Covered Arbitrage in Foreign Exchange Markets with Forward Forward Contracts in Interest Rates: Comment

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In a recent issue of this Journal,[1] Dilip Ghosh confuses the concept of a (known) forward interest rate with that of a (random) future short-term interest rate. As a result, both of his main conclusions—that combining interest rate forwards with foreign exchange (FX) forwards produces a new version of the Interest Rate Parity Theorem (IRPT), and that a sufficient condition for its validity is the Pure Expectations Hypothesis (PEH) of the term structure of interest rates—are invalid.

The traditional version of IRPT states that if investors can borrow or lend 6-month funds at ${}_0R_2$ domestically and at ${}_0R_2^*$ in a foreign market, then covered interest arbitrage will ensure that the relationship between

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the spot rate S_0 and the 6-month forward rate F_2 in the FX market will be[2]

$$S_0(1 + {}_0R_2/2) = F_2(1 + {}_0R_2^*/2) \tag{1}$$

Observing that the 6-month horizon can be decomposed into two consecutive 3-month periods, Ghosh considers the possibility that investors might instead borrow or lend for 3 months at ${}_0R_1$ domestically and at ${}_0R_1^*$ in the foreign market and, and rather than leave themselves exposed to interest rate risk for the second 3-month period, they might arrange to roll over the proceeds at known forward interest rates ${}_1R_2^f$ and ${}_1R_2^{f*}$. He suggests that investors can contract for these forward loans in the "forward forward" market, but even in the absence of an external market, it is well established that forward loans can be created synthetically by simultaneous lending and borrowing at deposit rates of different maturities. For example, by lending for 3 months at ${}_0R_1$ and borrowing the same zamount for 6 months at ${}_0R_2$, an investor has effectively contracted to borrow for a 3-month period beginning 3 months in the future. The domestic and foreign forward interest rates must therefore definitionally satisfy the relationships

$$(1 + {}_0R_1/4)(1 + {}_1R_2^f/4) = (1 + {}_0R_2/2) \tag{2a}$$

$$\text{and } (1 + {}_0R_1^*/4)(1 + {}_1R_2^{f*}/4) = (1 + {}_0R_2^*/2) \tag{2b}$$

Note that the values of these forward interest rates are determined definitionally by the relationship between the market 3-month and 6-month deposit rates. Moreover, given the transparency of these markets, it is virtually impossible for a forward interest rate quoted in the "forward forward" market or any other external market for forward loans (e.g., the forward rate agreement or the interest rate futures markets) to deviate significantly from these values. In contrast, the actual 3-month interest rates that will prevail in the domestic and foreign deposit markets in three months time are random variables, the values for which are not constrained by the values of the current 3-month or 6-month (and therefore forward) interest rates in these markets. One can, however, theorize about the relationship between these future interest rates and the corresponding forward interest rates for loans over that same future period. One such theory is the Pure Expectations Hypothesis of the term structure of

[2]The equivalent formula in Ghosh mistakenly fails to prorate the interest rates, which are conventionally quoted as annualized zero coupon deposit rates, to reflect the actual day counts for maturity periods of less than one year. This error produces a misleading numerical example of the potential gains from arbitrage in Section II of the Ghosh paper.

interest rates. If we denote the future 3-month interest rate as ${}_1\tilde{R}_2$ domestically and ${}_1\tilde{R}_2$ in the foreign market, PEH can be stated as

$$ {}_1R_2^f = E[{}_1\tilde{R}_2] \tag{3a} $$

$$ \text{and } {}_1R_2^{f*} = E[{}_1\tilde{R}_2^*] \tag{3b} $$

where E [·] is the expectation operator. The implication of this hypothesis is that, on average, forward interest rates are unbiased predictors of future short-term interest rates. As a result, over a long period of time, borrowers and lenders who are exposed to interest rate risk would not expect to benefit from lower borrowing costs or increased loan revenue by hedging with forward contracts.

In the Ghosh framework, if investors were to borrow and lend in the two deposit markets for two consecutive 3-month periods, the resulting statement of IRPT would become

$$ S_0(1 + {}_0R_1/4)(1 + {}_1R_2^f/4) = F_2(1 + {}_0R_1^*/4)(1 + {}_1R_2^{f*}/4) \tag{4} $$

It should be noted, however, that this is not an independent version of IRPT, because by substituting Eqs. (2), it is clear that Eq. (4) is exactly equivalent to the original version in Eq. (1). By contracting to roll over loans and borrowings at the market forward interest rates, investors with a 6-month horizon are indifferent between initial borrowing and lending for 6 months or for 3 months. Moreover, PEH is not sufficient to ensure that equation (4) will hold empirically if the FX forward market is not in equilibrium.

The situation Ghosh considers is slightly different. He posits that if investors chose initial domestic borrowings or loans of one maturity (3 or 6 months) and initial foreign borrowings or loans of the other maturity, a new version of IRPT would hold, taking the following form[3]:

$$ \begin{aligned} &S_0[(1 + {}_0R_1/4)(1 + {}_1R_2^f/4) - (1 + {}_0R_2/2)] \\ &\quad = F_2[(1 + {}_0R_1^*/4)(1 + {}_1R_2^{f*}/4) - (1 + {}_0R_2^*/2)] \end{aligned} \tag{5} $$

Ghosh further maintains that, although IRPT is an equilibrium condition

[3]Ghosh presents this as Eq. (6) of his paper in the form:

$$ \begin{aligned} &(1 + {}_0R_1)(1 + {}_1R_2^f) - (1 + {}_0R_1^*)(1 + {}_1R_2^{f*}) - ({}_0R_2 - {}_0R_2^*) \\ &\quad = [(F_2 - S_0)/S_0][(1 + {}_0R_1^*)(1 + {}_1R_2^{f*}) - (1 + {}_0R_2^*)] \end{aligned} $$

After correcting for the day count error, though, this is algebraically equivalent to Eq. (5) here.

that does not generally hold at all points in time (enabling investors to earn arbitrage profits by restoring equilibrium), if the PEH is valid, this new version of IRPT will always hold empirically. It is quite easy to show, however, that both of these conclusions are false.

First, by substituting the forward rate definition from Eqs. (2), it is clear that equation (5) is not a new version of IRPT at all but rather an arithmetic identity ($0 = 0$), devoid of any behavioral content. It follows that the second point is also false, because PEH is irrelevant to the validity of such an identity. A hint to the source of Ghosh's mistakes lies in his identification (in four places in his paper) of PEH not with Eqs. (3), as is appropriate, but erroneously with the forward rate definition in Eqs. (2). His discussion of whether "PEH", in the form of Eqs. (2), holds empirically implies that the 3 month interest rates 3 months hence assumed in these equations are not the forward interest rates ${}_1R_2^f$ and ${}_1R_2^{f*}$, but rather the future short-term interest rates ${}_1\tilde{R}_2$ and ${}_1\tilde{R}^*$hm-p3_2. Through this assumption, Eq. (5) no longer holds as an identity, but it would hold empirically only by chance. If, furthermore, one were to take expected values of both sides and assume the validity of PEH by substituting Eqs. (3), the identity would once again be restored. Although these algebraic manipulations may have inspired Ghosh's conclusions, nevertheless they do not constitute valid economic logic.

Although Ghosh's assumption leads to a flawed analysis of covered interest arbitrage, it could shed light on profitable strategies for uncovered arbitrage, or relative value trades designed to profit from systematic price relationship biases in the market. In both the FX and deposit markets, these trades leave the FX and interest rate risk unhedged and assume that future spot FX and short-term interest rate movements are not completely random. The first of these strategies applies to the FX market. In this relative value trade, investors still borrow and lend funds in the domestic and foreign deposit markets, but instead of repatriating the funds by procuring forward cover at the inception of the transaction, they instead reconvert the funds at the new spot FX rate at the conclusion of the trade. If M is the amount of the domestic currency bought to implement the strategy, the profitability criterion for a 6-month time horizon can be seen by modifying Eq. (1) to obtain

$$M[S_0(1 + {}_0R_2/2) - \tilde{S}_2(1 + {}_0R_2^*/2)] > 0 \tag{6}$$

where $\tilde{S}_2$ is the future spot FX rate in 6 months time. Using the IRPT equilibrium condition from Eq. (1), this criterion becomes

$$M(F_2 - \tilde{S}_2) > 0 \tag{7}$$

If the choice of M depends on the investor's expectation about ${}_1\tilde{S}_2$, this criterion can be restated as

$$M \gtreqless 0 \text{ as } F_2 \gtreqless E[\tilde{S}_2] \tag{8}$$

Investors will borrow foreign funds, sell foreign currency to buy domestic currency, and lend domestically if the 6-month forward FX rate overstates the FX spot rate expected to prevail in 6 months' time, and they will do the reverse if the 6-month forward rate underestimates the expected future spot FX rate.

To turn Eq. (8) into a systematic strategy, it is necessary to combine it with a theory of the future FX spot rate. One such theory is the Speculative Efficiency Hypothesis (SEH), which states that ${}_1\tilde{S}_2$ follows a random walk, so that

$$E[\tilde{S}_2] = S_0 \tag{9}$$

Using Eq. (9) in criterion (8) produces the decision rule to sell forward premium currencies and to buy forward discount currencies. The logic is that because the FX forward rate must satisfy IRPT to prevent arbitrage, if the spot FX rate follows a random walk, the FX forward rate will systematically overpredict the future spot rate when it is at premium to the current spot rate and underpredict it when it is at a discount.

The other relative value trade suggested by the Ghosh analysis is the choice of a maturity at which to lend or borrow in the foreign and domestic deposit markets without hedging with interest rate forwards. If N is the amount of 3-month funds lent domestically and N* the amount lent in the foreign market, the criterion for the profitable maturity choice is obtained from the following modification of equations (2):

$$N[(1 + {}_0R_1/4)(1 + {}_1\tilde{R}_2/4) - (1 + {}_0R_2/2)] > 0 \tag{10a}$$

$$\text{and } N^*[(1 + {}_0R_1^*/4)(1 + {}_1\tilde{R}_2^*/4) - (1 + {}_0R_2^*/2)] > 0 \tag{10b}$$

Substituting Eqs. (2), these become

$$N({}_1\tilde{R}_2 - {}_1R_2^f) > 0 \tag{11a}$$

$$\text{and } N^*({}_1\tilde{R}_2^* - {}_1R_2^{f*}) > 0 \tag{11b}$$

and assuming the choice of N and N* depends on the investor's expectation of future short-term interest rates, it further reduces to

$$N \gtreqless 0 \text{ as } E[{}_1\tilde{R}_2] \gtreqless {}_1R_2^f \tag{12a}$$

$$\text{and } N^* \gtreqless 0 \text{ as } E[{}_1\tilde{R}_2^*] \gtreqless {}_1R_2^{f*} \tag{12b}$$

There are several possible theories about the relationship between forward interest rates and expected future short-term interest rates which could be used to convert Eq. (12) into a usable decision rule. One of these, PEH, as given in Eqs. (3), suggests that investors would be indifferent about what maturity to borrow or lend, since, over time, there would be no net cash flow advantage from borrowing or lending for 3 months (with rolled-over funds exposed to interest rate risk) or for 6 months. Another theory, however, the Liquidity Premium Hypothesis (LPH), suggests that forward interest rates overstate future short-term interest rates on average. From criterion (12), the implication is that if LPH is valid, investors can systematically profit from a strategy of borrowing unhedged 3-month funds and lending 6-month funds in domestic and foreign deposit markets.