A Note on Estimating the Minimum Extended Gini Hedge Ratio

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The extended Gini coefficient, Γ, is a measure of dispersion with strong theoretical merit for use in futures hedging. Yitzhaki (1982, 1983) provides conditions under which a two-parameter framework using the mean and Γ of portfolio returns yields an efficient set consistent with second-order stochastic dominance. Unlike mean-variance theory, the mean-Γ framework requires no particular return distribution or utility function to yield this conclusion. However, Γ must be computed iteratively making it less convenient to use than variance. Shalit (1995) offers a solution to the computation problem by suggesting an instrumental variables (IV) slope estimator, β^{IV}, as the basis for the minimum extended Gini hedge ratio where the instruments are based on the empirical distribution function (edf) of futures prices. However, the validity of employing the IV slope coefficient as the basis for the minimum extended Gini hedge ratio requires the questionable assumption that the rankings of futures prices to be the same as those for the profits of the hedged portfolio. © 1999 John Wiley & Sons, Inc. Jrl Fut Mark 19:101–113, 1999

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INTRODUCTION

The purpose of this article is to empirically assess the effect of the ranking assumption. If the assumption is reasonable then we should find that the IV-based hedge ratio, X^{IV}, minimizes Γ. We compare hedge ratios using Shalit's method with true minimum extended Gini hedge ratios. We examine data from six equity indices and find that Shalit's hedge ratios perform inconsistently, both economically and statistically, as estimates of true minimum extended Gini hedge ratios.

THE EXTENDED GINI

Employing notation used in Shalit (1995), the Gini coefficient of portfolio profits, w, can be expressed as:

$$\Gamma(\nu) = -\nu Cov\{w, [1 - G(w)]^{\nu-1}$$

where G is the cumulative distribution function (CDF) of hedged portfolio profits, w, where $w = Qs_1 + X(f_0 - f_1)$ and ν is the parameter of risk aversion. Risk seekers have values of ν such that $0 < \nu < 1$ whereas risk neutral agents have a parameter value of $\nu = 1$ (Kolb and Okunev, 1992). Risk aversion increases monotonically for all $\nu > 1$ and for $\nu = 2$ we get the mean-Gini as a special case of the extended mean-Gini.

The empirical application of Γ requires that G be estimated since it is typically unknown. A common procedure, proposed in Lerman and Yitzhaki (1984), is to estimate G using its edf, which involves ranking the sample in ascending order and dividing each rank by the size of the sample. For example, to estimate the CDF of futures returns, $r = (f_0 - f_1)$, one would rank the observations of futures returns and divide by sample size yielding the sample $\{1/T, 2/T, \ldots, T/T\}$ corresponding to $r_1, r_2, \ldots, r_T$ where $r_1 < r_2 < \ldots < r_T$. These sample points then provide a discrete estimate of the CDF of r. This estimation method is followed in Kolb and Okunev (1992, 1993), and Cheung, Kwan, and Yip (1990).

We employ a different approach to estimate G. A nonparametric kernel estimation method, as used in Lien and Luo (1993), is followed. The kernel method is superior in that efficiency can be attained with a smaller sample. For a more complete treatment of kernel estimation see Silverman (1986) and Wand and Jones (1995). Once $\hat{G}$ is obtained, we calculate the extended Gini using the following sample representation:

$$\hat{\Gamma}(\nu) = -\frac{\nu}{T}\left\{\sum_{t=1}^{T} w_t(1 - \hat{G}(w_t))^{\nu-1} - \left(\frac{1}{T}\sum_{t=1}^{T} w_t\right)\left(\sum_{t=1}^{T} (1 - \hat{G}(w_t))^{\nu-1}\right)\right\}$$

A grid search procedure is used to search over values of X until a minimum extended Gini is found.

MINIMUM EXTENDED GINI HEDGING FRAMEWORK

Shalit (1995) shows that the minimum extended Gini hedge ratio can be expressed as:

$$X^* = Q\frac{\text{cov}\{s_1,[1 - G(w)]^{\nu-1}\}}{\text{cov}\{f_1,[1 - G(w)]^{\nu-1}\}}$$

If, in addition, the assumption is made that rankings for w and f_1 are identical, then the optimal MEG hedge ratio becomes $X^{IV} = Q\beta^{IV}$, where β^{IV} is the instrumental variables regression slope coefficient. β^{IV} can be expressed as:

$$\beta^{IV} = \frac{\text{cov}\{s_1,[1 - \hat{G}(f_1)]^{\nu-1}\}}{\text{cov}\{f_1,[1 - \hat{G}(f_1)]^{\nu-1}\}} = \frac{\sum_{t=1}^{T}(s_1 - \bar{s}_1)(z - \bar{z})}{\sum_{t=1}^{T}(f_1 - \bar{f}_1)(z - \bar{z})},$$

where $z = [1 - \hat{G}(f_1)]^{\nu-1}$, $\bar{z}$ is the mean of z, and $\hat{G}(f_1)$ is the empirical distribution of f_1. The ranking assumption implies the following relationship between w and f_1 will hold:

$$w_1 = Qs_1 + X(f_0 - f_{1,1})$$

$$w_2 = Qs_1 + X(f_0 - f_{1,2})$$

$$\vdots$$

$$w_T = Qs_1 + X(f_0 - f_{1,T}),$$

where $w_1 < w_2 < \ldots < w_T$ and $f_{1,1} < f_{1,2,} < \ldots < f_{1,T}$ are T samples of f_1. We argue that this assumption is not justified and that estimated hedge ratios based on this assumption will not in general be optimal.

NECESSARY CONDITIONS FOR IDENTICAL RANKING

Portfolio returns and futures prices will not, in general, be ranked identically. However, to see when such an assumption is justified, consider the profits of the hedged portfolio, w, where $w = Qs_1 + X(f_0 - f_1)$. Assuming s_1 is approximately equal to f_1, we can express w as: $w = Qf_1 + X(f_0 - f_1)$. Rearranging, we get:

$$w = (Q - X)f_1 + Xf_0.$$

In this form, it is clear that w will increase with f_1 only when $X < Q$, or the hedge ratio, h, is less than unity. When $X > Q$ ($h > 1$), an increase in f_1 will lead to a decrease in w, thus violating the ranking assumption. The qualification of $h < 1$ is only a necessary condition for equal rankings and thus will not guarantee such an outcome. The effect of f_0 easily leads to different rankings for w and f_1.

EMPIRICAL EVIDENCE[1]

In this article we examine $R_p = R_s - XR_f$, where $R_s = ln(s_1/s_0)$ and $R_f = ln(f_1/f_0)$. This formulation, following Lien and Luo (1993) is similar to $w = Qs_1 + X(f_0 - f_1)$ used by Shalit.[2] Upon appropriately substituting notation, we can derive the IV estimator for X:

$$X^{IV} = \frac{\mathrm{cov}\{R_s, [1 - G(R_p)]^{\nu-1}\}}{\mathrm{cov}\{R_f, [1 - G(R_p)]^{\nu-1}\}}.$$

We examine daily observations from the following six stock indices: Nikkei, Standard and Poor's, Tokyo Stock Price Index (TOPIX), Korea Composite Stock Price Index (KOSPI), Hang Seng, and IBEX. The futures price referred to is the price of the nearby contract. See Table I for a detailed data description. For each index the IV hedge ratios, X^{IV}, and the minimum extended Gini hedge ratios, X^{MEG}, are calculated; results are listed in Tables II and III respectively.

To examine the magnitude of the disparities between the IV and minimum extended Gini hedge ratios we calculate the absolute value of the differences between X^{IV} and X^{MEG}, and standardize them by the estimated standard deviation of X^{IV}. Explicitly, we calculate:

$$\delta = \frac{|X^{IV} - X^{MEG}|}{SD(X^{IV})},$$

where $SD(X^{IV})$ is the estimated standard deviation of X^{IV}. These stan-

[1]The identical ranking assumption can be empirically tested by the following procedure. First we derive X^{IV} and substitute into the equation of w to obtain w^{IV}. The distribution of w^{IV} is then compared to the distribution of f_1 through statistical hypothesis testing procedures. If the null hypothesis of identical distributions is rejected, the question remains how much efficiency is lost by using the IV estimator instead of the accurate estimator. To circumvent this problem we compare the two estimators directly.

[2]In the literature, the hedging problem is usually cast in R_p (instead of w). The choice depends upon the distribution for spot and futures prices. If both prices are normally distributed, then w is adopted. When they are log-normally distributed, R_p is adopted.

TABLE I

Data Description

INDEX	Country	N	Beginning Date	Ending Date
Nikkei	Japan	713	9/4/86	9/1/89
S&P	United States	758	4/22/82	4/19/85
TOPIX	Japan	901	4/4/90	12/30/93
KOSPI	South Korea	163	5/4/96	12/3/96
Hang Seng	Hong Kong	721	1/5/87	12/1/89
IBEX	Spain	496	4/7/93	3/31/95

TABLE II

Instrumental Variables Hedge Ratios

v	TOPIX	Nikkei	IBEX	S&P	KOSPI	Hang Seng
2	0.773	0.747	0.539	0.719	0.725	0.785
3	0.777	0.752	0.526	0.715	0.719	0.792
4	0.782	0.749	0.497	0.709	0.709	0.795
5	0.788	0.744	0.471	0.703	0.700	0.797
6	0.793	0.738	0.449	0.696	0.692	0.798
7	0.798	0.732	0.430	0.691	0.686	0.800
8	0.803	0.727	0.413	0.685	0.681	0.801
9	0.808	0.722	0.398	0.681	0.677	0.801
10	0.812	0.717	0.385	0.676	0.674	0.802
20	0.841	0.679	0.290	0.648	0.662	0.803
30	0.857	0.651	0.234	0.633	0.667	0.800
40	0.867	0.631	0.197	0.624	0.677	0.796
50	0.873	0.614	0.171	0.617	0.688	0.793
100	0.881	0.566	0.103	0.607	0.742	0.780
150	0.881	0.542	0.074	0.614	0.779	0.771
200	0.882	0.528	0.058	0.629	0.800	0.763
500	0.898	0.506	0.025	0.755	0.833	0.735

The instrumental variables hedge ratios are calculated using the following expression: $\sum_{t=1}^{T}(R_{st}-\bar{R}_s)(z_t-\bar{z}) \Big/ \sum_{t=1}^{T}(R_{ft}-\bar{R}_f)(z_t-\bar{z})$, where $z_t = [1-\hat{G}(R_{ft})]^{v-1}$, $\bar{z}$ is the mean of z_t, and $\hat{G}(R_f)$ is the empirical distribution of R_f.

TABLE III

Minimum Extended Gini Hedge Ratios

v	TOPIX	Nikkei	IBEX	S&P	KOSPI	Hang Seng
2	0.782	0.686	0.467	0.729	0.719	0.786
3	0.767	0.685	0.477	0.720	0.707	0.798
4	0.760	0.668	0.464	0.707	0.639	0.805
5	0.754	0.668	0.423	0.699	0.625	0.805
6	0.748	0.668	0.337	0.689	0.612	0.805
7	0.748	0.668	0.260	0.688	0.600	0.805
8	0.748	0.655	0.075	0.668	0.594	0.805
9	0.748	0.652	0.022	0.637	0.594	0.805
10	0.748	0.652	0.021	0.637	0.594	0.805
20	0.721	0.628	0.020	0.584	0.582	0.805
30	0.707	0.598	0.020	0.542	0.580	0.805
40	0.707	0.553	0.020	0.474	0.578	0.805
50	0.682	0.548	0.021	0.474	0.577	0.798
100	0.648	0.538	0.022	0.474	0.547	0.791
150	0.647	0.539	0.023	0.533	0.546	0.790
200	0.644	0.547	0.024	0.621	0.546	0.789
500	0.706	0.637	0.029	0.624	0.549	0.759

The extended Gini is calculated using the following sample representation:

$$\hat{\Gamma}(v) = -\frac{v}{T}\left\{\sum_{t=1}^{T} R_{pt}(1-\hat{G}(R_{pt}))^{v-1} - \left(\frac{1}{T}\sum_{t=1}^{T} R_{pt}\right)\left(\sum_{t=1}^{T}(1-\hat{G}(R_{pt}))^{v-1}\right)\right\}$$

A grid search procedure is used to search over values of X_f until a minimum extended Gini is found.

dardized differences are calculated for 17 different levels of risk aversion for each index, for a total of 102 results. To summarize these extensive results, we report the number of standardized differences exceeding threshold values as well as the mean, maximum, and minimum standardized differences. The results are provided in Table IV.

It is clear from Table IV that the performance of X^{IV} as an estimate of X^{MEG} varies substantially across indices. The poorest performance is found for the TOPIX and Nikkei contracts, where, for each contract, 14 of the 17 IV hedge ratios exceed two standard deviations from X^{MEG}. Additionally, for the TOPIX contract, 10 hedge ratios exceed three standard deviations, seven exceed four standard deviations, and six exceed five. For the Nikkei index, eight hedge ratios exceed three standard deviations and one exceeds five standard deviations. The mean differences are 3.593 and 2.932 for the TOPIX and Nikkei contracts, respectively.

TABLE IV

Standardized Differences between IV and Minimum Extended Gini Hedge Ratios

Panel A:

v	TOPIX	Nikkei	IBEX	S&P	KOSPI	Hang Seng
2	0.614	2.660	0.719	0.702	0.165	0.064
3	0.654	2.868	0.484	0.331	0.308	0.384
4	1.383	3.477	0.335	0.126	1.703	0.643
5	2.063	3.289	0.502	0.241	1.742	0.517
6	2.644	3.059	1.206	0.407	1.785	0.455
7	2.853	2.824	1.879	0.169	1.853	0.327
8	3.054	3.207	3.830	0.928	1.817	0.264
9	3.249	3.146	4.360	2.338	1.686	0. 265
10	3.385	2.946	4.315	2.021	1.584	0.201
20	5.304	2.455	3.816	2.766	1.323	0.142
30	5.858	2.640	3.411	3.493	1.285	0.373
40	5.681	3.970	3.084	5.264	1.346	0.696
50	6.276	3.408	2.801	4.666	1.413	0.398
100	5.922	1.492	1.867	3.414	2.006	0.958
150	5.069	0.162	1.319	1.801	2.100	1.730
200	4.583	1.031	0.947	0.161	2.089	2.426
500	2.481	7.219	0.132	1.876	1.899	2.265

Panel B:	TOPIX	Nikkei	IBEX	S&P	KOSPI	Hang Seng
#δ > 2 std. dev.	14	14	7	7	3	2
#δ > 3 std. dev.	10	8	6	4	0	0
#δ > 4 std. dev.	7	1	2	2	0	0
#δ > 5 std. dev.	6	1	0	1	0	0

Panel C:	TOPIX	Nikkei	IBEX	S&P	KOSPI	Hang Seng
Mean	3.593	2.932	2.059	1.806	1.536	0.712
Max	6.276	7.219	4.360	5.264	2.100	2.426
Min	0.614	0.162	0.132	0.126	0.165	0.064

The standardized differences measure the magnitude of the differences between the IV-based hedge ratios, X^{fV}, and the minimum extended mean-Gini hedge ratio, X^{MEG}. The standardized differences are calculated as follows: $\delta = |X^{fV} - X^{MEG}| / SD(X^{fV})$, where $SD(X^{fV})$ is the estimated standard deviation of X^{fV}.

The maximum (minimum) value is 6.276 (0.614) for the TOPIX and 7.219 (0.162) for the Nikkei. For these indices, the IV-based hedge ratios perform poorly as estimates of the minimum extended Gini hedge ratios, strongly suggesting that the ranking assumption is not justified.

Results for the IBEX and S&P also show that X^{IV} does not approximate X^{MEG} with regularity. For both indices, seven of the 17 differences exceed two standard deviations from zero. Six of these seven differences exceed three standard deviations for the IBEX contract and four exceed three standard deviations for the S&P contract. Two differences exceed four standard deviations for each index, while one exceeds five standard deviations for the S&P index. The mean differences are 2.059 and 1.806 for the IBEX and S&P contracts, respectively. The maximum (minimum) differences for the IBEX and S&P contracts are 4.360 (0.132) and 5.264 (0.126), respectively.

The IV hedge ratio performs best for the KOSPI and Hang Seng contracts. Three standardized differences exceed two standard deviations for the KOSPI and only two exceed two standard deviations for the Hang Seng. No differences exceed three standard deviations for either index. The mean difference is 1.536 for the KOSPI contract, whereas the maximum (minimum) difference is 2.100 (0.165). For the Hang Seng, the mean difference is 0.712 and the maximum (minimum) is 2.426 (0.064).

Considering all indices together, 47 of the 102 IV hedge ratios exceed two standard deviations from the corresponding minimum extended Gini hedge ratios. Moreover, 28 exceed three standard deviations, 12 exceed four standard deviations, and eight exceed five. The average standardized difference is 2.092, whereas the maximum (minimum) is 7.219 (0.064) standard deviations from zero. Interestingly, all hedge ratios satisfy the necessary condition ($h < 1.0$) for equal rankings to obtain; however, the evidence indicates that IV-based hedge ratios do not, in general, approximate the minimum extended Gini hedge ratios.

Across risk aversion levels, X^{IV} tends to perform better for low levels of risk aversion for all indices except for the Nikkei. For the latter index, large standardized differences are distributed among all levels of risk aversion, although performance is lowest for $\nu = 500$. Other patterns are index specific. For example, for the TOPIX contract, the IV hedge ratio performance declines to a minimum at $\nu = 50$. For the IBEX contract, IV hedge ratio performance is lowest at levels of risk aversion $\nu = 9$ and 10. Performance is lowest for the S&P contract at $\nu = 40$. For the KOSPI and Hang Seng indices, performance is lowest at very high risk aversion levels ($\nu \geq 100$).

The standardized differences, while providing a measure of the economic discrepancies between X^{IV} and X^{MEG}, ignore the fact that X^{MEG} is a random sample. When taking into account the latter property, to calculate the statistical significance of the differences between X^{IV} and X^{MEG}, we implement a bootstrap resampling procedure to calculate the

TABLE V

Bootstrap Results

Panel A: Nikkei

v	Test Statistic	p-value
2	23.9580	0.00
5	14.6113	0.00
10	8.7942	0.00

Panel B: KOSPI

v	Test Statistic	p-value
2	5.8482	0.00
5	10.7719	0.00
10	13.4567	0.00

These results are based on 100 bootstrap samples. The test statistic is calculated as $\sqrt{n}(\bar{X}^{IV} - \bar{X}^{EMG})/\hat{\sigma}^{BS}$, where n is the bootstrap sample size, $\bar{X}^{i}$ ($i = IV$ or MEG) is the bootstrap mean of each hedge ratio, and $\hat{\sigma}^{BS}$ is the bootstrap standard deviation of $X^{IV} - X^{MEG}$.

sample mean and variance. We obtain a test statistic that is asymptotically standard normal. The heavy computational requirements of this procedure limit our test to only two contracts (Nikkei and KOSPI) for three levels of risk aversion ($v = 2$, 5, and 10) for a total of six tests. Using 100 bootstrap samples we test the null hypothesis $X^{IV} - X^{MEG} = 0$. The results of these tests are summarized in Table V.

The results are highly significant. For the Nikkei index we obtain z-ratios of 23.96, 14.61, and 8.79 for risk aversion levels $v = 2$, 5, and 10, respectively—whereas for the KOSPI we get z-ratios of 5.85, 10.77, and 13.46. The maximum p-value for these results is approximately zero. The statistical results soundly reject the equality of X^{IV} and X^{MEG} and question the justification of the ranking assumption.

Last, the relative hedging effectiveness of the minimum extended Gini hedge ratio is examined. We calculate the extended Gini coefficient of a spot/futures portfolio employing the IV hedge ratio and denote this quantity $\Gamma(R^{IV})$. Next, the additional percentage risk reduction of using a minimum extended Gini hedge ratio is calculated, taking $\Gamma(R^{IV})$ as the base. Specifically, we calculate:

$$\frac{\Gamma(R^{IV}) - \Gamma(R^{MEG})}{\Gamma(R^{IV})} \times 100,$$

where $\Gamma(R^{IV})$ and $\Gamma(R^{MEG})$ are the extended Gini coefficients of spot/

TABLE VI

Relative Hedging Effectiveness (%)

v	TOPIX	Nikkei	IBEX	S&P	KOSPI	Hang Seng
2	0.010	0.760	0.371	0.076	0.105	0.005
3	0.050	0.962	0.090	0.014	0.100	0.033
4	0.161	1.103	0.122	0.012	0.479	0.022
5	0.323	1.160	0.119	0.047	1.141	0.017
6	0.479	1.191	0.406	0.033	1.836	0.008
7	0.687	1.168	1.129	0.024	2.405	0.020
8	0.926	1.136	2.345	0.092	3.004	0.017
9	1.153	1.145	4.071	0.231	3.622	0.007
10	1.323	1.180	5.563	0.384	4.260	0.013
20	3.318	1.130	12.924	1.754	12.517	0.017
30	4.971	0.833	14.806	2.176	22.908	0.023
40	6.402	0.814	15.037	2.748	33.659	0.013
50	7.614	1.005	14.526	3.463	43.263	0.021
100	10.144	0.770	10.316	3.087	74.572	0.327
150	10.736	0.016	6.962	0.545	87.950	0.677
200	11.079	0.589	4.789	0.131	94.135	0.562
500	14.305	7.365	0.272	18.322	99.910	2.481
Mean	4.334	1.313	5.520	1.949	28.580	0.251
Max	14.305	7.365	15.037	18.322	99.910	2.481
Min	0.010	0.016	0.090	0.012	0.100	0.005

Relative hedging effectiveness is calculated as: $100\text{x}\left(\Gamma(R^{IV}) - \Gamma(R^{MEG})\right)/\Gamma(R^{IV})$, where $\Gamma(R^{IV})$ and $\Gamma(R^{MEG})$ are the extended Gini coefficients of spot/futures portfolios with futures positions of X^{IV} and X^{MEG}, respectively. Relative hedging effectiveness measures the additional risk reduction attained using the minimum Gini hedge ratio, over and above that obtained from using the IV hedge ratio.

futures portfolios with futures positions of X^{IV} and X^{MEG}, respectively. If the IV hedge ratios are good estimates of the minimum extended Gini hedge ratios, we should see little reduction in risk. Relative effectiveness measures are calculated for 17 risk aversion levels across all six indices, for a total of 102 results. The findings are reported in Table VI.

The magnitude of the results vary substantially across indices. For the KOSPI contract, the mean relative hedging effectiveness is 28.580%. The maximum (minimum) relative hedging effectiveness measure is 99.910% (0.100%). The IBEX and TOPIX contracts have mean relative hedging effectiveness measures of 5.520% and 4.334%, respectively. The maximum (minimum) values are 15.037% (0.090%) for the IBEX contract and 14.305% (0.010%) for TOPIX. The mean value for the Nikkei index is 1.313%, whereas the maximum (minimum) is 7.365% (0.016). For the S&P index, the average is 1.949% with a maximum (minimum) of 18.322% (0.012%). The IV hedge ratios perform best for the Hang Seng index. The mean relative hedging effectiveness is 0.251% and the maximum (minimum) is 2.481% (0.005%).

Across levels of risk aversion, there is a tendency for relative hedging effectiveness to vary directly with risk aversion level. Relative hedging effectiveness is low for low levels of risk aversion and high for high levels. This pattern is present for all of the indices except for the Nikkei and Hang Seng. For the Nikkei, relative hedging effectiveness is relatively invariant with risk aversion. The same is true for the Hang Seng except at very high levels of risk aversion where relative hedging effectiveness increases.

Considering all 102 results in aggregate, the mean relative hedging effectiveness is 7.606%. This means that, on average, employing minimum extended Gini hedge ratios will reduce risk by 7.606% over and above the risk reduction achieved by IV hedge ratios. The maximum relative hedging effectiveness is an astounding 99.910%, whereas the minimum is a respectable 0.005%. These hedging effectiveness results indicate that IV based hedge ratios do not approximate minimum extended Gini hedge ratios with consistency.[3]

The totality of the empirical evidence presented soundly rejects the reasonableness of the ranking assumption needed to justify the use of the IV-based hedge ratio. Although in some instances the IV procedure performs well, it does not do so with regularity. Moreover, there are many instances in which the method completely fails. We conclude that IV-based hedge ratios do not serve, economically or statistically, as dependable estimates of minimum extended Gini hedge ratios.

CONCLUSIONS

The extended Gini coefficient as an alternative measure of dispersion has strong theoretical promise for use in futures hedging. The mean and extended Gini yield portfolios contained in the stochastic dominance efficient set, thus yielding optimal hedge ratios. This optimality holds for any

[3]We also examine whether there exist material differences between the Gini and minimum variance hedge ratios. If no substantial difference exists, then little is gained with the additional computational complexity of using Gini hedge ratios. We find substantial economic and statistical differences between the two hedge ratios. Our results are available upon request.

distribution of portfolio returns and does not require the restrictive requirement of quadratic utility functions. The main drawback of using the Gini coefficient is that it is notably more difficult to compute than variance. No closed-form expression or simple computational technique exists for the Gini coefficient.

Shalit (1995) proposed the IV slope estimator as the basis for the optimal minimum-extended Gini hedge ratio. If futures price rankings are identical to spot-futures portfolio profit rankings, then the IV-based estimator can be used as an explicit expression for calculating minimum extended Gini hedge ratios, overcoming the extended Gini's principal disadvantage. Without the ranking assumption, however, the IV-based hedge ratio is not justified.

In this paper we assess the effect of the ranking assumption and find that the IV-based hedge ratios do not approximate the minimum extended Gini hedge ratios with regularity. Forty-seven of the 102 differences calculated exceeded two standard deviations from zero, 28 exceeded three standard deviations, 12 exceeded four standard deviations, and eight exceed five standard deviations.

Our statistical tests conducted on two indices for three risk aversion levels reject the hypothesis that $X^{IV} - X^{MEG} = 0$ at levels of significance substantially exceeding the 1% level. The maximum p-value of all of the tests conducted was approximately zero.

We calculate relative hedging effectiveness measures which measure the additional risk reduction attained using the minimum Gini hedge ratio, over and above that obtained from using the IV hedge ratio. Aggregating all results, the average relative hedging effectiveness is 7.606% and the maximum is 99.910%. Whereas the minimum effectiveness measure is a respectable 0.005%, we find that there is much inconsistency among indices.

The entirety of our empirical evidence leads us to strongly reject the ranking assumption and conclude that IV-based hedge ratios are not consistent with minimum extended Gini hedge ratios.

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