
THE PROFITABILITY OF INDEX FUTURES ARBITRAGE: EVIDENCE FROM BID-ASK QUOTES

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Previous studies investigated the profitability of stock index futures based on transaction price data, and could overstate the frequency of arbitrage opportunities and size of arbitrage profits. This article obtains a data base for the Hong Kong index futures and index options market that contains both real-time transaction prices and bid-ask quotes; the article further examines the bias of identifying arbitrage opportunities based on transaction prices. The article finds the percentage of observations violating no-arbitrage bounds is significantly reduced when bid-ask quotes are employed instead of transaction prices. This suggests studies that implement arbitrage strategies based on transaction prices employ prices from the wrong side of the

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spread. This article finds a relationship between the frequency of violations (evaluated from transaction prices) and the size of bid-ask spreads in the futures and options markets. This phenomenon indicates that a larger mispricing, which may arise when the bid-ask spread is wider, does not necessarily imply profitable arbitrage opportunity. © 1998 John Wiley & Sons, Inc. *Jrl Fut Mark* 18:743–763, 1998

INTRODUCTION

This article studies the profitability of arbitrage strategies between stock index futures and stock index options in Hong Kong. Although these index futures and index options are quite new and their trading volumes are relatively light compared with their counterparts in the United States, the data base is interesting, as it contains real-time bid-ask quotes as well as transaction prices, so it is possible to study the effect of bid-ask spread on arbitrage profitability. The results have general implications for studies that examine profitability of arbitrage strategies for stock index futures and options markets.

Numerous studies have investigated the arbitrage profitability for stock index derivative markets, including S&P 500 index futures (MacKinlay and Ramaswamy, 1988), MMI index futures (Chung, 1991), Nikkei 225 futures (Bailey, 1989; Brenner, Subrahmanyam, and Uno, 1989; and Chung, Kang, and Rhee, 1993), and S&P 500 index futures and options (Lee and Nayar, 1993). All of these studies identify mispricings based on transaction prices. Despite the fact that these studies acknowledge the importance of transaction costs when evaluating the profitability of arbitrage strategies, none of them consider the direct effect of bid-ask spreads on arbitrage profitability simply because bid-ask quotations are not available.¹

There are two biases in evaluating arbitrage profitability based on transaction prices.² First, the frequency of arbitrage opportunities is overstated. Suppose that a futures transaction takes place at the bid price and, based on the bid price, we conclude that the futures is underpriced. Therefore the arbitrage strategy is to buy the underpriced futures. However, the price that we could buy at is the ask, not the bid. If we use the

¹While the New York Stock Exchange (NYSE) and the Chicago Board Options Exchange (CBOE) maintain historical records of all transaction prices and bid/ask quotes, futures exchanges such as the Chicago Board of Trade (CBT) and the Chicago Mercantile Exchange (CME) record "times and sales data," which contain only the time and price of a transaction if the price is different from the previously recorded price. Bid and ask quotes appear in this file only if the bid quote exceeds or if the ask quote is below the previously recorded transaction price. Therefore, studies that examine stock index futures do not have a complete record of bid-ask quotes.

²Phillips and Smith (1980) had a detailed discussion of the effect of bid-ask spread on arbitrage profits.

correct price (the ask), there might be no arbitrage opportunity. Second, the size of arbitrage profits is overstated. Suppose the futures is underpriced, so that the arbitrage strategy is to purchase futures (at the ask). If only transaction prices are observed, one might mistakenly use a sale price (at the bid) for a futures purchase, so that the purchase price is understated and the arbitrage profit is overstated. To correct for the bias, a common approach is to obtain an estimate of the bid-ask spread, and adjust the transaction cost for the estimated bid-ask spread. This method, however, assumes that the bid-ask spread is constant.

An interesting feature of the dataset for Hong Kong index options and futures is that it contains bid-ask quotes as well as transaction price data. This allows one to examine the size and frequency of arbitrage opportunities based on bid-ask quotations and compare with those based on transaction prices. The analysis is divided into three parts. In the first part, similar to previous studies, arbitrage profitability is evaluated based on transaction prices. This will be used as a benchmark case for comparison. The second part is to examine arbitrage profitability based on bid-ask quotations. Bid-ask spreads are considered explicitly, and arbitrage transactions are required to take place at the right side of the order (purchases at the ask and sales at the bid). In the third part, transaction prices are used, but the trades are classified as buyer- or seller-initiated based on quotation information. Since trades often take place inside the quotes, the transaction price an arbitrageur could trade at is not necessarily equal to the quoted prices. Evaluation of arbitrage opportunity will be based on transaction prices of the correct trade direction.

Results show that the frequency of arbitrage opportunities is very sensitive to different approaches. For example, assuming a transaction cost of 20 index points, the percentage of observations violating no-arbitrage bounds is 18.01% for short strategy (sell overpriced futures and create synthetic long futures) and 16.45% for long strategy (buy underpriced futures and create synthetic short futures) when transaction prices are used. The percentages decline to 2.27% (short strategy) and 2.15% (long strategy) when bid-ask quotations are used, and to 2.62% (short strategy) and 4.87% (long strategy) when transaction prices of correct trade direction are used. Therefore, using transaction prices generally overstates the frequency of arbitrage opportunities by several times.

This article finds a relationship between the likelihood of an arbitrage opportunity (evaluated based on transaction prices) and the size of bid-ask spreads in the futures and options markets. This suggests that a larger mispricing, which may arise when the spread is wider, does not necessarily imply profitable arbitrage opportunity.

TABLE I

Creation of Synthetic Futures through Index Options and Borrowing/Lending

<i>Creation of a Synthetic Long Futures Contract</i>		<i>Cash Flow on T</i>	
	<i>Cash Flow at t</i>		
1. Long futures	0	$S_T - F_t$	
		if $S_T \geq X$	if $S_T < X$
2. Long call	$-C_t$	$S_T - X$	0
Short put	P_t	0	$-(X - S_T)$
Borrow/lend	$\frac{F_t - X}{(1 + r)^{T-t}}$	$-(F_t - X)$	$-(F_t - X)$
	$-C_t + P_t + \frac{F_t - X}{(1 + r)^{T-t}}$	$S_T - F_t$	$S_T - F_t$

<i>Creation of a Synthetic Short Futures Contract</i>		<i>Cash Flow at T</i>	
	<i>Cash Flow at t</i>		
1. Short futures	0	$F_t - S_T$	
		if $S_T \geq X$	if $S_T < X$
2. Short call	C_t	$-(S_T - X)$	0
Long put	$-P_t$	0	$X - S_T$
Borrow/lend	$-\frac{F_t - X}{(1 + r)^{T-t}}$	$F_t - X$	$F_t - X$
	$C_t - P_t - \frac{F_t - X}{(1 + r)^{T-t}}$	$F_t - S_T$	$F_t - S_T$

A long (short) futures position can be replicated indirectly through index options and borrowing and lending. To prevent arbitrage opportunity, the initial cash flow of setting up a "replicating" portfolio equals zero.

PROFITABILITY OF ARBITRAGE BETWEEN INDEX FUTURES AND OPTIONS

Futures-Put-Call Parity

Both Hang Seng Index futures and options are based on the same underlying stock index and mature on the same expiration date. Therefore, a stock index futures contract can be replicated through a combination of the call, the put and the riskless asset. In an arbitrage-free market, the futures price equals the price of the "replicating" portfolio.

Suppose there is no transaction cost, the effects of marking-to-market procedures are negligible, and that the index futures and index options expire at time T . Consider a long position in the futures contract at time t with a futures price of F_t expiring at time T . The payoff at time T would be $(S_T - F_t)$, where S_T is the stock index price at time T . It can be shown

(Table I) that this payoff equals that of a portfolio consisting of a long position in the call option, a short position in the put option and a short position of $(F_t - X)/(1 + r)^{(T-t)}$ in the riskless bond,³ where X denotes the exercise price of both index call and put options and r denotes the risk-free rate. Likewise, the payoff at time T from a short position in the futures contract equals that of a portfolio consisting of a short position in the call option, a long position in the put option and a long position of $(F_t - X)/(1 + r)^{(T-t)}$ in the riskless bond.⁴ Since the payoffs of futures and the replicating portfolio are identical at time T , to prevent arbitrage, the initial cash flow of setting up the replicating portfolio (synthetic futures) should be equal to that of a futures position, i.e., zero. This results in the futures-put-call parity relation

$$-C_t + P_t + \frac{F_t - X}{(1 + r)^{(T-t)}} = 0 \quad (1)$$

where C_t and P_t are call and put option premiums, respectively. Eq. (1) could be rewritten as,

$$F_t = (C_t - P_t)(1 + r)^{(T-t)} + X. \quad (2)$$

A violation of eq. (2) implies arbitrage opportunity. If futures is underpriced (observed futures price is less than the theoretical price), one could arbitrage by buying the futures, selling the call and buying the put (hereafter called long-arbitrage strategy). If futures is overpriced (observed futures price is higher than the theoretical price), one could arbitrage by selling the futures, buying the call and selling the put (hereafter called short-arbitrage strategy).

An introduction of transaction costs will affect the no arbitrage relation in eq. (2) by creating a band within which arbitrage is not profitable. Denote TC as the transaction costs, a long-arbitrage strategy will generate profits (net of transaction costs) only if

$$F_t \leq (C_t - P_t)(1 + r)^{(T-t)} + X - TC. \quad (3)$$

Similarly, a short arbitrage strategy will generate profits (net of transaction costs) only if

$$F_t \geq (C_t - P_t)(1 + r)^{(T-t)} + X + TC \quad (4)$$

³If F_t is greater (less) than X , it is borrowing (lending).

⁴If F_t is greater (less) than X , it is lending (borrowing).

Bid-Ask Spread and Selection Bias

The arbitrage strategy in the above section does not take into account the bid-ask spread. The bid-ask spread is the difference between the price at which a trader can buy and sell in the market. Usually, the bid-ask quotes are maintained by the market makers who provide liquidity for the market, and the spread reflects the cost of immediacy. Market microstructure theory suggests that bid-ask spreads cover three cost components: order processing cost, inventory cost, and adverse information cost.⁵

Any trader who seeks to transact immediately in the market must incur the cost of the bid-ask spread. Therefore, an arbitrageur has to consider the bid-ask spread before he or she decides whether or not to take arbitrage positions. For a long-arbitrage strategy (buy a futures, sell a call, buy a put), since the purchase takes place at the ask, and the sale takes place at the bid, the long-arbitrage transactions will generate profits only if

$$F_t^a \leq (C_t^b - P_t^a)(1 + r)^{(T-t)} + X - TC \quad (5)$$

where superscripts *a* and *b* denote ask and bid prices, respectively. Eq. (5) demonstrates how the implementation of the arbitrage strategy based on transaction prices will easily lead to selection bias. Since the strategy attempts to pick out undervalued futures for purchase, and if it is assumed that the futures could be purchased at transaction prices, then the trading rule will systematically pick out more futures prices transacted at the bid rather than at the ask. In other words, the arbitrage trading rules systematically use prices from the wrong side of the bid-ask spread (Phillips and Smith, 1980).

On the other hand, the short-arbitrage strategy (sell a futures, buy a call, sell a put), will generate profits only if

$$F_t^b \geq (C_t^a - P_t^b)(1 + r)^{(T-t)} + X + TC \quad (6)$$

and the strategy attempts to pick out overvalued futures for sale. If we execute the arbitrage strategy based on transaction prices, the trading rule will systematically pick out more futures prices transacted at the ask rather than at the bid.

Without observing time-stamped bid-ask quotes, there is no satisfactory way to handle the selection bias problem. A common approach is to obtain an estimate of the bid-ask spread, adjust the transaction costs for

⁵See Demsetz (1968) and Tinic (1972) for order processing cost, Stoll (1978), Amihud and Mendelson (1980) and Ho and Stoll (1981) for inventory cost, and Copeland and Galai (1983) and Glosten and Milgrom (1985) for adverse information cost.

the estimated bid-ask spread, and then identify mispricings based on adjusted transaction costs. The bid-ask spread could be estimated based on a sample of bid and ask quotations (Phillips and Smith, 1980; Yadav and Pope, 1990). In the case where bid-ask quotations are not observed, estimators could be derived based on moments of transaction prices (Roll, 1984; Stoll, 1989; Smith and Whaley, 1994). A drawback of this approach is that it assumes the bid-ask spread is constant. If bid-ask spreads differ across transactions, then arbitrage trading rules systematically pick out transactions for which the bid-ask spread is large, when the arbitrage transactions may in fact be unprofitable. Therefore, even after adjusting the transaction costs with the average bid-ask spread, the frequency and size of mispricings may still be overstated.

The selection bias associated with using transaction prices can be avoided if we evaluate the arbitrage opportunity based on bid-ask quotations. Arbitrage profits will also be appropriately computed as arbitrage transactions take place at the right side of the spread. The frequency and size of mispricings are, thus, expected to be smaller when bid-ask quotes are used to generate arbitrage signals than when transaction prices are employed.

Using bid-ask quotations, however, may understate the size and frequency of mispricings. This is because trades often take place inside the quotes, rather than at the quotes. As a result, the spread that an arbitrageur actually incurs is smaller than the one that being quoted in the market. The effective spread is, therefore, lower than the quoted spread. To correct for this problem, we use transaction prices and attempt to classify the trades as buyer- or seller-initiated based on quotation information. Evaluation of arbitrage opportunity is based on transaction prices of the correct trade direction. Compared with other two approaches (based on transaction prices and bid-ask quotes), this approach should yield the smallest bias in estimating the frequency and size of mispricings.

HANG SENG INDEX FUTURES AND INDEX OPTIONS

Both Hang Seng Index (HSI) futures and options contracts represent derivatives claims with the HSI as the underlying index. The HSI is a value-weighted index composed of the 33 largest and most active listed companies in Hong Kong.⁶ The index is the most widely quoted in Hong Kong and is used as a proxy for the market as a whole. In recent years,

⁶The HSI is compiled by HSI Services Ltd, a subsidiary of Hang Seng Bank.

the HSI has represented 75% of total market value and 70% of total market turnover.

HSI futures were introduced by the Hong Kong Futures Exchange (HKFE) in 1986 and proved to be a huge success. In 1987 the new contract became the third most actively traded index futures contract in the world, with trading volume rising to 3.6 million contracts. The trading volume, however, shrank from the peak of 3.6 million to less than 150,000 contracts after the market crash of October 1987. As the stock market began to rise in 1992, futures trading became active again with 1.03 million contracts for the year, and witnessed the highest trading volume record of 4.9 million contracts in 1994. During that year the highest daily volume was about 42,000 contracts and the average daily volume was about 17,000 contracts.

HSI options contracts were launched in March 1993. The trading was relatively thin in 1993, with a daily average of only several hundred lots. The new contracts gained popularity in the following year and a total of 600,000 contracts were traded in 1994. The highest daily trading volume was about 9,000 contracts and the average daily volume was about 2,500 contracts.

Table II summarises basic features of the HSI futures and options contracts. There are four maturity months for both futures and options contracts: spot month, the next calendar month, and the next two calendar quarter months. The last trading day is the business day preceding the last business day of the month. The contract size is HK\$50 per index point. All settlement is in cash, with the settlement made on the first business day after the last trading day. All contracts are subject to a daily price limit of about 10 percent of the contract value. An initial margin is required when the contract is opened, and margins can be posted in the form of cash or interest-rate bearing securities.

There are two trading sessions for both contracts from Monday to Friday, 10:00 a.m.–12:30 p.m. and 14:30 p.m.–16:00 p.m. (local Hong Kong time). The trading of both futures and options is conducted in the same trading room, so that futures and options traders see the trading activities of each other. Trading in the futures market is conducted by the open outcry system, whereas trading in the options market is operated by the dealership system. The current bid, ask, and the last transaction prices are displayed in the exchange's computer system, being input by an HKFE official on a real-time basis from the quotes on the exchange floor.

Several contract features are noteworthy. First, the HSI options are European options. Therefore, there is no early exercise option that will

TABLE II

Contract Specifications of Hang Seng Index Futures and Options Contracts

	<i>HSI Futures</i>	<i>HSI Option</i>
Underlying index	The Hang Seng Index	The Hang Seng Index
Quotation	Index Points	Index points
Contract size	The HSI Futures $\times$ HK\$ 50	Option Premium (quoted in index points) $\times$ HK\$ 50
Price limits	500 Index Points per Trading Session above or below last Closing Quotation	N.A.
Trading method	Open Outcry	Market makers
Trading hours	Two Trading Sessions 10:00–12:30, 14:30–16:00 local Hong Kong Time	Two trading sessions 10:00–12:30, 14:30–16:00 local Hong Kong Time
Delivery months	Spot month, the next calendar month, and the next two calendar quarter months	Spot month, the next calendar month, and the next two calendar quarter months
Last trading day	The business day preceding the last business day of the month	The business day preceding the last business day of the month
Settlement day	The first business day after the last trading day	The first business day after the last trading day
Settlement method	Cash	Cash
Margin requirement	HK\$ 40,000	Margin procedures as set out in the Exchange rules
Position limit	Nil	Nil
Exercise style	Not applicable	European
Exercise price	Not applicable	Exercise Price Increment is 50 points for HSI level below 2000, 100 points for HSI level between 2000 and 8000, and 200 points for HSI level above 8000

destroy the futures-put-call parity relation. Second, the HSI futures and options contracts expire in the same month. The arbitrage position can be, therefore, unwind simultaneously for both of them. Third, both contracts are traded at the same time, and this avoids complications arising from the nonsynchronous trading hours.

Since the existence of transaction costs may reduce profitable arbitrage opportunities, this article considers the sensitivity of arbitrage profits to different levels of transaction costs in empirical tests. For futures trading, based on conversations with floor traders in the HKFE, one-way commission is estimated to be about \$HK80 per contract for institutional investors. The broker commission includes trading fees charged by the Securities and Futures Commission.⁷ For index options transactions,

⁷The current trading fees include: (i) Exchange Fee of \$HK 10.00, (ii) SFC Levy of \$HK 1.00, and (iii) Compensation Fund Levy of \$HK 0.50, totaling \$HK 11.50.

commission costs vary with the price of the option. Minimum commission will be payable on all trades at the lesser of (i) 1% of the contract value rounded up to the nearest Hong Kong dollar with a minimum of \$HK 30 and (ii) \$HK 100. Index option trading requires the payment of commissions twice: at the time of the initial trade and at the time of closing out the position. If an option position expires unexercised, only one-way commission is required. If an option is exercised on expiration day, an exercise fee of \$HK 10 per contract is required. As a conservative estimate, a maximum commission of \$HK 150 is assumed.

Given the above assumptions regarding brokerage commission, and ignoring market impact costs, each arbitrage strategy requires about \$HK 540 commission costs (10.8 index points), which include \$HK 80 for futures position, and \$HK 460 for call and put option positions.⁸ Since margins can be posted in the form of interest-bearing securities, we ignore the interest forgone on margin deposits. Empirical analysis on arbitrage profitability will be conducted for different levels of transaction costs ranging from 10 to 80 index points.

EMPIRICAL RESULTS

Data and Preliminary Statistics

The data are obtained from the Hong Kong Futures Exchange (HKFE) for Hang Seng Index (HSI) futures and option contracts for the sample period from 1 October 1993 to 30 June 1994. The database comprises trade-and-quotation records for the HSI futures and options of all different maturities and exercise prices. All trade-and-quotation records are time-stamped to the nearest second. The trade records contain the prices and number of shares traded for all transactions. The quotation records contain the complete history of bid and ask prices recorded in the sequence they are quoted. Since HKFE inputs transaction prices and bid-ask quotes into a computerized price reporting system as soon as they are called out on the market floor, the reporting lag is believed to be only a few seconds.

For the risk-free interest rate that is required for futures-put-call parity, we employ the Hong Kong Inter-Bank Offered Rate (HIBOR) maturing on the day closest to the expiration date of the futures and option contract.

⁸Since the arbitrage strategy calls for the exercise of either a call or put option alone, only one exercise fee is incurred.

TABLE III
 Percentage Distribution of Absolute Spreads in Index Points

<i>Spread</i>	<i>Futures</i>	<i>Put</i>	<i>Call</i>
0 <= spread <= 5	80.83	9.11	7.24
5 < spread <= 10	13.95	19.90	18.04
10 < spread <= 20	3.30	36.54	37.71
20 < spread <= 30	0.97	12.41	12.28
30 < spread <= 40	0.41	13.05	14.99
40 < spread <= 50	0.25	8.67	8.34
50 < spread <= 70	0.14	0.24	0.28
70 < spread	0.15	0.08	0.12
Total	100.00%	100.00%	100.00%
N	156,436	13,037	15,973
Mean	6.06	23.00	23.83
Median	5.00	20.00	20.00

Spreads (in index points) are computed from the bid-ask quotes that are nearest to the transaction prices of Hong Kong Futures Exchange index futures and options for the period of 1 October 1993 to 30 June 1994. To be included in the sample, the prevailing bid-ask quotes should not be more than 10 minutes apart from the transaction prices. Table figures are percentages of total observations in each category.

Table III reports the percentage distribution of bid-ask spreads (in index points) for HSI futures and options. For index futures, 80.83% of observations have a spread of less than 5 index points, and the median spread is 5 points. For both puts and calls, the distribution is more evenly distributed, and the median spread is 20 index points. Therefore, puts and calls have higher dollar spreads than futures. Since the prices of options are generally smaller than those of futures, this suggests that the percentage bid-ask spreads for puts and calls are much higher than for futures.

Arbitrage Strategies Based on Transaction Prices

Arbitrage profitability is first evaluated based on transaction prices. A sample of observations for futures, puts, and calls is created, matched in the expiration date and exercise price (for options). The sample is constructed so that whenever a new transaction price observation comes in from either a put or call, the price is matched with the most recent transaction prices of the other two markets. To mitigate the stale price problem, the matching transaction prices are required to be within ten minutes of each other. Suppose we have October futures traded at 2:58 p.m. and 3:08 p.m., October call traded at 2:59 p.m. and 3:10 p.m., and October put traded (with the same exercise price) at 3:04 p.m. and 3:26 p.m. There are, then, two valid observations: (i) put at 3:04 p.m., call at 2:59

TABLE IV

Percentages of Arbitrage Opportunities and Profits (Net of Transaction Costs)
to Arbitrage Strategies Based on Transaction Prices

Transaction Costs	Short Strategy			Long Strategy		
	Percentage Violations	Mean Profits	Median Profits	Percentage Violations	Mean Profits	Median Profits
None	52.81%	20.73	14.76	46.91%	22.48	14.98
10 index pt.	33.40	19.97	10.60	30.04	22.43	11.89
20 index pt.	18.01	22.97	10.00	16.45	26.69	14.77
30 index pt.	8.95	31.26	10.18	9.69	31.68	14.92
40 index pt.	4.69	44.99	14.91	6.13	37.05	14.12
50 index pt.	2.97	57.97	16.01	3.44	52.11	14.76
60 index pt.	1.76	84.57	29.60	1.91	79.40	29.79
70 index pt.	1.21	109.95	59.85	1.56	86.24	27.63
80 index pt.	0.94	130.67	62.91	1.09	111.13	37.83

A short (long) strategy takes advantage of overvalued (undervalued) futures contracts by short futures, long call, short put (long futures, short call, long put). The sample consists of transaction prices for futures, puts and calls that are matched in the expiration date and exercise price (for options), and is constructed so that whenever a new transaction price observation comes in from either a put or call, the price is matched with the most recent transaction prices of the other two markets. To mitigate the stale price problem, the matching transaction prices are required to be within ten minutes of each other. The matching process results in 2,560 observations. Profits are in terms of index points and net of transaction costs.

p.m., and futures at 2:58 p.m.; and (ii) call at 3:10 p.m., put at 3:04 p.m. and futures at 3:08 p.m. For the put transaction at 3:26 p.m., since there is no futures trade within 10 minutes of the trade, no valid observation is obtained. This matching process results in 2,560 observations, which will be examined for arbitrage opportunities. A long-arbitrage strategy will be implemented for undervalued futures by buying a futures, selling a call and buying a put, and the net profit (arbitrage profit net of transaction costs) is computed as

$$(C_t - P_t)(1 + r)^{(T-t)} + X - F_t - TC \quad (7)$$

where TC is the assumed transaction cost. A short-arbitrage strategy will be implemented for overvalued futures by selling a futures, buying a call and selling a put, and the net profit is computed as

$$F_t - (C_t - P_t)(1 + r)^{(T-t)} - X - TC. \quad (8)$$

Table IV reports the percentage violations and net profits of these arbitrage strategies for a range of transaction costs from 0 to 80 index points. Not surprisingly, when a zero transaction cost is assumed, the percentage violations for the long strategy and the short strategy add up to almost 100%. This is because unless the prices conform exactly to the futures-

put-call parity, the futures contract is either overvalued or undervalued. At transaction costs of 20 index points, the percentage violation is 18.01% for short strategy and 16.45% for long strategy. Even when transaction costs of 40 index points is assumed, the frequency of violations is still economically significant – 4.69% for short strategy and 6.13% for long strategy.

As for arbitrage profits, both mean and median are reported. Since there are some outliers, median profits are more reliable for inference. The median profits range from 14.76 points to 62.91 points for short strategy, and from 14.98 points to 37.83 points for long strategy, depending on levels of transaction costs assumed. Overall, arbitrage opportunities are abundant. This may reflect the fact that we do not consider the impact of bid-ask spread, and that arbitrage transactions take place at the wrong side of the spread. We will therefore evaluate the arbitrage strategies based on bid-ask quotations.

Arbitrage Strategies Based on Bid-Ask Quotes

This section examines arbitrage opportunity based on bid-ask quotations. To keep the sample size the same, for every transaction price used in the previous section of this article, we identify the nearest bid and ask quotes. Since a short arbitrage strategy requires selling a futures (at bid), buying a call (at ask) and selling a put (at bid), while a long arbitrage involves buying a futures (at ask), selling a call (at bid) and buying a put (at ask), the matching criteria are very specific about whether the bid or ask quote is used. A valid observation has to be either an observation comprising futures bid price, call ask price and put bid price, or an observation comprising futures ask price, call bid price and put ask price. The net profit to the long-arbitrage strategy is determined by

$$(C_t^b - P_t^a)(1 + r)^{(T-t)} + X - F_t^a - TC, \quad (9)$$

while the net profit to the short-arbitrage strategy is determined by

$$F_t^b - (C_t^a - P_t^b)(1 + r)^{(T-t)} - X - TC. \quad (10)$$

Table V reports the percentage violations and net profits to arbitrage strategies. Compared with Table IV, the frequency of arbitrage opportunities dramatically declines. At transaction costs of 20 index points, the percentage violation is 2.27% for short strategy and 2.15% for long strategy. At transaction costs of 40 index points, the percentage is 0.94% for short

TABLE V

Percentages of Arbitrage Opportunities and Profits (Net of Transaction Costs)
to Arbitrage Strategies Based on Bid-Ask Quotes

Transaction Costs	Short Strategy			Long Strategy		
	Percentage Violations	Mean Profits	Median Profits	Percentage Violations	Mean Profits	Median Profits
None	11.33%	22.92	9.58	9.49%	26.26	9.20
10 index pt.	4.73	38.45	9.95	3.87	48.49	12.42
20 index pt.	2.27	64.73	17.20	2.15	72.86	19.71
30 index pt.	1.45	89.02	25.09	1.37	101.56	36.04
40 index pt.	0.94	124.29	43.95	1.02	124.49	50.87
50 index pt.	0.78	138.56	45.41	0.86	135.74	114.74
60 index pt.	0.63	161.37	71.25	0.70	154.73	127.61
70 index pt.	0.55	173.38	98.46	0.63	163.10	141.19
80 index pt.	0.47	190.64	96.76	0.59	163.66	145.30

A short (long) strategy takes advantage of overvalued (undervalued) futures contracts by short futures, long call, short put (long futures, short call, long put). The sample consists of bid-ask prices for futures, puts and calls that are matched in the expiration date and exercise price (for options), and is constructed based on bid-ask quotes that are nearest to those transaction prices generated for Table IV. A valid observation has to be either one comprising futures bid price, call ask price and put price for the short strategy, or one comprising futures ask price, call bid price, and put ask price for the long strategy. The matching process results in 2,560 observations. Profits are in terms of index points and net of transaction costs.

strategy and 1.02% for long strategy. These represent about 80% reduction from those reported in Table 4.

We could compare Tables IV and V to examine the effectiveness of the approach that evaluates arbitrage opportunity based on transaction prices and takes into account the impact of bid-ask cost through estimated spread. Without observing real time bid-ask quotes data, previous studies (Phillips and Smith, 1980; Yadav and Pope, 1990; and Lee and Nayar, 1993) obtain an estimate of bid-ask spread, and adjust the one-way transaction cost for a half of the bid-ask spread. In this study, since the median bid-ask spreads are 5, 20 and 20 index points for futures, put, and call, one-half of the total spread is about 22.5 index points. Therefore, if we assume that the other transaction cost (e.g., commission cost) is 10 index points, then adding 22.5 index points will increase the total transaction cost to 32.5 index points. From Table IV, this implies percentage violations of 8.95% for short strategy and 9.69% for long strategy (30 index points). On the other hand, since using bid-ask quotations already accounts for the difference between the purchase and sale price, there is no need to add the bid-ask spread to the transaction cost in Table V. Assuming that the other transaction cost is 10 index points, this implies percentage violations of 4.73% for short strategy and 3.87% for long strategy. Therefore, the frequency of arbitrage opportunities is still higher

when transaction prices are used than when bid-ask quotes are used, even if transaction costs are adjusted for the estimated bid-ask spread. A similar conclusion is reached for other levels of transaction costs.

A possible reason for this result is that the size of the bid-ask spread is not constant but varies over time. Furthermore, when the spread is widened, arbitrageurs are unlikely to step into the market, so that the mispricing persists. Therefore, based on transaction prices, we are likely to pick out some seeming arbitrage opportunities which are, in fact, not profitable once we consider the cost of the spread. We will explore this possibility in later sections.

Arbitrage Strategies Based on Feasible Transaction Prices

Since trades could take place inside the quotes, rather than at the quotes, the spread that an arbitrageur actually incurs is smaller than that quoted in the market. This section examines arbitrage profitability based on feasible transaction prices, the transaction prices that take place at the right side of the spread and are feasible for arbitrage transactions. For each transaction price, we attempt to check the direction of the trades. Our trade-classification scheme is similar to those used by Lee and Ready (1991) and Easley, O'Hara, and Srinivas (1994). We compare trade prices with the bid-ask quotes in neighboring 10-minute periods. Trades occurring in the lower half of the spread, at the bid or below are classified as sells, and trades occurring in the upper half of the spreads at the ask or above are classified as buys. Trades occurring at the midpoint of the spread are not classified. A summary of the trade classification is reported in Table VI. It is interesting to note that for futures, more than 90% of the trades take place at the bid-ask quotes (46.41% at the ask and 48.98% at the bid). On the other hand, trades for the put and call often take place inside the spread. Compared with futures, call and put options have many transaction prices whose trade classification cannot be determined, since they are equal to the midpoint of the bid-ask spread (7.42% for the put and 7.62% for the call). This drastically reduces the number of feasible transaction prices for examination.

Table VII reports the frequency of arbitrage opportunities based on feasible transaction prices. Overall, there are only 267 valid observations to be examined for the short strategy, and 258 valid observations for the long strategy. Not surprisingly, the percentage violations are generally larger than those in Table V, but smaller than those in Table IV. For example, at transaction costs of 20 index points, the percentage is 2.62%

TABLE VI
Results of the Trade Classification

<i>Category</i>	<i>Futures</i>	<i>Put</i>	<i>Call</i>
Equal to the ask	46.41	36.52	34.92
Between ask and quote midpoint	1.09	8.13	7.81
Equal to the midpoint	2.58	7.42	7.62
Between bid and quote midpoint	0.94	8.01	7.50
Equal to the bid	48.98	39.92	42.15
Total	100.00%	100.00%	100.00%

The table categorizes the transaction prices of futures, puts, and calls used in Table IV by comparing the price of each trade to the bid/ask price of the prevailing quote. The transaction prices should not be more than 10 minutes apart from the prevailing quotes. Table figures represent the percentage of total trades in each category.

for short strategy and 4.87% for long strategy. At transaction costs of 40 index points, the percentage is 1.50% for short strategy and 2.25% for long strategy. Overall, when we use feasible transaction prices, arbitrage opportunities are still slim.

Sensitivity Tests

Table VIII reports some sensitivity tests to check the robustness of the results when transaction prices and bid-ask quotations of futures and options are matched based on finer intervals. The prices are required to be within 5 minutes and 1 minute of each other respectively. This obviously results in fewer observations. Nevertheless, the frequency of mispricing opportunities varies across different approaches in a pattern similar to before—the percentage violations are the highest for transaction prices, lower for feasible transaction prices, and the lowest for bid-ask quotations.

Relationship between Arbitrage Opportunities and the Size of the Bid-Ask Spread

A problem associated with using transaction prices in evaluating arbitrage opportunity is that the bid-ask cost is ignored. It is obvious that when bid-ask spread is wider, a larger deviation from futures-put-call parity is required to induce arbitrageurs to step into the market. Therefore, when using transaction prices, it is likely that arbitrage observations will be evident, that are not so when the cost of the spread is considered.

To examine this possibility, the arbitrage profits are regressed on bid-ask spreads in futures and options markets. The arbitrage profits are com-

TABLE VII

Percentages of Arbitrage Opportunities and Profits (Net of Transaction Costs)
to Arbitrage Strategies Based on Feasible Transaction Prices

Transaction Costs	Short Strategy			Long Strategy		
	Percentage Violations	Mean Profits	Median Profits	Percentage Violations	Mean Profits	Median Profits
None	19.48%	16.18	6.98	15.73%	38.53	9.94
10 index pt.	7.12	27.39	7.51	7.87	63.13	19.42
20 index pt.	2.62	56.83	29.49	4.87	89.31	14.68
30 index pt.	2.62	46.83	19.49	3.00	130.90	40.15
40 index pt.	1.50	66.18	12.53	2.25	163.69	50.55
50 index pt.	0.75	112.64	112.64	1.87	184.50	45.56
60 index pt.	0.37	210.19	210.19	1.50	219.44	123.67
70 index pt.	0.37	200.19	200.19	1.50	209.44	209.44
80 index pt.	0.37	190.19	190.19	1.50	199.44	103.67

A short (long) strategy takes advantage of overvalued (undervalued) futures contracts, that is, short futures, long call, short put (long futures, short call, long put). The sample consists of feasible transaction prices for futures, puts and calls that are matched in the expiration date and exercise price (for options). For each transaction price used in Table IV, the trade direction will be inferred from bid-ask quotes in the neighboring 10 minutes. Only observations of correct trade direction that are feasible for arbitrage strategy are retained. This sample construction process results in 267 and 258 observations for short and long strategies, respectively. Profits are in terms of index points and net of transaction costs.

puted based on transaction prices and net of transaction costs of 10 index points. Table IX reports regression results for several different specifications. Also included are number of days to maturity and moneyness of the options, to control for their possible effects on arbitrage profits. Evidence generally indicates that net arbitrage profits are positively and significantly related to bid-ask spreads in futures and options markets.

Table X reports logit regressions of the probability of arbitrage opportunity on the bid-ask spreads, where the binary choice variable equals one if the arbitrage profits net of transaction costs of 10 index points is positive and zero otherwise. Estimated coefficients for the bid-ask spreads are all positive and significant. This is consistent with the argument that the seeming arbitrage opportunity is due to the wider bid-ask spread in the markets.

CONCLUSION

This article examines the profitability of arbitrage strategies between index futures and index options in Hong Kong. A unique feature of this study is that it utilizes real time bid-ask quotes in addition to transaction prices in evaluating the profitability of arbitrage strategies. With bid-ask quotes, the bid-ask spread explicitly is taken account in identifying arbitrage

TABLE VIII

Percentages of Arbitrage Opportunities and Profits (Net of Transaction Costs) to Arbitrage Strategies Based on Prices of Futures, Puts, and Calls, Matched within 10-minute, 5-Minute and 1-Minute Interval

<i>Prices Used</i>	<i>10-Minute Interval Matching</i>					
	<i>Short Strategy</i>			<i>Long Strategy</i>		
	<i>Percentage Violations</i>	<i>Mean Profits</i>	<i>Median Profits</i>	<i>Percentage Violations</i>	<i>Mean Profits</i>	<i>Median Profits</i>
Transaction-price	8.95% (2560)	31.26	10.18	9.68% (2560)	31.68	14.92
Bid-ask quotes	4.73 (2560)	38.45	9.95	3.87 (2560)	48.49	12.42
Feasible transaction prices	7.12 (267)	27.39	7.51	7.87 (258)	63.13	19.42
<i>Prices Used</i>	<i>5-Minute Interval Matching</i>					
	<i>Short Strategy</i>			<i>Long Strategy</i>		
	<i>Percentage Violations</i>	<i>Mean Profits</i>	<i>Median Profits</i>	<i>Percentage Violations</i>	<i>Mean Profits</i>	<i>Median Profits</i>
Transaction prices	9.15% (1463)	32.03	9.79	9.02% (1463)	36.17	14.78
Bid-ask quotes	3.96 (1463)	50.64	9.89	3.41 (1463)	64.06	13.99
Feasible transaction prices	7.04 (142)	43.58	17.89	6.67 (150)	89.36	20.02
<i>Prices Used</i>	<i>1-Minute Interval Matching</i>					
	<i>Short Strategy</i>			<i>Long Strategy</i>		
	<i>Percentage Violations</i>	<i>Mean Profits</i>	<i>Median Profits</i>	<i>Percentage Violations</i>	<i>Mean Profits</i>	<i>Median Profits</i>
Transaction prices	8.72% (321)	81.96	9.90	14.01% (321)	31.13	13.39
Bid-ask quotes	3.73 (321)	158.93	22.21	4.04 (321)	72.34	44.76
Feasible transaction prices	3.22 (31)	4.25	4.25	12.50 (24)	28.27	20.39

A short (long) strategy takes advantage of overvalued (undervalued) futures contracts, that is, short futures, long call, short put (long futures, short call, long put). The sample is constructed employing 10-minute, 5-minute and 1-minute time interval in matching prices of index put and call options and futures. Figures in the table are net of transaction costs of 30 index points for transaction prices, and 10 index points for bid-ask quotes and feasible transaction prices. Figures in parenthesis are the total number of observations for each category of prices employed. Profits are in terms of index points and net of transaction costs.

TABLE IX
 OLS Regression of the Mispricings on Bid-Ask Spreads

<i>Model</i>	<i>Intercept</i>	<i>fsp</i>	<i>psp</i>	<i>csp</i>	<i>ttm</i>	<i>dummy</i>	<i>Adj-R²</i>
1	19.1063 (0.00)	0.6264 (0.00)					0.0034
2	14.4452 (0.00)		0.3349 (0.00)				0.0062
3	13.7951 (0.00)			0.3749 (0.00)			0.0058
4	9.5930 (0.00)	0.4582 (0.02)	0.2167 (0.01)	0.2253 (0.03)			0.0096
5	12.8721 (0.00)	0.5164 (0.01)			0.3099 (0.01)	1.8650 (0.44)	0.0053
6	6.2655 (0.07)		0.3438 (0.00)		0.3123 (0.01)	4.0229 (0.11)	0.0093
7	8.6219 (0.00)			0.3282 (0.00)	0.3080 (0.01)	0.8090 (0.74)	0.0075
8	4.2545 (0.23)	0.3900 (0.06)	0.2540 (0.00)	0.1662 (0.12)	0.2350 (0.06)	3.3919 (0.18)	0.0110

Estimates from the regression model are given by the following equation:

$$y = a + b_1 fsp + b_2 psp + b_3 csp + b_4 ttm + b_5 dummy + e.$$

The dependent variable is the absolute value of mispricing where the mispricing is computed as $F_t - (C_t - P_t)(1 + r)^{T-t} - X$ (based on transaction prices used in Table IV). Explanatory variables include the bid-ask spreads of futures (*fsp*), and put (*psp*) and call (*csp*) options, number of days to maturity (*ttm*), and the dummy variable that takes the value of one when index price is greater than exercise price and zero otherwise. Figures in parentheses are probability values.

trade opportunities. This circumvents the selection bias problem inherent in studies using transaction prices alone.

The frequency and size of arbitrage opportunities are evaluated based on three approaches: (i) transaction prices, (ii) bid-ask quotes, and (iii) transaction prices that are checked for trade direction using bid-ask quotes. Overall, the percentage of observations violating no-arbitrage bounds is dramatically reduced under (ii) and (iii) relative to (i). There is also found a relationship between the likelihood of arbitrage opportunity (evaluated based on transaction prices) and the size of bid-ask spreads in the futures and options markets. This suggests that the reason for the appearance of some arbitrage opportunities is that arbitrageurs would not step into the market when the spread is large. Therefore those seeming arbitrage opportunities might in fact be not profitable.

As far as is known, this study represents the first attempt to examine index arbitrage profitability based on bid-ask quotations. It has a broad implication for earlier studies that examine profitability of arbitrage strategies for stock index futures and options markets. The results underscore biases incurred when transaction prices are employed to investigate ar-

TABLE X
Logit Regression of the Probability of Mispricings on Bid-Ask Spreads

Model	Intercept	<i>fsp</i>	<i>psp</i>	<i>csp</i>	<i>ttm</i>	Dummy
1	0.2905 (0.00)	0.0515 (0.00)				
2	0.0483 (0.00)		0.0217 (0.00)			
3	-0.1174 (0.17)			0.0299 (0.00)		
4	-0.3642 (0.00)	0.0339 (0.00)	0.0101 (0.00)	0.0227 (0.00)		
5	0.1471 (0.16)	0.0498 (0.00)			0.0058 (0.18)	0.0839 (0.31)
6	-0.2273 (0.06)		0.0235 (0.00)		0.0060 (0.15)	0.2324 (0.00)
7	-0.1765 (0.11)			0.0293 (0.00)	0.0037 (0.38)	0.0025 (0.97)
8	-0.4456 (0.00)	0.0342 (0.00)	0.0120 (0.00)	0.0212 (0.00)	0.0002 (0.95)	0.1297 (0.15)

Estimates from the logit model are given by the following equation:

$$y = a + b_1 fsp + b_2 psp + b_3 csp + b_4 ttm + b_5 dummy + e.$$

The binary choice variable, y , takes the value of one if the absolute value of mispricing computed as $F_t - (C_t - P_t)(1 + r)^{t-T} - X$ (based on transaction prices used in Table IV) is greater than 10 index points and zero otherwise. Explanatory variables include the bid-ask spreads of futures (fsp), and put (psp) and call (csp) options, number of days to maturity (ttm) and the dummy variable that takes the value of one when index price is greater than exercise price and zero otherwise. Figures in parenthesis are probability-values.

bitrage opportunity. Therefore studies that evaluate arbitrage opportunity based on transaction prices should be cautious in interpreting their results.

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