Dynamic Hedging of Commercial Paper with T-Bill Futures

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Despite the growing importance of the commercial paper market there is no empirical work investigating the hedging performance of dynamic hedging strategies versus traditional static hedging strategies. This article proposes a dynamic hedging model for commercial paper that takes advantage of time dependencies present in the joint density of commercial paper and T-bill futures. The hedging effectiveness of the dynamic model is compared to that of the static regression model. There is clear evidence that dynamic hedging is superior to static hedging in terms of both total variance reduction and expected utility maximization. These results hold even when transactions costs are explicitly taken into account. © 1998 John Wiley & Sons, Inc. Jrl Fut Mark 18:925–938, 1998

INTRODUCTION

Commercial paper has evolved into an important source of short term financing. Large creditworthy corporations routinely sell commercial paper in order to meet their short term financing needs. Such financing provides a low-cost alternative to traditional bank loans. Because credit

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risk is very low, rates of return on commercial paper are highly correlated with comparable maturity T-bill rates. From an investor's perspective, commercial paper can be very attractive because maturity and denominations can be adopted to accommodate their specific needs. Because of the advantages for both investors and issuers, the dollar amount of commercial paper has grown rapidly in recent years, exceeding the dollar amount of any other money market financial instrument. For example, in the period 1970–1991 the annual growth has been 14% and from 1991 to 1995 the rate of growth has been 6.3%.[1] Calomiris, Himmelberg, and Wachtel (1995) argue that the rapid growth of commercial paper may have altered the monetary transmission mechanism by weakening the bank-loan mechanism.

Minimum denominations for commercial paper are rather high for individual investors to hold. Consequently, the typical investors in this market are money managers, institutional investors, dealers, and domestic and foreign banks. Individual investors hold commercial paper indirectly through money market mutual funds and trusts. Despite the high credit quality of commercial paper, risks due to either illiquidity or outright insolvency of the issuer cannot be ruled out. The bankruptcy of Penn Central in 1970 with $82 million of commercial paper outstanding and the inability of Olympia and York to roll over $1.1 billion of commercial paper clearly illustrate the need to hedge the interest rate risk exposure from holding commercial paper. Hedging interest rate risk is also important for corporate managers who anticipate issuing commercial paper in the future in order to meet working capital financing needs.

Interest rate risk hedging strategies have traditionally relied on (i) duration-based hedge ratios and (ii) regression-based hedge ratios. The former involves creating a portfolio of cash and futures with zero effective duration. The resulting hedge ratio is proportional to the ratio of the durations of the cash asset and the futures contract (see Hull, 1997). This approach assumes parallel shifts in the yield curve and constant yield spreads. Violation of these assumptions renders hedge ratios unreliable. In particular, the assumption of constant yield spreads is unlikely to hold in the case of cross-hedging, that is, when the asset underlying the futures is not the same as the asset to be hedged (asset mismatch). Maturity mismatch, that is, attempting to hedge for dates that no futures contract expires, can further complicate matters.

The second approach assumes neither parallel shifts in the yield curve nor stability of yield spreads. The optimal hedge ratio is the slope estimate of the regression

[1] See for example Hanh (1993) and Allen (1997).

$$\Delta S_t = \alpha + \beta \, \Delta F_t + \varepsilon_t, \tag{1}$$

where, Δ is the first difference operator, S_t is the price (or the logarithm of the price) of the cash asset at time t, F_t is the price (or the logarithm of the price) of the futures contract at t, ε_t is the random error term, assumed to be an i.i.d. process, and α and β are fixed intercept and slope coefficients respectively. The minimum-variance hedge ratio is the estimate for β and it defines the amount of dollars to go short in futures per dollar of investment in the spot asset. If futures prices follow a martingale process then β is also the expected utility maximizing hedge ratio for an agent with quadratic utility function, for example, Anderson and Danthine (1981), Ederington (1979), and Johnson (1960) among others.

The standard assumption of the regression approach is that the moments of the joint distribution of cash and futures prices are constant over time. This assumption, however, runs contrary to the findings of several studies, for instance, Anderson (1985), Malliaris and Urrutia (1991), Kroner and Sultan (1993), and Park and Bera (1987) among others. Consequently the use of constant regression-based hedge ratios may lead to inferior hedging decisions. Several recent studies find that time-varying hedge ratios lead to higher risk reduction than constant hedge ratios for such diverse assets as commodities (Ballie and Myers, 1991), Treasury bonds (Cecchetti, Cumby, and Figlewski, 1988), Canadian banker's acceptances (Gagnon and Lypny, 1995), foreign currency (Kroner and Sultan, 1993) and stock index futures (Park and Switzer, 1995).

Despite the growing evidence that dynamic hedging is, in several instances, superior to the traditional static hedging, no effort has been devoted to ascertaining the importance and feasibility of dynamic hedging techniques for commercial paper. This stands in sharp contrast to the growing importance of the commercial paper market and the attention commercial paper yields have received as leading economic indicators (see, for example, Stock and Watson, 1989 and Bernanke, 1990).

This article evaluates the performance of dynamic hedge ratios relative to the static hedge ratios for commercial paper. An Error Correction bivariate GARCH-type model (EC-GARCH) is used to estimate time-varying hedge ratios along the lines of Kroner and Sultan (1993) and Gagnon and Lypny (1995) among others. The EC-GARCH is estimated using weekly data on commercial paper with maturities of two, three, and six months. The hedging instrument used is the three-month T-bill futures contract. The hedging effectiveness of the dynamic model is compared to that of the static regression model. The evidence suggests that dynamic hedge ratios are superior to static ones in terms of both total variance reduction and expected utility maximization. These results hold true even when transactions costs are incorporated into the analysis.

ECONOMETRIC SPECIFICATION

The bivariate EC-GARCH model used to estimate dynamic hedge ratios can be described by the following set of equations:

$$r_{s,t} = \beta_{s,0} + \beta_{s,1}\,\mathrm{EC}_{t-1} + \varepsilon_{s,t} \tag{2}$$

$$r_{f,t} = \beta_{f,0} + \beta_{f,1}\,\mathrm{EC}_{t-1} + \varepsilon_{f,t} \tag{3}$$

$$\sigma^2_{s,t} = E_{t-1}(\varepsilon^2_{s,t}) = c^2_s + a^2_s\varepsilon^2_{s,t-1} + d^2_s u^2_{s,t-1} + b^2_s\sigma^2_{s,t-1} \tag{4}$$

$$\sigma^2_{f,t} = E_{t-1}(\varepsilon^2_{f,t}) = c^2_f + a^2_f\varepsilon^2_{f,t-1} + d^2_f u^2_{f,t-1} + b^2_f\sigma^2_{f,t-1} \tag{5}$$

$$\begin{aligned}\sigma_{s,f,t} = E_{t-1}(\varepsilon_{s,t}\varepsilon_{f,t}) &= c_s c_f + a_s a_f \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\ &\quad + d_s d_f u_{s,t-1}u_{f,t-1} + b_s b_f \sigma_{s,f,t-1},\end{aligned} \tag{6}$$

where eqs. (2) and (3) describe the conditional means and eqs. (4)–(6) describe the conditional variance-covariance of the joint distribution of the returns of the cash asset and the futures contract. EC_{t-1} is the error correction term, which ensures that deviation from the long-run equilibrium between the prices of the cash asset and the futures contract are temporary.[2] The conditional variances follow a GARCH(1,1) process whereby volatility at time t depends on last period's squared innovation $\varepsilon^2_{j,t-1}$ and last period's volatility $\sigma^2_{j,t-1}$, where, $j = s, f$ (s = cash asset, f = futures contract). Asymmetry in the conditional variances is introduced through the term $u^2_{j,t-1}$, where $u_{j,t} = \min(0, \varepsilon_{j,t})$ and the impact

[2]The error correction term EC_t is the residual from the regression $s_t = \delta_0 - \delta_1 f_t + \mathrm{EC}_t$, where, s_t and f_t are the natural logarithms of the spot and the futures price respectively. According to Engle and Granger (1987), if two variables are cointegrated then their short-term movements should be influenced by the deviations from the cointegrating relationship. For T-bills the cost-of-carry model states that ${}_Tf_0 = S_0 e^{rT}$, where ${}_Tf_0$ and S_0 are the futures and the spot price, T is time to expiration of the futures contract and r is the cost-of-carry. Taking logarithms on both sides yields $\ln({}_Tf_0) - \ln(S_0) = rT$. Thus the log of the spot and the futures will be cointegrated if the forward rate is stationary (see Brenner and Kroner, 1995). The case for cointegration between spot prices for commercial paper and futures prices for T-bills is less clear since the spot asset is not the underlying asset in the futures contract. However, if we redefine the cost-of-carry as $r + d$, where d is a risk premium, then commercial paper and T-bill futures will be cointegrated if $r + d$ is stationary. Assuming this is true, we can say that cointegration is the result of a fundamental equilibrium relationship.

of such asymmetries is captured by d_j.[3] The conditional covariance, given by equation (6), is a function of the product of past innovations, and its own lagged value. The term $d_s d_f u_{s,t-1} u_{f,t-1}$ captures potential asymmetries in the conditional covariance. At any given time t the dynamic hedge ratio b_t will be given by, $b_t = \sigma_{s,f,t}/\sigma^2_{f,t}$.[4]

The model outlined above is estimated using commercial paper with maturities of two, three and six months alternatively, as the cash asset, and three month T-bill futures as the hedging instrument. Assuming bivariate conditional normality for the error vector, maximum likelihood parameter estimates are obtained using the numerical algorithm of Berndt et al. (1974).

DATA AND EMPIRICAL FINDINGS

Data and Preliminary Statistics

The data set consists of weekly prices on three-month T-bill futures and weekly annualized yields on commercial paper with maturities of 60, 90 and 180 days to maturity. The sample period extends from 5 January 1985 to 3 August 1996 for a total of 545 observations. Futures prices are based on Wednesday's settlement price, and they were obtained from Tick Data Inc. Data on commercial paper are constant maturity annualized yields and have been made available by Merrill Lynch, Inc. The yields are used to obtain bond-equivalent prices. Delivery months for the 3-month T-bill futures are March, June, September, and December. To avoid problems related to thin trading we use observations from the most actively traded contracts, thus creating a time series of non-overlapping contracts.

[3]The contribution of a positive innovation will be equal to a_j^2 whereas, the contribution of a negative innovation will be equal to $a_j^2 + d_j^2$. Consequently, the ratio $a_j^2/(a_j^2 + d_j^2)$ can be used as an intuitive measure of the degree of asymmetry in the conditional variance. The justification for asymmetry in the conditional variance can be based on existing models of short-term interest rates. Most of these models imply that higher short term rates are associated with higher volatility (see Chan, Karolyi, Longstaff, and Saunders, 1992). Recent studies have documented such asymmetries in the conditional volatility of short term interest rates (see for example Brenner, Harjes, and Kroner, 1996 and Gagnon and Lypny, 1995). Because the price-yield relationship is one-to-one and onto, asymmetry in the conditional variance of yields implies reverse asymmetry in the conditional variance of prices.

[4]Using matrix notation, the variance-covariance matrix can be written compactly as follows:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + D'u_{t-1}u'_{t-1}D + B'H_{t-1}B$$

where ε_t the (1×2) vector of innovations assumed to follow bivariate normal distribution with zero mean and H_t variance-covariance matrix; u_t is the vector of negative innovations, that is, $u_t = \min(0, \varepsilon_t)$; A, B, and D are diagonal 2×2 matrices for the slope coefficients; and C is the 1×2 vector with the intercept coefficients. This particular parameterization of H_t is known as the BEKK model and it has the advantage of assuring positive semidefiniteness for all t (see Engle and Kroner, 1995).

TABLE I

Preliminary Statistics

	60-Day CP	*90-Day CP*	*180-Day CP*	*3-Month T-Bill Futures*
		Unit Root and Cointegration Tests		
PP	−1.1165	−1.1286	−1.2644	−1.7024
PP(differenced)	−23.4396[a]	−22.5391[a]	−22.0678[a]	−24.4043[a]
EG	−5.3053[a]	−5.4372[a]	−6.3932[a]	
		Preliminary Statistics		
μ	0.0007	0.0011	0.0023	0.0061
σ^2	0.0007	0.0014	0.0054	0.0467
S	1.2614[a]	0.7873[a]	0.2038[a]	0.8773[a]
K	12.2096[a]	6.6654[a]	3.1430[a]	13.6175[a]
LB(24)	25.1389	25.9647	26.5290	25.9748
$LB^2(24)$	77.2736[a]	95.8986[a]	90.2015[a]	31.4720[a]
LM	93.6229[a]	113.3419[a]	90.2015[a]	31.4720[a]

Notes: The sample period extends from 5 January 1985 to 3 August 1996 for a total of 545 observations. PP is the Phillips-Perron statistic for unit roots with a truncation lag of 4. The null hypothesis is that a unit root exists. EG is the Engle-Granger statistic for cointegration between cash and futures. The null hypothesis is no cointegration. PP and EG critical values at the 5% levels are −3.41 and −3.37 respectively. μ, σ, S, and K, are the sample mean, standard deviation, skewness and excess kurtosis respectively. LB(24) is the Ljung-Box χ^2 statistic for 24 lags calculated for returns and squared returns; the null hypothesis is that all autocorrelations up the 24th lag are jointly zero.
[a]Statistical significance at the 5% level.

Table I reports several preliminary statistics exploring the time series properties of the T-bill futures and the commercial paper prices. The first step is to test for integration, that is, whether a unit root is present in the series, and cointegration, that is, whether some linear combination of the futures and the commercial paper series is stationary. Unit root and cointegration tests are reported in Table 1. The Phillips and Perron (1988) statistic (PP), with truncation lag equal to five, fails to reject the null hypothesis of a unit root in the natural logarithm of commercial paper prices and the natural logarithm of the 3-month T-bill futures prices.[5] This implies that the logarithms of the prices are integrated of order one and first differencing is both necessary and sufficient to induce stationarity. The estimated PP statistics for the first differences (returns) reject the hypothesis of a unit root, thereby confirming that this is indeed the case. The next step involves testing whether the prices of the commercial paper series are cointegrated with the price of the futures contract, that is whether they share a common stochastic trend. This is done by means of the Engle-Granger test (EG).[6] This is essentially a test for a unit root in the residuals of a regression involving the variables to be tested for a

[5]For uniformity the level of significance is set at 5% for all hypotheses testing.
[6]See Engle and Granger (1987) and Engle and Yoo (1987).

common stochastic trend. The estimated EG statistics confirm that each one of the commercial paper price series is cointegrated with the T-bill futures price. The importance of this finding is that the short-term dynamics of price changes will, to some extent, be influenced by past deviations from the common stochastic trend. This in turn provides justification for the inclusion of the error correction term in eqs. (2) and (3). Table I reports some statistics on the first logarithmic difference of prices (returns). The sample means for both the commercial paper and the futures price changes are statistically zero. Statistics for skewness and excess kurtosis indicate significant deviations from normality. Such deviations can to a considerable degree be attributed to the presence of conditional heteroskedasticity, or volatility clustering (see, for example, Bollerslev, Chou, and Kroner 1992). The Ljung-Box statistic for 24 lags shows no significant autocorrelations in the returns. There are, however, significant autocorrelations in the squared returns implying that conditional heteroskedasticity is present in all series. This is also confirmed by the Lagrange Multiplier statistic (LM) suggested by Engle (1982). Such dependencies can be modeled by allowing second moments to be time varying. The use of time varying variances and covariances in hedging decisions is expected to produce better results than those obtained under the assumption of constant second moments.

Volatility specification test statistics, proposed by Engle and Ng (1993), are used to test for asymmetries in the second moments of the return series. These statistics are reported in Table II. At least one of the individual tests suggests the presence of asymmetric volatility across all maturities. The joint test is significant in all instances confirming the presence of asymmetries. Thus inclusion of the asymmetric terms in the variance-covariance matrix is justified.

Model Estimation

Parameter estimates and residual based diagnostics of the three bivariate EC-GARCH models are reported in Table III. Tests performed on the standardized residuals show that substantial nonnormality remains in the form of excess kurtosis. The implication is that the estimated standard errors under the assumption of conditional normality will be understated, even though the parameter estimates will be consistent and asymptotically normal. To deal with this problem, in addition to the usual asymptotic standard errors, robust standard errors based on Bollerslev and Wooldridge (1992) are estimated and reported. Even though robust standard errors are much higher, both sets produce the same inference, re-

TABLE II

Sign and Size Bias Tests

Individual Test	*60-Day CP*	*90-Day CP*	*180-Day CP*	*3-Month T-Bill Futures*
Sign Bias (*t*-tests)	7.2852[a]	3.4020[a]	−0.9477	−3.5064[a]
Negative Size Bias (*t*-tests)	−11.3114[a]	−0.5371	5.9317[a]	−1.3306
Positive Size Bias (*t*-tests)	−6.9901[a]	−1.6096	2.8954[a]	0.2961
Joint Test F(3,753)	52.7964[a]	4.7069[a]	17.2115[a]	7.3269[a]

Notes: This table reports volatility specification tests based on the news impact curve proposed by Engle and Ng (1993).
[a]Statistical significance at the 5% level at least.

garding parameter significance, with only three exceptions. The error correction term is statistically significant in all cases using asymptotic standard errors and in most cases using robust standard errors. Thus the time paths of the commercial paper and the T-bill futures are influenced by deviations from long-run equilibrium. The likelihood ratio statistic, LR, reported in Table III, tests the restriction that the error correction term is insignificant. The estimated statistics clearly reject the restriction.

The coefficients that describe the variance-covariance matrix are significant at almost any level. This in turn implies that tomorrow's forecast of the variance-covariance matrix will depend on its current value and the current vector of innovations. Such intertemporal dependencies in the second moments provide valuable information for forecasting purposes. Better forecasting of second moments, in turn, can be used to provide improved hedge ratios. In this respect, a test statistic for forecasting accuracy due to Diebold and Mariano (1995) confirms that the forecasting accuracy of the dynamic EC-GARCH model is statistically different to that of the static OLS model.[7] There is some evidence of asymmetric response of volatility to past innovations. The relevant coefficient is negative and statistically significant for the 90-day commercial paper and the T-bill futures contract.

Diagnostics on the standardized residuals show no evidence of misspecification of the variance-covariance matrix. For example, the Ljung-Box statistics up to twenty-four lags show no evidence of autocorrelation in the squared standardized residuals. The volatility specification tests suggested by Engle and Ng (1993) are also calculated for the bivariate EC-GARCH model standardized residuals and reported in Table IV. These tests show no evidence of any remaining asymmetry. These findings

[7]The Diebold-Mariano statistic is used primarily for first moment forecast accuracy. The version used here is the simple sign test based on the residuals obtained form the null and the alternative models.

TABLE III

Maximum Likelihood Estimates for the Error-Correction GARCH Models

	60-Day CP	*90-Day CP*	*180-Day CP*
$\beta_{s,0}$	0.0013	0.0022	0.0039
	(0.0010)	(0.0016)	(0.0032)
	[0.0008]	[0.0013]	[0.0026]
$\beta_{f,0}$	0.0052	0.0039	−0.0003
	(0.0089)	(0.0091)	(0.0088)
	[0.0063]	[0.0078]	[0.0076]
$\beta_{s,1}$	−5.0371	−6.7751	−9.6095
	(1.1730)[a]	(1.3218)[a]	(1.8699)[a]
	[2.1381][a]	[2.3407][a]	[2.8303][a]
$\beta_{f,1}$	30.1112	21.7484	14.5753
	(12.2662)[a]	(9.5733)[a]	(6.2645)[a]
	[16.1055]	[14.0723]	[7.2878][a]
c_s	0.0124	0.0158	0.0295
	(0.0008)[a]	(0.0011)[a]	(0.0024)[a]
	[0.0014][a]	[0.0048][a]	[0.0083][a]
c_f	0.0562	0.0505	0.0481
	(0.0045)[a]	(0.0039)[a]	(0.0039)[a]
	[0.0094][a]	[0.0129][a]	[0.0108][a]
a_s	0.5552	0.4186	0.3280
	(0.0500)[a]	(0.0411)[a]	(0.0351)[a]
	[0.1256][a]	[0.0970][a]	[0.0836][a]
a_f	0.2731	0.2416	0.2345
	(0.0287)[a]	(0.0271)[a]	(0.0244)[a]
	[0.0719][a]	[0.0865][a]	[0.0732][a]
d_s	0.2052	−0.2352	−0.1003
	(0.1365)	(0.0759)[a]	(0.0926)
	[0.3080]	[0.1987]	[0.1621]
d_f	−0.2864	−0.2772	0.2526
	(0.0361)[a]	(0.0289)[a]	(0.0268)[a]
	[0.0724][a]	[0.0870][a]	[0.0624][a]
b_s	0.6744	0.7697	0.8360
	(0.0392)[a]	(0.0286)[a]	(0.0230)[a]
	[0.0454][a]	[0.1084][a]	[0.0836][a]
b_f	0.8975	0.9147	0.9249
	(0.0071)[a]	(0.0055)[a]	(0.0059)[a]
	[0.0235][a]	[0.0315][a]	[0.0264][a]
S_{2a}	3.2705	3.1845	2.4959
LR	32.02	43.48	57.9
LB(24) for $z^2_{s,t}$	10.7031	7.1388	9.6754
LB(24) for $z^2_{f,t}$	7.8491	4.8813	4.7645

Notes: This table reports maximum likelihood estimates of the bivariate EC-GARCH model described by

$$r_{s,t} = \beta_{s,0} + \beta_{s,1}\,\mathrm{EC}_{t-1} + \varepsilon_{s,t}$$

$$r_{f,t} = \beta_{f,0} + \beta_{f,1}\,\mathrm{EC}_{t-1} + \varepsilon_{f,t}$$

$$\varepsilon_{j,t}|\Omega_{t-1} \sim \mathrm{N}(0,H_t)$$

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + D'u_{t-1}u'_{t-1}D + B'H_{t-1}B$$

where Ω_{t-1} is the information set at time $t-1$. The data are weekly log-differenced prices covering the period 5 January 1985 through 3 August 1996 for a total of 545 observations. Numbers in parentheses $(\cdot)$ are the asymptotic standard errors and numbers in brackets $[\cdot]$ are the Bollerslev-Wooldridge robust standard errors. LL is the sample log-likelihood function evaluated at the maximum. LR is the sample likelihood ratio test statistic testing the hypothesis that the error correction term is zero. It is distributes as a χ^2 and has 95% critical value 5.99. LB(24) are the Ljung-Box χ^2 statistic testing the null hypothesis of no serial correlation in the squared standardized residuals z^2_t. S_{2a} is the Diebold-Mariano statistic testing for equality of forecast accuracy (see Diebold and Mariano, 1995). In large samples it follows standard normal distribution.
[a]Statistical significance at the 5% level.

TABLE IV

Sign and Size Bias Tests

Individual Tests	*60-Day CP*	*3-Month T-Bill Futures*	*90-Day CP*	*3-Month T-Bill Futures*	*180-Day CP*	*3-Month T-Bill Futures*
Sign Bias (*t*-tests)	−0.0058	−0.557	0.4134	−0.3769	0.7215	0.0930
Negative Size Bias (*t*-tests)	−0.9216	−0.9344	−1.4287	−1.09	−1.1322	−1.3655
Positive Size Bias (*t*-tests)	−1.4092	−0.268	−1.3883	−0.2162	−1.5797	−0.1865
Joint Test F(3,753)	1.2593	1.0733	1.3731	1.0198	1.1929	0.9097

Notes: This table reports Engle-Ng volatility specification tests for the standardized residuals obtained from the bivariate EC-GARCH model (see also Table II notes).
[a]Statistical significance at the 5% level at least.

suggest that dynamic hedge ratios based on the time varying variance-covariance matrix of the EC-GARCH model should outperform the static OLS based hedge ratios.

Hedging Effectiveness

The hedge ratios based on the bivariate EC-GARCH models are evaluated in terms of variance reduction of the hedged returns relative to a) hedge ratios based on bivariate GARCH models (that is, ignoring the error correction term) and b) static OLS based hedge ratios. Both within-sample and out-of-sample evaluations are performed. Within-sample evaluations are reported in Table V. For the 60-day maturity the use of EC-GARCH hedge ratios reduces the variance of the hedged returns by an additional 1.8% over the GARCH based hedge ratios and an additional 6.89% over the OLS-based hedge ratios. For the 90-day maturity there is no improvement over the GARCH approach but a 3.03% improvement over the OLS approach. For the 180-day maturity dynamic hedge ratios and static hedge ratios produce the same results.

Out-of-sample evaluation is probably the best way to gauge the performance of different hedging techniques. The out-of-sample simulations for the three maturities are based on a hold out sample of one hundred weekly observations (that is, from 3 September 1994 to 3 August 1996). The models are reestimated by including successively one more observation from the holdout sample. At each step of the iteration the variances of the hedged portfolios are calculated. This process is repeated until the holdout sample is depleted. The results are reported in Table V. The smallest variance is achieved when dynamic hedge ratios based on the EC-GARCH model are used irrespective of maturity. The importance of the

TABLE V

Percentage Variance Improvements of EC-GARCH Hedge Compared to:

Type of Hedge	*60-Day CP*	*90-Day CP*	*180-Day CP*	*Average*
		Within-Sample		
GARCH	1.8181	0.0000	0.0000	0.6061
OLS	6.8965	3.0303	0.0000	3.3090
Unhedged	31.6456	46.4646	44.4444	36.1615
		Out-of-Sample		
GARCH	0.9345	1.1642	0.2489	0.7825
OLS	2.7779	1.6260	1.3862	1.9300
Unhedged	13.5876	23.8482	33.8108	23.7489

Notes: Table V compares the variance of (i) portfolios using EC-GARCH based hedge ratios, (ii) portfolios using GARCH based hedge ratios, (iii) portfolios using static OLS based hedge ratios, and (iv) unhedged portfolios. The upper portion reports within-sample performances, whereas the lower portion reports out-of-sample performance.

error correction mechanism can be seen by the additional variance reduction (0.78% on average) over and above to that achieved by the use of dynamic hedges based on a bivariate GARCH model. The advantage of using EC-GARCH based dynamic hedges rather than static ones is a 1.93% additional reduction in variance when we average across maturities. For the 60-day maturity the reduction is 2.78%. Clearly, exploiting long-run relationships (cointegration) and time variation in second moments and cross moments can produce tangible benefits for investors.

The benefits, in terms of variance reduction, are based on the assumption that the dynamic hedge ratios are updated every time period irrespective of transactions costs. In reality, however, there are transactions costs which rise proportionately with the frequency of portfolio rebalancing. From an economic perspective it is important to investigate whether the benefits from risk reduction outweigh the transactions costs from rebalancing the hedged portfolio. Assuming that the investor has a time separable mean-variance utility function, the dynamic hedging strategy can be considered superior, in an economic sense, if its usage results in higher average utility than the static hedging strategy, net of transactions costs. Table VI shows the total expected utility measures for the three different maturities along with the optimal number of portfolio rebalancings as in Kroner and Sultan (1993) and Gagnon and Lypny (1995).[8] The dynamic hedging strategies (including both EC-GARCH

[8]The expected utility calculations are based on: $E[U(r_h)] = E(r_h) - \theta\sigma_{h,t}^2$, where $E(r_h)$ and $\sigma_{h,t}^2$ are the expected return and the variance of the return of the hedged portfolio and θ is the coefficient of risk aversion. Assuming θ equals 4, and the expected return on the hedged portfolio is zero, the utility from hedging is $-y - 4\sigma_{h,t}^2$, where $-y$ is the percent reduction in expected return due to transactions costs (see Kroner and Sultan, 1993).

TABLE VI

Comparisons of Hedging Effectiveness: Total Expected Utility Comparisons

Hedge	*Transaction Cost*	*60-Day CP*	*90-Day CP*	*180-Day CP*
		Within-Sample		
EC-GARCH	0.0005	−2.1041(4)	−2.9223(11)	−9.9171(475)
GARCH	0.0005	−2.1533(5)	−2.8335(10)	−10.3544(412)
OLS	0.0005	−2.1523(0)	−3.0700(0)	−9.2588(0)
		Out-of-Sample		
EC-GARCH	0.0005	−0.1541(3)	−0.3165(5)	−1.0305(31)
GARCH	0.0005	−0.1618(2)	−0.3366(4)	−0.9063(6)
OLS	0.0005	−0.2107(0)	0.3774(1)	−1.1546(1)

Notes: This table reports within-sample and out-of-sample comparisons of the total expected utility derived assuming the investor rebalances when there is economic gain. The number of portfolio rebalancings is given in parentheses. Transaction costs are assumed to be $5 for a round-trip per $1,000,000 contract or 0.0005%.

and GARCH based hedging) produce the highest total utility across all three maturities both within-sample and out-of-sample. It is interesting to note that the presence of transactions costs prevents the investor from adjusting his hedge ratio very frequently. The number of out-of-sample rebalancings over the 100-week holdout period is considerably less than 10. The only exception is the 180-day maturity where the optimal number of rebalancings is 31. Presumably, transactions costs in this case prevent the EC-GARCH hedge from having higher total expected utility than the GARCH hedge.[9]

Overall the empirical findings suggest that on a total variance reduction basis and expected utility maximization basis the dynamic hedging strategies produce tangible benefits for investors who want to hedge interest rate risk in commercial paper.

CONCLUSIONS

This article has examined the feasibility and possible gains from using dynamic hedging strategies for commercial paper. Commercial paper with maturities of 60, 90, and 180 days is used along with T-bill futures as the hedging instrument. The dynamic hedge ratios are based on the time varying variance-covariance structure of cash and futures assets. Second moment time variation is modeled successfully via a version of the bivariate GARCH model suggested by Engle and Kroner (1995). The hedging effectiveness of the dynamic model is compared to that of the static re-

[9]Transactions costs for institutional investors range from $5 to $12 for a round trip per $1,000,000 contract. Using 0.0012% rather than 0.0005% does not alter the main conclusions.

gression model. Within-sample and out-of-sample evaluations reveal that dynamic hedge ratios are superior to static ones in terms of both total variance reduction and expected utility maximization. These findings hold in the presence of transactions costs.

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