VALUATION OF A EUROPEAN FUTURES OPTION IN THE BIFFEX MARKET

JOSTEIN TVEDT

INTRODUCTION

Thirteen years have passed since the Baltic International Freight Futures Exchange (BIFFEX) was initiated. Contrary to numerous prophesies, the BIFFEX is still in operation, and has until recently been the sole alternative for organized trading of freight rate risk.[1]

At present, there is informal trading in futures options in the BIFFEX market. Some practitioners use the Black formula (Black, 1976), or even the Black-Scholes formula (Black and Scholes, 1973) to price these options. As the Baltic Freight Index (BFI) most probably does not follow a geometric Brownian motion, even over short time intervals, practitioners should consider other pricing formulas than Black's. To investigate the possibility of deriving such a formula is the aim of this paper.

BIFFEX quotes futures on the BFI. The BFI is an arithmetic weighted dry bulk freight rate index. The index is compiled from actual observed freight rates on certain prespecified representative routes. These rates are provided daily by 8–12 major London shipbrokers, called, the panel. When insufficient data are available to the panel, the rates represent the panelists' views of the fair rate levels on the individual trades.

Knut K. Aase has contributed useful comments on earlier versions of this manuscript. Constructive suggestions and advice from three anonymous referees are also acknowledged.

[1]In January 1997, a new index, covering the handy size dry bulk market, was initiated. This index is intended for futures trading also.

■ *Jostein Tvedt is an Associate Professor at the Centre of International Economics and Shipping at the Norwegian School of Economics and Business Administration.*

The Journal of Futures Markets, Vol. 18, No. 2, 167–175 (1998)

CCC 0270-7314/98/020167-09

Eight contracts are traded at any time. There is always a contract for the current month, the "spot contract," and for the month to come, the "prompt contract." Further, the January, April, July, and October contracts are traded up to 18 months ahead. Settlement of the contracts occurs during the first trading day of the following month, and the settlement value is the average of the last five days' BFI values.

The initial value of the BFI was set at 1000 on 1 March 1995. Since then, the index has fluctuated between 553.5 in 1986 to above 2000 in 1995. According to Gray (1990, ch. 3.3), the low 1986 notation represents the lowest income level that a shipowner will accept before laying up the vessels. That is, an index value just above 500 represents a lower bound to the BFI. Historically, also according to Gray (1990), it seems as if 1650 is an upper resistance level to the BFI. Obviously, as the freight rate rises, the demand for bulk carriers is reduced. At the same time, more vessels are attracted by the dry bulk trades, i.e., the combined carriers leave oil for dry bulk. Further, the shipowners try to increase the efficiency of the dry bulk fleet if freight rates rise. In the long run, more capacity will be made available by the construction of new vessels. All this implies that the freight rate will be reverted. Consequently, the BFI is restricted downward and the extremely high levels are rare. Taking this into consideration, a mean reversion representation of the freight rate may be appropriate.

These market dynamics influence the value of a futures option. Bjerksund and Ekern (1995) value an option on a time charter shipping contract assuming that the underlying freight rate follows a Ornstein-Uhlenbeck process. That is, they assume that the freight rate process is normally distributed and that the freight rate, after any shock, is gradually reverted to a constant mean level. The Ornstein-Uhlenbech process has been used previously to describe the dynamics of the interest rate and has been applied subsequently as the basic underlying process for valuing interest rate derivatives. [See, e.g., Vasicek (1977).] Another use of mean reversion processes is found in Brennan and Schwartz (1985) where commodity prices are assumed to follow an Ornstein-Uhlenbech process. This approach is employed here, but a more tailor-made stochastic process is proposed to describe the dynamics of the underlying freight rate index.

THE FUTURES PRICE PROCESS

Due to short- and long-term capacity adjustments, it is assumed that an appropriate process for describing the BFI should have mean reversion properties. The appropriate process should be restricted downward by the lay-up level of dry bulk vessels. When freight rates are too low to cover variable sailing costs, a shipowner may consider mothballing his vessel. This has been discussed by Mossin (1968), Dixit and Pindyck (1994), and others where optimal lay-up in an uncertain environment is derived. Let λ be an absorbing level for the BFI process, i.e., the lay-up level for the dry bulk vessels. Postulating that the BFI less the absorbing level λ is log-normally distributed, it is assumed that the increment of the index is given by a mean reversion process with absorbing level (the MRA process):

$$dX_t = \kappa(\alpha - \ln(X_t - \lambda))(X_t - \lambda)dt + \sigma(X_t - \lambda)dZ_t \tag{1}$$

where X_t is the index value at time t; dZ_t is the increment of a standard Brownian motion; and κ, α and σ are constants.

The MRA process has mean reversion properties. If $\ln(X_t - \lambda)$ is above α, the trend of the process is negative, and if $\ln(X_t - \lambda)$ is below α, the trend is positive. Mean reversion is stronger for high than for low index values for the same absolute deviation of $\ln(X_t - \lambda)$ from α. Also, the MRA process exhibits increasing volatility as the index level rises.

The logarithm of $(X_t - \lambda)$ is Gaussian with conditional expectation of $\ln(X_\tau - \lambda)$ at time t, given by

$$E[\ln(X_\tau - \lambda)|\mathcal{X}_t] = e^{-\kappa(\tau-t)} \ln(x_t - \lambda) + \left(\alpha - \frac{1}{2}\frac{\sigma^2}{\kappa}\right)(1 - e^{-\kappa(\tau-t)}) \tag{2}$$

where $\mathcal{X}_t$ is a sigma field representing the available information at time t. Further, the conditional variance of the process is

$$Var[\ln(X_\tau - \lambda)|\mathcal{X}_t] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(\tau-t)}) \tag{3}$$

The variance is independent of λ and α, i.e., the "level" parameters. The standard deviation of the relative change of X_t, σ, the "speed of the adjustment" parameter, κ, and the time horizon, $(\tau - t)$, establish the variance.

Let Q be a certainty equivalent measure to the original probability measure P. Generally, the futures price at time t is the expected value at time t of the spot price at the time of settlement $T \geq t$, under the certainty equivalent measure Q, i.e., $\Phi_t = E_t^Q [X_T|\mathcal{X}_t]$. By the principle of convergence, Q is also a martingale measure to the futures price process, Φ_t. That is, since $\Phi_T = X_T$, almost surely, one obtains $\Phi_t = E_t^Q [\Phi_T|\mathcal{X}_t]\ \forall\ t$

$\leq T$, and Φ_t is a $(Q, \mathcal{F}_t)$ martingale. To establish an applicable representation of the certainty equivalent measure, Q, is challenging. Black (1976) assumes that the change in the futures price process is uncorrelated with the return on the market portfolio, i.e., there is no systematic risk. This implies no risk premium from investing in the futures market. Hence, the futures price process is given by the expectation of the spot process at the time of settlement under the original probability measure, P. In the rest of this paper the same assumption is made.

Solving the stochastic partial differential eq. (1), one calculates $\Phi_t = E_t^P [X_T|\mathcal{F}_t]$ and Itô's lemma on $E_t^P [X_T|\mathcal{F}_t]$ is used to derive the increment of the futures price process:

$$d\Phi_t = e^{-\kappa(T-t)}\sigma(\Phi_t - \lambda)dZ_t \tag{4}$$

The weight $e^{-\kappa(T-t)}$ determines to what degree volatility in the spot rate, i.e., the BFI, is transferred over to the futures price process. If the time horizon of the contract, $(T - t)$, is long, then $e^{-\kappa(T-t)}$ is small. This implies that present shocks to the spot rate to a restricted degree are transferred over to the futures price. The price process of a futures contract with a distant settlement date, T, is therefore mainly governed by the long-run mean level of the spot process. That is, the market recognizes that even though the present spot level may be far from the mean, the most probable spot level at the settlement date of the futures is closer to the mean.

As the futures contract approaches the settlement date, the volatility in the futures price process is approximately identical to the volatility in the spot rate. This follows by the convergence principle, and is easily seen from the last term of eq. (1) and from eq. (4) where $e^{-\kappa(T-t)}$ approaches unity. Consequently, the volatility of the futures price process increases as the contract approaches the settlement date. This is in accordance with the Samuelson hypothesis (Samuelson, 1965), also called the "maturity effect." Bessembinder, Coughenour, Seguin, and Smoller (1996) point out that the most credible reason for this phenomenon is that the underlying process is mean reverting. They interpret the term $e^{-\kappa(T-t)}$ as the elasticity of the expected spot price at the time of settlement T with respect to the spot rate at time t. From econometric studies of 11 futures markets they find for agricultural goods and crude oil, a strong relationship between mean reversion in the spot prices and increased volatility in the futures prices processes as time to settlement nears. In metal markets mean reversion is less strong and the Samuelson hypothesis has less support. For financial markets they conclude that mean reversion does not prevail and that the Samuelson hypothesis is not supported. There is a large literature that supports the existence of a maturity effect, especially in agricultural markets [see, e.g., Anderson (1985), Bessembinder, Coughenour, Seguin, and Smoller (1995), Castelino and Francis (1982), Galloway and Kolb (1996), and Milonas (1986)].

For commodities that can be stored, mean reversion in spot prices is consistent with a no-arbitrage cost-of-carry argument if there is time variation in inventory costs. These variations may include changes in the spot market risk premium and the convenience yield of holding the commodities. [See Bessembinder et al. (1996) for a detailed discussion.] The BFI is an index of prices of shipping services, and since services by nature cannot be stored, the cost-of-carry argument does not apply. Consequently, mean reversion in prices can prevail without being smoothed out by storage and can be explained without referring to changes in inventory costs.

Along the lines of the econometric studies of the Samuelson hypothesis, Chang, Choi, and Chang (1996) investigate the price behavior of the BIFFEX market. They conclude that a maturity effect exists in the BIFFEX market, in general, but for the January and October contracts the Samuelson hypothesis is not supported.

THE PRICE OF A EUROPEAN CALL OPTION ON A BIFFEX FUTURES

A European call option on a BIFFEX futures is a right, but not an obligation, to buy the futures at a given price at a given date. The option will only be exercised if the futures price process is above the agreed level at the settlement date of the option. The value of exercising the option at the settlement date, τ, is then

$$C_\tau = [(\Phi_\tau - \psi)\,\chi_A] \tag{5}$$

where χ_A is the indicator function of the event A, where $A \in \{\omega:\Phi_\tau(\omega) - \psi \geq 0\}$ and ψ is the exercise value. That is, the option is only exercised given that the transaction is favorable for the option holder. The present value of the option at time t will then be given by the expectation of the deflated C_τ, under the certainty equivalent probability measure where r is a constant risk-free interest rate:

$$C_t = e^{-r(\tau-t)}\, E^Q\,[(\Phi_\tau - \psi)\,\chi_A|\mathscr{F}_t] \tag{6}$$

Black (1976) offers a solution to eq. (6) in the case that the underlying futures process follows a geometric Brownian motion. That would be the case if the initial spot rate also follows a geometric Brownian motion, as is Black's assumption. As Black (1976) notes, the derivation of his option pricing formula is easily extended to allow for a time-dependent variance rate. This is used in deriving an option formula for the BIFFEX futures. Calculating eq. (6), using traditional arbitrage arguments and assuming no transaction costs or taxes, the value of a European option on a futures contract in the BIFFEX market is given by

$$C_t = e^{-r(\tau - t)} \{(\Phi_t - \lambda)\, N(d_1) - (\psi - \lambda)\, N(d_2)\} \tag{7}$$

where

$$d_1 = \frac{\ln\left(\dfrac{\Phi_t - \lambda}{\psi - \lambda}\right) + 1/2\, e^{-2\kappa(T-\tau)}\sigma^2\,(\tau - t)}{e^{-\kappa(T-\tau)}\sigma\sqrt{\tau - t}} \tag{8}$$

$$d_2 = \frac{\ln\left(\dfrac{\Phi_t - \lambda}{\psi - \lambda}\right) - 1/2\, e^{-2\kappa(T-\tau)}\sigma^2\,(\tau - t)}{e^{-\kappa(T-\tau)}\sigma\sqrt{\tau - t}} \tag{9}$$

This option pricing formula does not require knowledge of the mean level of the spot process to derive the value of the option. This follows from the martingale properties of Φ_t under the equivalent probability measure Q. The effects of the mean level, α, is already incorporated in Φ_t. Therefore, all one needs to know are the parameters which are embodied in the variance of the log of the index, κ, σ, and λ, the observable futures price, the corresponding time horizon, and the strike value, to price the option.

The relationship to Black's formula is quite clear. The variance of the log of the futures price process can be derived from eq. (4), and is given by

$$Var[\ln(\Phi_\tau - \lambda)|\mathcal{F}_t] = e^{-2\kappa(T-\tau)}\sigma^2\,(\tau - t) \tag{10}$$

The variance of the log of a geometric Brownian motion is given by $\hat{\sigma}^2(\tau - t)$, where $\hat{\sigma}$ is a constant variance rate. If one substitutes $\hat{\sigma}^2(\tau - t)$ for $e^{-2\kappa(T-\tau)}\sigma^2(\tau - t)$ in eqs. (8) and (9) and sets the lay-up level, λ, equal to zero, then the option value formula (7) is the Black formula. Hence, the formula differs from Black's formula in that the mean reversion property of the freight rate index is taken into account by letting the variance

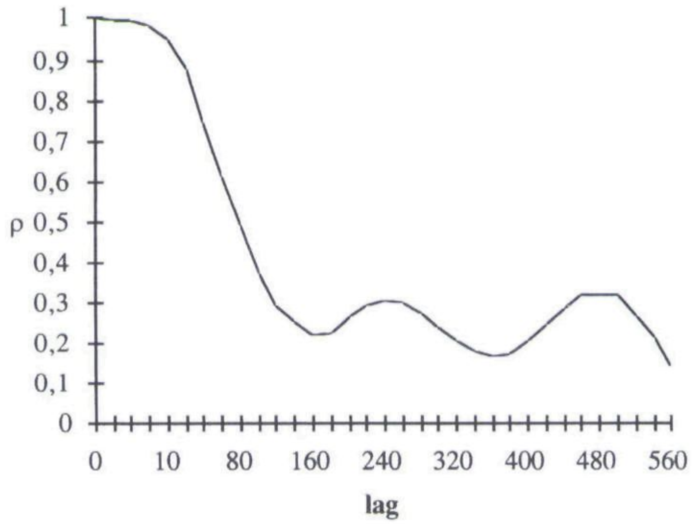

FIGURE 1
A sample autocorrelation function for the BFI with 560 lags.

of the futures process be time dependent, and the process is allowed to have an absorbing level different from zero.

CONCLUSIONS

By suggesting a stochastic process for describing the BFI a pricing formula for European futures options in the BIFFEX market is derived. Two main characteristics make this formula different from the futures option formula of Black (1976). Due to the possible lay-up of vessels, the BFI is never close to zero. Therefore, it is assumed that the BFI, and also the futures price process, are restricted downward by an absorbing level above zero. Further, it is recognized that the freight rates are reverted downward if they are above average and upward if they are below average. This mean reversion property is due to frictional capacity adjustments to changes in the demand for shipping services, and it influences the valuation of the futures options via the variance of the futures price process.

Further work is needed in this area. It is, for example, a well-known fact that the dry bulk markets are fairly dull during the summer months. The seasonal pattern is most easily seen in the autocorrelation function for the BFI, which is illustrated in Figure 1. The function is derived from the available BFI observations up until December 1993 and by applying 560 lags. There are about 250 observations each year. Therefore, note the peaks at 250 and 500 which clearly indicate an annual cycle in the BFI.

The fact that there are seasonal fluctuations in the BFI calls for a modification of the specification of the stochastic process. A possible improvement is to change the drift term of the MRA process to include a sine term. This implies a process with the following incremental change

$$dX_t = \kappa(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))(X_t - \lambda)dt + \sigma(X_t - \lambda)dZ_t \tag{11}$$

The added sine term makes the degree of convergence depend on time. The new parameters introduced are φ, γ, and θ. The magnitude of the cycle is given by φ, whereas γ and θ give the correct timing of the cycle.

As a description of the change in the BFI, eq. (11) is probably an improvement to eq. (1). However, the variances of the spot rate and the futures price process are not influenced by the introduction of the seasonal sine waves, since the sine waves are deterministic. Hence, the option pricing formula (7) still holds for deterministic seasonal patterns as in eq. (11). However, as Lo and Wang (1995) point out, the choice of underlying process can be vital, even though it does not influence the option pricing formula directly. In this case, this will be true if the volatility parameters are estimated using observations of the BFI. Then the hypothesis of the underlying trend term will influence the estimates of the volatility parameters. This problem can, however, be circumvented by estimating the volatility parameters directly from observations of the futures prices.

In addition to seasonality, a natural next step is to investigate the possibility of formulating an equilibrium model, from which a stochastic freight rate process like eq. (1) can be derived. In such a model the lay-up level, λ, should be endogenously determined. An attempt along these lines can draw on a long tradition of equilibrium modeling in shipping. See Beenstock and Vergottis (1993) for an overview.

Even though this study's model is inspired by special features related to the dry bulk shipping markets, the option pricing formula may also be applicable to other futures markets, after some adjustments. This should especially include markets for services and perishable commodities. Since these cannot be stored, or the cost of storage is prohibitively high, a mean reversion nature in prices can persist without being smoothed out by changes in stocks. High prices, or anticipation of high prices, trigger extension of production capacity, which, when in operation, increases supply and brings prices back to the long-run mean level. Too low demand and depressed prices are met by reduction in production capacity. Supply is reduced and prices increase toward a higher equilibrium level. In markets where mean reversion in prices persists, modifications to Black's formula should be considered. Equation (7) is such an attempt.

BIBLIOGRAPHY

Anderson, R. (1985): "Some Determinants of the Volatility of Futures Prices," *The Journal of Futures Markets,* 5:331–348.

Beenstock, M., and Vergottis, A. (1993): *Econometric Modelling of World Shipping,* in *International Studies in Economic Modelling 16,* Motamen-Scobie, H. (ser. ed.). London: Chapman & Hall.

Bessembinder, H., Coughenour, J. F., Seguin, P. J., and Smoller, M. M. (1995): "Mean Reversion in Equilibrium Asset Prices: Evidence from Futures Term Structure," *Journal of Finance,* 50:361–375.

Bessembinder, H., Coughenour, J. F., Seguin, P. J., and Smoller, M. M. (1996): "Is There a Term Structure of Futures Volatilities? Reevaluating the Samuelson Hypothesis," *The Journal of Derivatives,* Winter:45–58.

Bjerksund, P., and Ekern, S. (1995): "Contingent Claims Evaluation for Mean-Reverting Cash Flows in Shipping," in Trigeorgis, L. (ed.) *Real Options in Capital Investment, Models, Strategies and Applications,* Westport, CT: Praeger, pp. 208–219.

Black, F. (1976): "The Pricing of Commodity Contracts," *Journal of Financial Economics,* 3:167–179.

Black, F., and Scholes, M. (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 81:637–654.

Brennan, M. J., and Schwartz, E. S. (1985): "Evaluating Natural Resource Investments," *Journal of Business,* 58:135–157.

Castelino, M. G., and Francis, J. C. (1982): "Basis Speculation in Commodity Futures: The Maturity Effect," *The Journal of Futures Markets,* 2:195–206.

Chang, H.-B., Choi, J. Y., and Chang, Y.-T. (1996): "Price Behaviour in the Baltic Freight Futures Market." Paper presented at the International Association of Maritime Economists' 1996 International Conference, Vancouver.

Dixit, A., and Pindyck, R. S. (1994): *Investment under Uncertainty.* Princeton, NJ: Princeton University Press.

Galloway, T. M., and Kolb, R. W. (1996): "Futures Prices and the Maturity Effect," *The Journal of Futures Markets,* 16:809–829.

Gray, J. W. (1990): *Shipping Futures.* 2nd ed. London: Lloyd's of London Press.

Lo, A. W., and Wang, J. (1995): "Implementing Option Pricing Models when Asset Returns Are Predictable," *Journal of Finance,* 50:87–129.

Milonas, N. T. (1986): "Price Variability and the Maturity Effect in Futures Markets," *The Journal of Futures Markets,* 6:443–460.

Mossin, J. (1968): "An Optimal Policy for Lay-Up Decisions," *Swedish Journal of Economics,* 70:170–177.

Samuelson, P. (1965): "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review,* 6:41–49.

Vasicek, O. (1977): "An Equilibrium Characterization of the Term Structure," *Journal of Financial Economics,* 5:177–188.