# HOW TO FINANCE YOUR INVESTMENT OPPORTUNITY INTERNALLY: A NOTE

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## INTRODUCTION

From the theory on the value of waiting to invest, it is well known that a firm might gain by postponing an irreversible investment. A firm should only invest when the project value reaches a certain critical value (McDonald and Siegel, 1986; Dixit and Pindyck, 1994). When the project value depends on the price of natural resources, the investment rule is to invest when the price of the natural resource is higher than a certain threshold. This is quite intuitive. For example, an oil company will be more inclined to develop an oil well when the price of oil is high.

The costs of undertaking the irreversible investment are often very high. The stochastic nature of the project value complicates the financing of the investment opportunity as the time at which investment is optimal is stochastic. The aim of this note is to price instruments that guarantee the costs of the investment when it is optimal to invest and to analyze their main characteristics. Such instruments are important as internally generated cash flow has dominated as a source of funding an investment. Typically, between 70 and 90% of long-term financing comes from cash flows that corporations generate from operations (Ross, Westerfield, and Jaffe, 1996, chap. 14). Internal financing is at the top of the pecking order in financing (Myers, 1984). The pecking order reflects the managers' attempt to minimize issue costs and to avoid the adverse signals

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conveyed to investors when equity issues are announced (Myers and Majluf, 1984). Thus, the note integrates real option theory and the pecking order theory.

With the contracts, the firm profits from the high price of the output once investment is optimal. The costs of the instruments are fairly low because there is a risk that prices decrease. However, when prices decrease, the firm will not invest and thus it would not require any cash to undertake the investment. This makes the price of the instruments attractive. Two financing possibilities are considered: a derivative contract and a long position in the underlying asset. The price of the derivative contract appears to be the lowest. Therefore, when the firm does not profit from the convenience yield of holding the asset, the derivative contract is preferred.

The note is organized as follows. First, straightforward formulas for the derivative contract and the long position in the commodity are derived. Subsequently, I analyze the expected rate of return on both contracts. Though closed form solutions for the expected rate of return of complex derivative securities are rare, the derivative contract allows such a solution. It is shown that the expected rate of return is equal to the ratio of the trigger price at which it is optimal to invest and the investment costs. Finally, it is shown that the price of the contracts decreases with uncertainty. This is not a direct and intuitive implication of the result that the value of waiting to invest increases with uncertainty, since this result relates to reaching a higher project value by waiting. The price of the contract does not relate to this, since the payoff is fixed.

## THE MODEL

Suppose that the price ($P$) of a unit output of an investment project follows a geometric Brownian motion with drift $\alpha$ and standard deviation $\sigma$, i.e.,

$$dP = \alpha P dt + \sigma P dz \tag{1}$$

where $dz$ is the increment of a standard Brownian motion. Dixit and Pindyck (1994, chap. 6) show that a firm should undertake an irreversible investment when the price of one unit output generated by undertaking the irreversible investment exceeds a critical value $P^*$, where $P^*$ is given by

$$P^* = \frac{\beta\delta}{\beta - 1} I \tag{2}$$

in which

$$\beta = \frac{1}{2} - \frac{r - \delta}{\sigma_P^2} + \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{3}$$

where r is the risk-free rate and $\delta$ is the dividend yield of the output[1] or, in the case of a commodity, the convenience yield. It is shown (Dixit and Pindyck, 1994) that $\beta$ increases—and hence $P^*$ decreases—as uncertainty decreases, the convenience yield increases, and the risk-free rate increases. Waiting to invest becomes more attractive when uncertainty increases since the firm can attain a higher project value. Waiting to invest becomes less attractive when the convenience yield increases since the opportunity costs of not holding the project become higher. Also, an important finding is that $\beta > 1$.

The price (C) of a contract that pays off $I$ at the moment that $P = P^*$ can be obtained by risk-neutral valuation. C is given by discounting the cost of the investment against the risk-free rate.

$$C = \hat{E}[\exp(-rT)I] \tag{4}$$

where the head on the expectation operator indicates that the expectation is taken in a risk-neutral world. Since $T$ is the first exit time at which $P$ reaches the barrier $P^*$, $T$ is a random variable. Harrison (1990, chap. 3, formulas 22 and 25) provides a solution for C when $T$ is the first time that an arithmetic Brownian motion with a certain drift and variance hits a constant barrier. In order to calculate C, we can rewrite eq. (1) using Ito's lemma as

$$d \ln(P) = \left(\alpha_P - \frac{1}{2}\sigma_P^2\right) dt + \sigma_P dz_P \tag{5}$$

As a result, ln($P$) follows arithmetic Brownian motion and Harrison (1990) can be applied. Then, it follows that

$$C = \left(\frac{P_0}{P^*}\right)^{\gamma} I \tag{6}$$

where $\gamma = 1/2 - \hat{\alpha}/\sigma^2 + \sqrt{(1/2 - \hat{\alpha}/\sigma^2)^2 + 2r/\sigma^2}$ and $\hat{\alpha}$ denotes the drift rate in a risk-neutral world. Using risk-neutral valuation, the total (i.e., capital gains and dividends) expected return must equal the risk-

[1] $\delta$ can be calculated as the difference between the required rate of return and on the asset ($\mu$) and the growth rate of the asset ($\alpha$). The required rate of return can be calculated with an equilibrium model such as the CAPM. The CAPM states that $\mu = r + \varphi\rho\sigma$, where $\sigma$ is the market price of risk and $\rho$ is the correlation of the asset with the market.

free rate. Hence, $\hat{\alpha} = r - \delta$, and as a result of substituting $\hat{\alpha}$ in $\gamma$, it is true that $\gamma = \beta$.

## CHARACTERISTICS OF THE CONTRACT PRICE

From the equations, it follows that $C$ equals the cost of the investment when $P$ is equal to $P^*$. In this case, the firm will invest immediately and directly requires an amount of money equal to $I$. Hence, the cost of the contract is $I$. When $P < P^*$, it is obvious that $C < I$. Note that there are two effects of uncertainty on $C$. Suppose that uncertainty would not affect $P^*$. Then, an increase in uncertainty leads to an increase in the variance of the distribution of the first passage time. Since the exponential function is convex, it follows from Jensen's inequality that an increase in uncertainty leads to a higher $C$. However, an increase in uncertainty also leads to a higher $P^*$, thereby reducing the costs. Considering formula (6), an increase in uncertainty has a negative impact on $\beta$, leading to a higher $C$, but has a positive impact on $P^*$, leading to a lower $C$. Thus, the overall effect of an increase in uncertainty is ambiguous at this point. Next, the expected rate of return on the contract will be derived, showing that $C$ must decrease with uncertainty.

The attractiveness of the instrument especially arises when the firm cannot profit from ownership of the physical commodity. To illustrate this point, consider another means to fund the investment opportunity. A long position in the output of $(P/P^*)I$ ensures the firm the cost of the investment when investment is optimal. The expected rate of return on this strategy contains the convenience yield and the annualized relative change in the price of the commodity, i.e.,

$$\delta + E\left[\frac{1}{T}\ln(P^*/P)\right] \tag{7}$$

The expected rate of return on the derivative instrument is

$$E\left[\frac{1}{T}\ln(C/I)\right] = \beta E\left[\frac{1}{T}\ln(P^*/P)\right] \tag{8}$$

Since the payoffs match and are paid out at the same time, the expected rates of return in eqs. (7) and (8) must be equal. For example, when the dividend yield converges to zero, $\beta$ goes to 1 and hence eqs. (7) and (8) are equal. Since $\beta > 1$, however, if the firm cannot profit from the convenience yield when holding the asset, the expected rate of return in eq.

**TABLE I**

The Critical Value at Which It is Optimal to Invest, the Expected Time to Invest, the Expected Return on the Instrument, and the Costs of the Instrument for Different Volatilities

| $\sigma$ | $P^*$ | $E[T]$ | $\mu_i$ (%) | $C$ |
|---|---|---|---|---|
| 0.15 | 8.19 | 0.63 | 8.19 | 97.22 |
| 0.16 | 8.52 | 1.42 | 8.52 | 94.08 |
| 0.17 | 8.87 | 2.29 | 8.87 | 91.04 |
| 0.18 | 9.22 | 3.27 | 9.22 | 88.12 |
| 0.19 | 9.61 | 4.36 | 9.61 | 85.13 |
| 0.20 | 10.00 | 5.60 | 10.00 | 82.60 |
| 0.21 | 10.41 | 7.05 | 10.41 | 80.00 |
| 0.22 | 10.83 | 8.75 | 10.83 | 77.49 |
| 0.23 | 11.26 | 10.81 | 11.26 | 75.09 |
| 0.24 | 11.71 | 13.36 | 11.71 | 72.78 |
| 0.25 | 12.17 | 16.64 | 12.17 | 70.56 |

(8) is always higher than the expected rate of return in eq. (7). This is also clear from formula (6), since the price of the derivative contract is always lower than the price of the long position in the commodity when $\beta$ (and hence $\gamma$) exceeds 1.

By equating the expected rates of return, it follows that the expected rate of return ($\mu_i$) on the instrument equals

$$\beta\delta/(\beta - 1) \tag{9}$$

Note that $\mu_i = P^*/I$. Since $P^*$ increases as uncertainty increases, it is true that the expected rate of return increases with uncertainty. Since the expected time to invest also increases with uncertainty because of a higher critical value, $C$ must decrease with uncertainty.

Table I displays the expected time to investment, $E[T]$, where

$$E[T] = \ln(P^*/P)\Big/\left(\alpha - \frac{1}{2}\,\sigma^2\right) \tag{10}$$

$C$, $P^*$, and the expected rate of return for $0.15 < \sigma < 0.25$. Base case parameters are $\alpha = 0.05$, $r = 0.06$, $\varphi = 0.04$, $\rho = 0.5$, $P = 8$, and finally $I = 100$. For all parameter values of $\sigma$ waiting is optimal. Table I shows that for these parameter values the costs of the instrument are decreasing in the standard deviation.

## CONCLUSIONS

This article examines the cost of a contract that pays off a fixed amount when some underlying asset reaches a fixed boundary. The analysis is important for firms that internally finance an investment opportunity. It is shown that the costs of the contract decrease with uncertainty. Though the analysis originated from a means to fund an investment opportunity, it can also be applied to a stock or index reaching a certain barrier; e.g., to calculate the price of a contract that pays off a fixed amount when the Dow Jones reaches a predetermined threshold. In this case, (i) an increase in uncertainty raises the cost of the contract and (ii) the investment banker can hedge the instrument by investing the cost of the instrument and the dividends that accrue.

The first result is explained by the observation that the trigger value at which investment occurs is unaffected by uncertainty. Since $\gamma$ decreases with uncertainty, and $P^*$ is fixed, it follows from eq. (6) that an increase in uncertainty actually raises the cost of the contract when the trigger value is exogenously determined. The second result relates to the fact that dividends on stocks are paid in cash and can easily be reinvested in the stock. The investment bank cannot directly reinvest the convenience yield of the commodity, but can only do so indirectly, by the use of futures contracts on the commodity, and rolling these forward up to the optimal time of investment.

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