An Examination of the Relationship between Stock Index Cash and Futures Markets: A Cointegration Approach

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INTRODUCTION

The existence of price discovery, market efficiency, and market stability associated with spot and futures markets continues as a prominent discussion among academics, practitioners, and regulators. Numerous papers examine the role of price discovery in the futures markets for various types of commodities and financial assets. Generally, the studies by Garbade and Silber (1983), Herbst, McCormack, and West (1987), Kawaller, Koch, and Koch (1987), and Schroeder and Goodwin (1991) indicate that price discovery occurs more significantly in the futures market compared to the cash market.

The opinions expressed are those of the authors and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System.

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Different econometric techniques, such as regression analysis and spectral analysis, to test efficiency and price discovery have developed over time. Since the topics deal with short-run and long-run deviations from a presumed equilibrium relationship based on no arbitrage price bounds, the introduction of cointegration analysis with error correction models is fortuitous. The use of cointegration analysis and error correction models enables one to distinguish between short-run deviations from equilibrium indicative of price discovery and long-run deviations that account for efficiency and stability.

This paper examines price discovery in the Standard and Poor's 500 (S&P 500) cash index and its three-month and six-month stock index futures using intraday, minute-by-minute data. Cointegration analysis is used and several error correction models are developed. The data extend over a three-month contract's expiration period. Antoniou and Garrett (1993) used minute data and a cointegration model, but examined only two days of data. Stoll and Whaley (1990) used five-minute intervals for price announcements over a longer time period, but applied standard econometric analysis that failed to distinguish between short-run and long-run movements in indices. While Wahab and Lashgari (1993) introduced the use of cointegration techniques with stock price data, they used daily closing prices. By using the finer grid of per minute data, a more robust examination of price discovery is offered. Nonsynchronous trading issues are addressed similar to the testing procedures used by Stoll and Whaley (1990). Unlike other papers, this paper models two different contract expirations, the three- and six-month contracts, with the cash market to examine price discovery in each market. Lastly, this paper is unique in that it incorporates a finer grid over an entire contract period using cointegration techniques.

COINTEGRATION ANALYSIS AND ERROR CORRECTION MODELS

Evidence of price changes in one market generating price changes in the other market so as to bring about a long-run equilibrium relationship is given in eq. (1):

$$F_t - \beta_0 - \beta_1 S_t = e_t \tag{1}$$

where S_t and F_t are contemporaneous cash and futures prices at time, t; β_0 and β_1 are parameters; and e_t is the deviation from parity. Ordinary least squares (OLS) is inappropriate if S_t and/or F_t are nonstationary because the standard errors are not consistent. The inconsistency disallows hypothesis testing of the cointegrating parameter, β_1. According to Engle and Granger (1987), if S_t and F_t are nonstationary, which is suspected, but the deviations, e_t, are stationary, S_t and F_t are cointegrated. Thus, an equilibrium relationship exists between S_t and F_t. If the deviations are nonstationary, then either the markets are inefficient, in that an equilibrium relationship does not exist between them, or the two markets do not represent the same underlying asset.

For S_t and F_t to be cointegrated, they must be integrated of the same order. The order of integration is the number of times the series needs to be differenced to become stationary. Performing unit root tests on each univariate price series determines the order of integration. If each series is nonstationary in the levels, but the first differences and the deviations, e_t, are stationary, the prices are cointegrated of order (1,1), denoted CI(1,1), with β_1 as the cointegrating coefficient.

An error correction model exists for each series, which is not subject to spurious results, if the two series are CI(1,1). Wahab and Lashgari (1993, p. 713) state that because "cointegration implies that each series can be represented by an error correction model that includes last period's equilibrium error as well as lagged values of the first differences of each variable, temporal causality can be assessed by examining the statistical significance, and relative magnitudes, of the error correction coefficients and the coefficients on the lagged variables." The first differences of the cash and futures prices are ΔS_t and ΔF_t, respectively. Following Wahab and Lashgari (1993), these are then used in the error correction models of the form:

$$\Delta S_t = \alpha_1 + \alpha_S \hat{e}_{t-1} + \sum_{i=1}^{n} \alpha_{11}(i)\Delta S_{t-i} + \sum_{i=1}^{n} \alpha_{12}(i)\Delta F_{t-i} + \varepsilon_{St} \quad (2)$$

$$\Delta F_t = \alpha_2 + \alpha_F \hat{e}_{t-1} + \sum_{i=1}^{n} \alpha_{21}(i)\Delta S_{t-i} + \sum_{i=1}^{n} \alpha_{22}(i)\Delta F_{t-i} + \varepsilon_{Ft} \quad (3)$$

Equations (2) and (3) represent a near vector autoregression (VAR) in first differences; thus, all variables are held jointly endogenous and OLS is an appropriate method of estimation. Each equation is interpreted as having two parts. The first part, $\hat{e}_{t-1}$, is the equilibrium error. This measures how the left-hand-side variable adjusts to the previous period's deviation from long-run equilibrium. The remaining portions of the equations are the lagged first differences, which represent short-run effects of the previous period's changes in price on the current period's change in price.

The coefficients on the equilibrium errors, α_S and α_F, are the speed of adjustment coefficients. The speed of adjustment coefficients have important implications in an error correction model. At least one speed of adjustment coefficients must be nonzero for the model to be an error correction model. If the value of α_S in eq. (2) is zero, the current period change in the index does not respond at all to the last period's deviation from long-run equilibrium. If α_s is zero and all α_{12} (i) are zero, then ΔF_t does not Granger cause ΔS_t. Wahab and Lashgari (1993) state two purposes for the speed of adjustment coefficients. They serve the role of identifying the direction of the causal relationship and show the speed at which departures from equilibrium are corrected.

DATA AND ECONOMETRIC TESTING

Price data on the S&P 500 stock index, the three-month and six-month S&P 500 index futures contract, are from the Chicago Mercantile Exchange. The data are between January 1987 and March 1987. While the S&P 500 index is recalculated and transmitted to Chicago about every 15 seconds, futures contracts prices may not change as often, especially for the six-month expiration contract.

Given the nonuniform time periods in which price changes can occur, the mean prices are calculated for one-minute intervals. The data begin after 8:40 AM (CST) and end at 3:00 PM (CST). Although the exchanges are open and record transactions prior to 8:40 AM and after 3:00 PM, those prices are not used to eliminate stale price effects.

As Wahab and Lashgari (1993) point out, the lagged differences for the spot and futures prices, ΔS_t and ΔF_t, must be purged of serial correlation to eliminate the effects of infrequent trading and the bid–ask price effect. The methodology that follows is similar to Stoll and Whaley's (1990).

Taking the log of each variable and its first difference, the instantaneous relative price changes (returns) are

$$s_t = \ln S_t - \ln S_{t-1} \tag{4}$$

$$f_t = \ln F_t - \ln F_{t-1} \tag{5}$$

Stoll and Whaley (1990) demonstrate that the effects of infrequent trading in the stock index can be modeled in terms of a pure autoregressive (AR) process and that the bid–ask price effect can be modeled in terms of a pure moving average (MA) process. The cash market, which is subject to infrequent trading, is purged of serial correlation with an AR(28). The three-month and six-month futures indices, which potentially suffer from the bid–ask effects, require MA(25) and MA(30), respectively, to purge the effects.

Table I summarizes the serial correlation of the innovations in the transformed data. These innovations, s_t' and f_t', replace ΔS_t and ΔF_t in the error correction model's eqs. (2) and (3).

To determine the order of integration of each price series, unit root tests are computed for each day on the levels of each price series. Three unit root tests are utilized: the augmented Dickey-Fuller τ-test, the Phillips-Perron z-test, and the weighted symmetric τ-test. Performing all three tests on each day on the first differences of each series shows that the null hypothesis of a unit root is rejected for every day; thus, each series is I(1).

Since all three series are I(1), tests for cointegration with the following cointegrating regressions for the three-month and six-month futures, respectively, are given:

$$F_{3t} = \beta_0 + \beta_1 S_t + e_{3t} \tag{6}$$

$$F_{6t} = \beta_0 + \beta_1 S_t + e_{6t} \tag{7}$$

According to Enders (1995), for large sample sizes it is only necessary to compute cointegrating equations in which either the spot index level or the futures level is on the left-hand side; asymptotic theory states that in large samples the position of the variables in the cointegrating equation does not matter.[1]

The Engle-Granger τ-test is performed on the $\{e_{3t}\}$ and $\{e_{6t}\}$ from eqs. (6) and (7). The results reported in Table II show the spot price level and three-month futures price level are CI(1,1) and the spot price level and six-month futures price level are CI(1,1).

Since both residual sequences are stationary, the error correction models for the three- and six-month futures, respectively, are estimated using OLS regression:

$$s_t' = \alpha_1 + \alpha_{3s't}\hat{e}_{3t-1} + \sum_{i=1}^{30} \alpha_{11}(i)s_{t-i}' + \sum_{i=1}^{30} \alpha_{12}(i)f_{3t-i}' + \varepsilon_{s't} \tag{8}$$

$$f_{3t}' = \alpha_1 + \alpha_{3f't}\hat{e}_{3t-1} + \sum_{i=1}^{30} \alpha_{11}(i)s_{t-i}' + \sum_{i=1}^{30} \alpha_{12}(i)f_{3t-i}' + \varepsilon_{f't} \tag{9}$$

[1]The sample size is over 18,000 observations. The models, however, are tested using both the spot and futures as the left-hand-side variable. The results are identical. Only one set of results is reported.

TABLE I

Summary of Serial Correlations of Innovations of Transformed Data (2 January 1987 to 20 March 1987)

Variable (Transformed-Process)	*Ljung-Box* Lag(6)	Lag(12)	Lag(18)	Lag(24)	Lag(30)
Spot Index	0.05	0.34	3.04	4.65	18.3
AR (28)	(1.00)	(1.00)	(1.00)	(1.00)	(0.95)
3M future	1.38	2.58	5.37	9.72	30.31
MA(25)	(0.97)	(1.00)	(1.00)	(1.00)	(0.45)
6M future	0.86	3.70	6.45	8.32	13.53
MA (30)	(0.99)	(0.99)	(0.99)	(1.00)	(1.00)

Notes: The transformation process is applied to each day to avoid overnight effects. The test for serial correlation is applied to the residuals of the full series. Numbers in parentheses denote the probability of accepting the null hypothesis of no serial correlation.

TABLE II

Cointegration Tests between Spot and 3M and 6M Futures

Dependent Variable	*Coefficients on Independent Variables* Spot	3M Future	6M Future	*E-G (Tau) Test (Lags)*
With constant				
Spot		−1.018		−6.79[a] (51)
3M future	−1.019			−6.81[a] (51)
Spot			−0.993	−6.04[a] (58)
6M future	−0.995			−6.06[a] (58)
Without constant				
Spot		−0.998		−5.40[a] (58)
3M future	−0.998			−5.40[a] (58)
Spot			−0.992	−6.02[a] (58)
6M future	−0.992			−6.02[a] (58)

Notes: Cointegrating equations are bivariate models. E-G denotes the Engle-Granger test of the residuals of the cointegration equation. The null hypothesis: H_0 = unit root.
[a]Significant at the 1% level.

$$s'_t = \alpha_1 + \alpha_{6s'_t}\hat{e}_{6t-1} + \sum_{i=1}^{30} \alpha_{11}(i)s'_{t-i} + \sum_{i=1}^{30} \alpha_{12}(i)f'_{6t-i} + \varepsilon_{s't} \quad (10)$$

$$f'_{6t} = \alpha_1 + \alpha_{6f'_t}\hat{e}_{6t-1} + \sum_{i=1}^{30} \alpha_{11}(i)s'_{t-i} + \sum_{i=1}^{30} \alpha_{12}(i)f'_{6t-i} + \varepsilon_{f't} \quad (11)$$

Table III displays the estimates of the speed of adjustment coefficients.

TABLE III

Estimates of Coefficients

Equation 8		*Equation 9*		*Equation 10*		*Equation 11*	
$\alpha_{3s'_t}$	1.67E-7	$\alpha_{3f'_t}$	−9.53E-8	$\alpha_{6s'_t}$	1.64E-7	$\alpha_{6f'_t}$	−1.33E-7
t-Statistic	6.69[b]		−1.43		6.85[b]		−2.17[a]
$\Sigma\ \alpha_{11}$							
F-Statistic	195.97[b]		8.44[b]		180.90[b]		7.70[b]
$\Sigma\ \alpha_{12}$							
F-Statistic	401.14[b]		8.76[b]		385.45[b]		4.46[a]

[a]Significant at the 5% level.
[b]Significant at the 1% level.

For the three-month futures/cash index eqs. (8) and (9), the speed of adjustment coefficients ($\alpha_{3s'_t}$ and $\alpha_{3f'_t}$) indicate that the spot and three-month futures contract behave somewhat differently. The significance of $\alpha_{3s'_t}$ means the spot market responds to the previous period's deviation from equilibrium. A one standard deviation shock in the equilibrium error results in about a 2% change in the spot market innovation, indicating that the response is fairly large in magnitude.[2] The lack of significance of $\alpha_{3f_t'}$ indicates that the current period three-month futures innovation does not respond to the previous period's deviation from equilibrium. Therefore, any adjustment in the current period's futures innovation is caused by the lagged futures and cash market innovations.

Both speed of adjustment coefficients are significant in the error correction model using the six-month futures innovations. Both the current period spot and futures innovations respond to the previous period's deviation from equilibrium. Once again, a one standard deviation shock in the equilibrium error results in approximately a 2% change in magnitude of either innovation.

The results of the error correction models do not support the theory that there is unidirectional causation from either market. The insignificant speed of adjustment coefficient in eq. (9) does not mean that the spot market is not leading the futures market. All $\alpha_{11}(30)$ in eq. (10) would have to be individually and jointly equal to zero to conclude that the spot market never leads the three-month futures market. The F-statistic indicates the rejection of the null hypothesis that the coefficients

[2]The speed of adjustment coefficient size appears small because the error correction term is calculated as a residual from a regression on price levels, expressed with five digits (e.g., an S&P 500 price of 345 is 34500), and the innovations are residuals from an AR or MA model estimated on minute returns.

are jointly equal to zero. The first three lags of the index innovations [$\alpha_{11}(1)$, $\alpha_{11}(2)$, $\alpha_{11}(3)$] are statistically significant in eq. (10), indicating that the spot market leads the three-month futures by at least three minutes.[3] The last statistically significant index innovation occurs at lag 23 in eq. (9), suggesting that the spot market leads the three-month futures market by at least three minutes and by at most 23 minutes.

Equation (8) demonstrates the leadership effect of the three-month futures contract. The three-month futures innovation shows a much stronger tendency to lead with the first 20 lagged futures market innovations being significant. The last statistically significant coefficient appears at lag 29. Thus, the three-month futures market leads the spot market by at least 20 minutes and by at most 29 minutes.

Turning to the six-month futures contract, the evidence suggests that both markets are adjusting to long-run equilibrium via the significance of the speed of adjustment coefficients. Equation (10) shows that six-month futures innovation is significant to lag 20, with the last significant lag occurring at lag 29. This indicates that the six-month futures contract tends to lead the spot market by at least 20 minutes and by at most 29 minutes. It is rather striking that both the three-month futures and the six-month futures have the same leadership characteristics relative to the spot market.[4]

Equation (11) shows significant cash index innovations through lag 4, with the last significant coefficient occurring at lag 18. This implies that the spot market leads the six-month futures market by at least four minutes and by at most 18 minutes.[5]

CONCLUSIONS

This paper examines the relationship between the S&P 500 stock index and the three-month and six-month futures contracts' expiration over the same time period. The unit root tests conclude that each price series is nonstationary in the levels but stationary after first differencing.

Both the spot index and three-month futures and the spot index and six-month futures markets are tested for cointegration using the Engle-Granger two-step procedure. The Engle-Granger analysis indicates that

[3]The full output from the estimation of the error correction models is available from the authors upon request.

[4]The cross-maturity spread activities between the three-month and six-month futures, which are not directly modeled in this paper, may account for the significant coefficients in eqs. (10) and (11). This, however, is an area for future research.

[5]It should be noted that the residuals from eqs. (8)–(11) are examined via Yule-Walker methods for the presence of serial correlation. No significant serial correlation coefficients are found.

both the spot index and the three-month futures markets and the spot index and the six-month futures markets are cointegrated, indicating market efficiency. Thus, calculating the two appropriate error correction models is presented. The speed of adjustment coefficients indicates stability.

The results of these models show that both the three- and six-month futures markets lead the spot market by at least 20 minutes. The spot market leads the three-month futures by at least three minutes and the six-month futures by at least four minutes. While the futures market does tend to have a stronger lead effect, unidirectional causation of futures-to-spot is refuted.

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