.000750
February 1998	0.002380	0.003404	−0.001024	0.004550	0.005619	−0.001069
Contract						
Jun 1997	0.004519	0.003712	0.000807	0.003306	0.002716	0.000590
Sep 1997	0.010545	0.009573	0.000971	0.006075	0.005618	0.000456
Dec 1997	0.010090	0.009226	0.000864	0.009393	0.008752	0.000641
Mar 1998	0.009039	0.007952	0.001087	0.010956	0.011784	−0.000828
Jun 1998	0.008394	0.007830	0.000565	0.004542	0.005667	−0.001125

This table presents the average absolute pricing errors for in-the-money and out-of-the-money Eurodollar call options when priced using the Ho and Lee and Black, Derman, and Toy models. The difference (Ho and Lee minus Black, Derman, and Toy) in the average absolute pricing errors is also provided. The statistics are computed over the entire sample period, for options with less than 90 days remaining until maturity, for options with greater than or equal to 90 days remaining until maturity, for each month during the sample period, and for each contract maturity in the sample.

maturity), options with greater than or equal to 90 days remaining to maturity (90 to 132 days to maturity), each month during the sample period, and each contract maturity in the sample period. The results of the ANOVA test for differences in the mean pricing errors are given in Table V.

The average absolute errors in the pricing of Eurodollar futures options calculated over the entire sample period indicate that both models performed well. The average absolute pricing errors were less than 1 tick (0.01) in every case. When pricing out-of-the-money call options and in-the-money put options there were essentially no differences in the average absolute pricing errors. However, the average absolute pricing errors dif-

TABLE II

Average Absolute Errors Put Options

	In-the-Money			*Out-of-the-Money*		
Sample	*Ho and Lee*	*BDT*	*Difference*	*Ho and Lee*	*BDT*	*Difference*
All	0.006006	0.005936	0.000071	0.007424	0.006532	0.000892
Days to Maturity						
<90 Days to maturity	0.004480	0.004613	−0.000133	0.007259	0.006856	0.000403
≥90 Days to maturity	0.007649	0.007359	0.000290	0.007600	0.006183	0.001417
Month						
March 1997	0.003667	0.002593	0.001074	0.004878	0.002902	0.001977
April 1997	0.002580	0.003025	−0.000445	0.002404	0.002104	0.000299
May 1997	0.005407	0.004551	0.000857	0.008861	0.007118	0.001743
June 1997	0.006021	0.005282	0.000739	0.010804	0.009654	0.001151
July 1997	0.000161	0.000116	0.000045	0.008913	0.008695	0.000218
August 1997	0.008125	0.006873	0.001252	0.007927	0.006569	0.001358
September 1997	0.007534	0.006747	0.000787	0.010556	0.009734	0.000822
October 1997	0.005142	0.005119	0.000023	0.007861	0.007446	0.000416
November 1997	0.013051	0.014541	−0.001491	0.009715	0.007704	0.002011
December 1997	0.007690	0.007929	−0.000239	0.009517	0.008095	0.001422
January 1998	0.011052	0.011768	−0.000717	0.004656	0.005228	−0.000572
February 1998	0.003065	0.004322	−0.001257	0.002606	0.002813	−0.000207
Contract						
Jun 1997	0.003033	0.002720	0.000313	0.003559	0.002520	0.001039
Sep 1997	0.003824	0.003306	0.000518	0.009760	0.008715	0.001045
Dec 1997	0.007233	0.006573	0.000660	0.008721	0.007857	0.000865
Mar 1998	0.010281	0.011073	−0.000792	0.008038	0.007038	0.001000
Jun 1998	0.002957	0.004284	−0.001327	0.002589	0.002851	−0.000263

This table presents the average absolute pricing errors for in-the-money and out-of-the-money Eurodollar put options when priced using the Ho and Lee and Black, Derman, and Toy models. The difference (Ho and Lee minus Black, Derman, and Toy) in the average absolute pricing errors is also provided. The statistics are computed over the entire sample period, for options with less than 90 days remaining until maturity, for options with greater than or equal to 90 days remaining until maturity, for each month during the sample period, and for each contract maturity in the sample.

fered when applying the models to price in-the-money call options and out-of-the-money put options. In both cases, the average absolute errors calculated using the Black, Derman, and Toy model were less than those calculated using the Ho and Lee model.

Examination of the average absolute errors during each month in the sample period indicates that the differences in the pricing of in-the-money call options and out-of-the-money put options by the models were evident throughout 1997. In each of these months, the average absolute pricing errors calculated using the Black, Derman, and Toy model were less than those calculated using the Ho and Lee model. However, during January and February of 1998, this difference was reversed.

TABLE III

Summary Statistics Call Options

		In-the-Money				*Out-of-the-Money*			
		Ho and Lee		*BDT*		*Ho and Lee*		*BDT*	
Sample	*# Obs*	*Mean*	*Std. Dev.*	*Mean*	*Std. Dev.*	*Mean*	*Std. Dev.*	*Mean*	*Std. Dev.*
All	245	−0.00766	0.00615	−0.00643	0.00616	−0.00182	0.00857	−0.00285	0.00825
Days to Maturity									
<90 DTM	127	−0.00702	0.00666	−0.00630	0.00675	−0.00448	0.00575	−0.00497	0.00565
≥90 DTM	118	−0.00834	0.00550	−0.00657	0.00549	0.00104	0.01008	−0.00057	0.00988
Month									
March 1997	20	−0.00548	0.00425	−0.00348	0.00390	0.00480	0.00254	0.00340	0.00240
April 1997	22	−0.00210	0.00373	−0.00145	0.00365	−0.00104	0.00211	−0.00166	0.00202
May 1997	21	−0.00975	0.00565	−0.00802	0.00505	0.00424	0.00426	0.00267	0.00396
June 1997	21	−0.01076	0.00735	−0.00962	0.00732	0.00387	0.00953	0.00301	0.00887
July 1997	21	−0.00952	0.00405	−0.00933	0.00385	−0.00429	0.00055	−0.00439	0.00048
August 1997	20	−0.00963	0.00302	−0.00830	0.00277	0.00580	0.00926	0.00422	0.00860
September 1997	20	−0.01207	0.00565	−0.01127	0.00577	0.00189	0.01116	0.00096	0.01031
October 1997	23	−0.00874	0.00423	−0.00816	0.00434	−0.00419	0.00771	−0.00461	0.00752
November 1997	19	−0.01177	0.00381	−0.00956	0.00386	−0.01426	0.00208	−0.01579	0.00189
December 1997	21	−0.01047	0.00394	−0.00906	0.00389	−0.00686	0.00711	−0.00784	0.00670
January 1998	18	−0.00067	0.00564	0.00044	0.00601	−0.00981	0.00315	−0.01056	0.00343
February 1998	19	0.00006	0.00272	0.00186	0.00330	−0.00345	0.00367	−0.00524	0.00364
Contract									
Jun 1997	45	−0.00340	0.00430	−0.00218	0.00383	0.00182	0.00363	0.00084	0.00327
Sep 1997	62	−0.01046	0.00531	−0.00943	0.00513	0.00100	0.00726	0.00019	0.00661
Dec 1997	62	−0.01009	0.00460	−0.00918	0.00465	0.00082	0.01043	−0.00016	0.00972
Mar 1998	58	−0.00782	0.00661	−0.00623	0.00645	−0.01000	0.00562	−0.01108	0.00565
Jun 1998	18	0.00026	0.00265	0.00211	0.00319	−0.00338	0.00376	−0.00527	0.00374

This table presents the mean pricing errors and standard deviation of the mean pricing errors for in-the-money and out-of-the-money Eurodollar call options when priced using the Ho and Lee and Black, Derman, and Toy models. The statistics are computed over the entire sample period, for options with less than 90 days remaining until maturity, for options with greater than or equal to 90 days remaining until maturity, for each month during the sample period, and for each contract maturity in the sample.

The analysis of the average absolute pricing errors computed for each contract maturity in the sample yielded similar results. The errors calculated using the Black, Derman, and Toy model to price in-the-money call options and out-of-the-money put options were less than those calculated using the Ho and Lee model for the contracts that matured in June 1997, September 1997, December 1997, and March 1998.[10]

An ANOVA procedure was employed to test for differences in the mean pricing errors resulting from the application of the two models to

[10]The differences were not evident for the June 1998 contract, however, there were only 18 observations.

TABLE IV

Summary Statistics Put Options

		In-the-Money				*Out-of-the-Money*			
		Ho and Lee		*BDT*		*Ho and Lee*		*BDT*	
Sample	*# Obs.*	*Mean*	*Std Dev*	*Mean*	*Std Dev*	*Mean*	*Std Dev*	*Mean*	*Std Dev*
All	245	−0.00117	0.00782	−0.00216	0.00761	−0.00695	0.00532	−0.00571	0.00538
Days to Maturity									
<90 DTM	127	−0.00316	0.00609	−0.00360	0.00613	−0.00646	0.00603	−0.00572	0.00616
≥90 DTM	118	0.00097	0.00888	−0.00061	0.00869	−0.00747	0.00440	−0.00569	0.00443
Month									
March 1997	20	0.00356	0.00252	0.00218	0.00244	−0.00488	0.00272	−0.00287	0.00246
April 1997	22	−0.00233	0.00206	−0.00294	0.00196	−0.00211	0.00210	−0.00143	0.00210
May 1997	21	0.00372	0.00505	0.00218	0.00490	−0.00886	0.00383	−0.00712	0.00335
June 1997	21	0.00480	0.00683	0.00400	0.00617	−0.01080	0.00585	−0.00965	0.00583
July 1997	21	0.00016	0.00023	0.00012	0.00017	−0.00891	0.00361	−0.00870	0.00340
August 1997	20	0.00572	0.00821	0.00416	0.00762	−0.00793	0.00266	−0.00657	0.00244
September 1997	20	0.00333	0.00838	0.00245	0.00751	−0.01056	0.00570	−0.00973	0.00587
October 1997	23	−0.00232	0.00812	−0.00270	0.00807	−0.00775	0.00429	−0.00715	0.00440
November 1997	19	−0.01305	0.00214	−0.01454	0.00194	−0.00971	0.00382	−0.00750	0.00390
December 1997	21	−0.00580	0.00607	−0.00673	0.00569	−0.00952	0.00330	−0.00809	0.00326
January 1998	18	−0.01105	0.00336	−0.01177	0.00366	0.00046	0.00548	0.00157	0.00584
February 1998	19	−0.00253	0.00259	−0.00430	0.00251	−0.00178	0.00286	0.00003	0.00337
Contract									
Jun 1997	45	0.00058	0.00364	−0.00038	0.00330	−0.00342	0.00267	−0.00218	0.00230
Sep 1997	62	0.00284	0.00529	0.00207	0.00480	−0.00976	0.00442	−0.00871	0.00430
Dec 1997	62	0.00178	0.00920	0.00085	0.00859	−0.00868	0.00454	−0.00775	0.00463
Mar 1998	58	−0.00960	0.00520	−0.01064	0.00524	−0.00645	0.00629	−0.00486	0.00617
Jun 1998	18	−0.00239	0.00260	−0.00426	0.00258	−0.00172	0.00293	0.00015	0.00343

This table presents the mean pricing errors and standard deviation of the mean pricing errors for in-the-money and out-of-the-money Eurodollar put options when priced using the Ho and Lee and Black, Derman, and Toy models. The statistics are computed over the entire sample period, for options with less than 90 days remaining until maturity, for options with greater than or equal to 90 days remaining until maturity, for each month during the sample period, and for each contract maturity in the sample.

the pricing of Eurodollar futures options. The tests were carried out for in-the-money call options, out-of-the-money call options, in-the-money put options, and out-of-the-money put options over the entire sample period, for options with less than 90 days remaining until maturity, and for options with greater than or equal to 90 days remaining until maturity. The results are provided in Table V.

Significant differences in the mean pricing errors were detected for in-the-money call options and out-of-the-money put options over the entire sample period and for options with greater than or equal to 90 days remaining until maturity. In each of these cases, the mean pricing errors,

TABLE V

ANOVA Results

Sample	*In-the-Money Calls*	*Out-of-the-Money Calls*	*In-the-Money Puts*	*Out-of-the-Money Puts*
All	4.86[b]	1.82	2.02	6.60[a]
<90 Days to Mat.	0.74	0.46	0.33	0.94
≥90 Days to Mat.	6.10[b]	1.54	1.91	9.60[a]

This table presents the F-statistics for the test to determine whether the mean pricing errors computed using the Ho and Lee and Black, Derman, and Toy pricing models differed. The statistics are computed for in-the-money call options, out-of-the-money call options, in-the-money put options, and out-of-the-money put options over the entire sample period, for options with less than 90 days remaining until maturity, and for options with greater than or equal to 90 days remaining until maturity.

[a]Significant at the 1% level; [b]Significant at the 5% level.

presented in Tables III and IV, as well as the average absolute pricing errors for the Black, Derman, and Toy model are closer to zero than those for the Ho and Lee model. This supports the earlier findings and indicates that the Black, Derman, and Toy model outperforms the Ho and Lee model for in-the-money call and out-of-the-money put options. The difference in pricing ability is most evident when pricing options with more than 90 days remaining to maturity.

An examination of the mean pricing errors provides an indication of the models' tendency to overprice or underprice Eurodollar futures options. The results indicate that both models tend to overprice in-the-money call options and out-of-the-money put options. The mean pricing errors for out-of-the-money call options and in-the-money put options were very close to zero, although generally negative, indicating that there is very little bias in the pricing of these options.

ANALYSIS OF RESULTS

The results of the empirical analysis of the pricing ability of the Ho and Lee and Black, Derman, and Toy models indicate that both models perform very well. The average absolute pricing errors and mean pricing errors over the sample period were less than one tick in every case. The results indicate that the Black, Derman, and Toy model slightly outperforms the Ho and Lee model in the pricing of in-the-money call options and out-of-the-money put options over the period studied. The difference was most apparent when the models were applied to price options with 90 days or more remaining until maturity.

The two models specify differently the manner in which interest rates evolve over time on a binomial lattice and, thus, imply different forms for the short rate volatility along the lattice. It can be shown that the short rate volatility is constant at all nodes on the lattice in the Black, Derman, and Toy model. In the Ho and Lee model, the short rate volatility, if defined similarly, at any time on the lattice can be shown to be greater in the states where the short rate is lower.[11] It is likely that this difference in short rate volatility, which results from the different stochastic processes specified by the models, is an important factor influencing the relative pricing ability of the models, especially for options with 90 days or more remaining until maturity.

Because differences were detected in the pricing of in-the-money call options and out-of-the-money put options by the two models, it is interesting to note that the upper part of the binomial lattice is more important in determining the prices of in-the-money call options than it is in determining the prices of out-of-the-money call options. The upper portion of the lattice is also more important in the pricing of out-of-the-money put options than it is for in-the-money put options. Thus the different stochastic processes implied by the models may have their greatest impact on the upper portion of the binomial lattice.

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[11]The short rate volatility measure used as the basis for these statements was calculated as the standard deviation of the natural log of the short rates. This is the same measure used by Black, Derman, and Toy [see eq. (12)]. It is important to note that the form of the short rate volatility along the lattice in the two models will differ regardless of the measure used.