# HEDGING TIME-VARYING DOWNSIDE RISK

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## INTRODUCTION

One of the major functions of derivative instruments is risk reduction. A long-standing tradition in the finance literature treated the risk with a two-sided notion. Standard deviation or variance are employed to measure risk. A recent survey by Adams and Montesi (1995), however, indicated that corporate managers are more concerned with variability in losses but not so much with variability in gains. This finding is consistent with Mao (1970). One may name the former variability "downside risk" and the latter "upside potential" (Lee and Rao, 1988). Adopting this viewpoint, if a derivative instrument is to reduce the risk, it should result in a smaller downside risk instead of a smaller standard deviation or variance.

An appropriate measure of the downside risk is the Fishburn risk measure (Fishburn, 1977) or the so-called lower partial moments (Bawa, 1975, 1978). Herein risk is measured by a probability-weighted power function of the shortfalls from a specific target return. The power applied in the calculation is called the "order" of the lower partial moment (LPM). A larger order corresponds to a greater risk aversion toward large losses.

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This article is concerned with the optimal futures hedge ratio in downside risk reduction. Besides the fact that downside risk is a more relevant concept for corporate managers, the minimization of LPM is consistent with the expected utility hypothesis (Bawa, 1975, 1978).

In many cases futures on certain securities are available but options are not. Whenever both are available, it appears that options (puts and calls) may be the better derivative instruments for hedging the downside risk as large losses are effectively eliminated with options. The considerations of small and medium losses (or gains short of the target return) in calculating LPM, however, rebut the above conjecture. Ahmadi, Sharp, and Walther (1986) found that futures hedge reduces more downside risk than options on currency markets such as the British pound, deutsche mark and Japanese yen. Adams and Montesi (1995) also found that corporate managers prefer futures to options, citing the large transaction costs in options trading as the main reason.[1]

Lien and Tse (1997) applied the analysis to the Nikkei Stock Average futures traded in SIMEX (Singapore International Monetary Exchange). It was found that the minimum LPM hedge strategy differs from the minimum variance hedge strategy when the target return is small and when the hedger is more risk averse to large losses. Their approach, however, relies upon the assumption that the return distribution is time invariant; henceforth the optimal LPM hedge ratio is also time invariant.

Recent findings on the conditional heteroskedasticity (and stochastic volatility) in many futures and spot price series imply that the optimal hedge strategy should be time varying. Assuming that the stochastic process of the price series is adequately captured by a bivariate GARCH (Generalized Autoregressive Conditional Heteroskedasticity) or EGARCH (Exponential GARCH) model, the conditional variance and covariance of the futures and spot series will be time varying. The minimum variance hedge strategy should, therefore, also be time varying. Obviously this concept also applies to the lower partial moment hedge strategy.

In fact, by allowing the variance-covariance matrix to be time varying, GARCH models specify a time-varying joint distribution for the spot and futures prices. Hansen (1994) extended the GARCH framework to autoregressive conditional density models. Within the context of univariate

[1]We would like to compare futures with options, unfortunately we do not have an option contract that matches the time period of our study. SIMEX has an option on the Nikkei 225 with a much shorter history. Moreover, even if such a contract exists, it would be very thin (as we have experienced in comparing currency futures with currency options), and a continuous time series would not be available for the study of time-varying hedging. In addition, the choice of a nearby option would be problematic for options.

time series, he postulated a skewed Student-$t$ distribution and assumed that all parameters are time varying. Although it may be possible to generalize Hansen's framework to multivariate time series, we do not pursue this approach. Instead, we restrict ourselves to the GARCH family. More specifically, we adopt a more flexible bivariate APARCH-M (Asymmetric Power ARCH in Mean) model to estimate the joint distribution of the Nikkei Stock Average (NSA) spot and futures series. The estimated distribution is then applied to determine the optimal LPM hedge ratio.

We found that the model fits the data well. There is a significant leverage effect in the conditional variance equations of the series. In fact, all parameters in these two equations are statistically significant at the 1% level. The futures return reacts only to the underlying risk, whereas the spot return responds to the risk, lagged spot and futures returns, and the cointegration residual. In terms of Granger causality, the NSA futures market leads the spot market. This finding is consistent with Tse (1995). The risk parameters in both mean equations are negative; that is, a smaller return is associated with a larger conditional variance.

Recall that LPM is characterized by two parameters: the order and the target return. For negative (positive) target returns, the LPM hedge ratios are generally smaller (larger) than the MV ratio and the difference increases (decreases) with increasing order. When the (daily) target return is between $-0.5\%$ and $1.0\%$, the two hedge ratios are highly correlated, except when the order of the LPM is one. For large negative returns, say $-1.5\%$, the correlation is low.

## LOWER PARTIAL MOMENTS

Consider a one-period framework in which a representative individual holds a given portfolio. The return of the portfolio is summarized by a random variable $X$. Let $F(\cdot)$ denote the probability distribution function of $X$. For any positive integer $n$, the $n$-th order lower partial moment (LPM) of $X$ is defined as follows:

$$l(c, n, X) = \int_{-\infty}^{c} (c - x)^n \, dF(x), \tag{1}$$

where $c$ is the target return. The LPM calculation disregards the returns exceeding $c$ and takes into account only the returns falling short of $c$; a concept congruent with the downside risk. A larger $c$ implies a more optimistic individual who expects better returns from the portfolio. The value of $n$ determines the attitude toward large losses. When $n$ is large, the individual is more risk averse to large losses and assigns greater weights to these losses. If we choose $n = 2$ and $c = 0$, we obtain the semivariance of $X$ (when the mean of $X$ is zero); a risk measure proposed by Markowitz (1959, 1990) to replace variance (and standard deviation).

Fishburn (1977) risk measure takes the same form as equation (1) but allows $n$ to be a rational number. Some properties of LPM are summarized as follows:

1. For a given pair of $c$ and $n$, LPM is uniquely determined by the probability distribution function. That is, if $X_1$ and $X_2$ have the same distribution, then
$$l(c, n, X_1) = l(c, n, X_2). \tag{2}$$
2. As $c$ increases, LPM also increases. That is, when $c_1 > c_2$,
$$l(c_1, n, X) > l(c_2, n, X). \tag{3}$$
3. Suppose that the density function of $X$ is symmetric at $c$. Let var($X$) denote the variance of $X$, then
$$l(c, 2, X) = \mathrm{var}(X)/2. \tag{4}$$
If $c = 0$, LPM is identical to the semivariance which is a half of the variance.

To incorporate hedging decisions into the framework, we consider a hedger with initial wealth $W_0$ and a given nontradeable spot position $Q$ at time 0. To reduce risk exposure, the hedger establishes $kQ$ futures positions. Both spot and futures positions are liquidated at time 1. Let $\Delta p$ and $\Delta f$ denote the changes in spot and futures prices from time 0 to time 1, respectively. The hedger's end-of-period wealth is then

$$W_1 = W_0 + (\Delta p + k\Delta f)Q = W_0 + (r_p + \theta r_f)p_0Q, \tag{5}$$

where $p_0$ and $f_0$ are, respectively, the spot and futures prices at time 0; $r_p = \Delta p/p_0$ and $r_f = \Delta f/f_0$ are, respectively, the returns from spot and futures positions; and $\theta = kf_0/p_0$ is the (adjusted) hedge ratio.

The expected utility hypothesis presumes a utility function $U(\cdot)$ and a subjective probability distribution, both defined over the end-of-period wealth, $W_1$. The hedger chooses the value of $\theta$ to maximize the expected utility. Lower partial moment methods require only the probability distribution and search for the value of $\theta$ that minimizes the lower partial moment. The congruence between these two approaches are discussed in Bawa (1975, 1978) for the general case and repeated in Lien and Tse (1997) for the hedging framework.

In this paper, we consider the lower partial moment of $(r_p + \theta r_f)$ instead of the lower partial moment of $W_1$, and identify the value of $\theta$

giving rise to the minimum LPM as the LPM hedge ratios.[2] This approach removes the effects of the initial wealth $W_0$ and the nontradeable spot position $Q$. Suppose the target return is $c$, by equation (1), the $n$-th order LPM of $r_p + \theta r_f$ (denoted by $m$) is

$$l(c, n, m) = \mathrm{E}[(\max(0, c - r_p - \theta r_f))^n], \tag{6}$$

where $\mathrm{E}(\cdot)$ is the expectation operator. Consequently, when $n > 1$, the LPM hedge ratio, $\theta$ satisfies the following first order condition:

$$-n\mathrm{E}[(\max(0, c - r_p - \theta^* r_f))^{n-1} r_f] = 0. \tag{7}$$

It can be easily shown that the second order condition is always satisfied. Equation (7) does not lead to any analytical solution. One must rely upon numerical methods for further analysis of the LPM hedge ratios.[3]

Direct evaluation of $l(c, n, m)$ requires the knowledge of the joint distribution of $r_p$ and $r_f$. Alternatively, one needs to know the distribution function of $m$ for every possible $\theta$. Based upon the historical data, empirical distributions of $m$ are calculated and substituted into equation (6); see Price, Price, and Nantell (1982), Homaifar and Graddy (1990), and Harlow (1991). Lien and Tse (1997) adopted the Gaussian kernel method to estimate the distribution function. Reiss (1981) demonstrated the better performance of the kernel method over the empirical distribution approach in density estimation.

In either approach, the underlying distribution is assumed to be the same for the historical sample data. Recent findings in the conditional heteroskedasticity of financial asset returns clearly violate this assumption. In fact, the discovery of conditional heteroskedasticity revolutionizes the academic research in optimal hedge ratios. Essentially, the minimum variance hedge ratios must be time varying. This criticism applies to the LPM hedge ratios as well. Herein one expects the joint distribution of $r_p$

[2]For given $W_0$, $p_0$ and $Q$, the ranking of $r_p + \theta r_f$ is identical to the ranking of $W_1$. Therefore, the lower partial moment method based upon the former is consistent with the expected utility hypothesis.

[3]Lee and Rao (1988) considered the co–lower partial moment between two assets where each asset is subject to an independent target return. In our case, the target return is imposed on the linear combination of the two assets; that is, $r_p + \theta r_f \leq c$. When modifying the notation of Lee and Rao (1988), one can express the optimal hedge ratio as the ratio of the co–lower partial moment of spot and futures returns to the lower partial moment of the futures return. Because the target return is imposed on $r_p + \theta r_f$, the integral required for the co–lower partial moment calculation has an upper bound that is a function of $\theta$ (which is different from the case in Lee and Rao, 1988). As a result, we do not have analytical expressions. Numerical methods are required to derive the optimal hedge ratio. For any given $\theta$ and the joint distribution of spot and futures returns, the lower partial moment can be calculated. A grid search is then adopted to find the optimal value of $\theta$ that minimizes the lower partial moment.

and $r_f$ to vary over time. As a consequence, the LPM hedge ratios must be time-varying. To account for this property, parametric specifications of the joint distribution deem necessary.

## LPM HEDGE RATIOS UNDER THE BIVARIATE NORMALITY ASSUMPTION

Suppose that $r_p$ and $r_f$ are bivariate normal variables.[4] The joint distribution is characterized by the two means $\mu_i = E(r_i)$, $i = p, f$, and by the variance–covariance terms $\sigma_{ij} = \text{cov}(r_i, r_j)$, $i, j = p, f$. Let $f(.,.)$ denote the joint density function, then equation (1) can be rewritten as follows:

$$l(c, n, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{c-\theta r_f} (c - r_p - \theta r_f)^n f(r_p, r_f) dr_p dr_f. \qquad (8)$$

Upon taking the partial derivative of $l(c, n, m)$ with respect to $\theta$, we conclude that the LPM hedge ratio, $\theta^*$, must satisfy the following condition:

$$-\int_{-\infty}^{\infty} \int_{-\infty}^{c-\theta^* r_f} n(c - r_p - \theta^* r_f)^{n-1} r_f f(r_p, r_f) dr_p dr_f = 0. \qquad (9)$$

Let $\mu = (\mu_p + \theta\mu_f - c)\gamma$ where $\gamma^{-2} = \theta^2\sigma_{ff} + 2\theta\sigma_{pf} + \sigma_{pp}$. After some algebraic manipulation (see the Appendix), equation (9) implies

$$\int_{\mu}^{\infty} \gamma^{-1}(w - \mu)^n (\theta\sigma_{ff} + \sigma_{pf}) \exp(-w^2/2) dw$$

$$= \int_{\mu}^{\infty} (w - \mu)^{n-1}[(c - \mu_p)(\theta\sigma_{ff} + \sigma_{pf})$$

$$+ \mu_f(\theta\sigma_{pf} + \sigma_{pp})] \exp(-w^2/2) dw, \qquad (10)$$

when evaluated at $\theta = \theta^*$.

In the case of an unbiased futures market, $E(r_f) = \mu_f = 0$. Equation (10) implies that the LPM hedge ratio is established at $\theta^* = -\sigma_{pf}/\sigma_{ff}$, the same as the minimum variance (MV) hedge ratio. This result is consistent with Lence (1995). In fact, given the normality assumption and given the unbiasedness of the futures market, $-\sigma_{pf}/\sigma_{ff}$ is the optimal hedge ratio for expected utility maximization, regardless of the utility

[4]The bivariate normality assumption is adopted for operational purpose. Other multivariate distributions can be considered. The estimation will be much more complicated. In many cases, bivariate normality distributions are good approximations.

functional form. On the other hand, if the futures market is biased, so that $\mu_f$ is nonzero, the LPM hedge ratio will not be equal to the MV hedge ratio.

However, to derive the LPM ratio, it is easier to evaluate LPM directly. Specifically, equation (8) can be rewritten as follows (see the Appendix):

$$l(c, n, m) = \int_{\mu}^{\infty} \gamma^{-n}(w - \mu)^n \phi(w)dw, \tag{11}$$

where $\phi(\cdot)$ is the probability density function of a standardized normal variable. Equation (11) can be evaluated recursively (Patel and Read, 1982). More specifically, let

$$I_n(\mu) = \int_{\mu}^{\infty} w^n \phi(w)dw. \tag{12}$$

Then we have $I_0(\mu) = 1 - \Phi(\mu)$ and $I_1(\mu) = \phi(\mu)$, where $\Phi(\cdot)$ is the cumulative distribution function of a standardized normal variable. Moreover, for $n \geq 2$,

$$I_n(\mu) = \mu^{n-1} \phi(\mu) + (n - 1) I_{n-2}(\mu). \tag{13}$$

For a given hedge ratio $\theta$, we can compute $\gamma$ and $\mu$ when the joint distribution of $r_p$ and $r_f$ is known. After expanding the integrand of equation (11) into a sum of polynomials of $w$, equation (13) can be directly applied to calculate $l(c, n, m)$.

## DATA DESCRIPTION AND MODEL SPECIFICATIONS

We consider futures hedging in the Nikkei Stock Average (NSA) futures markets. Our data consist of daily observations of spot and futures prices of NSA from January 1989 through August 1996. The NSA futures is traded in the Singapore International Monetary Exchange (SIMEX) and is one of the most liquid contracts in the exchange. The regular contract months are March, June, September, and December, with maturity on the third Wednesday of the contract month. For the futures series, the settlement prices of the nearest contract month are used, with transition to the contract for the next regular contract month occurring at about ten days before contract expiry.[5] For the spot series, the closing index values are used. Thus, the data set consists of 1861 daily observations.

Let $s_t$ and $f_t$ denote the logarithmic spot and futures prices, respectively. Defining $z_t = s_t - f_t$, we consider the following bivariate error correction model:

$$\Delta s_t = \alpha_{s1} + \alpha_{s2} z_{t-1} + \alpha_{s3} \Delta s_{t-1} + \alpha_{s4} \Delta f_{t-1} + \alpha_{s5} (\sigma_{11,t})^{1/2} + \varepsilon_{st}, \qquad (14)$$

$$\Delta f_t = \alpha_{f1} + \alpha_{f2} z_{t-1} + \alpha_{f3} \Delta s_{t-1} + \alpha_{f4} \Delta f_{t-1} + \alpha_{f5} (\sigma_{22,t})^{1/2} + \varepsilon_{ft}, \qquad (15)$$

where $\Delta$ is the backward difference operator, and $\sigma_{11,t}$ and $\sigma_{22,t}$ are, respectively, the conditional variance of $\varepsilon_{st}$ and $\varepsilon_{ft}$. The error-correction models have been used to estimate the hedge ratios by, among others, Lien and Luo (1993) and Wahab and Leshgari (1993). Antoniou and Foster (1994) and Brenner and Kroner (1995) discuss the possibility of cointegration relationship between spot and futures prices.[6] Lien (1996) analyzes the effects of the cointegration relationship on minimum variance hedge ratios and hedging effectiveness.

In the formulation above the conditional standard deviations are allowed to have effects on the conditional means, resulting in an GARCH-M type of model. Denoting $\varepsilon_t = (\varepsilon_{st}\ \varepsilon_{ft})'$, we assume that the conditional distribution of a $\varepsilon_t$ follows a bivariate normal distribution. Let $\Psi_{s-1}$ denote

[5]A few recent articles expressed concerns over the popular adoption of nearby futures price series in the literature. Geiss (1994) and Rougier (1996) argued that a combination of the prices of a few futures contracts with different maturities may better track the spot price. Barrett, Li, and Thilmany (1996) demonstrated wheat futures price series corresponding to different maturities display different statistical properties. Thus, the homogeneity assumption implicitly adopted in constructing nearby futures price series is invalid. Although these issues are genuine, there are merits for adopting nearby futures prices. In analyzing the statistical behavior, a large sample size is required to ensure reliability of the results. If one considers a particular contract, the number of eligible observations would be small. In the early period there is little trading. At the maturity month, the maturity effect prevails. Elimination of these two periods usually leaves a researcher with 3 to 4 months of possible data points. Moreover, the nearby futures contract represents the most liquid contract. A weekly return series based upon nearby contracts represents the case of going to the market and buying (or selling) the most liquid futures contract and then lifting the position a week later.

[6]For a specific futures contract, the cost-of-carrying model dictates a cointegration relationship between spot and futures prices with a trend term that is related to carrying charge. When considering spot and nearby futures price series, there is conflicting evidence. Strictly speaking, the nearby futures price series consists of heterogenous data points, and one may not expect to find a cointegration relationship between spot and nearby futures prices. However, Lien, and Luo (1993) argued that the effect of heterogeneity may be rather small and, consequently, cointegration is established in many empirical cases. For our study, statistical tests are adopted to validate the prevalence of the cointegration relationship.

the information set a time $t - 1$, then $\varepsilon_t \mid \Psi_{s-1} \sim N(0, \Omega_t)$, where $\Omega_t$ is a $2 \times 2$ matrix with elements $\sigma_{ij,t}; i, j = 1, 2$. Several conditional variance models of the GARCH type have been proposed in the literature. These include the models due to Engle, Ng, and Rothschild (1990) and Baillie and Myers (1991). In this paper we adopt the parsimonious model suggested by Bollerslev (1990). This model assumes that the correlation coefficients of the error terms remain constant through time.[7] It has been applied by Park and Switzer (1995) in their work on estimating the hedge ratios of stock index futures. Furthermore, to incorporate the possible asymmetric effects in the conditional volatility of stocks as shown by Engle and Ng (1993), we extend the asymmetric power ARCH (APARCH) model due to Ding, Granger, and Engle (1993) to the bivariate case. Let $\sigma_{i,t} = (\sigma_{ii,t})^{1/2}$. We consider the following conditional variance equations:

$$\sigma_{1,t}^{\delta_s} = \beta_{s1} + \beta_{s2}\,(|\varepsilon_{s,t-1}| - \beta_{s3}\varepsilon_{s,t-1})^{\delta_s} + \beta_{s4}\sigma_{1,t-1}^{\delta_s}, \tag{16}$$

$$\sigma_{2,t}^{\delta_f} = \beta_{f1} + \beta_{f2}\,(|\varepsilon_{f,t-1}| - \beta_{f3}\varepsilon_{f,t-1})^{\delta_f} + \beta_{f4}\sigma_{2,t-1}^{\delta_f}, \tag{17}$$

$$\sigma_{12,t} = \rho\sigma_{1,t}\sigma_{2t}, \tag{18}$$

where $\delta_i$, $\beta_{i1}$ $\beta_{i2}$, and $\beta_{i4}$ are nonnegative, and $\beta_{i3}$ and $\rho$ are between $-1$ and 1 for $i = s, f$. This formulation nests the standard deviation form ($\delta_i = 1$) as well as the variance form ($\delta_i = 2$) as special cases. When $\beta_{i3}$ is positive, negative return shocks have larger effects on the volatility than positive return shocks of the same magnitude. This is termed the leverage effect for equities.

We first applied the Augmented Dickey Fuller (ADF) tests to confirm the unit roots in the two logarithmic price series. Then the Granger-Engle two-step procedure is applied to establish the cointegration relationship. The resulting cointegration variable is $s_t - 0.9877\,f_t$, which is very close to $z_t$. Note that $z_t$ is the so-called basis, the behavior of which is well studied in the futures market literature. Following the convention, we treat $z_t$ as the outright cointegration variable. Table I displays some statistical properties of the return series.

Both spot and futures returns have negative means. The variances are nearly the same, about 2.0. The two returns are positively skewed with excessive kurtosis. The $Q$ statistics for the squared returns and the absolute returns indicate the possibility of GARCH effects. Overall, the

[7]The assumption of a constant correlation coefficient imposes restrictions on the time path of optimal hedge ratios. However, this assumption is well accepted in the GARCH literature as a first attempt. Without this assumption, a time-series process needs to be specified for the correlation coefficient or the covariance. Any specification must satisfy the requirement that the variance-covariance matrix is positive definite. As a result, more parameters need to be estimated.

**TABLE I**

Summary Statistics of the Spot and Futures Returns

| *Statistics* | *Spot Returns* | *Futures Returns* |
|---|---|---|
| Mean | −0.0216 | −0.0380 |
| Variance | 2.0622 | 2.1984 |
| Standardized skewness | 7.5812 | 6.9152 |
| Standardized kurtosis | 50.878 | 45.213 |
| $Q$ | 46.286 | 39.790 |
| $Q^2$ | 413.68 | 450.71 |
| $Q^a$ | 1554.3 | 1429.7 |
| Number of observations | 1861 | 1861 |

Note: $Q$ is the Box-Pierce portmanteau statistic for the correlation of the return series. Similarly, $Q^2$ is the statistic for the squared return series and $Q^a$ is statistic for the absolute return sales. All portmanteau statistics are evaluated with 24 lags.

**TABLE II**

Estimation Results of the Bivariate APARCH-M Model

| *Parameter* | *Estimated Value* | *Standard Error* |
|---|---|---|
| $\alpha_{s1}$ | 0.2117 | 0.0628 |
| $\alpha_{s2}$ | 0.1355 | 0.0084 |
| $\alpha_{s3}$ | −0.3959 | 0.0136 |
| $\alpha_{s4}$ | 0.4486 | 0.0131 |
| $\alpha_{s5}$ | −0.1260 | 0.0563 |
| $\alpha_{f1}$ | 0.1063 | 0.0612 |
| $\alpha_{f5}$ | −0.1299 | 0.0543 |
| $\delta_s$ | 1.2173 | 0.1081 |
| $\beta_{s1}$ | 0.0544 | 0.0064 |
| $\beta_{s2}$ | 0.0879 | 0.0075 |
| $\beta_{s3}$ | 0.4311 | 0.0486 |
| $\beta_{s4}$ | 0.8897 | 0.0092 |
| $\delta_f$ | 1.1655 | 0.1007 |
| $\beta_{f1}$ | 0.0453 | 0.0054 |
| $\beta_{f2}$ | 0.0743 | 0.0063 |
| $\beta_{f3}$ | 0.4535 | 0.0517 |
| $\beta_{f4}$ | 0.9090 | 0.0078 |
| $\rho$ | 0.9580 | 0.0014 |

choice of variables in equations (14), (15), and the APARCH-M model specifications seem appropriate.

Estimation results for the NSA spot and futures returns (after eliminating statistically insignificant parameters) are presented in Table II. The expected futures return is not affected by the lagged spot and futures returns, nor by the cointegrating variable. Whereas the constant term for the futures return equation is positive at the 10% level, the standard deviation has a negative impact at the 1% level. Thus, the more risky (in the conventional sense) the futures market, the lower is the expected return.[8] For our study, the most important result is that the conditional futures return does not have a zero mean. Consequently, we expect the LPM hedge ratio to differ from the MV ratio.

The expected spot return, on the other hand, is affected by all explanatory variables at the 1% level. Like the futures series, the constant term is positive but the coefficient associated with the standard deviation is negative. Lagged spot and futures returns have opposite effects; the former leads to a larger current return but the latter a smaller current return. That is, there is mean reversion in the spot index. The cointegrating variable affects the spot return adversely.

From the above discussions, we conclude that the futures return causes but is not caused by the spot return in terms of Granger causality. That is, the NSA futures is a dominant market and the NSA index itself serves as a satellite market, following the terminology of Garbade and Silber (1983).

All parameters in the two conditional variance equations are significant at the 1% level. The estimated powers ($\delta_s$ and $\delta_f$) are both close to one, in favor of the standard deviation form. The leverage effect prevails in both series. Negative return shocks to the NSA spot and futures produce larger impact than positive shocks of the same magnitude. The result is consistent with other findings for the equity markets in the literature. Both $\beta_{s4}$ and $\beta_{f4}$ are close to one (0.8897 and 0.9090, respectively), indicating strong persistence in conditional volatility. Finally, as expected, the two returns are highly correlated. The correlation coefficient is 0.958.

## TIME-VARYING HEDGE RATIOS

The estimation results are applied to derive the conditional joint distributions of the spot and futures returns. For every sampled observation, we calculate the estimated means and variance–covariance matrix. Be-

[8]Although the result appears to be at odds with usual perceptions, there are possible explanations. For example, if risk aversion is positively related to the volatility of the market, then it is possible for the return to be negatively related to the volatility. Also, as we emphasize the importance of the downside risk, the tradeoff should be between the return and the lower partial moment (not the variance). In the empirical literature, it is not unusual to find that return and risk are negatively related. The result is named the "Bowman Paradox"; see Bowman (1980) and Ruefli (1990). Possible explanations are provided in Ruefli (1990) and Brockett and Graven (1994). In general, it is agreed that the negative coefficient derived from the econometric estimation does not conflict with the risk-return tradeoff envisioned in the theoretical literature.

**TABLE III**

Summary Statistics of the Time-Varying Hedge Ratios

| *Hedge Strategy* | *−1.5%* | *−1.0%* | *−0.5%* | *0%* | *0.5%* | *1.0%* | *1.5%* |
|---|---|---|---|---|---|---|---|
| LPM1 | 0.8251 | 0.9124 | 0.9306 | 0.9418 | 0.9964 | 1.1514 | 1.3239 |
| | (0.1207) | (0.0694) | (0.0579) | (0.0573) | (0.0821) | (0.1804) | (0.2856) |
| LPM2 | 0.7905 | 0.8950 | 0.9285 | 0.9327 | 0.9403 | 0.9521 | 0.9658 |
| | (0.1340) | (0.0809) | (0.0582) | (0.0575) | (0.0569) | (0.0571) | (0.0586) |
| LPM3 | 0.7680 | 0.8767 | 0.9268 | 0.9304 | 0.9344 | 0.9398 | 0.9461 |
| | (0.1494) | (0.0933) | (0.0587) | (0.0577) | (0.0573) | (0.0571) | (0.0573) |
| LPM4 | 0.7677 | 0.8688 | 0.9245 | 0.9293 | 0.9319 | 0.9353 | 0.9393 |
| | (0.1775) | (0.1086) | (0.0597) | (0.0578) | (0.575) | (0.0573 | (0.0572) |
| MV | 0.9230 | | | | | | |
| | (0.0581) | | | | | | |

Note: The hedge strategy LPM1 is the sequence of hedge ratios that minimizes the lower partial moment of order 1. Similar definitions apply to LPM2, LPM3, and LPM4. The minimum variance hedge strategy is denoted by MV. The two numbers in each cell correspond to the mean and the standard deviation (i.e., the number within the parenthesis). For example, when the target return is −1.0%, the mean of the LPM3 hedge ratios is 0.8767 and the standard deviation is 0.0933.

cause of the normality assumptions, these parameters fully characterize the joint distribution. The MV hedge ratios can be calculated directly whereas the method discussed in the above section helps determine the LPM ratios.[9] Note that, due to the normality assumptions, in our study the two hedge ratios differ from each other due to a distribution that is symmetric with respect to the mean, which is, incidently, not equal to the target return. Also note that the hedge ratios we calculated are ex post within sample hedge ratios. They are derived with the knowledge of the whole sample.

We consider four lower partial moments ($n = 1, 2, 3, 4$) and seven target returns ($c = -1.5\%, -1.0\%, -0.5\%, 0\%, 0.5\%, 1.0\%, 1.5\%$). Table III provides a summary of the estimated time-varying LPM and MV hedge ratios. For positive target returns, the LPM1 (lower partial moment of order 1) hedge ratio behaves quite differently from others and is much larger. When $c = -1.5\%$ or $-1.0\%$, all LPM hedge ratios are on average smaller than the MV ratio, regardless of the order. Also, the difference is larger for a higher order LPM. For other values of $c$, the LPM ratios are on average larger than the MV ratio, and the gap shrinks as the LPM order increases.

That is, based upon the mean comparisons, the largest difference between the LPM and MV hedge strategies occurs when the LPM order

[9]By applying the maximum likelihood estimation, the parameter estimates for equations (14) through (18) are asymptotically efficient. Because LPM ratios are functions of these parameters, the estimated LPM ratios derived from the estimated parameters are also asymptotically efficient.

**TABLE IV**

Correlation Coefficients between the LPM and MV Hedge Ratios

| *Hedge Ratios* | *−1.5%* | *−1.0%* | *−0.5%* | *0%* | *0.5%* | *1.0%* | *1.5%* |
|---|---|---|---|---|---|---|---|
| LPM1 LPM2 | 0.9860 | 0.9530 | 0.9964 | 0.9495 | 0.5359 | 0.3007 | 0.2896 |
| LPM1 LPM3 | 0.9612 | 0.8674 | 0.9897 | 0.9384 | 0.4785 | 0.2095 | 0.1695 |
| LPM1 LPM4 | 0.9166 | 0.7756 | 0.9666 | 0.9332 | 0.4573 | 0.1777 | 0.1257 |
| LPM1 MV | 0.3533 | 0.8106 | 0.9873 | 0.9112 | 0.3989 | 0.1047 | 0.0285 |
| LPM2 LPM3 | 0.9869 | 0.9707 | 0.9963 | 0.9992 | 0.9964 | 0.9894 | 0.9782 |
| LPM2 LPM4 | 0.9257 | 0.9013 | 0.9752 | 0.9982 | 0.9929 | 0.9798 | 0.9586 |
| LPM2 MV | 0.2881 | 0.6571 | 0.9917 | 0.9892 | 0.9723 | 0.9364 | 0.8802 |
| LPM3 LPM4 | 0.9021 | 0.9429 | 0.9879 | 0.9998 | 0.9994 | 0.9984 | 0.9968 |
| LPM3 MV | 0.2398 | 0.5258 | 0.9851 | 0.9940 | 0.9879 | 0.9764 | 0.9582 |
| LPM4 MV | 0.1979 | 0.4194 | 0.9586 | 0.9958 | 0.9925 | 0.9868 | 0.9779 |

is large and the target return is negative with a large absolute value, or when the LPM order is two and the target return is large. Similar results are obtained in Lien and Tse (1997) when the hedge ratios are assumed to be time invariant.

However, due to large variations in the hedge ratios, statistical tests fail to reject the null hypothesis that the LPM ratio (of any order) and the MV ratio have the same mean, regardless of the $c$ value.

When $c$ is greater than $-1.0\%$, the variance of the LPM ratios (with order greater than one) and that of the MV ratio are very similar. In fact, the two series are usually highly correlated. The smallest correlation coefficient, with the value of 0.8802, is obtained when $n = 2$ and $c = 1.5\%$. For the other two cases, the LPM ratios exhibit greater variations than the MV ratio. Also, the variance of the LPM ratios increases with increasing order. These ratios are only slightly correlated with the MV ratio. When $c = -1.0\%$, the correlation coefficient is 0.8106, 0.6571, 0.5258, and 0.4191, for $n = 1, 2, 3,$ and 4 respectively. The coefficient becomes even smaller, namely, 0.3533, 0.2881, 0.2398, and 0.1979 respectively when $c = -1.5\%$. Thus, when $n$ is large and $c$ is small and negative, we expect the LPM hedge ratio to differ from the MV ratio not only in mean

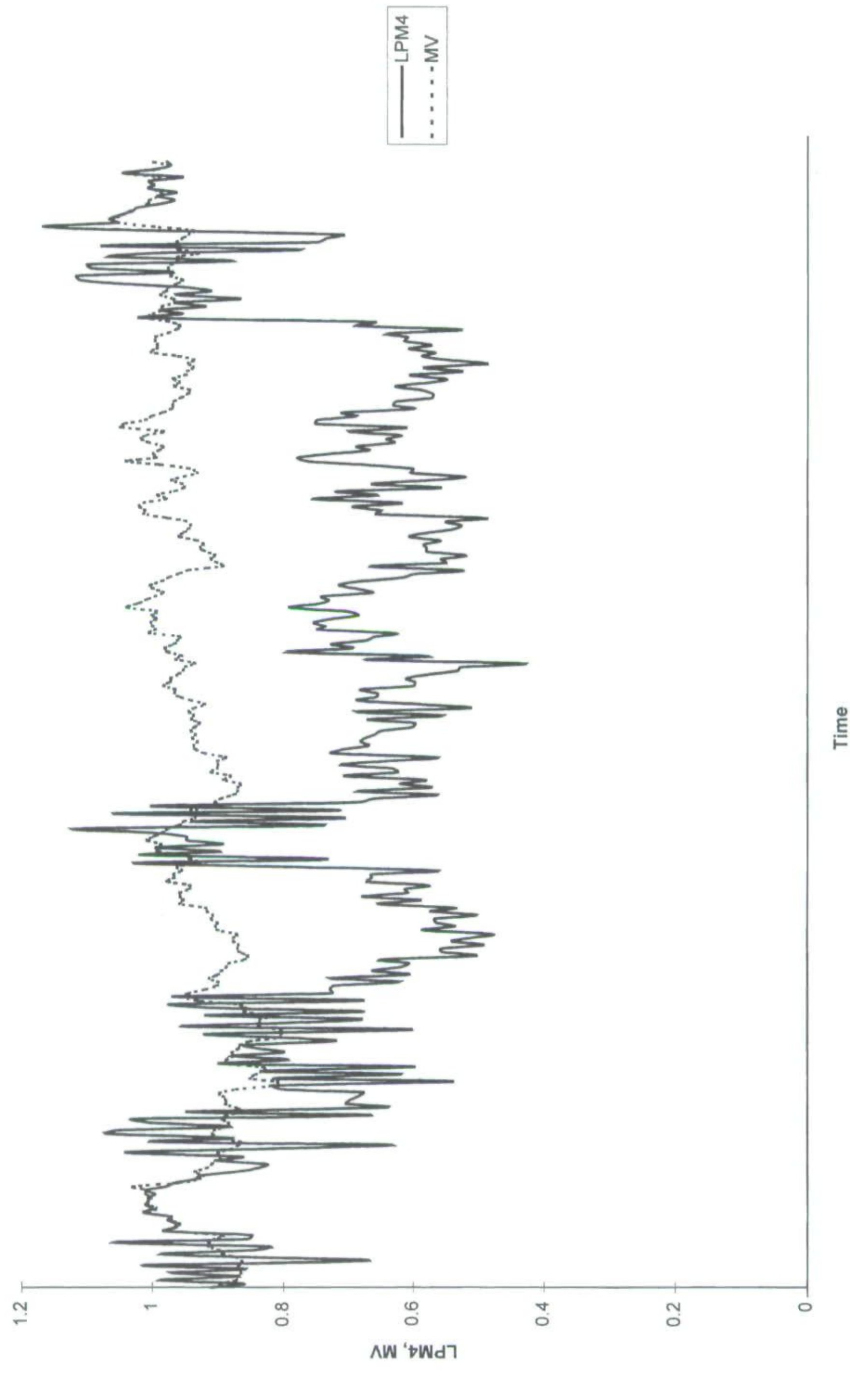


**FIGURE 1**
Observations 1200–1500.

but also in the time-varying pattern. Figure 1 displays the LPM ratios with $n = 4$ and $c = -1.5\%$ versus the MV ratios. To demonstrate the different movements of the hedge ratios, we consider only a subsample. This graph is, however, representative of the general pattern.

## CONCLUSIONS

This article constructs a bivariate APARCH-M model for the spot and futures returns and derives the time-varying lower partial moment hedge ratios. The method is applied to the Nikkei Stock Average index. The resulting LPM ratios exhibit strong variations (especially when the target return falls below $-1.0\%$), indicating the assumption of a time-invariant LPM hedge ratio cannot be accepted. When compared to the minimum variance hedge ratio, we found that the major diversion appears when the order of the lower partial moment is large and the target return is negative and small. For example, the hedge ratio derived for the fourth-order lower partial moment, with the target return being $-1.5\%$, displays little correlation with the MV hedge ratio. These cases exemplify the importance of the distinction between the downside risk and the usual two-sided risk.

## APPENDIX

Beginning with equation (9), we apply the variable transformation method by changing the variables from $r_p$ and $r_f$ to $x$ and $y$, where $x = c - r_p - \theta r_f$ and $y = r_f$. As a result, equation (9) can be rewritten as follows:

$$\int_0^{\infty}\int_{-\infty}^{\infty} nyx^{n-1}f(c - x - \theta y, y)dydx = 0. \qquad \text{(A1)}$$

The joint density $f(c - x - \theta y, y)$ can be decomposed as $(1/(2\pi\Delta))AB$, where $\Delta^2 = \sigma_{pp}\sigma_{ff} - (\sigma_{pf})^2$, and

$$A = \exp((-1/2)(y - (c - x - \mu_p)(\theta\sigma_{ff} + \sigma_{pf})\gamma^2 - \mu_f(\theta\sigma_{pf} + \sigma_{pp})\gamma^2)\gamma^{-2}\Delta^{-2}), \qquad \text{(A2)}$$

$$B = \exp((-1/2)(c - x - \mu_p - \theta\mu_f)^2\gamma^2). \qquad \text{(A3)}$$

For the definition of $\gamma$, see the text. Upon substituting the above decomposition into equation (A1) and completing the integration with respect to y, we obtain

$$\int_0^\infty n\gamma^2 x^{n-1}((c - x - \mu_p)(\theta\sigma_{ff} + \sigma_{pf}) + \mu_f(\theta\sigma_{pf} + \sigma_{pp}))$$

$$\exp\left[\frac{(c - x - \mu_p - \theta\mu_f)^2\gamma^2}{-2}\right] dx = 0. \tag{A4}$$

Next, we define $w = (x - c + \mu_p + \theta\mu_f)\gamma$. After a change of variable from $x$ to $w$, the above equation reduces to equation (10).

The above procedure is applied to derive equation (11). Herein we begin with equation (8) and apply the variable transformation from $(r_p, r_f)$ to $(x, y)$. The result is

$$l(c, n, m) = \int_0^\infty \int_{-\infty}^\infty x^n f(c - x - \theta y, y) dy dx. \tag{A5}$$

Using the decomposition of $f(\cdot,\cdot)$ and completing the integration with respect to $y$, the above equation can be written as follows:

$$l(c, n, m) = (2\pi)^{-1/2} \int_o^\infty x^n \gamma \exp\left[\frac{(c - x - \mu_p - \theta\mu_f)^2\gamma^2}{-2}\right] dx. \tag{A6}$$

A change of variable from $x$ to $w = (x - c + \mu_p + \theta\mu_f)\gamma$ then leads to eq. (11).

## BIBLIOGRAPHY


Adams, J., and Montesi, C. J. (1995): *Major Issues Related to Hedge Accounting.* Newark, CT: Financial Accounting Standard Board.

Ahmadi, H. Z., Sharp, P. A., and Walther, C. H. (1986): "The Effectiveness of Futures and Options in Hedging Currency Risk," *Advances in Futures and Options Research,* 1: 171–191.

Antoniou, A., and Foster, A. J. (1994): "Short Term and Long Term Efficiency in Commodity Spot and Futures Markets," *Financial Markets, Institutions & Instruments,* 3: 17–35.

Baillie, R. T., and Myers, R. J. (1991): "Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge," *Journal of Applied Econometrics,* 6: 109–124.

Barrett, C. B., Li, J., and Thilmany, D. D. (1996): "The Fallacy of Nearby Contract Commodity Futures Price Analysis: Intramarket Futures Contracts Are Not Identically Distributed," mimeo, Utah State University, Logan, Utah.

Bawa, V. S. (1975): "Optimal Rules for Ordering Uncertain Prospects," *Journal of Financial Economics,* 2: 95–121.

Bawa, V. S. (1978): "Safety-First, Stochastic Dominance and Optimal Portfolio Choice," *Journal of Financial and Quantitative Analysis,* 33: 255–271.

Bollerslev, T. (1990): "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH model," *Review of Economics and Statistics,* 72: 498–505.

Brenner, R. J., and Kroner, K. F. (1995): "Arbitrage, Cointegration, and Testing the Unbiasedness Hypothesis in Financial Markets," *Journal of Financial and Quantitative Analysis,* 30: 23–42.

Bowman, E. H. (1980): "A Risk/Return Paradox for Strategic Management," *Sloan Management Review,* 17–31.

Brockett, P. L., and Graven, J. R. (1994): "A Reexamination of the Relationship between Preferences and Moment Orderings by Rational Risk Averse Investors," mimeo, University of Texas, Austin, Texas.

Ding, Z., Granger, C. W. J., and Engle, R. F. (1993): "A Long Memory Property of Stock Market Returns and a New Model," *Journal of Empirical Finance,* 1: 83–106.

Engle, R. F., Ng, V. K., and Rothschild, M. (1990): "Asset Pricing with a Factor-ARCH Covariance Structure: Empirical Estimates for Treasury Bills," *Journal of Econometrics,* 45: 213–237.

Engle, R. F., and Ng, V. (1993): "Measuring and Testing the Impact of News on Volatility," *Journal of Finance,* 48: 1749–1778.

Fishburn, P. J. (1977): "Mean-Risk Analysis with Risk Associated with Below-Target Returns," *American Economic Review,* 67: 116–126.

Garbade, K. D., and Silber, W. L. (1983): "Price Movement and Price Discovery in Futures and Cash Markets," *Review of Economics and Statistics,* 65: 289–297.

Geiss, C. (1995): "Distribution-Free Futures Price Series," *Journal of Futures Markets,* 15: 805–831.

Hansen, B. E. (1994): "Autoregressive Conditional Density Estimation," *International Economic Review,* 35: 705–730.

Harlow, W. V. (1991): "Asset Allocation in a Downside-Risk Framework," *Financial Analysts Journal,* 47: 28–40.

Homaifar, G., and Graddy, D. B. (1990): "Variance and Lower Partial Moment Betas as Alternative Risk Measure in Cost of Capital Estimation: A Defense of the CAPM Beta," *Journal of Business, Finance and Accounting,* 17: 677–688.

Lee, W. Y., and Rao, R. (1988): "Mean Lower Partial Moment Valuation and Lognormally Distributed Returns," *Management Science,* 34: 446–453.

Lence, S. H. (1995): "On the Optimal Hedge under Unbiased Futures Prices," *Economics Letters,* 47: 385–388.

Lien, D., and Luo, X. (1993): "Estimating Multiperiod Hedge Ratios in Cointegrated Markets," *The Journal of Futures Markets,* 13: 909–920.

Lien, D. (1996): "The Effect of the Cointegration Relationship on Futures Hedging: A Note," *Journal of Futures Markets,* 16: 773–780.

Lien, D., and Tse, Y. K. (1997): "Hedging Downside Risk with Futures Contracts," mimeo, University of Kansas, Lawrence, Kansas.

Mao, J. (1970): "Models of Capital Budgeting, E-V vs E-S," *Journal of Financial and Quantitative Analysis,* 4: 657–675.

Markowitz, H. M. (1959): *Portfolio Selection.* New Haven: Yale University Press.

Markowitz, H. M. (1990): "Foundations of Portfolio Theory," *Journal of Finance,* 46: 469–477.

Park, T. H., and Switzer, L. N. (1995): "Bivariate GARCH Estimation of the Optimal Hedge Ratios for Stock Index Futures: A Note," *Journal of Futures Markets,* 15: 61–68.

Patel, J. K., and Read, C. B. (1982): *Handbook of the Normal Distributions.* Marcel Dekker: New York.

Price, K., Price, B., and Nantell, T. J. (1982): "Variance and Lower Partial Moment Measures of Systematic Risk: Some Analytical and Empirical Results," *Journal of Finance,* 37: 843–855.

Reiss, R. D. (1981): "Nonparametric Estimation of Smooth Distribution Functions," *Canadian Journal of Statistics,* 8: 116–119.

Rougier, J. (1996): "An Optimal Price Index for Stock Index Futures Contracts," *Journal of Futures Markets,* 16: 189–199.

Ruefli, T. W. (1990): "Mean-Variance Approaches to Risk-Return Relationships in Strategy: Paradox Lost," *Management Science,* 36: 368–380.

Tse, Y. K. (1995): "Lead-lag Relationship between Spot and Futures Price of the Nikkei Stock Average," *Journal of Forecasting,* 14: 553–563.

Wahab, M., and Leshgari, M. (1993): "Price Dynamics and Error Correction in Stock Index and Stock Index Futures Markets: A Cointegration Approach," *Journal of Futures Markets,* 13: 711–742.