Regime Switching and Cointegration Tests of the Efficiency of Futures Markets

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Many researchers have found that spot and futures prices are not cointegrated in some commodity markets, or they are cointegrated but not with a cointegrating vector (1, −1). One interpretation is that disturbances to excess returns have a unit root persistence, which implies that spot and futures prices do not move together one-for-one in the long run. To provide an alternative explanation for this finding, this article proposes a regime switching model of spot prices that can be viewed in the same framework as Fama and French (1988). Based on this model, Monte Carlo experiments are performed to show that tests for cointegration and estimates of the cointegrating vector are likely to be biased when a sample contains infrequent changes in regime. Taking these shifts into account, the null hypothesis that spot and futures prices are cointegrated and move together one-for-one in the long run can no longer be rejected. © 1998 John Wiley & Sons, Inc. Jrl Fut Mark 18:871–901, 1998

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INTRODUCTION

Recent research in commodity futures markets has documented a feature that appears to be at odds with conventional wisdom.[1] In particular, the k-period futures price at time t and the realized spot price at time $t + k$ are not cointegrated with a cointegrating vector $(1, -1)$ for many commodities, i.e., they do not move together one-for-one in the long run. Consequently, disturbances to the excess return between the realized spot price and the futures price seem to have the same level of persistence as do disturbances to the futures price. In a cost-of-carry framework, Brenner and Kroner (1995) present an argument that, in testing the efficiency of commodity futures markets, the k-period nominal interest rate at time t should be included in the cointegration analysis to account for such persistence. However, Beck (1994) has argued that the excess return is the premium to risk bearing after accounting for carrying cost and convenience yield. This premium should not include the risk-free rate, because speculators can obtain a position in futures markets by posting interest-bearing Treasury securities as a margin. Therefore, assuming the risk premium is stationary, there is no reason to expect a cointegrating system of the realized spot price, the futures price, and the risk-free interest rate.

Further to the arguments of Beck (1994), this article investigates the impact of changes in economic regimes on the cointegration tests of the efficiency of the futures market for precious metals. The changes in macroeconomic and financial regimes that investors and firms face are often discrete. These regime shifts may be caused by changes in fiscal and monetary policy, jumps in investors' confidence, transitions in global political and economic relationships, or technological shocks. A common example of regime shifts is the change in the behavior of interest rates in the late 1970s and early 1980s following the change in the operating procedures of the Federal Reserve. Such regime changes, and the expectations that they may occur, are likely to have a profound effect on asset prices, interest rates and investment decisions. Yet most of the existing literature on commodity markets does not account for the probability of such regime shifts.

In testing the theory of storage based on the hypothesis that the marginal convenience yield declines at higher inventory levels but at a decreasing rate, Fama and French (1988) note that the variation in metals spot and futures prices has a strong business cycle component. In partic-

[1]Following much of the literature, the distinction between forward and futures contracts is ignored here.

ular, their evidence suggests that metal production does not adjust quickly to positive demand shocks, so the resulting fall in inventories increases the convenience yield of holding the inventories. The spot price is hence more variable than the futures price when there is an increase in demand.

The positive demand shocks can be interpreted as a state of "high convenience" in holding the commodity, whereas the "low convenience" state is associated with the times when the demand is lower. If the spot price switches between "low convenience" and "high convenience" states due to general business conditions as implied in Fama and French (1988), rational market traders will incorporate this possibility into their forecasts of the future spot price. To the extent that these expectations are contained in the futures price, the spot and futures prices may deviate due to small sample serial correlations in forecast errors, thus creating a "peso problem."

The "peso problem" was first discussed in the context of foreign exchange markets by Krasker (1980). Since then, the effects of the "peso problem" have been studied in many other financial markets. Intuitively, when market participants anticipate a change in the underlying economic determinants, this anticipation creates a "peso problem" in the expected spot price during periods when switches do not materialize. This "peso problem" may induce a great deal of persistence in the deviation between the future realized spot price and the expected spot price during these periods, which appears to contradict conventional assumptions under rational expectations.

For the purpose of understanding how a "peso problem" affects the excess returns in metal futures markets, consider the following example in the framework of Fama and French (1988). Suppose there is an increase in the demand for metals *now.* The relative impact of such a shock on current and expected spot prices will depend on the inventory levels and the associated convenience yields. In addition, if the market perceives that this increase in demand will be sustained, i.e., an increase in the *future* demand, then the convenience yield will rise in the future.[2] This will be reflected in a larger change in the expected spot price component of the futures price. On the other hand, if the market anticipates a decrease in future demand, then the change in the expected spot price component of the futures price will be smaller, because the market an-

[2]Consider an extreme case: If gold were some day monetized, this would cause inventories to fall dramatically and produce a substantial rise in the convenience yield of holding gold permanently. Another example considered in Salant and Henderson (1978) is the effects upon the gold price from the market's assessed probabilities that the government might discretely sell its gold holdings. In this case, the convenience yield from holding gold will be low (high) if there are (no) government sales of gold.

ticipates future demand and supply responses partially to offset the current demand shock with little change in the future convenience yield.

Rational forecasts of the spot price thus include forecasts of the price conditional on different processes of the convenience yield. If a commodity spot price switches infrequently between different states such as "low convenience" and "high convenience," rational expectations of switches that are not realized over significant periods of time will result in systematic differences between the expected and realized spot prices. These differences may in turn generate persistent deviations between the realized spot and futures prices, giving the appearance of a permanent component in excess returns when no such component exists. In a small sample, this term may be quite persistent if the difference in expectations under the regimes are serially correlated. Using a Markov regime switching framework, this study explores the implications of the "peso problem" in futures markets, and presents an alternative explanation to the seemingly anomalous findings in the recent literature.

COINTEGRATION IN SPOT AND FUTURES MARKETS

Many studies have tested the efficiency of futures markets in the cointegration framework, but the issue of efficiency still remains unresolved, due in part to the use of different methodologies and sample periods. Researchers such as MacDonald and Taylor (1988), Sephton and Cochrane (1990, 1991), Chowdhury (1991), Crowder and Hamed (1993), and Krehbiel and Adkins (1993) have found evidence of either noncointegration between commodity spot and futures prices, or cointegration but not with a cointegrating vector $(1, -1)$. These results violate a necessary condition of the Efficient Market Hypothesis (EMH) under the traditional risk premium hypothesis. In this section, using a new data set, similar results in the futures market for precious metals are presented.

Long-Run Relationship between Spot and Futures Prices

Let s_t be the (logarithmic) spot price of a commodity purchased at time t, $f_{t+k|t}$ be the (logarithmic) price of a k-period futures contract at time t, and $\mathrm{E}_t[\cdot] \equiv \mathrm{E}[\cdot|F_t]$ denote the mathematical expectations conditional on the information set F_t available to economic agents at time t. For the purpose of exposition, consider $k = 1$ and simplify the notation by $f_{t+1|t} \equiv f_t$.

In general, if market participants are endowed with rational expectations but are risk averse, then the futures price can be thought of as differing from the expected future spot price by an amount representing the perceived risk of the contract:

$$f_t = \mathrm{E}_t[s_{t+1}] - \pi_t. \tag{1}$$

Eq. (1) is no more than a particular definition of the risk premium component (π_t) of the futures price as indicated in Fama and French (1987). The actual realization of s_{t+1} will differ from the expected level by a rational expectations forecast error ε_{t+1}, i.e.,

$$s_{t+1} = \mathrm{E}_t[s_{t+1}] + \varepsilon_{t+1}. \tag{2}$$

From eqs. (1) and (2), we have

$$s_{t+1} - f_t = \pi_t + \varepsilon_{t+1} \tag{3}$$

where $s_{t+1} - f_t$ is the premium or excess return on the time t futures contract to buy a commodity at time $t + 1$. Eq. (3) shows why empirical studies typically treat excess returns as stationary processes, or $I(0)$ in the recent econometrics literature. The risk premium has been considered stationary on theoretical grounds, and the forecast error is assumed to follow a stationary process under rational expectations. Since the sum of stationary variables is also stationary, the sum of the risk premium and the forecast error must also be stationary under the above assumptions.

On the other hand, the spot and futures prices have been found to follow processes with very persistent shocks, well approximated as disturbances with unit roots, or $I(1)$ in the literature. The requirement that both sides of eq. (3) be stationary therefore restricts the long-run relationship between spot and futures prices. Specifically, if spot and futures prices are $I(1)$, then the excess return will only be $I(0)$ when (a) s_{t+1} and f_t are cointegrated in the sense of Engle and Granger (1987), and (b) the cointegrating vector is $(1, -1)$. Therefore, if s_{t+1} and f_t are $I(1)$, we can examine whether the excess return is stationary by estimating the following cointegrating regression:

$$s_{t+1} = \beta_0 + \beta_1 f_t + u_{t+1}, \tag{4}$$

and testing whether $\beta_1 = 1$. Note that β_0 need not be equal to zero under the EMH if either $\mathrm{E}[\pi_t] \neq 0$ or there is a non-zero marking-to-market effect as argued in Brenner and Kroner (1995).

Under the null hypothesis of $\beta_1 = 1$, eq. (3) implies the residual in eq. (4) equals the sum of the time-varying component of the risk premium and the forecast error, i.e., $u_{t+1} = \pi_t - \beta_0 + \varepsilon_{t+1}$. If π_t and ε_{t+1} are indeed stationary, their sum must be asymptotically independent of the permanent innovations to s_{t+1} and f_t. That is, they cannot "contaminate" the cointegrating relationship and the estimate of β_1. Rejecting $\beta_1 = 1$ has a major impact on the inferred statistical properties of the risk premium, since

$$\pi_t = \mathrm{E}_t[s_{t+1}] - f_t = \beta_0 + (\beta_1 - 1) f_t + \mathrm{E}_t[u_{t+1}]$$

or equivalently,

$$\pi_t + \varepsilon_{t+1} = s_{t+1} - f_t = \beta_0 + (\beta_1 - 1)f_t + u_{t+1}. \tag{5}$$

If $\beta_1 \neq 1$ then the risk premium will be nonstationary, as it will contain a permanent component from the futures price, since ε_t is stationary by assumption.

Data

Four precious metals—gold, silver, palladium, and platinum—are chosen here, as they have a lengthy history of futures contract delivery periods from which to construct samples on a monthly frequency. Since the prices of precious metal futures are generally increasing functions of the time to maturity, it can be shown that upon maturity the benefits from holding the asset (including convenience yield and net of storage costs) are less than the risk-free rate. Therefore, it is assumed that contracts mature on the first Wednesday of the delivery month as in Pindyck (1993). A contract price may sometimes be constrained by exchange-imposed limits on daily price moves. In those cases, prices for the preceding Tuesday are used. If those prices likewise hit the limit-move, prices for the following Thursday are used, or if those are constrained, the preceding Monday and then the following Friday are chosen. In all cases, the settlement price of the gold and silver contracts traded on the Commodity Exchange Inc., as well as the palladium and platinum futures contracts traded on the New York Mercantile Exchange, are collected from the Commodity System Inc. databank and the *Wall Street Journal*.

Constant-to-maturity futures contracts of one-month maturity are collected for gold and silver, and three-month maturity are collected for palladium and platinum. That is, the futures prices $f_{t+k|t}$ are obtained for $k = 1$ (gold and silver) and 3 (palladium and platinum). Following the tradition of the literature, the spot price s_t is measured by the price of the corresponding maturing contract. Thus the spot and futures prices pertaining to exactly the same commodity and the time interval between the two delivery dates are known. The sample period is January 1975 to September 1997 for gold (273 observations), January 1970 to September 1997 for silver (333 observations), March 1977 to September 1997 for palladium (83 observations), and January 1970 to July 1997 for platinum (111 observations).[3] The price series used in this study are natural logarithmic values of the actual prices. Time series plots of the data are presented in Figure 1, while Figure 2 depicts the excess returns in the markets considered.

Cointegration Test Results

Tests of Cointegration

Cointegration tests are first conducted to assess the adequacy of the cointegrating regression model (4) over the sample period.[4] Although the maximum likelihood approach of Johansen (1991) is often used in the literature to test for cointegration, Moore and Copeland (1995) have argued that the Johansen's approach does not have any obvious advantage in the present context. So the residual-based ADF(p), Z_t and Z_a tests of Phillips and Ouliaris (1990) are carried out for each metal to test the null hypothesis of *no* cointegration. Since the primary interest is the hypothesis of cointegration, it is often argued that cointegration would be a more natural choice of the null hypothesis. Hence to test cointegration as the null hypothesis, the variable-addition $H(0, q)$ test of Park (1990) and the residual-based C_μ test of Shin (1994) are used. Compare with the Johansen's tests, the tests used here have the advantages that they allow for heterogeneously distributed, and possibly dependent, innovation sequences in the error terms.

The Akaike information criterion (AIC) is used in choosing p for the ADF(p) tests. Similar results are obtained when the Schwarz criterion (SBC) and the final prediction error (FPE) criterion are used. The "long-run" variances in the Z_t, Z_a, $H(0, q)$ and C_μ tests are computed using the

[3]The standard delivery months are February, April, June, August, October, and December for gold; January, March, May, July, September, and December for silver; March, June, September, and December for palladium; and January, April, July, and October for platinum. In addition, there are monthly supplemental contracts in gold and silver to fill in the months between the standard contracts.

[4]Prior to these cointegrating regressions, unit root and stationarity tests were conducted. The hypothesis of a unit root could not be rejected in any of the series considered.

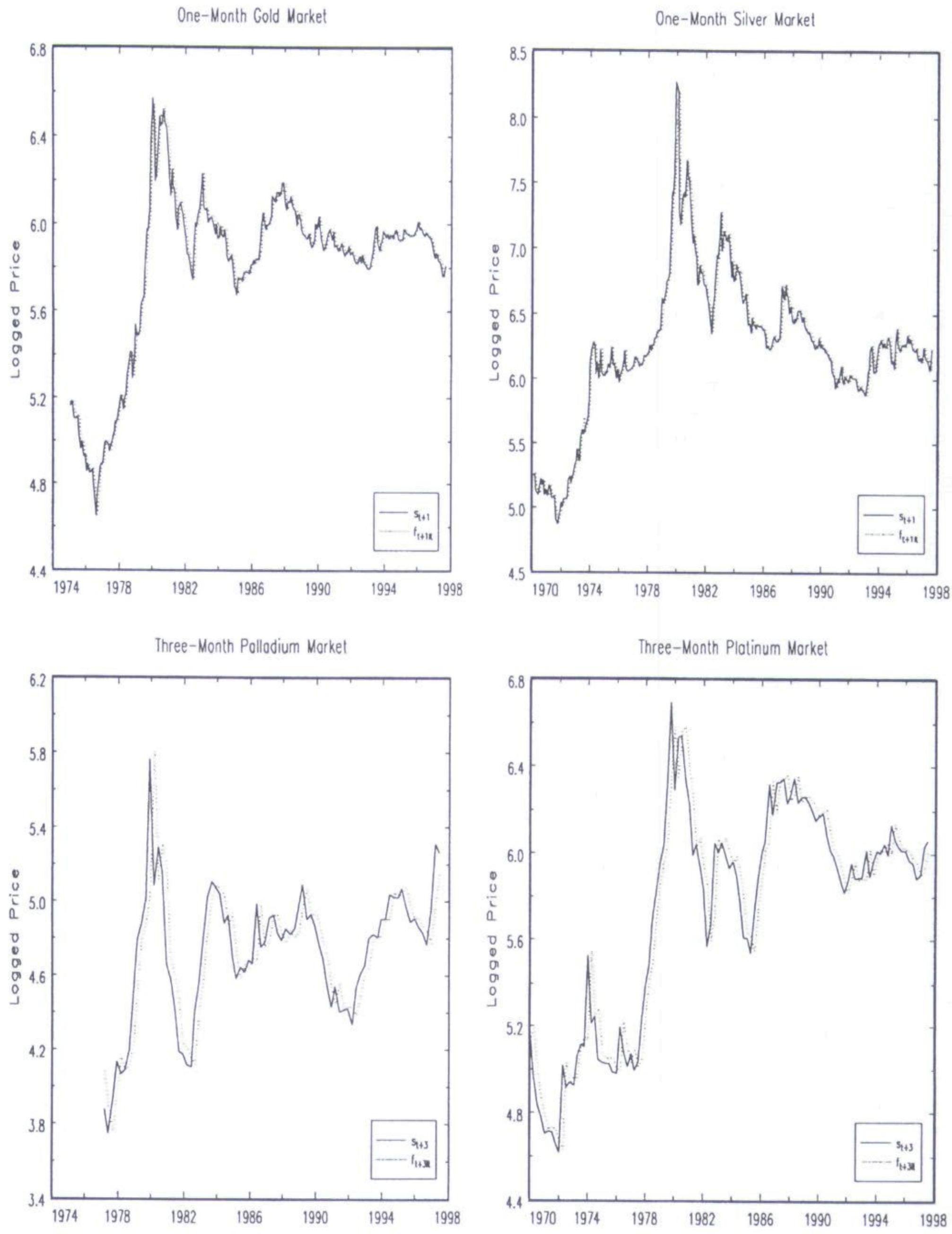

FIGURE 1
Time series plots of the data.

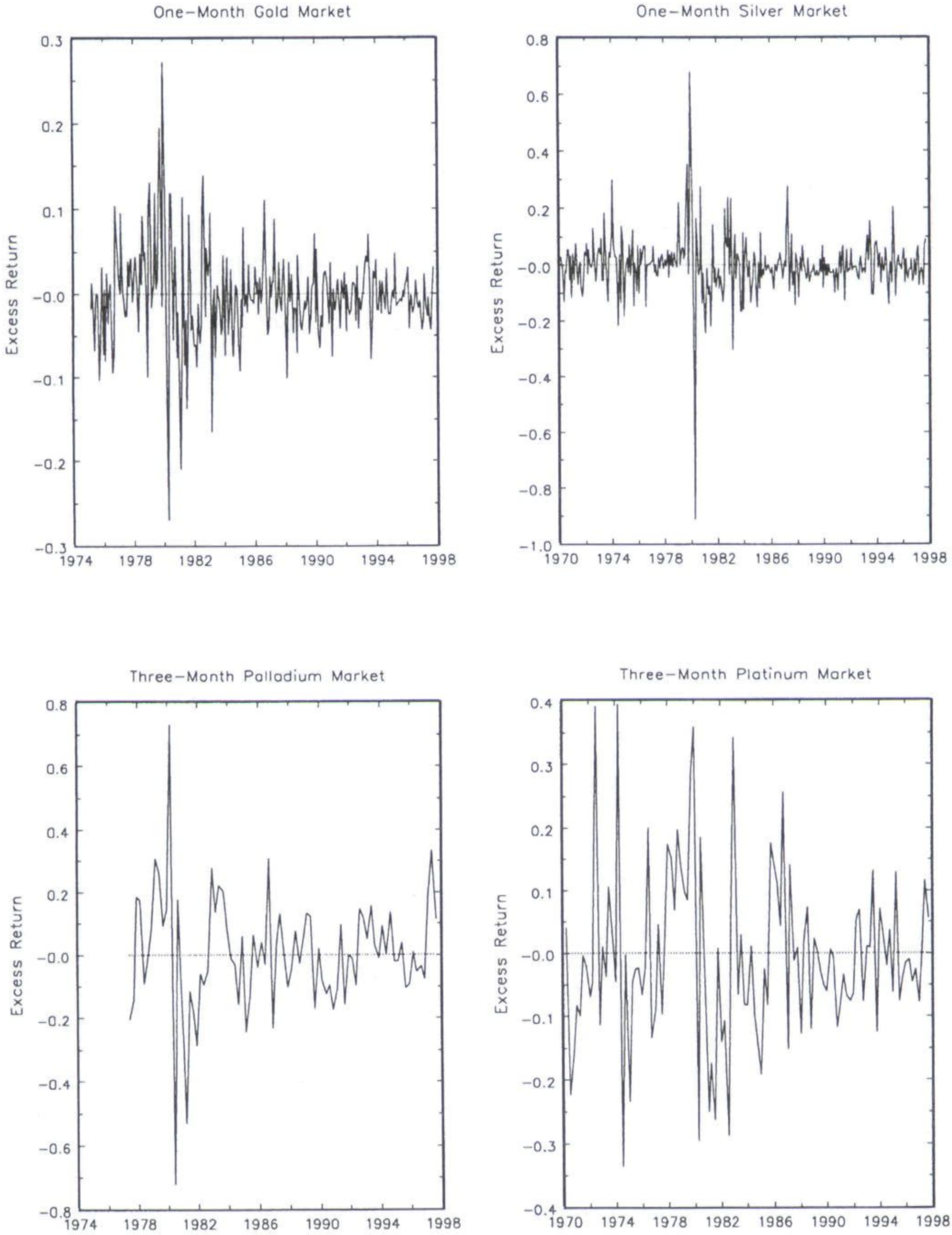

FIGURE 2

Time series plots of the excess returns.

TABLE I

Tests of Cointegration Between Spot and Futures Prices

Equation (4): $s_{t+k} = \beta_0 + \beta_1 f_{t+k|t} + u_{t+k}$

	H_0: *no* cointegration			H_0: cointegration	
Market	ADF(p)	Z_t	Z_a	$H(0, q)$	C_μ
Gold	−4.883*	−13.317*	−221.901*	1.950	2.008*
Silver	−9.388*	−15.164*	−278.264*	4.703	0.237‡
Palladium	−3.872†	−8.639*	−74.315*	1.044	0.524†
Platinum	−6.079*	−10.253*	−107.689*	2.347	0.612*

	Hansen (1992) Tests		
Market	L_C	MeanF	SupF
Gold	1.243*	65.884*	136.968*
Silver	0.614†	20.279*	56.041*
Palladium	0.682*	5.345†	7.668
Platinum	0.243	3.180	14.453†

*, † and ‡ indicate the rejection of the null hypothesis at the 1%, 5% and 10% levels. The critical values for ADF(p), Z_t, Z_a, C_μ, L_C, MeanF and SupF come from MacKinnon (1991), Shin (1994) and Hansen (1992). $H(0, q)$ has an asymptotic $\chi^2(q)$ distribution under the null hypothesis, where $q = 3$ when $k = 1$ and $q = 1$ when $k = 3$ as suggested in Han and Ogaki (1997).

quadratic spectral (QS) kernel, and the corresponding optimal bandwidths are computed by the "plug in" method as in Andrews and Monahan (1992). In particular, the prewhitened QS kernel estimator is used for Z_t and Z_a, since Maekawa (1994) shows that the prewhitened kernel estimator significantly improves the performance of these non-parametric tests. On the other hand, Shin (1994) argues that the C_μ test statistic for cointegration using a prewhitened kernel estimator is *not* consistent against the alternative of no cointegration. So the $H(0, q)$ and C_μ statistics are computed with the non-prewhitened QS kernel.

The results of the above tests are reported in the upper panel of Table I. The null hypothesis of *no* cointegration is rejected in all three tests. On the other hand, the direct tests of cointegration give conflicting results: The $H(0, q)$ statistics do not reject cointegration in all markets, while C_μ tests reject cointegration in all markets at conventional levels. With these conflicting results, it may be informative to plot the outcome of the cointegration tests as one moves progressively through the data sample. Figure 3 and Figure 4 show such recursive calculations for the residual-based ADF(p) test and the C_μ test for each metal. The recursive ADF(p) and C_μ statistics are calculated using the data-based estimators as before, so that the p and the bandwidth parameters are adjusted automatically on a period-by-period basis as the recursive calculations proceed.

Figure 3 and Figure 4 reveal some useful information about the data. The ADF(p) statistics show evidence of shifting against the existence of cointegration around 1974 for the silver market. In addition, both the ADF(p) and the C_μ statistics are quite volatile and shift against cointegration around 1979 to 1982 in all markets. Finally, the C_μ statistics seem to indicate a breakdown in the cointegration in all markets since the late 1980s. These observations coincide with the business cycle peaks, notably around 1973–1974 and 1979–1980, and the apparent shifts in the U.S. monetary regime between late 1979 and late 1982. Indeed, Fama and French (1988) have provided evidence that the metals spot and futures prices showed significant variations around the business cycle peaks. Furthermore, if interest rates do affect the cointegrating relationship as argued in Brenner and Kroner (1995), such effects should be reflected in these plots. It is noteworthy that the apparent swings in the recursive tests follow a pattern generally consistent with the recent empirical findings of regime shifts in the economy.

There is currently no theory available to allow for statistical inferences based on the recursive statistics as shown in Figure 3 and Figure 4. On the other hand, research such as Hansen (1992) has developed procedures to test parameter constancy in cointegrating regressions as a direct test of cointegration. The L_C, MeanF, and SupF tests of Hansen (1992) are performed for all markets here, with results presented in the lower panel of Table I and graphically in Figure 5.[5] Despite significant evidence against cointegration in both the gold and silver markets through the instability of the cointegrating parameter, the evidence against cointegration in other markets can vary, depending upon which statistics are chosen. The possibility of such conflicting results is discussed in Hansen (1992, p. 332): The three test statistics (L_C, MeanF, and SupF) are looking in different directions and will have more power against some alternatives than others. However, all three tests will have asymptotic power against the same set of alternatives.

[5]All the statistics are calculated using the trimming region [0.15, 0.85] as in Hansen (1992). For example, in the case of gold market, which has a sample of 273 observations (January 1975 to September 1997), the test statistics are calculated sequentially from the 41st observation (May 1978) to the 232nd observation (March 1994).

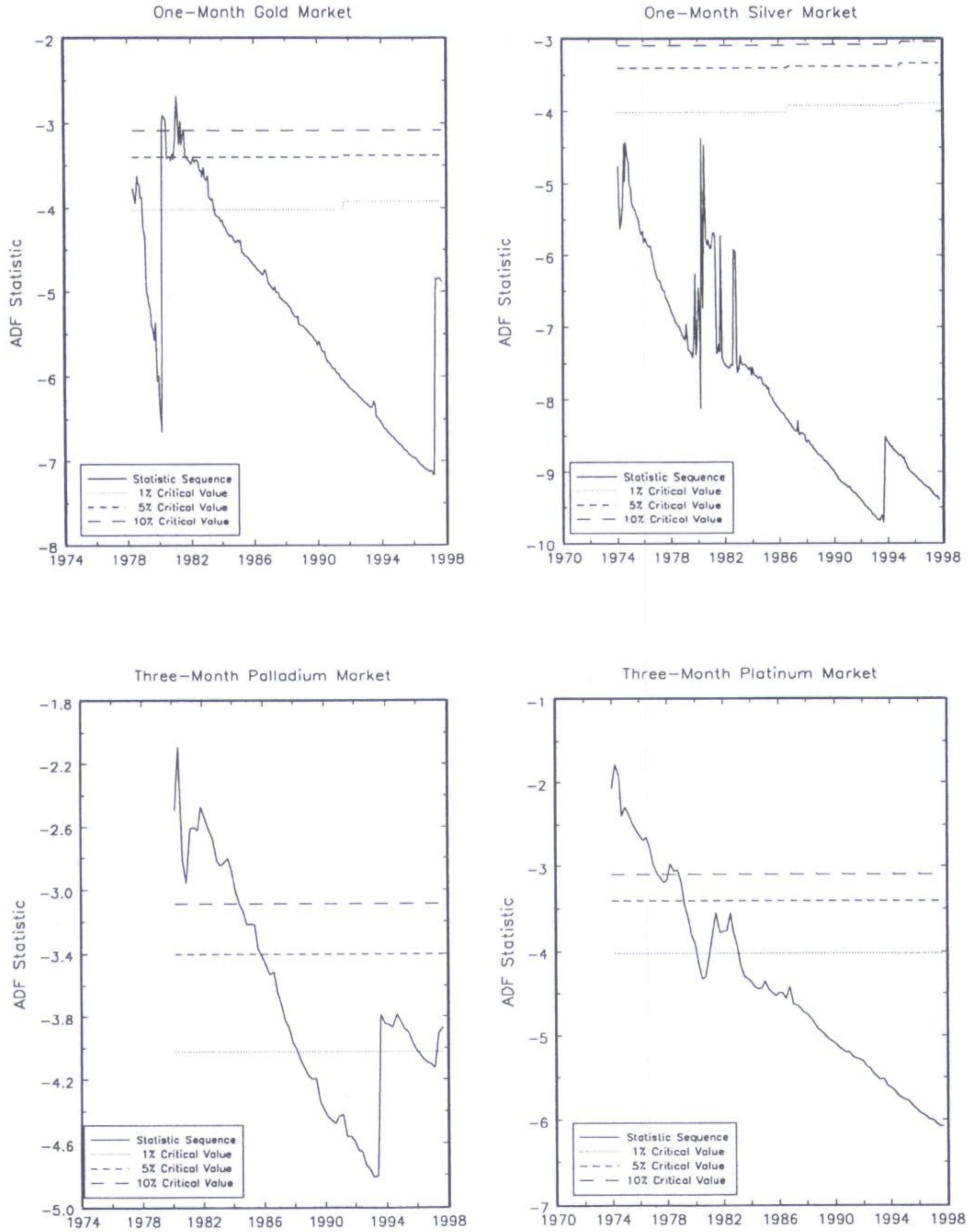

FIGURE 3
Recursive plots of residual-based ADF(p) cointegration tests.

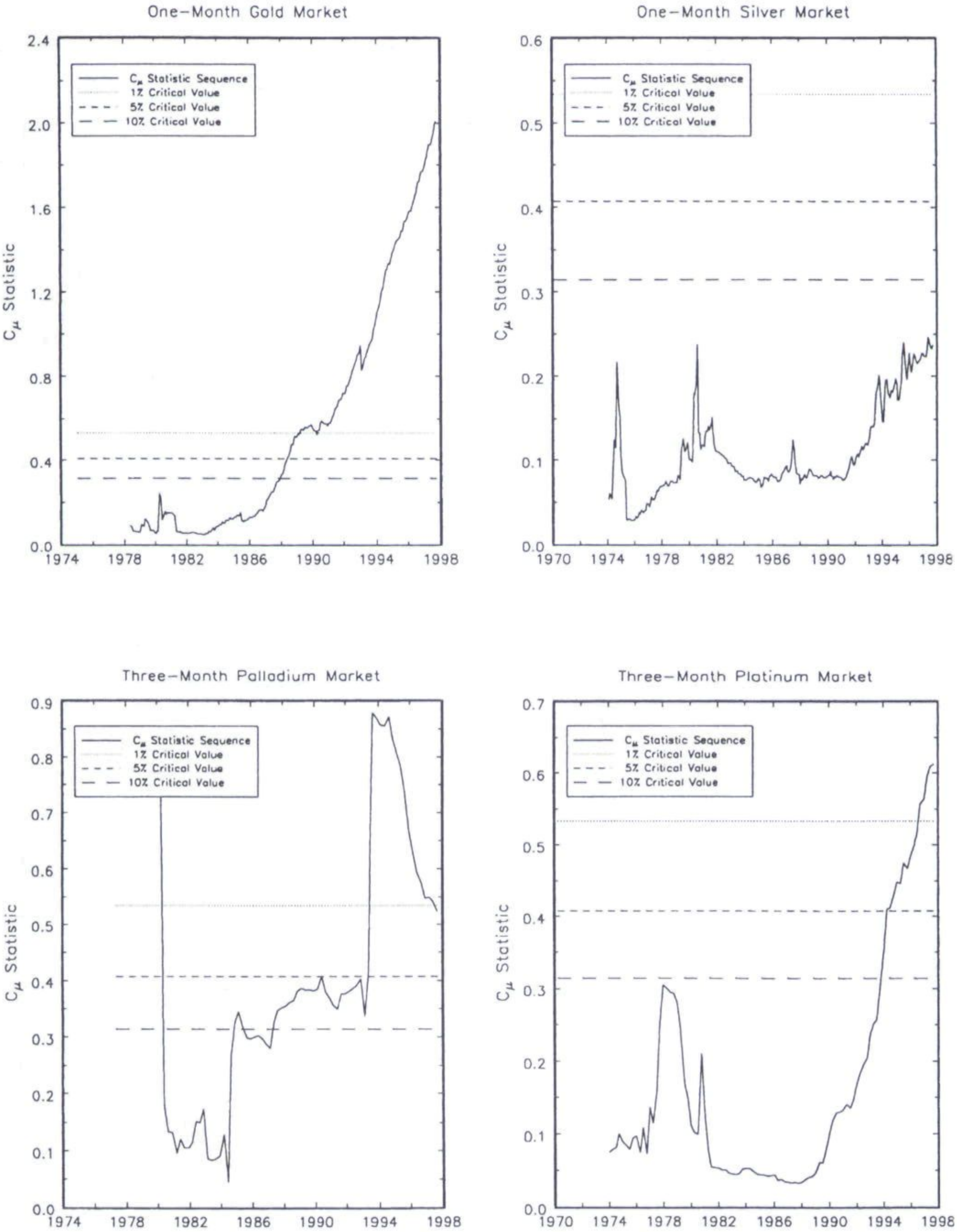

FIGURE 4
Recursive plots of residual-based C_μ cointegration tests.

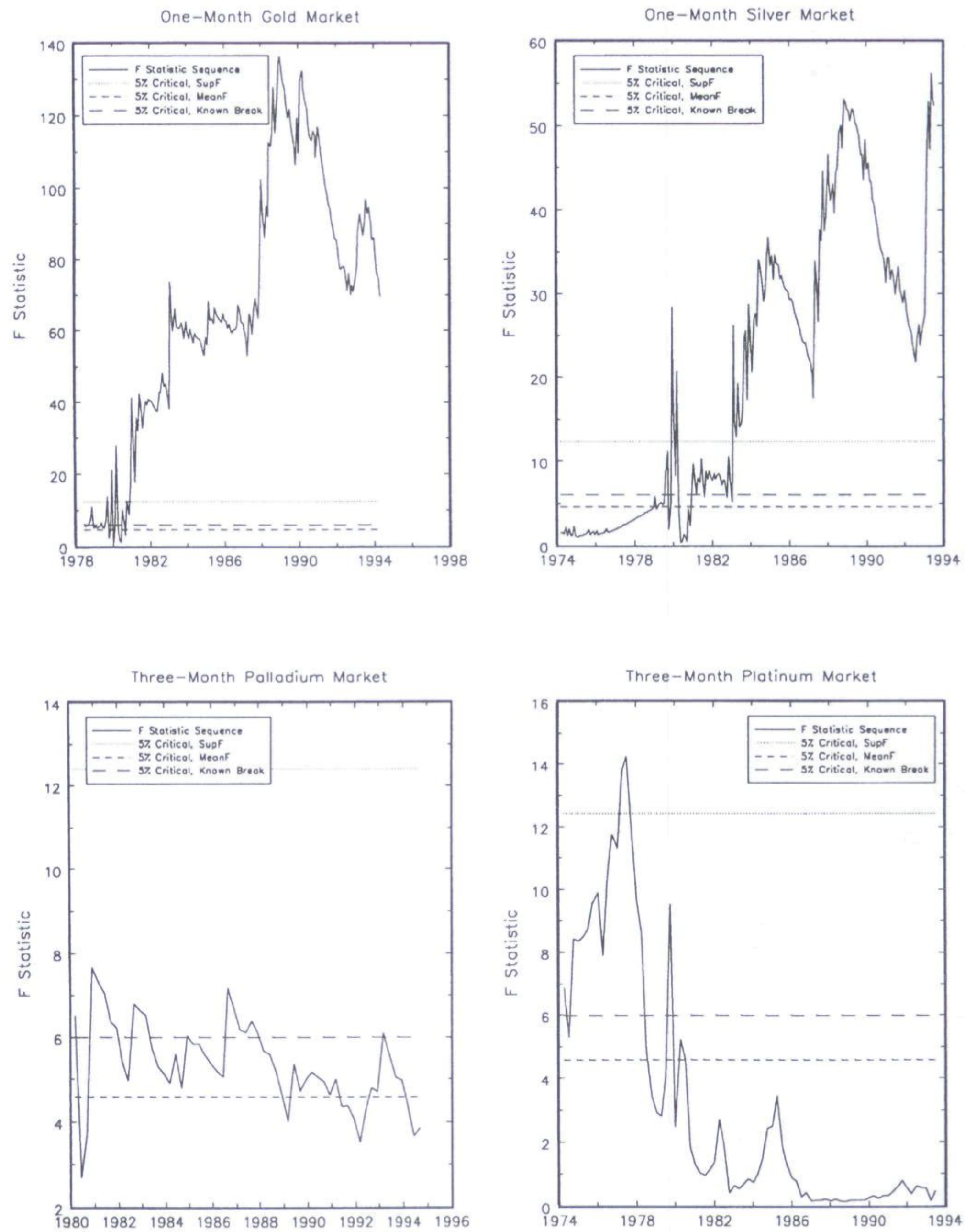

FIGURE 5

F-statistic sequence for parameter instability in cointegrating regressions.

From these results, it can be seen that the rejection of cointegration may well be due to a few one-time, but highly persistent, shocks in the system, thus "contaminating" the cointegrating relationship in a finite sample. This is consistent with much of the recent research on the impact of structural change in the economy on the unit root tests. For example, Perron (1989) has argued that a change in the economic regime will bias the unit root tests to favor the *null* hypothesis of $I(1)$. In the current context, a regime shift in the economy may bias the cointegration tests to favor the *alternative* hypothesis of *no* cointegration. It is reasonable to believe that the futures and the realized spot prices considered here are cointegrated, i.e., they do not deviate apart without bound, which is consistent with a necessary condition for the futures market to be informationally efficient: The futures price has predictive power regarding the movement of the future spot price.

Tests of the Cointegrating Parameter

To test whether there is a nonstationary risk premium component in the excess return, i.e., if the estimate of β_1 is significantly different from one, it may be useful to consider eq. (5). The motivation is as follows: If the excess returns are indeed stationary, i.e., s_{t+k} and $f_{t+k|t}$ are cointegrated with cointegrating vector $(1, -1)$, then a regression of $s_{t+k} - f_{t+k|t}$ on *any* $I(1)$ nonstationary variable, in particular $f_{t+k|t}$, should reveal a zero correlation between these variables asymptotically. A non-zero coefficient in such a regression would lead to the rejection of the stationarity of $s_{t+k} - f_{t+k|t}$.

It should be noted that the ordinary least squares (OLS) estimates of the cointegrating vector in cointegrating regressions are known to be biased in finite samples. Furthermore, the asymptotic distribution of OLS estimates is nonnormal, thus invalidating usual inferences. For these reasons and the results of Moore and Copeland (1995), two asymptotically efficient estimation procedures are used in this study, namely the canonical cointegrating regression (CCR) of Park (1992), and the super-consistent dynamic OLS (DOLS) estimator of Stock and Watson (1993).

The results are presented in Table II. All estimates are less than zero at about the 5% significance level, except in the silver markets, where the estimates are less than zero at about the 10% significance level. These results indicate that $\beta_1 < 1$ for all markets. One might conclude that, contrary to what has been implicitly or explicitly assumed in previous studies, the spot and futures prices in these markets are not cointegrated one-for-one. Since conventional assumptions require the rational expec-

TABLE II

Tests of Cointegrating Parameter between Spot and Futures Prices

Equation (5): $s_{t+k} - f_{t+k|t} = \beta_0 + (\beta_1 - 1)f_{t+k|t} + u_{t+k}$

	Estimate of $(\beta_1 - 1)$	
Market	CCR	DOLS
Gold	$-0.007^{\ddagger}$	$-0.003^{\dagger}$
	(0.063)	(0.020)
Silver	$-0.002^{\ddagger}$	$-0.001^{\ddagger}$
	(0.070)	(0.068)
Palladium	$-0.039^{\dagger}$	$-0.038^{\dagger}$
	(0.016)	(0.037)
Platinum	$-0.026^{\dagger}$	-0.010^{*}
	(0.043)	(0.003)

*, † and ‡ indicate the rejection of the null hypothesis that $\beta_1 - 1 = 0$ at the 1%, 5% and 10% levels. The p-values (with finite-sample corrections) are reported in parentheses.

tations forecast errors to be stationary and uncorrelated with all current information, one interpretation is that the risk premium is subject to permanent shocks and has the same level of persistence as the futures price. However, as shown in the following section, there is another interpretation of these results based upon the presence of switches in the spot price process.

A SWITCHING MODEL OF THE SPOT PRICES

The Model

This section presents a simple switching model for the spot price similar to that in Evans and Lewis (1995). Although the aim is not to develop a fundamental-based model for the commodity price as in, say, Pindyck (1993), such a framework can be used to consider the general origins and consequences of commodity price switching. Again, for the purpose of exposition, consider the case of $k = 1$.

Assume that the commodity price depends upon its current and discounted expected future payoffs, or benefits, from ownership of the asset:

$$s_t = \frac{E_t[s_{t+1}] + \psi_t}{1 + \delta} = \rho E_t[s_{t+1}] + x_t \tag{6}$$

where s_t is the spot price, ψ_t is a composite effect of variables that affect the commodity price such as convenience yield and/or interest rate, δ is the commodity-specific one-period discount rate (i.e., the expected rate of return an investor would require to hold a unit of the commodity), $\rho \equiv (1 + \delta)^{-1}$ and $x_t \equiv \rho\psi_t$.

While the general business conditions can be classified into many different states, we shall assume that x_t follows two different processes, namely the "low convenience" state and the "high convenience" state. Switches between these processes are governed by a state variable z_t that takes on a value of either zero ("low convenience") or one ("high convenience"), and realizations of x_t from the process in state z are denoted by $x_t(z)$. The state variable z_t is assumed to follow an independent first-order Markov process with transition probabilities $p_i = \Pr(z_{t+1} = i \mid z_t = i)$ for $i = 0, 1$. It should be stressed that although the switching behavior is not modeled endogenously, switches in the x_t process can be thought of as reflecting such influences as the phases in business cycles or shifts in the behavior of monetary regimes.

Under rational expectations, the spot price must satisfy eq. (6) regardless of the current process generating x_t. That is, the expectation of the next period's spot price, $E_t[s_{t+1}]$, will depend upon the probability weighted average of each state next period. Algebraically, suppose that x_t is currently in the state $z_t = i$ at time t, then

$$E_t[s_{t+1}] = p_i \, E_{t|i}[s_{t+1}(i)] + (1 - p_i) \, E_{t|i}[s_{t+1}(j)]$$

for $j \neq i$, where $E_{t|z_t}[s_{t+1}(z)]$ is the time t conditional expectation of the spot price at $t + 1$. Note that while z_t is assumed to be in the information set at time t, it is made explicit in the expectation operator to emphasize the switching nature of the underlying process. Substituting these expectations into eq. (6) gives

$$\begin{pmatrix} s_t(0) \\ s_t(1) \end{pmatrix} = \rho \begin{pmatrix} p_0 & 1 - p_0 \\ 1 - p_0 & p_1 \end{pmatrix} \begin{pmatrix} E_{t|z_t}[s_{t+1}(0)] \\ E_{t|z_t}[s_{t+1}(1)] \end{pmatrix} + \begin{pmatrix} x_t(0) \\ x_t(1) \end{pmatrix}.$$

More compactly, in matrix form we have

$$S_t = \rho P \, E_{t|z_t}[S_{t+1}] + X_t \qquad (7)$$

where P is the transition probability matrix, S_t, $E_{t|z_t}[S_{t+1}]$ and X_t are the vector of state-contingent spot prices, expected future spot prices and variables that influence the spot prices respectively.

Eq. (7) describes the dynamics of the spot price under different processes of x_t. It is obvious that the current spot price $s_t(z)$ depends on the weighted average of expected future spot prices from both regimes and x_t

realized from the given state, $x_t(z)$. As time progresses, the x_t process will switch and the observed spot price will be equal to either $s_t(1)$ or $s_t(0)$, depending on the value of the state variable z_t:

$$s_t = (1 - z_t)s_t(0) + z_t s_t(1). \tag{8}$$

Eqs. (7) and (8) provide a tractable framework for examining how switches in x_t affect spot prices. However, to examine how switching in x_t affects the spot price in detail, it is useful to focus on an example in which x_t switches between simple processes. For this purpose, consider the case where each of the x_t processes follows a random walk with drift, as in Evans and Lewis (1995).[6] That is, the vector $X_t \equiv (x_t(0), x_t(1))$ follows

$$X_t = A + X_{t-1} + \eta_t$$

where A and η_t are 2×1 vectors of drift coefficients and innovations respectively, with $E_{t-1}[\eta_t] = 0$. With this specification, eq. (7) can be solved for S_t, giving

$$S_t = (I_2 - \rho P)^{-1} \rho P(I_2 - \rho P)^{-1} A + (I_2 - \rho P)^{-1} X_t \tag{9}$$

where I_2 is a 2×2 identity matrix. Eq. (9) shows that spot prices depend upon the discounted present value of the probability-weighted averages of drift terms in all future periods, and the present value of the current level of x_t. Observe that even if current x_t are the same across states, i.e., $x_t(0) = x_t(1)$, the spot prices in each state will not be equal. That is, $s_t(0)$ will not equal $s_t(1)$ because the expected future evolution between the two x_t processes will differ across states as reflected by $(I_2 - \rho P)^{-1}$.

Taking the first difference of eq. (9) highlights the dynamics of the spot price within a regime:

$$\Delta S_{t+1} = (I_2 - \rho P)^{-1} A + (I_2 - \rho P)^{-1} \eta_{t+1}. \tag{10}$$

Changes in the spot prices are equal to the change in x_t weighted by their discounted present value effect in all future periods, including the transition probabilities of switching between states as summarized in P.

The effects of the "peso problem" will be apparent when comparing the spot price process in eq. (9) to the conventional present value solution. Note that eq. (7) can be written as

[6]While x_t can be modeled to switch between processes with more general time series properties, Evans and Lewis (1995) show that the more general solution for the spot price shares similar features to the simple model presented here.

$$\begin{aligned} S_t &= \rho E_{t|z_t}[S_{t+1}] + X_t + \rho(P - I_2)\, E_{t|z_t}[S_{t+1}] \\ &= \rho E_{t|z_t}[S_{t+1}] + X_t + \rho \nabla_{z_t} S_t. \end{aligned} \tag{11}$$

The first two terms on the right side of eq. (11) correspond to the standard spot price equation for each state in eq. (6). The last term,

$$\nabla_{z_t} S_t = \begin{pmatrix} p_0 & - & 1 \\ 1 & - & p_1 \end{pmatrix} E_{t|z_t}[s_{t+1}(0) - s_{t+1}(1)],$$

is an expected "jump" in the spot price resulting from switches in the x_t process. To see the implications of this term within a regime, eq. (11) is iterated forward to give

$$S_t = E_{t|z_t}\left[\sum_{j=0}^{\infty} \rho^j \,(X_{t+j} + \rho \nabla_{z_{t+j}} S_{t+j})\right]. \tag{12}$$

Eq. (12) shows that the spot price $s_t(z)$ is equal to the present value of x_t in the current process z, and the present value of expected future changes in the spot price caused by shifts in this process. This equality means that two different factors affect spot prices. First, innovations to the x_t process affect the present value of future x_t. Second, new information affects the expected future size of the jump in spot prices when switches in the process occur. In the solution for the spot price in eq. (10), these effects are identified by the diagonal elements of $(I_2 - \rho P)^{-1}$.

Estimation Results and Specification Tests

It has been shown that shifts in the behavior of the underlying determinants of the spot price will lead to accompanying jumps in the spot price. These jumps need to be taken into consideration in estimating the switching model. Combining the process for the spot price in eq. (8) with the solution in eq. (10), the equilibrium process of the spot price can be written in a non-stationary state-space model with the state equation

$$\begin{aligned} s_{t+1} &= (1 - z_{t+1}\; z_{t+1}) \begin{pmatrix} s_{t+1}(0) \\ s_{t+1}(1) \end{pmatrix} \\ &= (1 - z_{t+1}) s_{t+1}(0) + z_{t+1} s_{t+1}(1), \end{aligned} \tag{13}$$

and the transition equation

$$\begin{pmatrix} s_{t+1}(0) \\ s_{t+1}(1) \end{pmatrix} = \begin{pmatrix} \mu_0 \\ \mu_1 \end{pmatrix} + \begin{pmatrix} s_t(0) \\ s_t(1) \end{pmatrix} + \begin{pmatrix} v_{t+1}(0) \\ v_{t+1}(1) \end{pmatrix}. \tag{14}$$

Note that the drift parameters μ_i and the innovations $v_t(z)$ depend on the underlying drift and innovations to the x_t processes in eq. (10).

The specification in eqs. (13) and (14) differs from usual switching models by allowing the spot price to jump when there is a switch in regime, i.e., when $\Delta z_t \neq 0$. Specifically, eq. (13) can be expressed as

$$\begin{aligned} \Delta s_{t+1} &= (1 - z_{t+1})\Delta s_{t+1}(0) + z_{t+1}\Delta s_{t+1}(1) \\ &\quad - \Delta z_{t+1}(s_t(0) - s_t(1)). \end{aligned} \tag{15}$$

The first two terms on the right side represent the dynamics of the spot price in respective regimes, while the last term indicates a jump from one process to another when there is a switch in regime.

The information in the futures price can be used to identify the jumps in the spot price associated with a change in regime. Calculating the expected future spot price from eq. (15), one can substitute for $E_t[s_{t+1}]$ using equation (1), i.e., $E_t[s_{t+1}] = f_t + \pi_t$, and rearrange the resulting expressions to obtain

$$s_t(0) - s_t(1) = \Gamma(z_t)^{-1} (f_t - s_t + \pi_t - \Psi(z_t)), \tag{16}$$

where $\Gamma(0) = p_0 - 1$, $\Gamma(1) = 1 - p_1$, $\Psi(0) = p_0\mu_0 + (1 - p_0)\mu_1$, and $\Psi(1) = (1 - p_1)\mu_0 + p_1\mu_1$.

Eq. (16) implies that the size of the jump varies in response to changes in both the basis ($f_t - s_t$) and the risk premium (π_t). Thus, given an assumption about the behavior of the risk premium, the basis together with the parameters of the model can be used to identify $s_t(0) - s_t(1)$. While the risk premium may be time varying within regimes, we only need to know the risk premium when a change in regime occurs. That is, we only need to know the values of π_t when $\Delta z_t \neq 0$, since all other variables in eq. (15) are determined by the spot price switching model. Therefore, the simple identifying assumption that the risk premium $\pi_t = \pi(0)$ during switches from regime 0 to 1, and $\pi_t = \pi(1)$ during switches from regime 1 to 0, will be made for estimation purposes.

The proposed model is estimated using the data described earlier. In order to express the results in units of percentage change, the data are first differenced and multiplied by 100. Table III shows the estimates of the model given by eqs. (15) and (16). The rows p_0 and p_1 show the estimates of the transition probabilities of staying in each regime. The

TABLE III

Maximum Likelihood Estimates of the Switching Model

$$s_t = (1 - z_t)s_t(0) + z_t s_t(1)$$

$$p_i = \Pr(z_{t+1} = i|z_t = i), \quad i = 0, 1$$
$$s_t(i) = \mu_i + s_{t-1}(i) + \eta_t(i), \quad i = 0, 1$$

$$\eta_t(i) \sim N(0, \sigma_i^2), \quad i = 0, 1$$

$$f_{t+k|t} = E_{t|z_t}[s_{t+k}] - (1 - z_t)\pi(0) - z_t\pi(1)$$

	$k = 1$		$k = 3$	
Parameter	Gold	Silver	Palladium	Platinum
p_0	0.965	0.947	0.895	0.935
	(0.003)	(0.034)	(0.126)	(0.009)
p_1	0.904	0.801	0.869	0.814
	(0.010)	(0.021)	(0.012)	(0.022)
μ_0	−1.535	−1.819	−3.084	−5.157
	(0.697)	(0.889)	(1.532)	(1.978)
μ_1	1.522	2.203	3.978	7.701
	(0.650)	(1.247)	(1.736)	(3.038)
σ_0^2	3.104	2.859	12.959	8.116
	(1.450)	(0.742)	(5.370)	(3.869)
σ_1^2	5.417	8.468	13.734	11.043
	(1.857)	(2.751)	(7.227)	(4.182)
$\pi(0)$	−0.606	−0.400	−2.458	−1.084
	(0.285)	(0.226)	(1.031)	(1.036)
$\pi(1)$	0.607	0.488	2.583	1.759
	(0.225)	(0.466)	(2.066)	(1.381)

The asymptotic standard errors are reported in parentheses.

point estimates are all above 0.80. These estimates indicate that if the spot price is in either state, it is also likely to stay in that state in the next period.

The rows μ_0 and μ_1 report the estimates of the drift term in each state. All of the μ_0 estimates are negative and all of the μ_1 estimates are positive, reflecting the "low convenience" and "high convenience" states respectively. That is, the maximum likelihood estimates associate state 0 with 1.5% and 1.8% monthly decreases in the spot gold and silver prices,

TABLE IV

Some Diagnostic Tests of the Switching Model

$$s_t = (1 - z_t)s_t(0) + z_t s_t(1)$$

$$p_i = \Pr(z_{t+1} = i|z_t = i), \quad i = 0, 1$$

$$s_t(i) = \mu_i + s_{t-1}(i) + \eta_t(i), \quad i = 0, 1$$

$$\eta_t(i) \sim N(0, \sigma_i^2), \quad i = 0, 1$$

$$f_{t+k|t} = E_{t|z_t}[s_{t+k}] - (1 - z_t)\pi(0) - z_t\pi(1)$$

Test Statistic	$k = 1$ Gold	$k = 1$ Silver	$k = 3$ Palladium	$k = 3$ Platinum
LM_1	0.334	0.001	1.626	0.332
	(0.563)	(0.980)	(0.202)	(0.564)
LM_2	2.079	2.524	2.154	0.836
	(0.149)	(0.112)	(0.142)	(0.360)
LM_3	0.756	1.395	0.695	2.212
	(0.384)	(0.237)	(0.404)	(0.136)
LM_4	1.649	1.322	1.359	1.562
	(0.199)	(0.250)	(0.243)	(0.211)
LM_5	0.184	0.876	0.811	0.834
	(0.667)	(0.349)	(0.367)	(0.361)
LR_1	3.390	5.598	3.757	3.169
	(0.066)	(0.018)	(0.053)	(0.075)
LR_2	4.512	4.122	3.275	3.117
	(0.034)	(0.042)	(0.070)	(0.077)

The asymptotic p-values are shown below the test statistics. LM_1 and LM_2 are the tests of no serial correlation in the errors $\eta_t(i)$, while LM_3 and LM_4 are the tests of no first-order ARCH in $\eta_t(i)$, for states 0 and 1, respectively. LM_5 is the test of the cross-equation restrictions as in equation (17). LR_1 is the test for Markov independence ($p_0 = 1 - p_1$), and LR_2 is the test for $\mu_0 = \mu_1$ allowing $\sigma_0 \neq \sigma_1$.

and 3.1% and 5.1% quarterly decreases in the spot palladium and platinum prices. In state 1, the spot prices rise by 1.5% and 2.2% per month for gold and silver, and 4.0% and 7.7% per quarter for palladium and platinum.

The estimates of the variance in each state are reported in rows σ_0^2 and σ_1^2. It is apparent that, in all markets, the variance of the "high convenience" state is larger than that of the "low convenience" state. While these variances are not compared with the variances of the corresponding futures prices, such a pattern is consistent with the arguments in Fama and French (1988): When the level of inventory is low, i.e., when the convenience yield is high, demand and supply shocks will cause spot prices to be more variable.

The last two rows report the estimates of the risk premium during the period when the spot price process switches from one regime to another. The estimates of $\pi(0)$ are all negative while the estimates of $\pi(1)$ are all positive. These estimates provide an intuitive interpretation to eq. (1), although some of them are associated with high standard errors. During periods before a switch to a "high convenience" state, traders selling the commodity forward would require a higher futures price relative to $E_t[s_{t+k}]$ in order to be compensated for the risk of an upward jump in the spot price, hence a negative $\pi(0)$. Similarly, during periods before a switch to a "low convenience" state, traders willing to buy the commodity forward would require a lower futures price relative to $E_t[s_{t+k}]$ to compensate for the risk that the future spot price may fall, thus a positive $\pi(1)$.

Table IV reports the results of a series of diagnostic tests on the specification of the above model. LM_1 and LM_2 report the Lagrange Multiplier (LM) tests for first-order serial correlation in states 0 and 1 respectively. These statistics have an asymptotic distribution of $\chi^2(1)$ under the null hypothesis of no serial correlation. From the reported p-values, none of these statistics is significant at conventional levels. The rows with LM_3 and LM_4 report the LM tests for the exclusion of a first-order autoregressive conditional heteroskedastic (ARCH) process in states 0 and 1 respectively. The asymptotic distribution is also $\chi^2(1)$ and again none of the statistics is significant.

Note that the model places restrictions on the relationship between the futures price and the future spot price when there are changes in regime. This relationship can be seen by substituting for $s_t(0) - s_t(1)$ in eq. (15) with (16) to obtain

$$\Delta s_{t+1} = \begin{cases} \alpha_0(f_t - s_t + \pi(0) - p_0\mu_0) + v_{t+1}(1) & \text{when } \Delta z_{t+1} = 1 \\ \alpha_1(f_t - s_t + \pi(1) - p_1\mu_1) + v_{t+1}(0) & \text{when } \Delta z_{t+1} = -1 \end{cases} \tag{17}$$

where $\alpha_i = (1 - p_i)^{-1}$ under the null hypothesis. Since $E_t[\Delta s_{t+1}] = f_t + \pi_t - s_t$, one can therefore examine whether the expectations implicit in futures prices are consistent with the Markov switching process estimated for the spot prices. The LM test for this restriction is reported as LM_5 in Table IV. Again, none of the statistics is significant at conventional levels. Therefore, the hypothesis that market participants rationally included the effects of changes in regime when forecasting future spot prices cannot be rejected.

As Engle and Hamilton (1990) and others have noted, testing whether a Markov switching model includes an appropriate number of states poses some difficult econometric problems. Rather than directly testing whether specifications with one or three states are preferable to the above two-state model, two related hypotheses that can be tested without running into these problems are considered.[7] First, consider the test of whether the distribution of z_t is independent of z_{t-1}, i.e., the probability that $z_t = 0$ is p_0 regardless of whether $z_{t-1} = 0$ or 1. As the LR_1 row in Table IV shows, this hypothesis can be rejected at conventional levels for all markets. Second, the null hypothesis of $\mu_0 = \mu_1$ allowing for $\sigma_0 \neq \sigma_1$ is tested. The results of LR_2 row show that this hypothesis is again rejected at conventional levels for all markets.

Finally, to show some of the implications of the estimated model graphically, the smoothed probabilities of regime 1 (the "high convenience" state) for all markets are shown in Figure 6. It is worth noting that these probabilities correspond to the dates of business cycle peaks (1973–1974 and 1979–1980) and the changes in the U.S. monetary regime (1979–1982) very closely. Again, this result is consistent with the implications of Fama and French (1988).

REGIME SWITCHING AND COINTEGRATING REGRESSIONS

The results in the second section provide some indication that the futures and realized spot prices are not cointegrated one-for-one in the precious metals markets, implying that the shocks to excess returns appear to have the same degree of persistence as the futures prices. Since conventional assumptions require rational forecast errors to be stationary, one interpretation of this finding is that the innovations in the risk premium have close to unit root persistence.

The regime switching framework, however, provides an alternative explanation. In particular, while rationality by market participants implies that the persistence in the forecast errors caused by the previously discussed "peso problem" should disappear over long samples, the forecast

[7] Engle and Hamilton (1990) argue that these tests are more general and avoid a singular information matrix under the null. Both hypotheses are tested using a likelihood ratio test, since Gallant (1987, p. 219) argues that the likelihood ratio test is apt to be more robust than Wald tests in a nonlinear model.

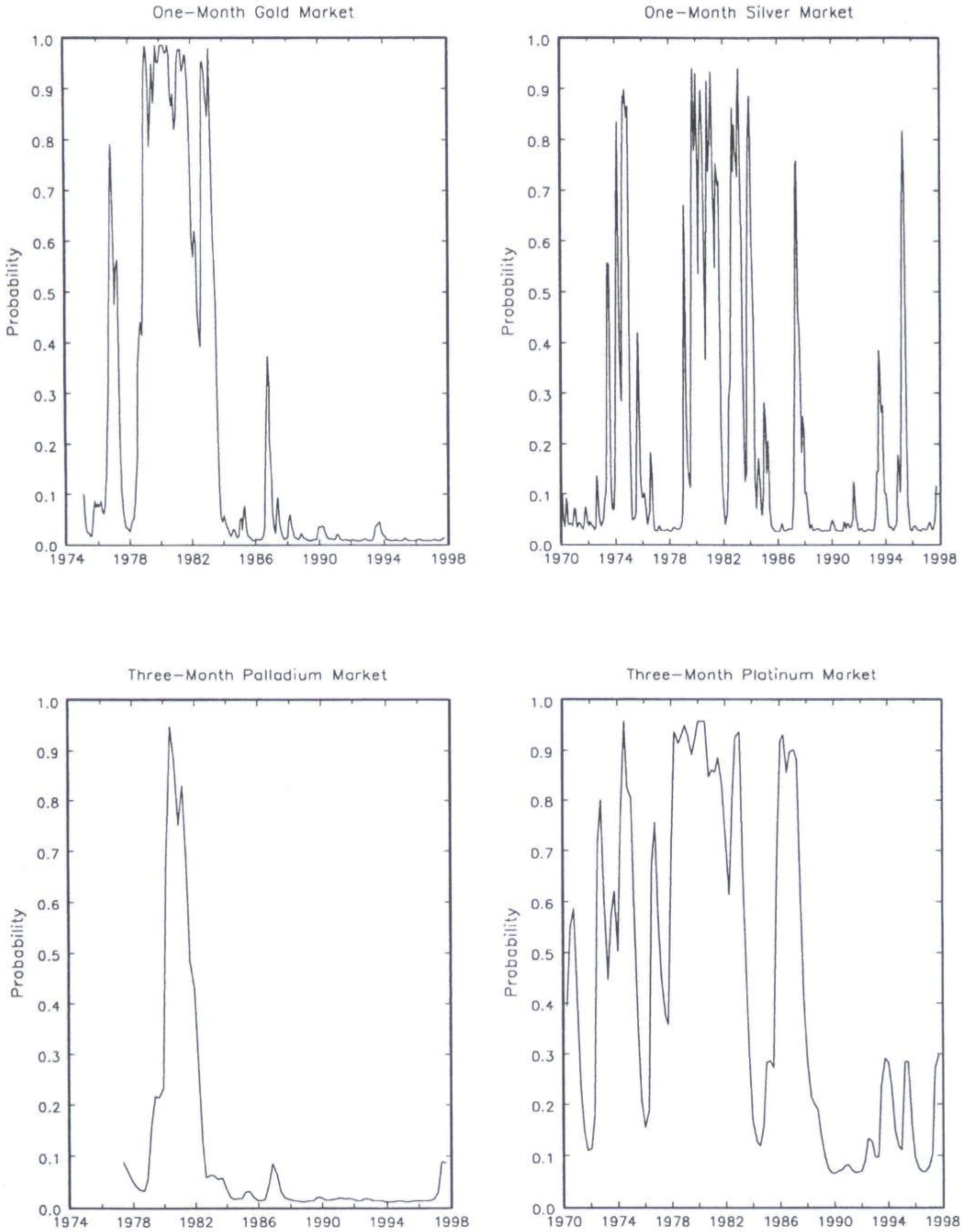

FIGURE 6
Probabilities of the "high convenience" state.

errors during samples with few regime changes may be sufficiently persistent to account for the noncointegration results above. To see this more clearly, note that the forecast errors are the differences between the realized spot price and the expected spot price, i.e., $\varepsilon_{t+1} \equiv s_{t+1} - \mathrm{E}_t[s_{t+1}]$. In the regime switching framework considered above, this can be expressed as

$$\begin{aligned}\varepsilon_{t+1} &= (1 - z_{t+1}\, z_{t+1})\,(S_{t+1} - P\mathrm{E}_{t|z_t}[S_{t+1}])\\ &= (1 - z_{t+1}\, z_{t+1})\,(S_{t+1} - \mathrm{E}_{t|z_t}[S_{t+1}]) + (I_2 - P)\,\mathrm{E}_{t|z_t}[S_{t+1}])\\ &= (1 - z_{t+1}\, z_{t+1})\,(S_{t+1} - \mathrm{E}_{t|z_t}[S_{t+1}]) - (1 - z_{t+1}\, z_{t+1})\,\nabla_{z_t} S_t.\end{aligned}$$

The first term on the right side is the *within regime* forecast error, $s_{t+1}(z) - \mathrm{E}_t[s_{t+1}(z)]$, which is uncorrelated with time t information. The second term is the time t expectation of the jump in the spot price that would accompany a change in regime. This term is the source of the "peso problem" in the forecast error. In small samples, the expected jump will generally be correlated with information in the time t information set, including past forecast errors.

In the context of the switching model (15), the forecast error is

$$\begin{aligned}s_{t+1} - \mathrm{E}_t[s_{t+1}] &= (1 - z_{t+1})v_{t+1}(0) + z_{t+1}v_{t+1}(1)\\ &\qquad - (\Delta z_{t+1} + \Gamma(z_t))\mathrm{E}_{t|z_t}[s_{t+1}(0) - s_{t+1}(1)].\end{aligned}$$

Observe that even in the periods when there is no change in regime, i.e., $\Delta z_{t+1} = 0$, the forecast error will include the "peso problem" component $-\Gamma(z_t)\,\mathrm{E}_{t|z_t}[s_{t+1}(0) - s_{t+1}(1)]$. This component will be serially correlated as long as $p_i < 1$. Over sufficiently long periods, however, the sample will include many regime changes so that the errors in forecasting changes in the regime will be serially uncorrelated. This can be seen from the fact that $\mathrm{E}_{t|z_t}[\Delta z_{t+1}] = -\Gamma(z_t)$, so the "peso problem" term $(\Delta z_{t+1} + \Gamma(z_t))$ $\mathrm{E}_{t|z_t}$ $[s_{t+1}(0) - s_{t+1}(1)]$ has a zero unconditional mean and is serially uncorrelated in large samples.

To examine whether such a "peso problem" in the forecast residuals could have affected the cointegration results in the second section, a series of Monte Carlo experiments are carried out. In the first set of experiments, the estimates of the above switching model are used to investigate the impact on the cointegration tests in the cointegrating regression (4) for all markets. First, a sequence of $s_t(z)$ equal to the length of the corresponding sample is generated based on eq. (13) using the estimates of the drift term (μ_i) and the variance (σ_i^2) in each state. Realizations for z_t are then drawn based upon the estimated transition prob-

TABLE V

Monte Carlo Cointegrating Regressions (I)

Equation (4): $s_{t+k} = \beta_0 + \beta_1 f_{t+k|t} + u_{t+k}$

Regime switching model

Market	C_μ	Monte Carlo p-value	Monte Carlo 5%
Gold	2.008*	0.149	0.434
Silver	0.237‡	0.192	0.288
Palladium	0.524†	0.132	0.450
Platinum	0.612*	0.169	0.384

Conditional homoscedastic model

Market	C_μ	Monte Carlo p-value	Monte Carlo 5%
Gold	2.008*	0.004	0.063
Silver	0.237‡	< 0.001	0.052
Palladium	0.524†	0.055	0.072
Platinum	0.612*	< 0.001	0.056

Conditional heteroscedastic model

Market	C_μ	Monte Carlo p-value	Monte Carlo 5%
Gold	2.008*	0.006	0.037
Silver	0.237‡	< 0.001	0.072
Palladium	0.524†	0.069	0.077
Platinum	0.612*	0.006	0.053

The C_μ column reports the respective statistics in the regression (4) as in Table I. The column of p-value gives the corresponding p-values based on the empirical distribution calculated from the Monte Carlo simulations, while the last column gives the estimates of the empirical size of the nominal 5% test. All simulation results are based on 10,000 replications.

ability matrix and then used to generate a series for the observed spot price process. The futures prices are constructed from these series using eq. (16) under the assumption that the risk premium is equal to the estimates of $\pi(z_t)$ within each state. The cointegrating regression (4) is then run with the generated spot and futures prices with the C_μ statistic and estimates of β_1 saved. This whole procedure is repeated 10,000 times to generate an empirical distribution and the results are reported in the top panel of Table V and Table VI.

It is obvious from Table V that the C_μ statistic suffers from severe size distortions with a regime switching model. At the nominal 5% level, the probability of rejecting the null hypothesis of cointegration ranges

TABLE VI

Monte Carlo Cointegrating Regressions (II)

Equation (5): $s_{t+k} - f_{t+k|t} = \beta_0 + (\beta_1 - 1) f_{t+k|t} + u_{t+k}$

Regime switching model

Market	CCR	F.S.C. p-value	M.C. p-value	DOLS	F.S.C. p-value	M.C. p-value
Gold	$-0.007^{\ddagger}$	0.063	0.370	$-0.003^{\dagger}$	0.020	0.326
Silver	$-0.002^{\ddagger}$	0.070	0.320	$-0.001^{\ddagger}$	0.068	0.247
Palladium	$-0.039^{\dagger}$	0.016	0.350	$-0.038^{\dagger}$	0.037	0.200
Platinum	$-0.026^{\dagger}$	0.043	0.311	-0.010^{*}	0.003	0.181

Conditional homoscedastic model

Market	CCR	F.S.C. p-value	M.C. p-value	DOLS	F.S.C. p-value	M.C. p-value
Gold	$-0.007^{\ddagger}$	0.063	0.093	$-0.003^{\dagger}$	0.020	0.072
Silver	$-0.002^{\ddagger}$	0.070	< 0.001	$-0.001^{\ddagger}$	0.068	< 0.001
Palladium	$-0.039^{\dagger}$	0.016	< 0.001	$-0.038^{\dagger}$	0.037	0.005
Platinum	$-0.026^{\dagger}$	0.043	< 0.001	-0.010^{*}	0.003	0.014

Conditional heteroscedastic model

Market	CCR	F.S.C. p-value	M.C. p-value	DOLS	F.S.C. p-value	M.C. p-value
Gold	$-0.007^{\ddagger}$	0.063	0.084	$-0.003^{\dagger}$	0.020	0.071
Silver	$-0.002^{\ddagger}$	0.070	< 0.001	$-0.001^{\ddagger}$	0.068	< 0.001
Palladium	$-0.039^{\dagger}$	0.016	0.003	$-0.038^{\dagger}$	0.037	0.006
Platinum	$-0.026^{\dagger}$	0.043	< 0.001	-0.010^{*}	0.003	0.013

The CCR and DOLS columns report the respective estimates of $\beta_1 - 1$ in the regression (5) as in Table II. The columns of F.S.C. p-value give the corresponding p-values (with finite sample correction) for testing the hypothesis H_0: $\beta_1 - 1 = 0$ from the actual data, while the columns of M.C. p-value give the corresponding p-values based on the empirical distribution calculated from simulations.

from about 28% to 45%. Also, from the empirical distributions, the probabilities of observing the C_μ statistics from the data range from 13% to 19%. Therefore, one need not be surprised to see a result of non-cointegration between the commodity spot and futures prices if the spot price switches between different states infrequently. The top of Table VI shows the probabilities of observing the CCR and DOLS estimates of $\beta_1 - 1$ when the data are assumed to be generated by the switching model. Such probabilities range from 18% to 37%. Thus, if market participants rationally anticipate switches in the state of convenience yield and the risk premium is stationary, a researcher would quite likely find the "biased" estimates as in the sample data. Therefore, the cointegration results in

Table II need not imply that the risk premium is subject to permanent shocks.

As mentioned above, the "peso problem" is essentially a small sample problem. It is reasonable to investigate whether the results in Table II could also be due to other small sample effects. This issue is addressed by performing two other experiments, which exclude the possibility of regime switching. The results are shown in the other two panels in Table V and Table VI. The first experiment assumes conditional homoskedasticity in the data generating process, while the second experiment allows for generalized conditional heteroskedasticity. Observe that the C_μ cointegration test has reasonable finite sample sizes from Table V. Furthermore, the empirical p-values of observing the C_μ statistics from the actual data are much lower than those of a switching model. As well, the middle and bottom panels of Table VI show that the p-values based on empirical distributions of the CCR and DOLS estimates of $\beta_1 - 1$ are typically smaller than the p-values (with finite sample corrections) based on the data. Therefore, the cointegration results in Table I and Table II do not seem to be explained by these small sample problems.

CONCLUSION

Contrary to standard theoretical models, many recent studies in the commodity markets have found that the futures and realized spot prices do not move one-for-one in the long run, implying that the risk premium may be non-stationary or that the model is misspecified.[8] The results of this study support the arguments of Beck (1994) that these seemingly anomalous findings are not necessarily the results of a model misspecification as argued in Brenner and Kroner (1995). Specifically, it is argued that such findings in the literature can be explained by rational expectations about changes in the state of "convenience yields" in the spot price process. If the spot price follows a regime switching model reflecting different states of the economy, then the results in Table V and Table VI show that the standard cointegrating regression tests are likely to find that commodity spot and futures prices are not cointegrated or move one-for-one in the long run.

Intuitively, the expectation of a switch from a "low convenience" state to a "high convenience" state, or vice versa, implies that forecast errors will be serially correlated during periods when the switch in spot price processes does not materialize. Since the processes within each regime

[8]Of course, the results could also be consistent with an inefficient futures market.

differ persistently, this "peso problem" induces a persistent small sample serial correlation in forecast errors. Therefore, apparent evidence that the risk premium has the same persistence as the futures price is an artifact of a persistent small sample serial correlation in forecast errors. When these expectations are taken into account, the simulation experiments in this study show that the hypothesis that spot and futures prices are cointegrated one-to-one, and thus the risk premium is stationary, cannot be rejected.

BIBLIOGRAPHY

Andrews, D. W. K., and Monahan, J. C. (1992): "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica,* 60:953–966.

Beck, S. (1994): "Cointegration and Market Efficiency in Commodities Futures Markets," *Applied Economics,* 26:249–257.

Brenner, R. J., and Kroner, K. F. (1995): "Arbitrage, Cointegration and Testing the Unbiasedness Hypothesis in Financial Markets," *Journal of Financial and Quantitative Analysis,* 30:23–42.

Chowdhury, A. R. (1991): "Futures Market Efficiency: Evidence from Cointegration Tests," *The Journal of Futures Markets,* 11:577–589.

Crowder, W. J., and Hamed, A. (1993): "A Cointegration Test for Oil Futures Market Efficiency," *The Journal of Futures Markets,* 13:933–941.

Engle, C., and Hamilton, J. D. (1990): "Long Swings in the Dollar: Are They in the Data and Do the Markets Know It?" *American Economic Review,* 80:689–713.

Engle, R. F., and Granger, C. W. J. (1987): "Co-Integration and Error Correction: Representation, Estimation and Testing," *Econometrica,* 55:251–276.

Evans, M. D. D., and Lewis, K. K. (1995): "Do Long-Term Swings in the Dollar Affect Estimates of the Risk Premia?" *Review of Financial Studies,* 8:709–742.

Fama, E. F., and French, K. R. (1987): "Commodity Futures Prices: Some Evidence on Forecast Power, Premiums and the Theory of Storage," *Journal of Business,* 60:55–73.

Fama, E. F., and French, K. R. (1988): "Business Cycles and the Behavior of Metals Prices," *Journal of Finance,* 43:1075–1093.

Gallant, A. R. (1987): *Nonlinear Statistical Models.* New York: Wiley.

Han, H. L., and Ogaki, M. (1997): "Consumption, Income and Cointegration: Further Analysis," *International Review of Economics and Finance,* 6:107–117.

Hansen, B. E. (1992): "Tests for Parameter Instability in Regressions with $I(1)$ Processes," *Journal of Business and Economeic Statistics,* 10:321–335.

Johansen, S. (1991): "Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Autoregressive Models," *Econometrica,* 59:1551–1580.

Krasker, W. S. (1980): "The 'Peso Problem' in Testing the Efficiency of Forward Exchange Markets," *Journal of Monetary Economics,* 6:269–276.

Krehbiel, T., and Adkins, L. C. (1993): "Cointegration Tests of the Unbiased Expectations Hypothesis in Metals Markets," *The Journal of Futures Markets,* 13:753–763.

MacDonald, R., and Taylor, M. (1988): "Metals Prices, Efficiency and Cointegration: Some Evidence from the London Metal Exchange," *Bulletin of Economic Research,* 40:235–239.

MacKinnon, J. G. (1991): "Critical Values for Cointegration Tests," in Engle, R. F., and Granger, C. W. J. (eds.), *Long-Run Economic Relationships: Readings in Cointegration.* Advanced Texts in Econometrics, chap. 13, pp. 267–276. Oxford: Oxford University Press.

Maekawa, K. (1994): "Prewhitened Unit Root Test," *Economics Letters,* 45:145–153.

Moore, M. J., and Copeland, L. S. (1995): "A Comparison of Johansen and Phillips-Hansen Cointegration Tests of Forward Market Efficiency: Baillie and Bollerslev Revisited," *Economics Letters,* 47:131–135.

Park, J. Y. (1990): "Testing for Unit Roots and Cointegration by Variable Addition," in Rhodes, G. F., and Fomby, T. B. (eds.), *Co-Integration, Spurious Regressions, and Unit Roots,* vol. 8 of *Advances in Econometrics,* pp. 107–133. Greenwich, CT: JAI Press.

Park, J. Y. (1992): "Canonical Cointegrating Regressions," *Econometrica,* 60:119–143.

Perron, P. (1989): "The Great Crash, the Oil Shock and the Unit Root Hypothesis," *Econometrica,* 57:1361–1401.

Phillips, P. C. B., and Ouliaris, S. (1990): "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica,* 58:165–193.

Pindyck, R. S. (1993): "The Present Value Model of Rational Commodity Pricing," *Economic Journal,* 103:511–530.

Salant, S., and Henderson, D. (1978): "Market Anticipations of Government Policies and the Price of Gold," *Journal of Political Economy,* 86:627–648.

Sephton, P. S., and Cochrane, D. K. (1990): "A Note on the Efficiency of the London Metals Exchange," *Economics Letters,* 33:341–345.

Sephton, P. S., and Cochrane, D. K. (1991): "The Efficiency of the London Metals Exchange: Another Look at the Evidence," *Applied Economics,* 23:669–674.

Shin, Y. (1994): "A Residual-Based Test of the Null of Cointegration Against the Alternative of No Cointegration," *Econometric Theory,* 10:91–115.

Stock, J. H., and Watson, M. W. (1993): "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica,* 61:783–820.