Stochastic Volatility Functions Implicit in Eurodollar Futures Options

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INTRODUCTION

In finance literature, most models written to capture the dynamics of the short-term interest rate are continuous time diffusions based on the specification

$$dr = \mu(r,t)dt + \sigma(r,t)dz_1 \tag{1}$$

[e.g., Vasicek (1977) and Cox, Ingersoll, and Ross (1985)], with various specifications for the functions μ and σ. This article presents tests of five models for the short-term interest rate based on equation (1). The article augments current literature on tests of the short-term interest rate process by discussion posed along three lines of inquiry. First, whether moving from a deterministic volatility single factor model to a stochastic volatility specification improves overall pricing ability (in terms of prediction and in-sample errors in interest rate option data). Second, whether stochastic volatility can explain the observed pattern of implicit volatilities in interest rate option data by reducing pricing biases related to strike price and maturity. Third, whether time-series data based volatility estimates are consistent with implied volatility parameters (interpreted as

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market forecasts of expected future volatility). Since most practitioners use implied volatility estimates in hedging and pricing applications, it is important to see whether our chosen model has implied volatility estimates that are consistent with the realized time series estimates of volatility.

Empirical investigation has revealed salient features of interest rate behavior that shed light on the validity of some specifications for the functions μ and σ in equation (1). Chan, Karoyli, Longstaff, and Sanders (CKLS, 1992) estimate parameters in equation (1) using various functional forms for the drift and volatility. They show that conditional and unconditional moments of changes in the interest rate, obtained from analysis of Treasury bill data, point to volatility that is time varying and dependent on the level of current interest rates [see also Anderson and Lund (1996) and Chacko (1997)]. Extensive literature on ARCH and GARCH modeling provides further corroboration of time varying and stochastic volatility for the changes in the short term interest rate [see Bollerslev, Chou, and Kroner (1992) for a survey]. Models in the Heath-Jarrow-Morton (HJM, 1992) class have been tested under various volatility specifications for the forward rates [Flesaker (1992)]. For reasons of tractability, the empirical work on this class of models is usually restricted to deterministic volatility or to volatility specifications that are dependent on the level of forward rates. In this article, stochastic volatility is incorporated in the specification for short-term interest rates based on eq. (1).

A second notable empirical observation from the interest rate option markets is evidence of a "volatility smile" or U-shaped pattern of implicit volatility across different strike prices. This indicates excess skewness and kurtosis in the terminal distribution of interest rates. This observation needs to be reconciled with existing models of the short-term interest rate. Although deterministic volatility models can accurately price at the money options, they show significant pricing biases with respect to strike price and maturity. Different specifications have been proposed to correct these observed biases, where some specifications depart from the functional form in eq. (1). Most recently, Merton's jump diffusion model, coupled with a stochastic volatility model has been advanced to explain the volatility smile in the currency markets (Bates, 1996). Other examples include application of Student's t (Rogalski and Vinso, 1978) and fat-tailed distributions of the stable Paretain type (Westerfield, 1977). This article investigates the impact of a stochastic volatility specification on the pricing of deep out-of-the-money and in-the-money options.

In this article, five models for the short-term interest rate are tested, and model parameter values implied by prices in Eurodollar futures options are estimated. The models specified in CKLS (1992) are augmented by adding a stochastic volatility specification. This article estimates model parameters implied in three-month Eurodollar futures options prices, while CKLS (1992) use historical Treasury-bill data for their tests. Using implicit distributions potentially exploits a richer information set than that available from time series analysis. According to Bates (1996), "implicit volatilities from option prices should theoretically summarize all relevant information regarding expected future volatilities whereas univariate ARCH and GARCH approaches can exploit only the subset of that information inferable from the past history of option prices." Data is obtained from the Commodities and Futures Trade Commission (CFTC). For each model, a time series of the implied volatility parameters is tabulated. Using these estimated parameters, the abilities of these models to predict the next day's and next week's option prices are tested. Some systematic discrepancies are noted between the various model and market prices relative to option type, moneyness and maturity for the in-sample data and out-of-sample data. It is shown that a stochastic volatility model reduces observed pricing biases in out-of-the-money and at-the-money options and explains a part of the volatility smile. However, the Geometric Brownian Motion specification has the best out-of-sample performance in terms of prediction errors and parameter stability.

The paper is organized as follows: The upcoming section describes the stochastic volatility models; the subsequent section is a note on Eurodollar futures; the next section is a description of data, followed by two sections that describe the tests; the final two sections are a discussion and a conclusion.

STOCHASTIC VOLATILITY MODELS

Introduction of stochastic volatility models represents a significant growth in our understanding and modeling of derivative markets. Published advances in this area are represented by the work of Hull and White (1987), Wiggins (1987), Chesney and Scott (1989), Stein and Stein (1991), and Heston (1992), among others. Ball (1993) provides a comprehensive survey of the topic. A stochastic volatility characterization assumes that volatility itself evolves as a stochastic process. The standard framework assumes a bivariate diffusion process for the interest rate process, r.

$$dr = \mu(t,r)dt + \sigma(r,t)dz_1 \tag{2}$$

and its volatility specification, $v = \sigma^2$:

$$dv = m(v)dt + k(v)dz_2 \tag{3}$$

Here, z_1 and z_2 are standard Brownian motion processes, possibly correlated. Previous work on the subject has characterized the differential equation that options satisfy when (2) and (3) are the underlying processes. However, the solution presents some difficulties due to the presence of a market price of risk, which in turn depends on investor preferences. The solution is independent of risk preferences only if either volatility is a traded asset or is uncorrelated to aggregate consumption. In the Hull and White (1987) model, squared volatility, $v = \sigma^2$, follows a lognormal diffusion:

$$dv = mvdt + kvdz_2. \tag{4}$$

Wiggins (1987) introduces the following diffusion for volatility:

$$d\sigma(t) = f(\sigma(t))dt + k\sigma(t)dz_2 \tag{5}$$

and in a third approach, Stein and Stein (1991) model stochastic volatility with the mean reverting diffusion:

$$d\sigma = -\delta(\sigma - \theta)dt + kdz_2. \tag{6}$$

Stochastic volatility can be interpreted as the introduction of an error term in an attempt to explain deviations in model and market prices (errors are the residual between fitted prices and model prices). Even though this approach is of recent vintage in finance literature, the introduction of "neglected heterogeneity" through an unobservable random characteristic belongs to a long tradition in econometrics (Renault, 1997). As noted earlier, a stochastic volatility specification could lead us away from the umbrella of the Harrison and Kreps (1979) approach to modeling derivative securities. To avoid the introduction of a market price of risk that depends on individual preferences, this paper assumes that volatility risk is diversifiable.

The spot rate of interest is directly modeled in this paper. Pricing is accomplished assuming the existence of a risk neutral measure. The model specifications in CKLS (1992) are included to benchmark the degree of improvement achieved by moving to an ostensibly more complicated model. Many specifications, besides the ones listed, were tested but dropped due to unsatisfactory convergence properties. The interest rate models tested are:

Model 1	Merton	$dr = \mu dt + \sigma dz_1$
Model 2	Vasicek	$dr = k(\mu - r)dt + \sigma dz_1$
Model 3	CIR-SR	$dr = k(\mu - r)dt + \sigma\sqrt{r}dz_1$
Model 4	GBM	$dr = \mu r dt + \sigma r dz_1$
Model 5	CIR-SR +	$dr = k(\mu - r)dt + \sigma\sqrt{r}dz_1, d\sigma = mdt + \sigma dz_2$
		$cor(z_1,z_2) = \rho.$

Model 1 is used by Merton (1979) to derive a model of discount bond prices. Model 2 is the Ornstein-Uhlenbeck process used by Vasicek (1977) to derive equilibrium bond prices. This Gaussian process has been used extensively in modeling bond options (Jamshidian, 1989; Gibson and Schwartz, 1990). Model 3 is the Cox, Ingersoll, and Ross (1985) single-factor general-equilibrium term structure model. Model 4 is the familiar geometric Brownian motion process. Model 5 incorporates stochastic volatility in the CIR model. Such stochastic volatility models have been proposed, as discussed earlier, for stock price processes. The models offer a rich and flexible distribution structure. Skewed distributions can arise due to correlations between the interest rate and volatility shocks. In addition, excess kurtosis can arise from volatile volatility. Stochastic volatility implies a direct relationship between the length of a holding period and the magnitude of conditional skewness and excess kurtosis. Such stochastic volatility models can arise in a general equilibrium production economy as demonstrated by Cox, Ingersoll, and Ross (1985).

The interest rate specification in selected models allows the interest rate to be negative, and the volatility specifications chosen permit a negative volatility. To prevent this, a reflecting barrier is inserted at the level of zero interest rates. If the interest rate reaches zero, it moves into positive territory with a positive probability in the next step. A check reveals that the barrier is not reached in any one of the estimations. The volatility parameters are also restricted to being positive during the estimations.

Consider the valuation of interest rate claims under these specifications of the short term rate. Under the risk neutral measure, the instantaneous expected rate of return on every traded security equals the spot rate of interest. Therefore, the futures price for a continuously marked to market futures contract follows a martingale. Further, if the price of a European option at date t is represented by $C(t)$ and this option matures at date T, then

$$C(t) = E_t\left[C(T)\exp\left(-\int_t^T r(u)du\right)\right] \tag{7}$$

where $r(u)$ is the spot interest rate at date u. Given the value of the option at the maturity date it is possible to compute its value at any prior date. American options can be similarly valued by the equation:

$$C(t) = \sup_{\theta \in F} E_t\left[G_\theta(\cdot)\exp\left(-\int_t^\theta r(u)du\right)\right] \tag{8}$$

where $G_\theta(\cdot)$ is the payoff of the option when it is exercised at date θ and F is the class of all early exercise strategies in $[t, T]$. In practice, the processes in models 1 through 5 need to be discretized by constructing a binomial-type tree representation (see Estimation of Implied Volatility Functions, later in this article).

EURODOLLAR FUTURES AND FUTURES OPTIONS

Eurodollar futures began trading in December 1981, on the Chicago Mercantile Exchange (CME). Identical future contracts are also traded on the London International Futures Exchange (beginning in 1982) and the Singapore International Monetary Exchange (beginning in 1984). Eurodollar futures options have been traded on the CME since 20 March 1985. The trading hours for both Eurodollar futures and options are 7:20 AM CST to 2:00 PM CST on the CME.

Trading volume for Eurodollar futures is approaching that of the Treasury bond (T-bond) futures and future option contracts, which are the most heavily traded interest rate futures and futures options contracts, respectively. In fact, the open interest in Eurodollar futures and options is now much higher than that of T-bond futures and options. Besides being extremely liquid, the Eurodollar contract is well suited for this study, since a cash-settled Eurodollar contract does not involve the complicated delivery and timing options which are inherent in the T-bond futures contract. [See Gay and Manaster (1986) for a description of implicit options.]

Eurodollar futures trade with up to five years until maturity and with almost the entire trading volume in contracts that expire in March, June, September, and December. The last trading day for each contract is the second London bank business day before the third Wednesday of the contract month. At maturity date T, the quoted futures settlement price is

$$F_T(T) = 100[1 - y(T)], \tag{9}$$

where $y(T)$ is the three-month annualized add-on yield on Eurodollar time deposits (three-month LIBOR). The minimum change in the quoted futures price is 0.01, and corresponds to a basis point or tick. Each futures contract has a $1 million notional amount. Each basis point represents a change of $25 in the contract value.

The options on this contract are American with the same maturity date as the futures. Upon exercise, the owner of a call option essentially receives the cash difference between the current futures price and the exercise price. The owner of a put option receives the cash difference between the exercise price and the current futures price. One option covers one futures contract, and like the futures contract, has a minimum quoted price change of 0.01 (one basis point or tick), equal to $25. The trading volume is concentrated in options with maturities of less than one year. Since January 1992, options with up to two years in maturity have also been traded. However, the trading volume in these long-term options is quite small.

DATA

CFTC data containing closing price data of Eurodollar futures and Eurodollar futures options from 1 January 1991 to 31 January 1994 is used for the tests. For each type of contract, the data base consists of a record for the last transaction that occurred. The data also contains daily open, high, low, close, and settlement prices. From this data base, everyday closing prices for all listed futures and futures options are extracted. The three-month Eurodollar interest rates for the corresponding day and time are obtained from the Bloomberg terminal.

Two exclusionary criteria are further applied to the data. First, options with more than 180 days to maturity are eliminated. Options with more than 180 days to maturity are infrequently traded, and therefore lead to errors due to nonsynchroneity of data. A test using options with maturity of up to one year revealed that the results did not change in any significant manner.

Second, options whose absolute moneyness is more than one hundred basis points are excluded. These options, which are very deep in and out-of-the-money, also have very little trading activity. They are prone to large errors because of price discreteness. In addition, the lack of trading activity exacerbates the problem of nonsynchroneity of prices with other data for the same day. Table I gives summary statistics of the data used. There are a total of 12,252 observations (15 observations for each day). The observations are evenly divided between call and put options, and are evenly distributed across the maturity spectrum. The out-of-the-money short duration calls and puts are low priced (average 2.1 and 1.7 basis points respectively) compared with the other sample points. These observations drive the results when fractional errors are computed (fractional error is the difference between model-determined and market price expressed as a fraction of market price of an option). In general, the average price of puts and calls of similar maturity and moneyness are comparable, and will not bias any inferences. The prediction tests are done using the same sample. However, lagged parameter estimates are used for the prediction errors.

ESTIMATION OF IMPLIED VOLATILITY FUNCTIONS

For each day and each contract, a tree is constructed to estimate the model parameters. For model 5 (the stochastic volatility model), each tree node has four branches emanating from it, while models 2 and 3 have two branches emanating from each node, and models 1 and 4 have a recombining tree. Model 5 has a total of 5 time steps, or 1024 final nodes, models 2 and 3 have a total of 10 time steps, or 1024 final nodes, and models 1 and 4 have 1023 time steps, or 1024 final nodes. To incorporate the possibility of correlations in model 5, the axes for (z_1,z_2) are rotated. (Note that (z_1,z_2) are the two underlying Brownian motions as described in (2) and (3)). Define a sequence of uncorrelated random variables (Y_1,Y_2) such that $Z_1(t) = Y_1(t)$ and $Z_2(t) = \rho Y_1(t) + \sqrt{(1 - \rho^2)}Y_2(t)$. Figure 1 is the probability distribution under a risk neutral measure. The risk neutral probability of each state is 0.25. The values of the random variables (Y_1, Y_2) in each state are used to construct the correlated random variables (Z_1, Z_2). Because the initial interest rate is known, the interest rates at each node of the tree can be computed using initial values for the drift and volatility parameters. These parameters are updated on each iteration of the estimation (details follow). Once the tree is set up, valuation of the American-style options is accomplished by backward induction.

Each day, the parameters of the implied volatility function are estimated by starting with the previous day's parameters as the initial value. The Levenberg-Merquardt[1] procedure is used to compute the parameter

[1]See Press et al. (1988) for a description. The Levenberg-Merquardt procedure is an efficient numerical procedure to minimize weighted sum of squares by using properties of the variance-covariance matrix of the estimates.

TABLE I

Summary Statistics on Number of Eurodollar Futures Options, Average Price, and Standard Deviations of Prices Classified by Maturity and Moneyness

Maturity	*Moneyness**	*Number*	*Average Price*	*Standard Deviation of Price*
		Calls		
	Out	327	2.1	1.2
Short	At	774	9.9	7.5
0—60 days	In	243	59.2	37.8
	Out	930	3.8	2.4
Medium	At	933	16.8	8.4
61–120 days	In	426	58.1	32.9
	Out	825	7.3	4.4
Long	At	909	23.6	9.2
121–180 days	In	459	63.2	27.6
		Puts		
	Out	428	1.7	1.1
Short	At	801	9.1	6.7
0-60 days	In	166	44.2	13.2
	Out	1174	4.5	2.7
Medium	At	1002	16.4	7.8
61–120 days	In	425	43.9	17.1
	Out	1219	6.5	4.3
Long	At	858	22.3	7.7
121–180 days	In	293	43.7	7.0

NOTES: All prices are reported in basis points (multiples of $25). Total number of observations = 12,252. Average number of options per day = 15. Of this number, calls per day = 7.1 and puts per day = 8. The sample period is January 1, 1991, to January 31, 1994. Options with maturity greater than 180 days were eliminated from the sample.

*Out of the money options are at least 25 basis points out of the money, at the money options are with |strike price–Futures price| < 25 basis points and in the money options are at least 25 basis points in the money.

values such that the sum of squares between the model futures options prices and market futures options prices is minimized. More specifically, the objective is to minimize the sum of squared errors

$$\sum_{l=1}^{n} [V_i(\theta) - \bar{V}_i]^2 \tag{10}$$

where $V_i(\theta)$ is the model price of the ith option and θ is the vector of model parameters, $\bar{V}_i$ is the corresponding market price, and n is the number of options in that day's sample. The nonlinear least squares methodology gives the maximum likelihood estimates of the parameter estimates. Setting the derivative of (10) with respect to θ equal to zero gives

$$\sum_{1}^{n} \frac{\delta V_i(\theta)}{\delta\theta} [V_i(\theta) - \bar{V}_i] = 0. \tag{11}$$

State	Value of (Y_1, Y_2)	Probability
1	(1,1)	0.25
2	(-1,1)	0.25
3	(1,-1)	0.25
4	(-1,-1)	0.25

FIGURE 1
Example of probability distribution under the risk neutral measure.

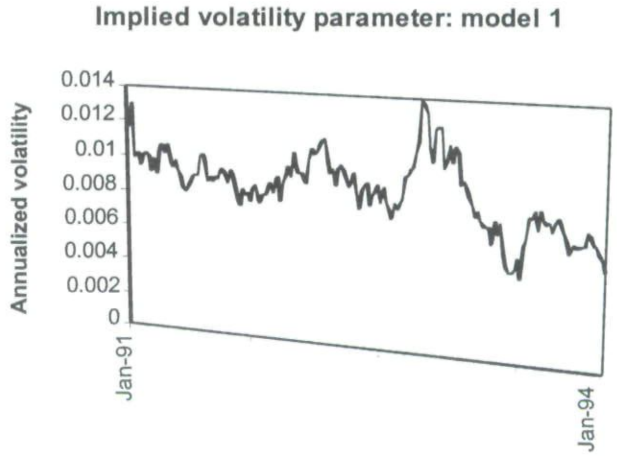

FIGURE 2
Daily values of the implied volatility parameter of model 1 for the sample period Jan. 1991–1994.

Using this procedure, the time series of the implied volatility parameters is computed. Other methods of computing the implied volatility parameters are possible. One could use a weighting scheme wherein the deep in the money options are weighted by a smaller factor than other options. As noted by Amin and Morton (1994), "only a small part of the large price of an in-the-money option is sensitive to volatility" and would not unduly influence the results. To check for consistency in the results, a weighting scheme is also used. The parameter estimates for each of the models in a sample period gave results that are fairly consistent with those obtained by the scheme used in the paper.

Using the scheme described above, the daily time series of the implied volatility parameters for the five different models is estimated. Figure 2 is a graphical depiction of the implied volatility parameter for the model 1 (note that the parameters for all models are not directly comparable). This figure plots the weekly time-series of the estimated implied volatility parameter σ (annualized) for model:

TABLE II

Implied Parameter Estimates

	μ	σ	m	n	ρ	k
Model 1 $dr = \mu dt + \sigma dz_1$						
Mean	0.00598	0.00893				
Standard deviation	0.00619	0.00168				
Lagged correlation	0.85	0.90				
Model 2 $dr = k(\mu - r)dt + \sigma dz_1$						
Mean	0.05831	0.008622				0.22
Standard deviation	0.04445	0.002143				0.25
Lagged correlation	0.34	0.60				0.75
Model 3 $dr = k(\mu - r)dt + \sigma\sqrt{r}dz_1$						
Mean	0.05581	0.04478				0.20
Standard deviation	0.04033	0.01056				0.35
Lagged correlation	0.00	0.33				0.05
Model 4 $dr = \mu r dt + \sigma r dz_1$						
Mean	0.13792	0.20198				
Standard deviation	0.14457	0.05784				
Lagged correlation	0.71	0.76				
Model 5 $dr = k(\mu - r)dt + \sigma\sqrt{r}dz_1$ $d\sigma = mdt + ndz_2$ $\rho(z_1 z_2) = \rho$						
Mean	0.05643	0.03393	−0.00609	0.004096	0.087	0.19
Standard deviation	0.05568	0.02288	0.03521	0.005776	0.66	0.16
Lagged correlation	0.09	0.60	0.66	0.00	0.10	0.04

Each day of the sample, the parameters of a given model are estimated by minimizing the sum of squared errors between market price and model-determined price of each option. In each portion, the first row reports the average of daily parameter estimates of a model for the sample period. The second row reports the standard deviation of daily parameter estimates. The third row reports the autocorrelation in the time-series of each parameter estimate. The sample period is 1 January 1991 to 31 January 1994.

$$dr = \mu dt + \sigma dz_1.$$

Each day in the sample, the parameters of the model are estimated by minimizing the sum of squared errors betwen market price and model-determined price of each option. Due to the large sample size, the estimate for every fifth day has been plotted. This figure demonstrates that there is significant variation in volatility through time. This is not particularly surprising, since the implied volatility represents the market's conditional expectation of future volatility, and realized volatility of rates fluctuates significantly (Engle, Ng, and Rothschild, 1991).

Table II reports the mean, standard deviation, and first-order correlations of the implied volatility parameters in each of the five models. The drift parameter μ, for model 1 and model 4 (Merton and GBM), has a high positive correlation (0.85 for model 1 and 0.71 for model 4) with lagged values of parameter estimates. This implies a mean reversion in the drift parameter. Theoretically, a mean reversion in the drift parameter can be due to a mean reversion in the short-term interest rate to some long-term mean. This behavior in the short-term interest rate has been suggested by CIR (1985), and is documented by CKLS (1992) and Sahalia (1996), among others. A high positive autocorrelation (0.9 for model 1 and 0.7 for model 4) is also seen in estimates of the implied volatility parameter σ for each of the models. This shows that the volatility parameter is also mean reverting. Model 2 (Vasicek model) incorporates mean reversion of interest rates into its specification. Parameter estimates of κ (0.22) and μ (0.058) show that this model accounts for the mean reverting drift in model 1. In this model, the additional variable results in a lower lagged-correlation of the drift parameter μ (0.34 for model 2) even though the mean reversion parameter κ exhibits an autocorrelation of 0.75. Estimates of the mean reversion parameter κ and the long-run mean μ are consistent with those obtained by CKLS (1992), but show substantial variation through the sample period. The standard deviation of κ is equal to the order of the estimate itself. In general, it is difficult to accurately estimate a mean reversion parameter from 180-day option data. This could be due to a greater impact of short-term events on the near-maturity option prices. Model 3 (CIR-SR) adds to the Vasicek model by including the square root of the current interest rate in the volatility specification. The spot-rate volatility is now dependent on the current level of interest rates. This reduces lagged correlation in parameter estimates for both the drift parameters k and μ, and the volatility parameter σ, in comparison to the Vasicek model. The reduction may be because some of the time variation in volatility is captured by the square root term. There is a substantial change in the volatility parameter σ through time (mean of .033 and standard deviation of 0.022 for the sample period) for model 3 (CIR-SR). Since spot-rate volatility is a function of interest-rate levels, in times of high interest rates, the model may overvalue volatility and vice versa. This accounts for a time variation in the volatility parameter that is further corroborated by a negative correlation (-0.35) between the volatility parameter estimates and the spot interest rate (not reported in the table). To overcome this problem, a free parameter was inserted instead of the square root of r (also referred to as the CEV model) and estimated. The model shows poor convergence properties and results in unstable parameter values. Model 5 incorporates stochastic volatility in the CIR-SR model. The drift and mean reversion parameters are similar to those obtained in model 3. The correlation between lagged values of the drift parameter is nearly zero. In addition, the mean reversion parameter k also has a near-zero lagged correlation. The stochastic volatility factor z_2 has a low and positive correlation with the first factor z_1. The correlation varies substantially through time as evidenced by its standard deviation. The estimate of n (volatility of volatility) is about 0.4%. Note that volatility drift (m) is mean reverting, suggesting a mean reversion in volatility of volatility. All the volatility parameters for model 5 have standard deviations that are of the same order or larger than the parameter estimates themselves.

Table III investigates time-series properties of the parameter estimates. The table has mean and standard deviation of first differences of parameter estimates, and Dickey-Fuller test statistics of the parameter estimates. The Dickey-Fuller unit root tests show that the implied volatility parameters vary over time, but are not unstable in general. Notable exceptions are the mean-reversion parameter k for model 2 and the correlation parameter ρ for Model 5. The Dickey-Fuller tests are rejected with a probability of 95% for each parameter in the case of models 1 and 4. The first differences and the standard deviation of first differences in the volatility estimate σ in models 1 and 4 are of a lower order than any of the other models. Parameter robustness and stability is of paramount importance in hedging applications. For example, the correlation estimate ρ of model 5 and the mean reversion parameter k for models 2, 3 and 5 have large standard deviations in their first difference estimates. Using a model in which the parameters vary substantively from one period to another may lead to hedging errors.

Table IV reports in-sample absolute valuation errors for the entire sample period. In-sample errors are the difference between model (fitted) prices and market prices (these errors are also referred to as residuals). Average absolute error (average of the absolute value of residuals) is reported instead of root mean square error for conformity with literature, and also because average absolute error is a more relevant measure for the purpose at hand. The overall average absolute error is the lowest for model 5 (1.5 basis points). A surprising result is that model 4 (GBM) has an in-sample error (1.9 basis points) that is lower than either model 1, 2, or 3. Average fractional errors for each of the models are computed as follows: Fractional error is the absolute error divided by the market price for each option. These errors are of the order of 19.4% for model 5 to 33.1% for model 1. The large fractional errors are driven by the low-priced options. Table IV also reports the average absolute one-day and one-week prediction errors for each of the five models. One-day prediction errors range from 24% for model 5 to 38.2% for model 2. The performance of model 5 for in-sample and one-day prediction errors is superior to the performance of any of the other models. Surprisingly, the performance

TABLE III

First Differences and Unit Root Test Statistics of Parameter Estimates

	μ	σ	m	n	ρ	k
Model 1 $dr = \mu dt + \sigma dz_1$						
Mean Δ	0.00008	−0.00003				
Standard deviation	0.00333	0.00071				
Unit root test	−3.1	−3.9				
Model 2 $dr = k(\mu - r)dt + \sigma dz_1$						
Mean Δ	0.00077	0.00012				0.01
Standard deviation	0.02764	0.00146				0.07
Unit root test	−4.5	−4.3				−0.51*
Model 3 $dr = k(\mu - r)dt + \sigma\sqrt{r}dz_1$						
Mean Δ	0.00001	0.00009				0.023
Standard deviation	0.02615	0.00081				0.645
Unit root test	−3.81	−3.28				−4.6
Model 4 $dr = \mu r dt + \sigma r dz_1$						
Mean Δ	0.00274	−0.00246				
Standard deviation	0.10411	0.014455				
Unit root test	−3.7	−3.8				
Model 5 $dr = k(\mu - r)dt + \sigma\sqrt{r}dz_1\ d\sigma = mdt + ndz_2\ \rho(z_1z_2) = \rho$						
Mean Δ	−0.00032	0.00011	−0.0004	−0.0014	−0.029	−0.008
Standard deviation	0.02749	0.01800	0.0391	0.0082	0.080	0.42
Unit root test	−2.81*	−2.63*	−3.2	−3.17	−0.29*	−4.23

Each day of the sample, the parameters of a given model are estimated by minimizing the sum of squared errors between the market price and the model-determined price of each option. In each panel, the first row reports the first differences of the time series of parameter estimates and the second row reports standard deviations of first differences of the parameter estimates for the period January 1, 1991, to January 31, 1994. The last row in each panel reports the Dickey-Fuller unit root test statistics as the t-statistic of the β coefficient of the regression

$$\Delta\theta_t = \alpha_0 + \beta\theta_{t-1} + \sum_{i=1}^{5}\alpha_i\Delta\theta_{t-i} + \varepsilon_t$$

wbere θ represents each parameter and Δ represents a first difference. The unit root is rejected with a probability of 0.95 in most cases.

*Unit root cannot be rejected at a significance of 95%.

of model 4 compares favorably with that of model 2 or model 3 for one day forecasts and is superior to all other models for a prediction horizon of one week.

Table V reports the average absolute in-sample error (average of absolute value of residuals) as a function of time to maturity and moneyness for both call and put options. The upper panel reports average absolute errors as a function of maturity, whereas the lower panel reports the errors as a function of moneyness. The errors are tabulated separately for call and put options. The in-sample absolute error for call options in model 5 is lower than each of the other models through maturities of 0 to 90

TABLE IV

Overall Absolute Error for the Sample Period

Model	*Merton* 1	*Vasicek* 2	*CIR-SR* 3	*GBM* 4	*CIR-SR+* 5
	Absolute In-Sample Errors				
Average absolute error	2.6	2.0	2.1	1.9	1.5
Average absolute percent error	33.1%	20.2%	29.3%	25.9%	19.4%
	Absolute One Day (one week) Prediction Errors				
Average absolute error	3.3 (3.6)	3.3 (4.0)	2.9 (3.9)	2.9 (3.4)	.4 (3.6)
Average absolute percent error	35.1% (36.4%)	38.2% (39.5%)	35.5% (37.5%)	28.2% (31.1%)	24.1% (33.7%)

This table reports the average absolute error (in basis points) and percent error for each of the five models for the full data set. For a given model, compute the absolute in-sample error of an option as the absolute value of the difference between the market price and the model-determined price (fitted price). Then sum this error for all options over the entire sample period and divide by the total number of observations. This gives the absolute in-sample error. The average absolute percent error is the absolute error expressed as a fraction of the market price of each option, summed for all options and divided by the total number of observations. Prediction error is the absolute value of the difference between the predicted model-determined price and market price. For one-day prediction errors use implied parameter estimates from the previous day to compute model-determined prices. Now compute the average prediction error for all observations by summing the errors and dividing by the number of observations. For one-week prediction errors use parameter estimates from 5 periods prior to the current date to compute model-determined prices.

days, but model 4 performs better for maturities of more than 90 days. For put options model 5 has the best performance across the maturity spectrum (excluding 90–120 days). The performance of model 4 (GBM) compares favorably with model 3 (CIR-SR) and model 2 (Vasicek) with lower absolute errors for most of the maturity categories. The in-sample absolute error for put options as a function of moneyness is lower in model 5 than any of the other models. In the case of call options, the performance of models 4 and 5 is comparable. Note that for the first two categories of out of the money puts (moneyness < -50 and -50 to -25), the pricing error is 0.8 and 0.9 basis points for model 5, as opposed to 1.7 and 2.1 basis points for model 1. The terminal distribution generated by model 5 has fatter tails, and therefore accounts for a part of the volatility smile. Note that model 4 (GBM) again compares favorably with model 3 (CIR-SR) in terms of absolute errors with moneyness through the entire range for both call and put options.

It is possible to cull out some reasons why hypothesized and observed options transaction prices deviate. Imperfect synchronization with the underlying futures prices suggests one source. A major issue for implicit parameter estimation is specification error. Any parsimonious time-series model imposes a structure on option prices that can capture only some

TABLE V

In-Sample Errors (Absolute Error with Time to Maturity and Moneyness)

Model	*1*	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*
			Calls					Puts		
Maturity										
0–30	2.7	1.7	1.8	1.7	1.6	3.1	2.9	2.7	2.0	1.3
30–60	2.8	2.0	2.5	2.0	1.7	3.4	3.1	2.9	2.9	1.5
60–90	3.4	2.5	2.6	2.8	2.0	3.7	3.7	3.3	3.1	1.4
90–120	1.5	0.9	1.3	0.7	1.3	1.7	1.2	1.4	0.8	1.2
120–150	1.6	1.0	1.4	0.9	1.6	2.0	1.8	1.8	1.5	1.5
> 150	2.2	1.6	1.8	1.4	1.6	3.0	3.0	2.8	2.0	1.2
Moneyness										
< −50	0.8	0.8	0.8	0.8	0.7	1.7	1.8	1.6	0.9	0.8
−50–−25	1.2	0.9	1.0	0.9	0.9	2.1	2.0	1.8	1.6	0.9
−25–0	1.9	1.3	1.7	1.4	1.5	3.2	2.7	2.6	2.5	1.4
0–25	2.8	1.8	2.3	1.9	2.2	4.2	3.5	3.6	3.3	2.2
25–50	3.8	2.4	2.9	2.9	2.4	4.4	2.6	3.2	3.0	2.3
> 50	5.5	3.9	3.9	2.7	2.5	5.6	4.5	5.6	2.8	2.4

For a given model compute the absolute in-sample error of an option as the absolute value of the difference between the market price and the model-determined price. Then sum this error for all options in a particular category for the entire sample period and divide by the total number of observations. This gives the absolute in-sample error for that category. The errors are categorized as a function of maturity in the first panel. The second panel reports the errors as a function of moneyness. Moneyness is defined as the futures price minus the strike price for call options, and as the strike price minus futures price for put options.

features of the data-generating process. Specification error implies that option pricing residuals of comparable moneyness and maturity will be contemporaneously correlated if the conditional-risk neutral distribution evolves gradually over time in fashions not captured by the model (Bates, 1996). In specification of the models, there is an obvious decision to be made between a model wherein the two random factors affecting the interest rate are additive, versus a case wherein they are multiplicative (like the ones tested here). In support for a multiplicative factor, Dybvig (1990) states that "the second factor if any should be related to the variance or other distributional features of the interest rates, not additive in levels of interest rates as is usually assumed".

To test for any significant in-sample pricing biases, a regression is run between market prices and model prices for each of the five models (Table VI). The upper portion reports regression estimates for call options, while the lower portion reports estimates for put options. The model price coefficient is greater than 1, and the constant term is negative for models 1 through 3, suggesting that the model tends to underprice high-priced calls and overprice low-priced call options. The coefficient is less than one for model 5, suggesting an overpricing of high-priced call

TABLE VI

Regression Analysis of In-Sample Pricing Errors

	Constant		*Model Price*			
Model	*Estimate*	*t-Stat*	*Estimate*	*t-Stat*	*R-sq.*	*F-Stat**
			Calls			
1	−0.0867	−0.8	1.002	343	0.97	25
2	−0.3215	−2.84	1.012	270	0.98	73
3	−0.1674	−2.315	1.004	440	0.98	12
4	0.0948	0.992	1.003	369	0.98	7.7
5	0.5010	2.58	0.9675	203	0.97	12
			Puts			
1	1.21	13.0	0.924	184	0.93	27
2	0.8227	5.9	0.956	138	0.95	19
3	1.001	12.2	0.936	205	0.93	42
4	0.6139	7.3	0.953	216	0.94	46
5	1.06	9.2	0.93	72	0.84	23

Estimates of OLS regressions reported in this table are based on the equation:

MARKET PRICE = CONSTANT + a*(MODEL PRICE)

for each of the five models. The independent variable is the market price of an option and the dependent variable is the model-determined (fitted) price for that option. The upper panel has estimates of the regression for all the call options in the sample and the lower panel has estimates for all put options in the sample. The estimates show systematic pricing biases in some of the models. The *t*-statistics and the *F*-statistics for each of the regressions are also reported.
*Joint test of the null hypothesis const = 0 and $a = 1$.

options. In the case of put options, the estimates reveal an underpricing of low-priced puts by models 1 and 5. Further, a regression is run between the valuation error as the dependent variable, and the amount by which the option was in-the-money and the maturity of the option (Table VII). The regression equation is

$$\begin{aligned} \textit{Market-Model} &= \textit{constant} + a(\textit{futures price-strike price}) \\ &\quad + b(\textit{maturity}) + \textit{error}. \end{aligned}$$

The regression is run separately for call and put option data (upper and lower portion of Table VII respectively). The moneyness coefficient and maturity coefficients show significant biases in the pricing of put options. For all models, the moneyness coefficient for put options is negative (estimates are significant with a probability of 99%), whereas the maturity coefficient is positive (estimates are significant with a probability of 99%). The R-square statistic is 0.17 for model 1, 0.25 for model 2, and 0.21 for model 3. However, for the stochastic volatility specification (model 5), regression estimates have low explanatory power, suggesting a lower systematic bias in pricing ability.

TABLE VII

Regression Analysis of In-Sample Pricing Errors

	Constant		*Moneyness*		*Maturity*			
	Estimate	*t-Stat*	*Estimate*	*t-Stat*	*Estimate*	*t-Stat*	*R-sq*	*F-Stat*
				Calls				
1	0.543	2.9	0.011	6.7	−0.004	−3.0	0.02	30
2	0.374	1.79	0.009	4.5	−0.003	−1.9	0.02	12
3	−0.222	−1.70	0.005	4.3	0.002	1.5	0.007	9
4	1.107	6.3	0.008	5.3	−0.008	−5.2	0.02	32
5	1.009	2.77	0.011	3.69	−0.013	4.1	0.02	14
				Puts				
1	−3.25	−19.0	−0.013	−8.7	0.029	19	0.17	29
2	−3.07	−15.5	−0.014	−8.65	0.033	15.65	0.25	17
3	−3.35	−23.9	−0.012	−9.7	0.029	23.8	0.21	41
4	−2.79	−19.01	−0.008	−6.061	0.024	18.2	0.14	23
5	−0.54	−1.4	−0.013	−4.11	0.021	6.14	0.04	22.8

Estimates of OLS regressions reported in this table are based on the equation:

MARKET PRICE-MODEL PRICE = CONST + a*(MONEYNESS) + b*(MATURITY)

for each of the five models using in-sample data. The independent variable is the option price and the dependent variables are the moneyness and maturity of that option. The upper panel has estimates of the regression for all call options in the data set and the lower panel has estimates for all put options in the data set. The estimates show systematic pricing biases in some models with respect to moneyness and maturity. The *t*-statistics and the *F*-statistics for each regression are also reported.

PREDICTIVE ERRORS

For predictive tests, the previous weeks' volatility parameters are used to compute the model value for each option. This value is the simple model forecast. The absolute forecast error is equal to the absolute value of the difference between the true market price observed on a given day and the forecasted value. Table VIII is the average absolute one-week prediction errors with maturity. The upper portion reports average absolute errors as a function of maturity, and the lower portion reports the errors as a function of moneyness. The errors are tabulated separated for call and put options. Moneyness is defined as the futures price less the strike price for calls, and strike price less the futures price for puts. A surprising result is that model 4 has the best performance in terms of average one-week absolute prediction error across the entire maturity spectrum (upper portion) and for all categories of moneyness (lower portion). The prediction ability of model 4 (GBM) is superior to that of model 3 (CIR-SR), even for a one-day forecast (not reported here). An increased number of factors improve the cross-sectional fit of the data, because they nest the single-factor models. However, they may not perform well out-of-sample due to

TABLE VIII

One Week Prediction Errors (Absolute Prediction Error with Time to Maturity and Moneyness)

Model	*1*	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*
			Calls					Puts		
Maturity										
0–30	2.7	2.6	2.5	2.1	2.1	2.9	2.3	3.1	2.6	2.7
30–60	3.5	3.7	3.6	3.1	3.1	3.6	3.6	3.4	3.5	3.4
60–90	4.1	3.3	3.2	3.3	3.2	3.5	3.7	3.5	3.4	3.8
90–120	3.0	3.5	3.1	3.0	3.1	4.4	4.2	4.1	4.0	4.4
120–150	3.7	3.9	3.8	3.6	3.6	2.9	3.2	3.2	3.0	3.0
>150	6.3	6.1	5.7	5.3	5.8	4.0	4.1	3.5	3.2	3.2
Moneyness										
<−50	2.1	2.3	2.1	1.9	1.9	2.1	1.9	1.9	1.9	2.0
−50 to −25	2.6	2.9	2.4	2.3	2.3	3.3	3.4	3.3	2.8	2.9
−25 to 0	3.1	3.3	3.2	3.1	3.2	3.3	3.6	3.4	3.2	3.4
0 to 25	5.2	5.2	5.4	5.0	5.4	5.4	5.6	5.9	5.2	5.5
25 to 50	5.5	5.7	5.4	5.2	5.6	5.4	6.7	5.6	5.0	5.2
>50	8.3	8.5	6.2	5.7	5.9	6.4	5.4	5.1	5.4	5.6

the possibility of overfitting noise. This statement is supported by tests discussed in this article. To study systematic biases, a cross-sectional regression was run between the market price and the model price separately for calls and puts, and the results are similar to those reported for the in-sample case as observed in Tables VI and VII. This article demonstrates that there is a substantive improvement in in-sample pricing ability using a stochastic volatility model, but that there should be considerable unease if parameter values vary drastically through time. For model 4, the time variation in implied parameters is not as pronounced as in the case of model 5. Thus, model 4 may be the model of choice for hedging and prediction applications.

DISCUSSION: SOME MORE ISSUES

Implied volatility is often interpreted as a forecast of realized volatility. The first part of this section is a test of the information content of implied volatility in our chosen model (GBM). Implied volatility parameters are compared with the realized volatility over the sample period. The second part is a discussion of using additive vs. multiplicative random factors in modeling the short-term interest rate.

Historical vs. Implied Volatility

Rubinstein (1994) states: "implied volatility models may be best suited for in-sample pricing of derivative assets but may not be suitable for hedging." This section examines whether implied volatility estimates of our preferred model (GBM) contain any information about future realized volatility. To accomplish this, the realized time series of Eurodollar interest rates is split into successive 180 day intervals, i.e., the first sample is from 1 January 1991 to 31 July 1991; the second sample is from 2 January 1991 to 1 August 1992—and so on. The GBM model is fitted to each sample point in the data to obtain a time-series–based estimate of volatility for the GBM model. These parameter estimates are then compared to the implied volatility parameter obtained from option data (under Estimation of Implied Volatility Functions above), such that the implied volatility estimate corresponds to the first day of the time-series sample. A regression is run on the implied and estimated volatility parameter σ.

$$(\textit{time series estimate of } \sigma) = \textit{constant} + b\ (\textit{implied estimate of } \sigma).$$

The regression estimate b has a value of 0.09 (t-statistic of 1.3) and the *constant* term equals 19.69 (t-statistic is 6.16). The regression has low explanatory power with an R-square of one percent. Figlewski (1997) conjectures that implied volatility estimates contain little information about the true or realized volatility in the future. This test reveals that, although the mean implied volatility and mean realized volatility are comparable, the variation in realized volatility cannot be explained by implied volatility estimates for the sample period.

Additive vs. Multiplicative Factors

The short-term interest rate in the stochastic volatility model is driven by two factors that are multiplicative. As noted earlier, there is a choice between a model where the two random factors are additive:

$$dr = \mu(t,r)dt + \sigma_1(r,t)dz_1 + \sigma_2(r,t)dz_2 \text{ and } \rho(z_1 z_2) = \rho,$$

or one in which they are multiplicative (such as the ones tested). Some researchers have suggested that the second factor is possibly one that influences the long end of the yield curve. The simplest specification tested here is one in which the volatility coefficient of the first factor is constant and the volatility coefficient of the second factor is dependent on the maturity of the futures contract. As expected, the prediction errors for the additive model are in all cases worse than the corresponding multiplicative models, or in the simple random walk with drift. The parameter estimates are sensitive to the starting values of the estimation procedure. Instability in estimates arises because the system is under-identified, and a potentially infinite number of solutions to each estimation is possible. Therefore the results obtained are sensitive to the initial values of the iterative procedure.

CONCLUSIONS

Five interest-rate models are tested in a setting where two random factors affect the interest-rate process. The models are fitted to Eurodollar futures and options data for the period between 1991 and 1994. Incorporating stochastic volatility in the Cox-Ingersoll-Ross (CIR, 1985) model reduces in-sample errors. The fitted option prices show that stochastic volatility models can explain a part of the volatility smile. Stochastic volatility model estimates vary considerably through time and are unstable for some of the estimated parameters. It is shown that a Geometric Brownian Motion (GBM) provides the best performance in terms of prediction errors for horizons of one week and is the model of choice for hedging applications. Implied volatility estimates of the GBM specification are consistent with mean realized volatility over the sample period but contain little information about variation in the realized volatility.

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