Extracting Market Views from the Price of Options on Futures

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INTRODUCTION

The price of an option on a futures contract is often quoted in terms of Black's implied volatility. Use of this volatility in Black's (1976) formula results in the market option price. It is convenient quoting implied volatility as opposed to price because it puts options at various strikes and expiry dates on a common basis and encourages easier comparison of value.

Implied volatilities are not usually the same across various strike prices even for options with similar maturity dates. A matrix of implied volatilities for each strike price and option term is required to reproduce a set of market option prices. This is frequently referred to as a *volatility smile* or skew because volatilities get higher as the strike moves away from the at-the-money level both lower and higher. Supply and demand tends to increase or decrease implied volatilities as the strike price moves away from the at-the-money level. This is especially noticeable in markets dominated by participants with similar positions and directional biases for the underlying securities.

The volatility skews seen for options on the S&P 500 index futures, for instance, probably reflect the fact that the majority of market participants are investors who are naturally long the underlying. This pushes up volatilities for puts, as there is similar demand for protection, and

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implied volatilities are pushed down for calls, as there is similar supply by investors wishing to write covered calls against their holdings of the underlying.

It is redundant to quote option prices by using volatility, because each option requires an implied volatility. It would be more straightforward to just quote price rather than volatility. However, the widespread use of implied volatility to quote option prices, rather than the prices themselves, is a strong endorsement of Black's model at an intuitive level. It suggests that option traders are more concerned with the volatility of the underlying futures contract than on the absolute futures price level.

Black's model, regardless of its appeal, is not an accurate model of the evolution of a futures price in a risk-neutral world. It cannot reproduce market option prices without resorting to a matrix of implied volatilities. If it were an accurate description of market behavior then only one implied volatility would be required to price all options on a particular futures contract.

The appeal of Black's model is derived largely from three features:

1. Options can be valued without consideration of investor's risk preferences (risk-neutral valuation).
2. The uncertainty in an option position can be hedged with the underlying futures.
3. The option valuation formula is easy to use, and the assumptions behind the model are clear.

The model is founded on essentially the same premise as the Black–Scholes (1973) model of a stock price. The variance rate of the return on a futures position is constant. Black's stochastic differential equation governing the evolution of a futures price in a risk-neutral world is

$$dF = \sigma F\, dz \tag{1}$$

where F is the futures price, σ is the volatility of the futures price, and z is a one-dimensional Markov process with stationary transition probability. σ is used in the markets as the implied volatility. Equation (1) can be solved with standard techniques. The price of a European call option is

$$c = e^{-r(T-t)}[FN(d_1) - KN(d_2)]$$

$$d_1 = \frac{\ln(F/K) + \sigma^2(T-t)/2}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t} \tag{2}$$

where c is the price of a European call option on the underlying futures contract, K is the strike price, T is the time the option expires, t is the current time, and r is the risk-free interest rate to the expiration of the option. $N(\cdot)$ is the cumulative standard normal distribution. The implied volatility is the value of σ used in Eq. (2) to reproduce an option price.

Proposed variations of the original Black–Scholes model generally fit market option prices better than the original model. However, none of these seem to be widely used, possibly because additional sources of risk, such as stochastic volatility and price jumps, are introduced. These prevent hedging the option with the underlying. Hull (1997) and Rubinstein (1985) give summaries of the main models.

Developing an option pricing model that is consistent with observed implied volatility structures is of considerable interest. Recent research progressing toward this goal has focused on building implied pricing trees for equity options. The basic assumption common to implied tree models is that the stochastic process followed by the stock price S in a risk-neutral world is governed by

$$dS = rS\,dt + S\sigma(S, t)\,dz \tag{3}$$

Equation (3) is closely related to the original Black–Scholes model except that local volatility $\sigma(S, t)$ is no longer constant but depends on stock price and time. This improvement has the advantage of retaining all the desirable features of the original model, except the ease of calculating option prices. However, this is a small cost for the improvement in accuracy. No functional form is prescribed for local volatility in the implied tree technique. Instead, special rules are developed for calculating the probabilities of stock price movements in the tree from one time level to the next. These are devised in such a way that the market price of options can be reproduced with the tree used in an arbitrage-free fashion. Accurate implied tree models are given by Dupire (1995), Derman and Kani (1995), and Rubinstein (1995).

Implied tree models, though capable of accurately reproducing option prices, give little insight into the stochastic process controlling the price evolution of the underlying asset. It is important to understand this behavior, because the price of the underlying has a profound effect on the value of options against it. The main objective of this article is to use option prices to extract market views about the price of an underlying futures contract. It is proposed that this can be accomplished by developing a stochastic price process whose transition probabilities can be interpreted in terms of the collective perceptions of market participants as to possible price movements of the underlying futures contract.

This notion is related to the concept of risk-neutral valuation. In a risk-neutral world actual asset returns are irrelevant for pricing options on the asset. This research assumes that in a risk-neutral world the actual process followed by the underlying futures contract is irrelevant for the purpose of pricing options. Market participants do not know what the actual stochastic process is and consequently their collective perceptions of the process become important for valuing options. Without doubt options are priced based on where the market thinks the price of the underlying can go during the term of the option.

This appears to be a departure from the usual approach taken when building stochastic price models. The more usual approach is to focus attention on the asset and attempt to identify factors that can influence its price. It is proposed in this research that the perceived price process determines option prices in a risk-neutral world, not the actual price process.

Market views can be extracted from the price of options in the model presented here because various market views are built into the model. If the market has a view similar to one of the views built into the model, then this fact can be extracted from the price of options using the model. A key is to identify important market views, then to develop stochastic processes that accurately reflect these views in the transition probabilities. An obvious benefit besides being able to extract market views is the ability to price and hedge options with a model that is consistent with the market price of options.

The proposed model is founded on the simple idea that the market prices options based on a view of the volatility of the underlying futures contract, which is consistent with the research of Derman, Kani, and Zou (1995). The market chooses a futures price at which it feels that price volatility should be minimum. The value of the futures price at which volatility is minimum corresponds to the technical level; the propensity of the futures price to stick to this level once attained results in the minimization of volatility. In addition to choosing the technical level, the market is also allowed to express a view on the appropriateness of the technical level. Market view strength can range anywhere from strong to weak.

If at some time during the life of the futures contract the market thinks that the current futures price is too high, for example, then it would choose a technical level below the current price, indicating it thinks the futures price will decrease. Alternatively, the market may think the current futures price is too low, in which case it would choose a technical level above the current price, indicating it thinks the futures price will increase. The market may also think that the current futures price is approximately correct; that is, the current price is approximately equal to the closing price at contract expiration, in which case it would choose a technical level approximately equal to the current price.

Although the stochastic process developed does not necessarily reflect the true process followed by the futures price, it does reflect aspects of what the market thinks is the true stochastic behavior of the futures price.

The model allows the technical level and the strength of view to be extracted from the price of options on futures. In addition to extracting market views the model is capable of reproducing option prices with a fair degree of accuracy. This is demonstrated with options on S&P 500 index futures, Yen futures and D-Mark futures.

The article will be of interest to researchers and traders. Researchers will find a promising new direction in efforts to understand the stochastic behavior of futures prices. Traders can benefit immediately by applying the model to options on futures to extract the market's collective perceptions of not only volatility but also the technical level of the futures price and the strength of the view on the technical level.

THE MODEL

A stochastic model in the Black–Scholes framework is developed founded on the idea that the market prices options on futures based on a view of the volatility of the underlying futures contract. Market participants buy and sell options and hedge their exposures with the underlying contract. For this reason it is important for them to estimate possible movements in the futures price, because these generally have the largest effect on the value of option positions.

Futures price variability is measured by its volatility. Trading options is often referred to as trading volatility. Therefore, it is reasonable to assume that market participants express their views about the futures price through option prices. Three important views that can be held by the market about the current futures price and are captured in the proposed stochastic price process are as follows. First, in an upward trending market, the current futures price may be different from the price at contract expiration, but there is definitely an upper limit on the price, and the final price will most probably fall between the current price and the upper price limit.

Second, in a downward trending market the current futures price may be different from the price at expiration, but there is definitely a lower limit on the price, and the final price will most probably fall between the current price and the lower price limit. Finally, in a low volatility market, the current futures price is probably close to the price anticipated at contract expiration.

The simplest stochastic price model that captures the desired market views is

$$dF = BF\left[\frac{1}{A} + \left(1 - \frac{F}{M}\right)^2\right] dz \tag{4}$$

where M is the technical level of the futures price, which, in the extremes, can be a lower price limit, an upper price limit, or the appropriate futures price. A is the strength of the market's view on the appropriateness of the technical level, and B is the volatility parameter, which indicates the propensity of the futures price to move away from the technical level.

Consider the behavior of the market view (MV) model given in eq. (4) when the view is very strong, that is, when A approaches infinity and $1/A$ approaches zero. In the strong view limit eq. (4) becomes

$$dF = BF\left(1 - \frac{F}{M}\right)^2 dz \tag{5}$$

Equation (5) has two special points, one at $F = 0$ and one at $F = M$. At both points the change in futures price is zero. If the futures price could equal either 0 or M then it would remain so forever. However, the futures price could never equal these values because if F is between 0 and M then it will always remain in this range; in this case M is an upper limit on F. Likewise, if F is greater than M then it will always be greater than M; in this case M is a lower limit on F. In contrast Black's model has $F = 0$ as the lower limit on the futures price with no reference to a limit relative to the current futures price.

The upper and lower limit effects are illustrated in Figures 1 and 2. Figure 1 shows futures contract price distributions at option expiration. Distributions are calculated from the prices of European options. Option prices are calculated with the finite-difference scheme given in the Appendix. Probabilities are calculated to be consistent with the MV model using eq. (9). The distributions in Figure 1 correspond to the parameter set, $F = 60$, $T = 0.2$, $r = 0.05$, $B = 10$, $M = 70$, and $A = 50$, 100 and 200. There is nothing special about the parameter set; many alternative sets exhibit similar behavior. Here the technical level is well above the futures price and it acts like an upper limit. This is clearly seen as the market view strengthens. At the lowest market confidence, $A = 50$,

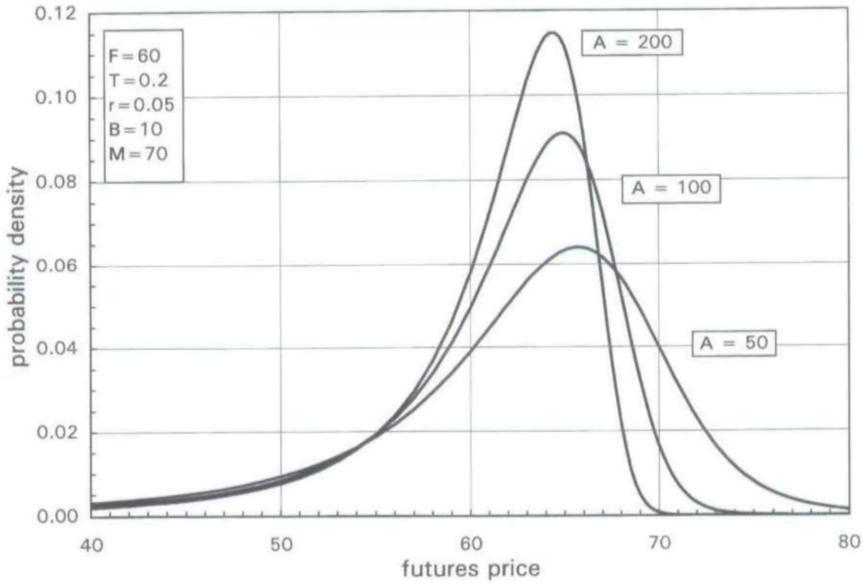

FIGURE 1

Terminal futures price distributions calculated from the price of European call options. The effect of increasing market confidence at constant technical level and volatility parameter is demonstrated. ($M > F$ upward-trending market.)

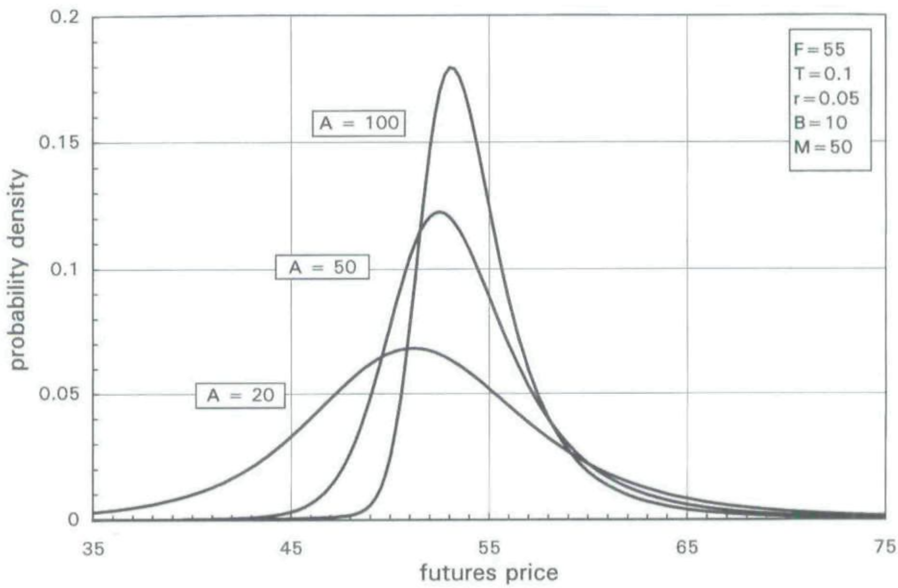

FIGURE 2

Terminal futures price distributions calculated from the price of European call options. The effect of increasing market confidence at constant technical level and volatility parameter is demonstrated. ($M < F$ downward-trending market.)

there is a 15% chance of the terminal futures price being above the technical level, a 33% chance of it being less than the current price, and a 52% chance that it will be between the current price and the technical level. At $A = 100$ the chance of being above the level is less than 2%, the chance of being below the current price has decreased slightly to 32%, and the chance of a final price being between the current price and the technical level has increased to 66%. At the highest market confidence, $A = 200$, there is virtually no chance that the terminal futures price will be above the level, the chance of it being below the current price has dropped slightly to 31%, and the chance of it ending between the current price and the technical level has increased to 69%. It is evident, in this example, that the market feels that the appropriate range for the futures price is between the current price and the technical level.

Increasing the strength of the market view causes the technical level to act like an upper limit. It also increases the probability that the terminal futures price will be between the current futures price and the technical level. At the same time the probability of a terminal price lower than the current price remains approximately constant and less than the probability of being between the current price and the technical level. The market holds the view that the futures price at option expiration is most probably going to be between the current price and the technical level.

Figure 2 illustrates the case when the futures price is above the technical level. The behavior is similar to that seen in Figure 1, except now the technical level acts as a lower limit on the price. Increasing the strength of the view again increases the probability that the terminal futures price will be between the current price and the technical level. The parameters are $F = 55$, $T = 0.1$, $r = 0.05$, $B = 10$, $M = 50$, and $A =$ 20, 50, and 100. The relationships presented in Figure 2 are typical of many parameter sets.

When the technical level is close to or equal to the current futures price an increase in the strength of the market's view increases the probability that the current price is the correct terminal futures price. This behavior is illustrated in Figure 3. In this case the current price is equal to the technical level. The model parameters are $F = 50$, $T = 0.2$, $r =$ 0.05, $B = 10$, $M = 50$, and $A = 25$, 50, and 100; again, there is nothing special about the parameters. At the lowest confidence level, $A = 25$, there is only a 26% chance that the futures price at expiration will be between 48 and 52. At $A = 50$ this increases to 41% and at $A = 100$ there is a 66% chance that the terminal price will be between 48 and 52.

This behavior allows A to be interpreted as the strength of the market's view about the appropriateness of the current futures price. The

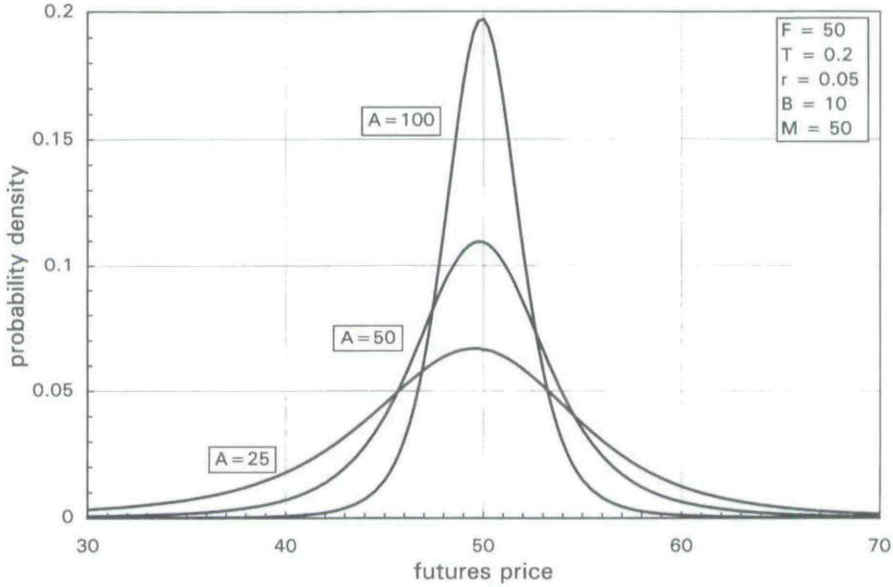

FIGURE 3
Terminal futures price distributions calculated from the price of European call options. The effect of increasing market confidence at constant technical level and volatility parameter is demonstrated. ($M = F$ low-volatility market.)

chances of the terminal price falling within a specified range increases as A increases; the market is confident about where it thinks the futures price can go during the life of the option.

The image evoked by the proposed model is a partition of the range of possible futures prices into two regions. The first region is between 0 and M and the second region is between M and infinity. Price movements from one region to the other are difficult because of the minimum in local volatility at the technical level. When the futures price breaks through the technical level in either direction volatility increases.

When F equals M eq. (4) becomes

$$dF = \frac{B}{A} F\,dz \tag{6}$$

According to the MV model the propensity of the futures price to move away from the technical level is determined by the magnitude of the volatility parameter, B, for a given confidence level, A. It is easier for F to move away from M as B increases.

The MV local volatility can be approximately related to Black's implied volatility by generating option prices with the use of the MV model

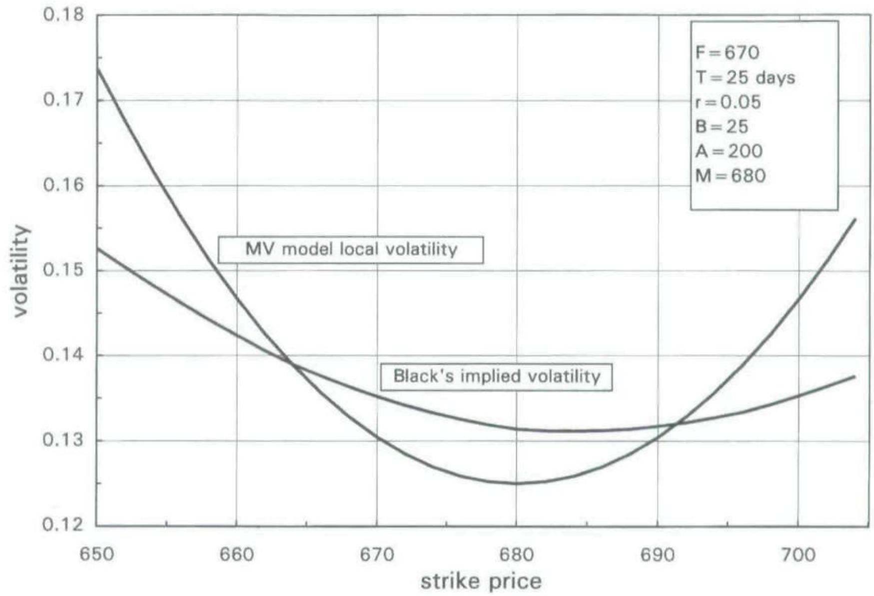

FIGURE 4
Volatility versus strike price. Black's implied volatility for call option prices generated with the MV model [eq. (7)] for the parameters in the inset.

and calculating the implied Black model volatility required to reproduce these prices. The local volatility function $\sigma(F)$ in the MV model is

$$\sigma(F) = B\left[\frac{1}{A} + \left(1 - \frac{F}{M}\right)^2\right] \tag{7}$$

This can be written in terms of strike price instead of futures price by resorting to the Fokker–Plank forward form of the partial differential equation governing the value of a call option on the futures. The forward equation can be derived with the use of the same procedure outlined by Derman and Kani (1995) for stock options. When this is done the result is

$$\frac{\partial c}{\partial T} = \frac{1}{2}\,\sigma(K)^2 K^2\,\frac{\partial^2 c}{\partial K^2} - rc \tag{8}$$

In Black's model $\sigma(K)$ is constant. Black implied volatility for option prices generated with the MV model is related to $\sigma(K)$. Figure 4 is a plot of Black implied volatility and $\sigma(K)$ against strike price for the parameter set, $F = 670$, $T = 25/365$, $r = 0.05$, $B = 25$, $M = 680$, and $A = 200$. The parameters are similar to those found in the S&P data. It is worth noticing that the local volatility minimum, defined as the technical level in the MV model, is close to the minimum Black implied volatility. The MV model technical level is 680, whereas the minimum Black implied volatility is 686. This example is not special; all parameter sets examined exhibited similar behavior. Therefore, the minimum Black implied volatility is approximately equal to the MV model technical level. This is especially true when the technical level is very close to the current futures price. It is only a crude approximation to say that the technical level is equal to the minimum implied Black volatility; nonetheless it allows important information to be taken directly from the volatility smile. It is easy to see if the technical level is above, below, or close to the current futures price. This bit of information gives a first crude picture of how the market views the current futures price; that is, the current price is either too high, too low, or just about right.

The probability density of the futures price at option expiration is obtained from the derivative of option price at expiration with respect to option maturity assuming European exercise. Denote by C the value of a European call option at T. The probability, $p(K, T; F, t)$, that the futures price is equal to K at T starting from F at t is given by

$$p(K, T; F, t) = \frac{2\partial C/\partial T}{K^2B^2[1/A + (1 - K/M)^2]^2} \tag{9}$$

Terminal price distributions are generated with eq. (9). Equation (9) is obtained from eq. (8) with the use of the well-known equation for probability; that is,

$$p(K, T; F, t) = e^{r(T-t)} \frac{\partial^2 c}{\partial K^2} \tag{10}$$

Equation (9) generates smoother probability distributions than does eq. (10) because of the noise introduced by the numerical solutions.

MARKET APPLICATION

The MV model is tested with call options on S&P 500 index futures. These are liquid instruments traded on the Chicago Mercantile Exchange. The options examined are against the September 1996 futures contract. The size of one futures contract is $500 multiplied by the index level, where each index point is worth $500. The minimum move in the futures price is 0.05 and this is worth $25. The contract first traded on September 18, 1995, the last trade date was September 19, 1996, and

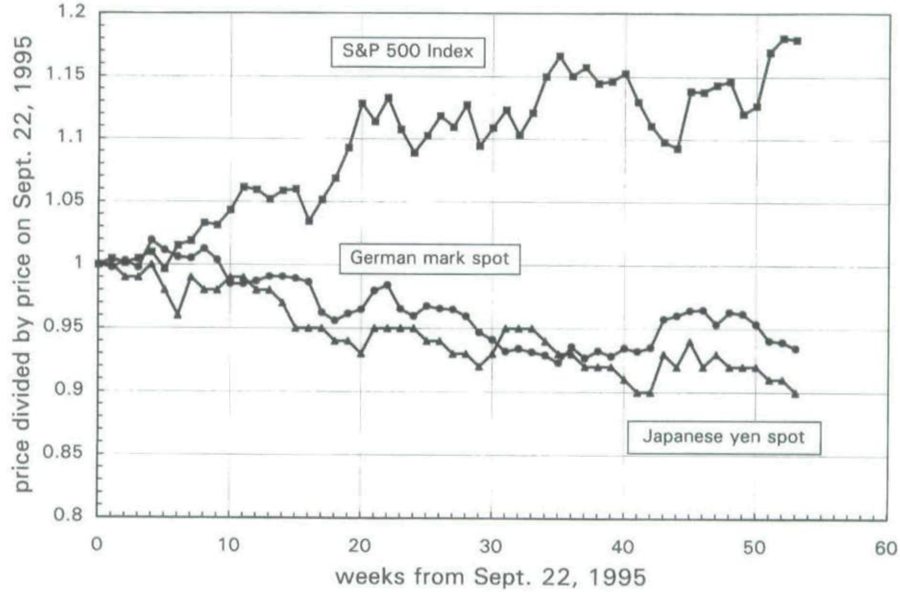

FIGURE 5

Spot price trends for the three futures contracts examined. The symbols represent the ratio of weekly closing spot price divided by the closing spot price on September 22, 1995, for the 52 weeks prior to the expiration of the September 1996 futures contracts.

the contract valuation date was September 20, 1996. Call options are written on one futures contract. The first trade for the call option was July 24, 1995, and the last trade date and expiration was September 19, 1996. These are American-style options. The open interest on the futures contract was steady at about 150,000 contracts between the end of June 1996 and the beginning of September 1996. Open interest dropped off significantly outside of these dates. The daily closing prices, available on Bloomberg, of call options at all strike prices from June 26, 1996 to September 16, 1996 are used to test the model. The data are reproduced in Table III of the Appendix.

At the expiration of a futures contract the spot and futures prices usually converge. Therefore, it is useful to follow the trend in the spot market to get a sense of where the futures price may be at expiration because of the convergence of the two prices. The S&P 500 spot index is an example of an upward-trending market. The MV model predicts that in this type of market the technical level will be above the current futures price. Application of the MV model in the S&P 500 futures market is therefore a good first test of its validity. Figure 5 shows the upward trend in the spot index for the 52 weeks prior to expiration of the September 1996 futures contract. The symbols represent the weekly closing values of the index divided by the index value on September 22, 1995. Figure 5 also shows the comparable data for the spot values of the German Mark and Japanese Yen. Both of these are downward trending, and will be used to illustrate application in downward trending markets. In downward trending markets the MV model predicts that the technical level will be below the current futures price.

Closing prices are used because these are easy to obtain and are sufficient to demonstrate the usefulness of the model. The risk-free interest rate is held constant at 5.2% for convenience, because the analysis is not very sensitive to the actual rate and because the actual risk-free rate was approximately 5.2% over this period.

The values of the technical level, strength of view, and volatility parameter are determined by minimizing the squared percentage error between the closing prices of the options and the values calculated with the MV model. Estimates of the MV model parameters are calculated for 56 sets of closing prices. The percentage pricing errors at all strikes are treated equally, even though the contribution from deep out-of-the-money options could be overwhelming. It is sometimes necessary to remove one or more of these options from the data when there is an obvious problem with the price. A typical example is when options at consecutive strikes have the same price. Another example is when an option is mispriced by one tick, which is the minimum price move of 0.05 index points. A one-tick move can easily represent a greater than 20% price change for some of these options.

Options are assumed to be European-style exercise because this allows much faster calculation than by assuming American-style exercise. Assuming European exercise to fit the model is not unreasonable, because since out-of-the-money options make the largest contribution to the pricing error and the prices of these options do not depend strongly on the exercise style. Most of the options used to fit the model are out of the money; even when American-style exercise is assumed it does not improve the fit significantly, except for deep-in-the-money options. Performing a least-squares fit assuming American exercise does improve the overall fit, but does not result in estimated parameters that are very different from those obtained assuming European exercise.

American option prices are calculated by first rewriting the governing partial differential equation in linear complementarity form, discretizing with the use of a finite difference scheme, and then solving the algebraic equations with the use of a projected successive over relaxation algorithm. Wilmott, Dewynne, and Howison (1995) outline numerical procedures for solving the governing differential equation assuming American exer-

TABLE I

August 2, 1996 Closing Prices of Call Options on the September 1996 S&P 500 Index Futures. The Euro and Amer Columns are the Calculated Prices of European and American Options Using the Same Parameters Set. MV Model Parameters Are Given in Table II.

Strike	Market	Euro	Amer	Strike	Market	Euro	Amer
510	156.35	155.70	156.45	655	20.95	20.92	20.95
530	136.35	135.98	136.57	660	17.60	17.59	17.61
550	116.45	116.32	116.78	665	14.55	14.53	14.54
580	87.15	87.10	87.38	670	11.70	11.77	11.78
600	68.00	67.97	68.16	675	9.30	9.33	9.33
610	58.60	58.62	58.76	680	7.25	7.23	7.24
620	49.50	49.48	49.58	685	5.50	5.48	5.48
625	45.05	45.01	45.11	690	4.05	4.04	4.05
630	40.75	40.65	40.72	695	2.90	2.91	2.91
635	36.50	36.39	36.45	700	2.05	2.05	2.05
640	32.25	32.26	32.31	705	1.45	1.41	1.41
645	28.30	28.29	28.33	710	1.00	0.96	0.96
650	24.50	24.50	24.53	715	0.70	0.64	0.64

cise. The prices of American options given in Table I are calculated with the use of their procedures.

Table I contains the August 2, 1996 closing prices for call options at various strike prices and the corresponding MV model prices of European and American options. The model parameters are given in Table II. The same parameters were used to calculate the prices of both American and European options. The closing futures price for the data in Table I is 666.35 index points, and the time to maturity is 48 days. The resulting American-style option prices are approximately 0.2% higher than the European-style option prices at strikes greater than 650 index points and approximately 0.5% higher at strikes less than this. American option prices, for the options studied, are not much more expensive than the comparable European options. Therefore, the error introduced by assuming European exercise is probably small.

Not all option sets exhibit clear minimums in the implied volatility plots. The volatility smile is most pronounced close to contract expiration and flattens out further away from expiration. The steepening of the smile toward expiration is illustrated in Figure 6, which is a plot of the implied volatility ratio versus strike price ratio. The Black implied volatility ratio is calculated by dividing the implied volatility by the at-the-money volatility. The strike price ratio is calculated by dividing the strike price by the at-the-money strike price. Data for four dates are shown: July 8, August

TABLE II

Futures Price, *F*, Days to Expiration, *T*, and MV Model Parameters for the S&P 500 Index Futures Data

Date	T	F	B	M	A	Date	T	F	B	M	A
26-Jun-96	85	668.65	3.368	786.94	52.488	7-Aug-96	43	666.20	6.102	737.00	70.027
27-Jun-96	84	673.05	3.241	791.78	55.190	8-Aug-96	42	666.20	8.899	715.25	90.535
28-Jun-96	83	676.80	4.021	767.70	56.783	9-Aug-96	41	662.45	11.37	705.04	110.91
1-Jul-96	80	680.80	4.530	751.36	52.686	12-Aug-96	38	667.75	11.43	709.15	116.95
2-Jul-96	79	678.80	6.623	725.43	66.685	13-Aug-96	37	661.80	9.008	709.81	88.569
3-Jul-96	78	676.10	6.431	724.82	63.175	14-Aug-96	36	663.60	10.31	705.09	99.851
5-Jul-96	76	661.10	4.617	735.79	48.717	15-Aug-96	35	663.60	13.96	699.53	144.18
8-Jul-96	73	656.30	5.486	722.15	54.585	16-Aug-96	34	668.10	18.76	692.92	182.08
9-Jul-96	72	659.15	5.619	722.29	56.586	19-Aug-96	31	668.20	13.73	700.50	134.67
10-Jul-96	71	661.70	6.366	713.50	61.535	20-Aug-96	30	667.95	21.64	689.69	206.65
11-Jul-96	70	648.35	5.312	713.40	50.881	21-Aug-96	29	666.00	20.37	688.85	204.11
12-Jul-96	69	648.05	6.026	705.60	55.770	22-Aug-96	28	671.85	22.52	690.89	222.19
15-Jul-96	66	630.25	8.022	676.33	60.605	23-Aug-96	27	668.80	17.57	695.31	182.66
16-Jul-96	65	632.10	8.118	675.90	60.903	26-Aug-96	24	664.20	26.04	684.02	233.84
17-Jul-96	64	637.80	6.729	686.67	52.541	27-Aug-96	23	666.70	26.38	684.92	242.26
18-Jul-96	63	647.45	6.767	703.84	64.391	28-Aug-96	22	665.70	25.66	684.93	238.15
19-Jul-96	62	642.00	6.350	699.63	57.014	29-Aug-96	21	657.15	21.93	683.50	211.50
22-Jul-96	59	637.55	5.865	703.93	51.086	30-Aug-96	20	651.35	22.72	677.83	200.82
23-Jul-96	58	628.15	6.727	691.65	54.738	3-Sep-96	16	655.85	35.05	673.45	265.43
24-Jul-96	57	630.70	5.249	717.55	49.892	4-Sep-96	15	656.60	31.87	676.14	240.10
25-Jul-96	56	635.20	5.158	717.52	50.733	5-Sep-96	14	649.00	36.25	670.32	256.39
26-Jul-96	55	639.35	8.165	691.06	73.577	6-Sep-96	13	658.25	43.22	671.22	355.41
29-Jul-96	52	631.00	8.442	687.76	72.063	9-Sep-96	10	665.05	52.17	676.14	430.44
30-Jul-96	51	637.70	10.25	688.77	87.575	10-Sep-96	9	665.10	56.76	676.43	481.09
31-Jul-96	50	642.40	7.169	711.08	72.812	11-Sep-96	8	667.00	64.78	677.30	576.90
2-Aug-96	48	666.35	10.35	706.32	96.179	12-Sep-96	7	672.15	72.22	681.62	651.50
5-Aug-96	45	661.70	11.07	700.29	97.278	13-Sep-96	6	682.85	125.1	687.42	1237.3
6-Aug-96	44	664.25	9.113	711.20	84.950	16-Sep-96	3	684.70	218.8	688.58	1821.8

15, August 27, and September 10. The smile does not become clear until early August, when a minimum is first measurable.

Figure 7 is a plot of pricing bias versus strike price for the August 27 data. The solid symbols represent the difference between the market price and Black's price expressed as a percentage of the market price. Black's price is calculated with the use of the at-the-money implied volatility. If the Black model price is greater than the market price, for example, then this pricing bias is negative. The curve represents the difference between the model price and Black's price expressed as a percentage of the model price. Again, if the Black price is greater than the model price, then this pricing bias is negative. If the MV model captures all of the Black model pricing bias then the curve will pass through all symbols.

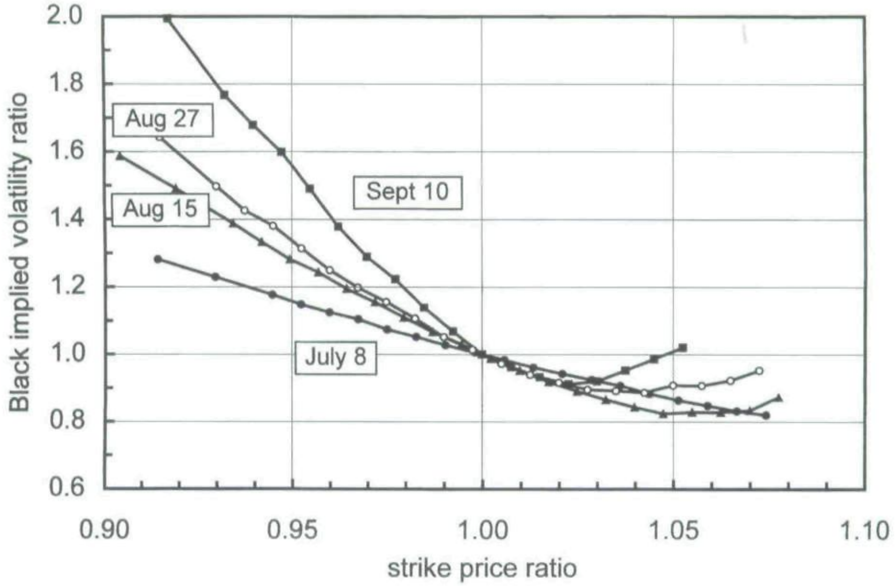

FIGURE 6
Black implied volatility ratio versus strike price ratio for call options on the September 1996 S&P 500 index futures contract. The ratios are calculated by dividing by the at-the-money values.

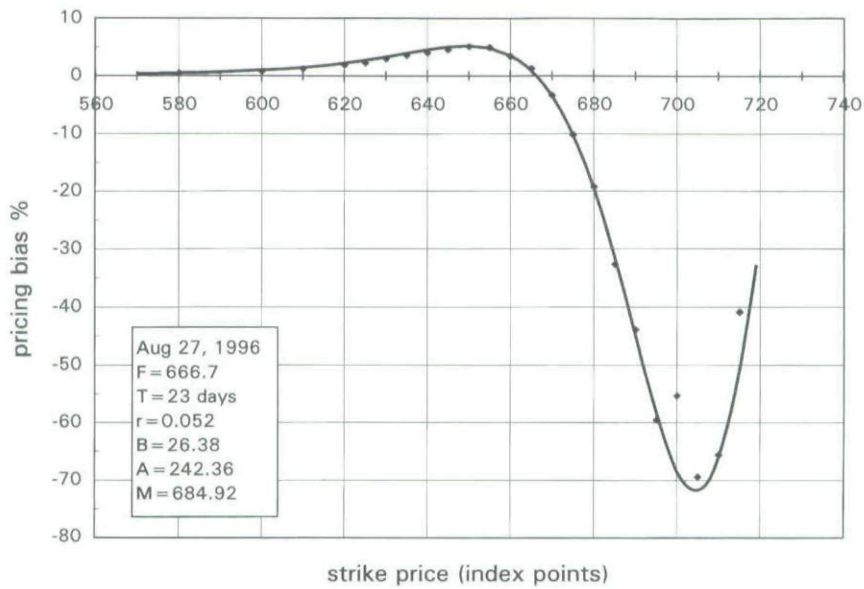

FIGURE 7
Pricing bias versus strike price for the August 27, 1996 closing prices of call options on the September 1996 S&P 500 index futures contract. The symbols represent market data and the curve represents MV model values.

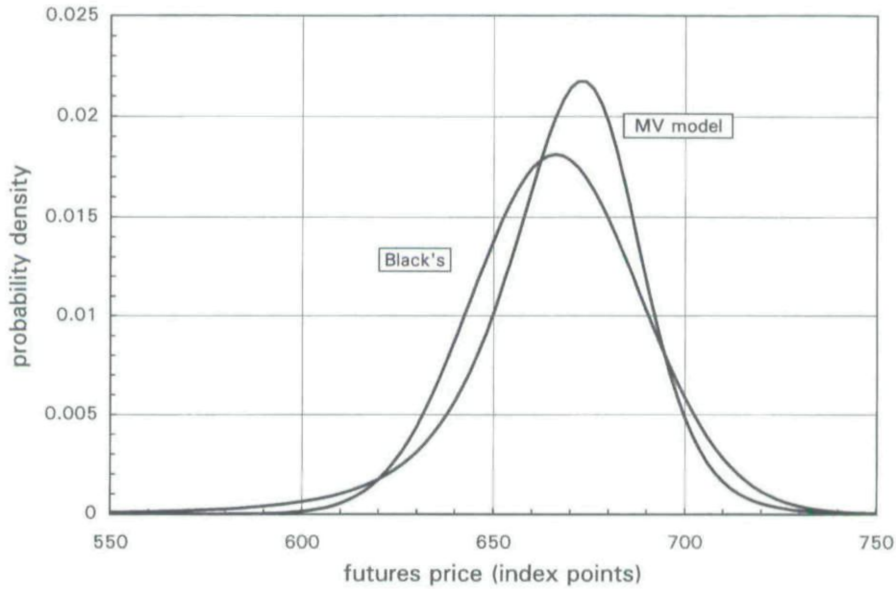

FIGURE 8

Terminal futures price distribution based on the August 27, 1996 closing prices of call options on the September 1996 S&P 500 index futures contract. Black's distribution is calculated with the use of the at-the-money implied volatility.

Because the model most probably does not capture the market's view exactly, and because there is noise in the market data, there is some mismatch between the market prices and the MV model prices. However, the model fits the data reasonably well. The same quality fit is typical of all the S&P data sets examined. If some of the deep out-of-the-money options are excluded from the least-squares calculation, then the resulting fit is greatly improved and in some cases nearly exact.

Futures price probability distributions at option expiration are shown in Figure 8 for the August 27 data, for the MV and Black models. Both distributions are calculated assuming European-style exercise. The distribution associated with the Black model is calculated with the use of the at-the-money implied volatility. It is lognormal and skewed to the right, whereas the actual distribution, calculated with the MV model, is skewed to the left. All calculated actual distributions for the S&P data are left skewed. The MV model distributions are used as a check of the numerical procedures by calculating the area under the curve and the average futures price. The areas are all between 0.99 and 1.02, and the means are within 0.05 of the futures price.

Figure 9 shows the technical level, M, calculated with the use of the MV model, and futures price plotted against days to option expiration for

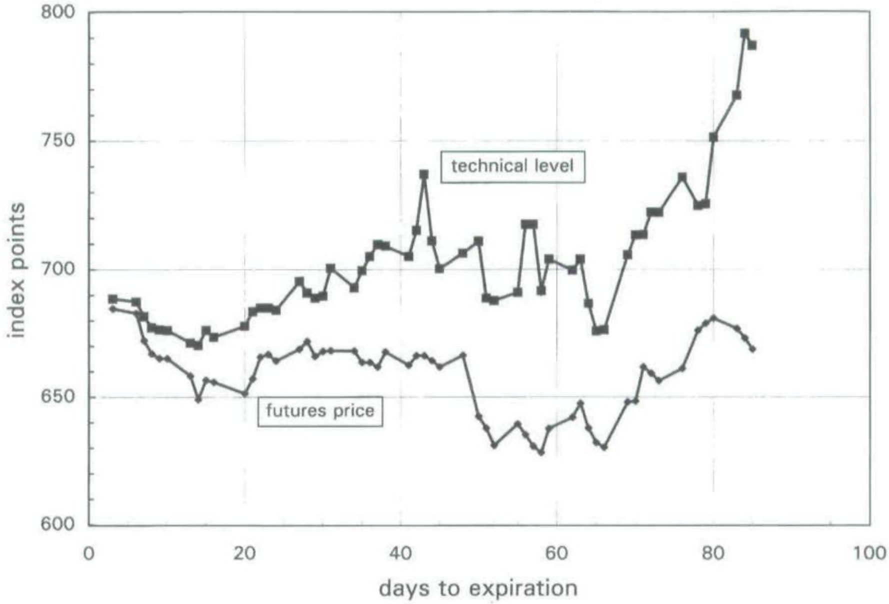

FIGURE 9
Calculated technical level, M, and closing futures price for the September 1996 S&P 500 index futures contract versus days to expiration between June 26 and September 16, 1996.

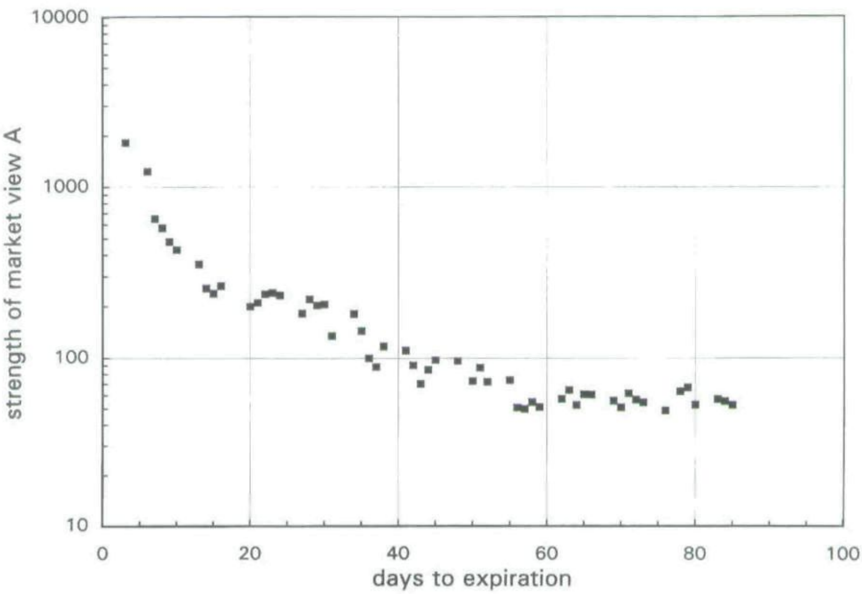

FIGURE 10
Strength of market view A versus days to expiration for the September 1996 S&P 500 index futures contract versus days to expiration between June 26 and September 16, 1996.

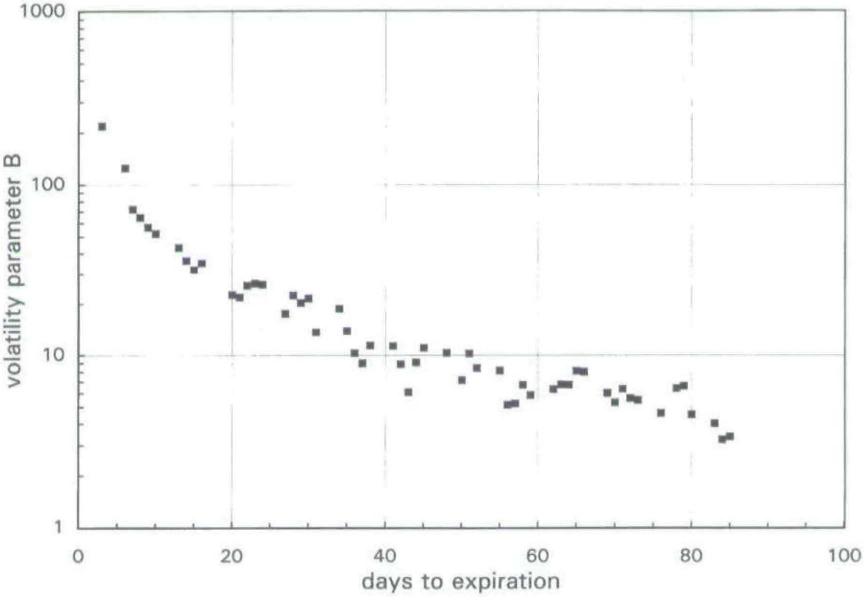

FIGURE 11

Volatility parameter B versus days to expiration for the September 1996 S&P 500 index futures contract versus days to expiration between June 26 and September 16, 1996.

all 56 S&P 500 data sets. As anticipated, the technical level and futures price converge as the expiration of the option gets closer. Figure 10 is a plot of the strength of market view, A, versus days to expiration. The trend indicates a steady increase in market confidence about the appropriateness of the technical level as the days to expiration decrease. Figure 11 shows the volatility parameter plotted against days to expiration. The volatility parameter increases as the days to expiration decrease, which is consistent with market folklore about accelerating information flow to the market as the expiration of the underlying futures contract draws closer.

Figure 9–11 create a picture of how market makers and market participants viewed the futures price of the S&P 500 index during this period. Far away from expiration, the market's confidence about the appropriateness of the futures price, as measured by A, is low and the market places an upper limit on the price. The technical level, M, behaves like an upper limit. This view gradually changes toward expiration; close to expiration the market's confidence, A, about the appropriateness of the futures price increases and the futures price is more readily accepted as being appropriate relative to the closing price of the futures at contract expiration. This causes the technical level and futures price to converge

FIGURE 12
Daily change in variance versus daily change in futures price for the September 1996 S&P 500 index futures versus days to expiration between June 26 and September 16, 1996.

as expiration approaches. The terminal probability distributions measured far from expiration are similar to the ones in Figure 1, in which there is a definite upper limit. Toward expiration the distributions are closer to the ones in Figure 3, in which the technical level is equal to the current futures price.

Figure 12 shows the daily change in variance of the terminal futures price distribution versus the daily change in futures price. Daily variance change is calculated with the use of variances from successive closing prices. In Table II, for example, closing prices are available on July 8 and 9, 1996. On each date the variance of the terminal distribution is calculated. Variance change values are calculated as the difference in variances from successive data sets. In this example the variances on the successive dates are 2,154 and 2,037, respectively. The corresponding futures prices are 656.3 and 659.15, respectively. Consequently, there is a data point (2.85, -117) in Figure 12. The results in Figure 12 indicate that when the futures price decreases, the variance increases and vice versa. When variance changes are calculated with the use of data from any two random days, for example, July 8 and September 12, the negative relationship seen in Figure 12 is not as strong.

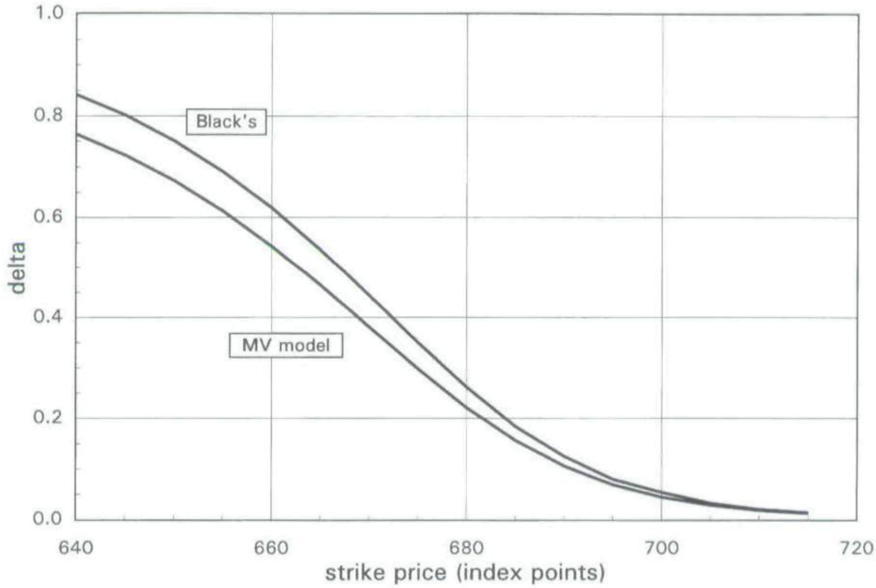

FIGURE 13
Call option delta versus strike price for the August 27, 1996 closing prices of call options on the September 1996 S&P 500 index futures contract. Black's delta is calculated with the use of the correct implied volatility at all strikes.

Black's model predicts the opposite effect because of the right skewness of the lognormal terminal distribution. The implication is that Black's model requires a larger delta hedge than as indicated by the market. Figure 13 is a plot of call delta versus strike price for the August 27 data. Black's delta is calculated with the use of the implied volatility for the option price associated with each strike. Black deltas are approximately 10% larger than MV model deltas.

The expected local volatility, $\sigma(F)$, of the futures price at expiration of the option obtained from eq. (7) is

$$E[\sigma(F)] = \sigma_0 + \frac{B}{M^2} V[F] \tag{11}$$

where $E[\cdot]$ is the expectation operator, σ_0 is the value of the local volatility when the current futures price is used in eq. (7), and $V[\cdot]$ is the variance operator. Figure 14 is a plot of expected terminal local volatility versus days to expiration. The symbols represent calculated values and the horizontal line is the average of all expected values. Expected local volatility does not increase toward expiration; instead, it appears to be mean reverting around 17%.

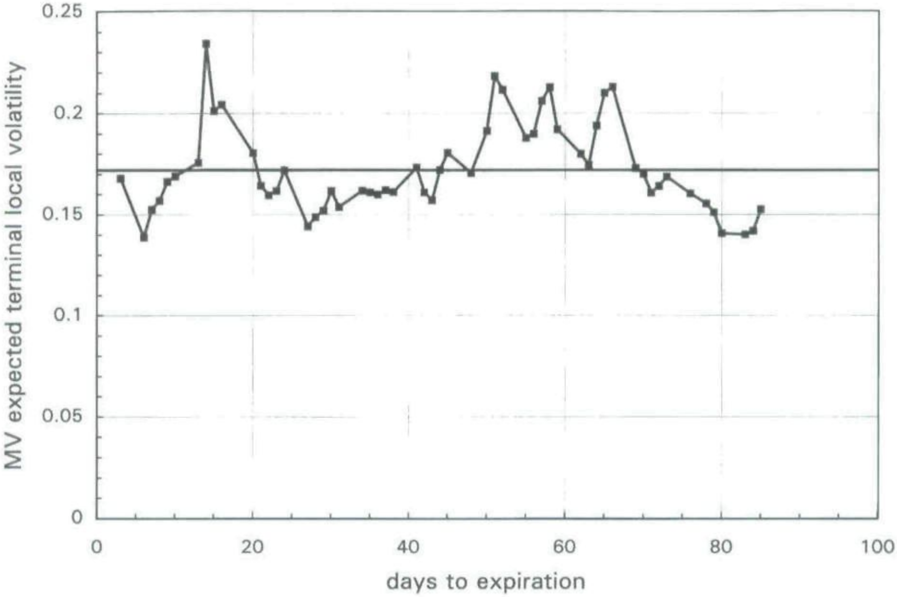

FIGURE 14
Expected terminal local volatility versus days to expiration for the September 1996 S&P 500 index futures contract versus days to expiration between June 26 and September 16, 1996.

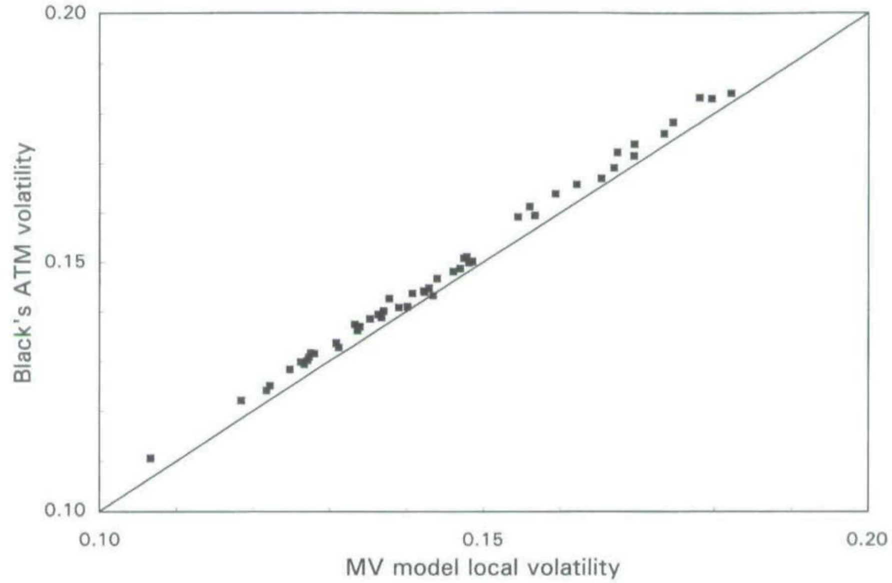

FIGURE 15
Black's at-the-money implied volatility versus MV model local volatility [eq. (7)] for the September 1996 S&P 500 index futures contract between June 26 and September 16, 1996.

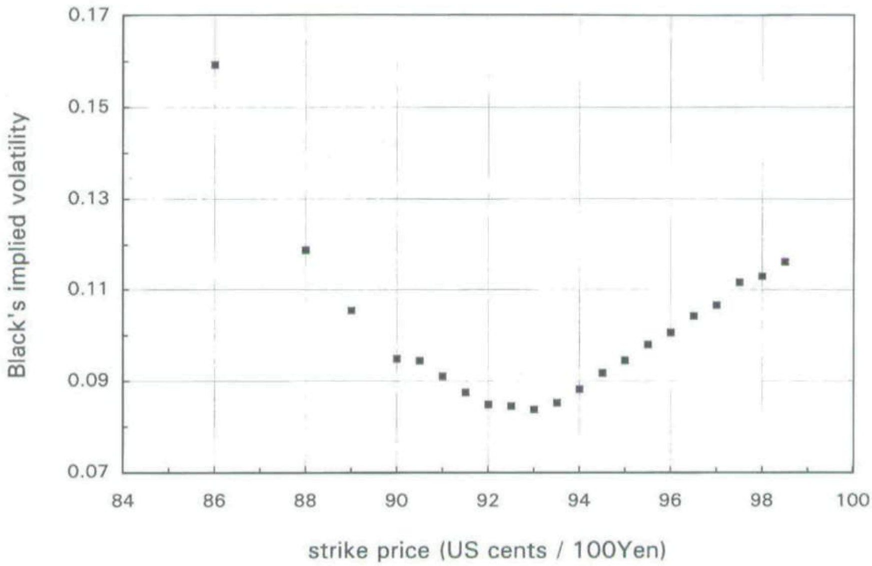

FIGURE 16
Black's implied volatility versus strike price for the August 6, 1996 closing prices of call options on the September 1996 Yen futures contract.

Figure 15 is a plot of Black's at-the-money implied volatility versus MV model local volatility $\sigma(F)$ for all the S&P data sets. If these values were equal, then the data points would fall along the 45° line. The Black implied volatility is consistently larger than MV model local volatility by approximately 5%. It is an interesting relationship that is useful in fitting the model to market data.

Figures 16–18 show the results of applying the model to the August 6, 1996 closing prices of call options on the September 1996 Yen futures contract. There is nothing special about the data; the data is chosen at random. The price of the futures contract converges to the spot price of the Japanese Yen at the expiration of the futures contract. The data analyzed in this example are taken from a downward trending market. The MV model predicts, as shown below, that the technical level should be below the current futures price. Figure 5 shows the downward trend in the Yen spot market for the 52 weeks prior to the expiration of the September 1996 futures contract. The symbols represent the ratio of weekly closing prices to the spot closing price on September 22, 1995.

Figure 16 is a plot of implied volatility versus strike price. The futures price is 94.14¢/100 Yen, and minimum volatility is at approximately 92.8¢/100 Yen. The model parameters are $F = 94.14$, $T = 29$ days, $r = 0.051$, $B = 27.44$, $A = 342.8$, and $M = 92.77$. The technical level

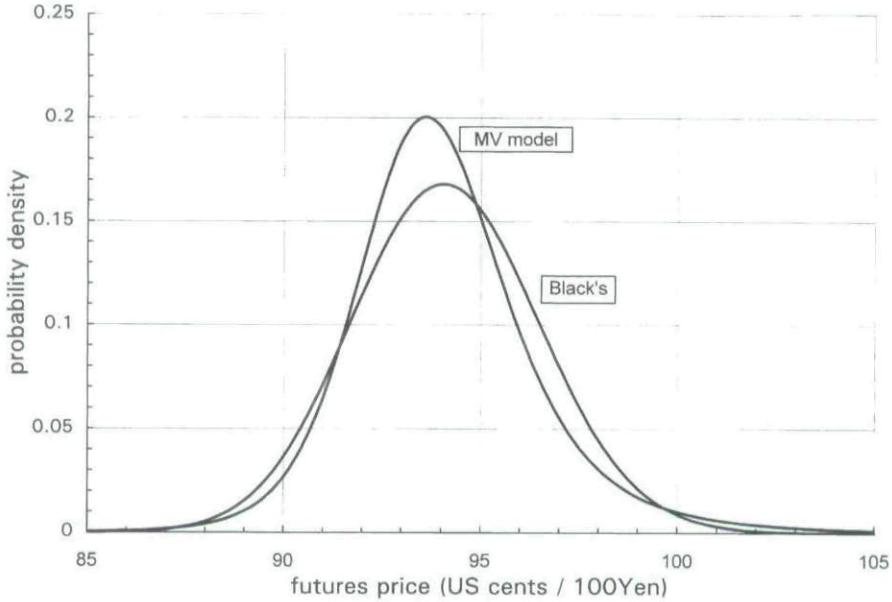

FIGURE 17
Terminal futures price distribution based on the August 6, 1996 closing prices of call options on the September 1996 Yen futures contract. Black's distribution is calculated with the use of the at-the-money implied volatility.

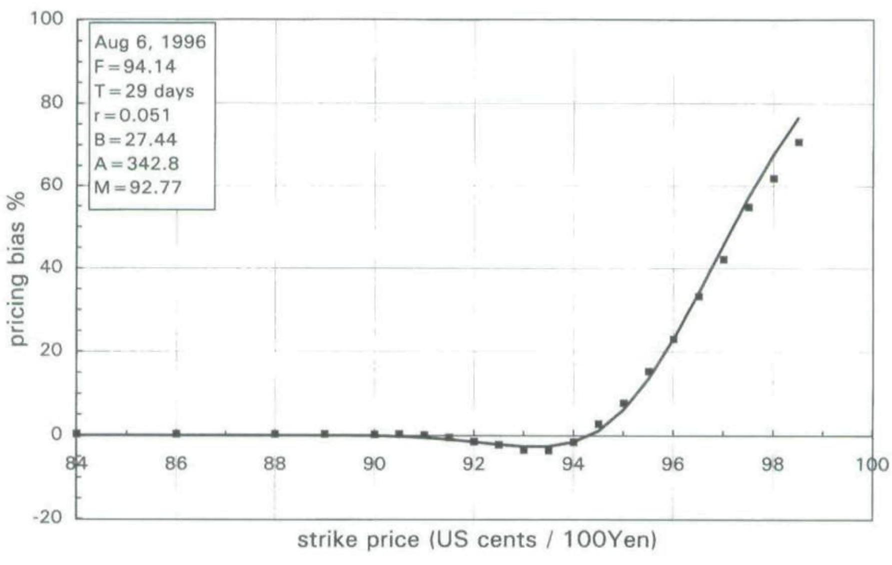

FIGURE 18
Pricing bias versus strike price for the August 6, 1996 closing prices of call options on the September 1996 Yen futures contract. The symbols represent market data and the curve represents MV model values.

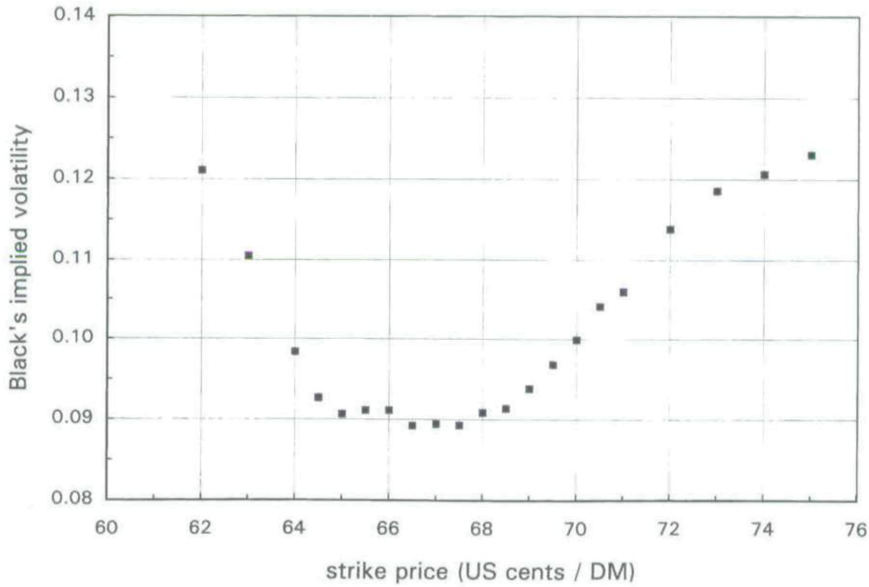

FIGURE 19
Black's implied volatility versus strike price for the July 18, 1996 closing prices of call options on the September 1996 D-Mark futures contract.

coincides with the minimum Black implied volatility. The technical level is below the futures price, which causes the MV model terminal futures price distribution to be slightly right skewed, as seen in Figure 17. Black's terminal distribution with the use of the at-the-money volatility is also shown. Pricing bias is shown in Figure 18. The bias is opposite to the S&P bias, because of the downward-trending market.

The last application is to the July 18, 1996 closing prices of call options on the September 1996 D-Mark futures contract. The price of the futures contract converges to the spot price of the German mark at the expiration of the futures contract. The German mark is also an example of a downward-trending market. The model predicts, as shown below, that the technical level should be below the current futures price. The D-mark spot price for the 52 weeks prior to the expiration of the September 1996 futures contract is shown in Figure 5. The symbols represent the ratio of weekly closing spot prices to the closing price on September 22, 1995.

The results are shown in Figures 19–21. Figure 19 is the implied volatility plot. The minimum occurs at 66.8¢/DM, which again coincides with the technical level. The level is also closer to the current futures price than in the Yen example, so the terminal distribution is closer to the ones shown in Figure 3, as shown in Figure 20. It is reasonably sym-

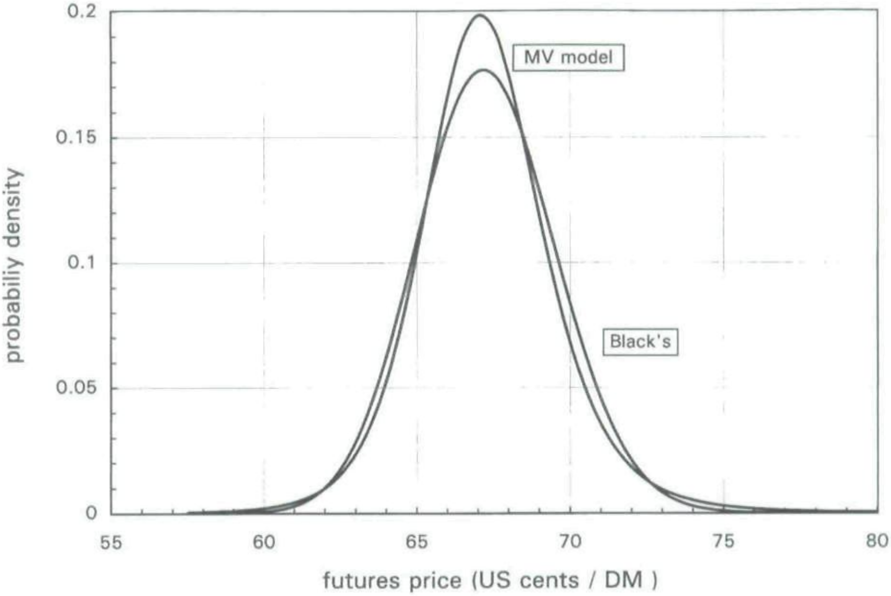

FIGURE 20

Terminal futures price distribution based on the July 18, 1996 closing prices of call options on the September 1996 D-Mark futures contract. Black's distribution is calculated with the use of the at-the-money implied volatility.

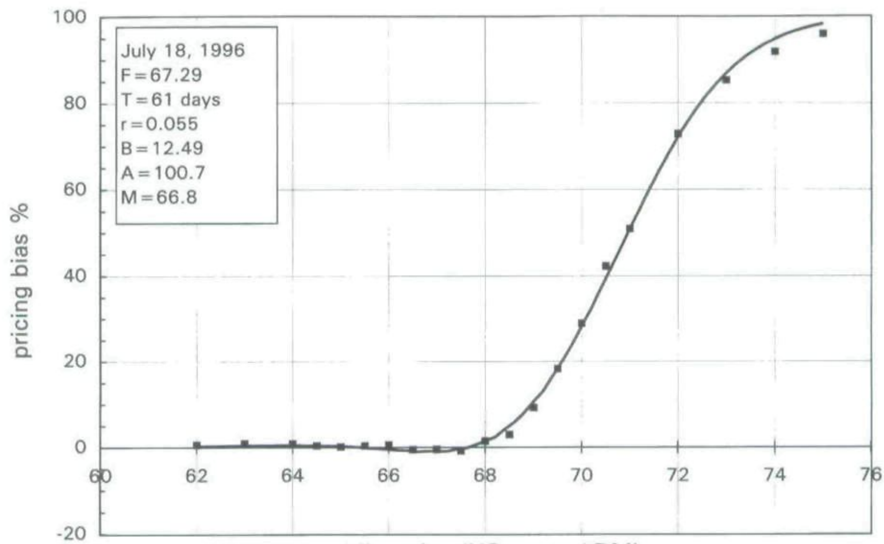

FIGURE 21

Pricing bias versus strike price for the July 18, 1996 closing prices of call options on the September 1996 D-Mark futures contract. The symbols represent market data and the curve represents MV model values.

metric. Pricing bias is shown in Figure 21, and it is opposite to that seen in the S&P data. Again, the model fits the data reasonably well. The calculated MV model parameters are $F = 67.29$, $T = 61$ days, $r = 0.055$, $B = 12.49$, $A = 100.7$, and $M = 66.8$.

The initial results of applying the market view model to market data are certainly encouraging. More testing is required to further establish the validity of the proposed method and model. Options on S&P 500 index futures should be used for further testing. This could involve simply repeating the initial tests with the use of option data from a different contract expiration date.

The limited results obtained with the currency options indicate that the model may also apply to these markets. The results are consistent with the MV model in the sense that both markets were downward trending, and the model predicts that the technical level should be below the current futures price, and this is found. The results are also consistent in the sense that option prices are reproduced with reasonable fidelity.

CONCLUSIONS

A stochastic model for the evolution of a futures price can be used to extract useful information from the price of options on futures contracts. Because market makers price options based on a view of the volatility of the underlying futures contract, it is possible to extract useful information about this view with a stochastic model. The stochastic model must be based on a set of possible variations of the futures price as collectively perceived by market participants. The actual futures price may not follow the proposed stochastic process, but this does not matter. What does matter is the fact that the market believes the futures price should follow the assumed stochastic process and, therefore, prices options according to this belief. The results show that information extracted about market participants' views on the S&P 500, USD/¥, USD/DM futures prices is consistent with the market view model.

The method used to develop the market-view model, and the model itself should be of interest to researchers and traders. Researchers can use the approach outlined here to develop new stochastic processes based on interpreting transition probabilities in terms of how market makers view possible price movements of the underlying asset. This is a significant departure from the previous approach of modeling the actual stochastic behavior of the underlying asset. It is also easier to develop models this way because opinions can be elicited directly from market makers, whereas it is very difficult to uncover the factors controlling the actual stochastic process followed by the futures. The method should also apply to other assets.

Traders can benefit immediately by applying the market-view model to options on S&P 500 index futures to extract the market's collective perceptions of not only volatility but also the technical level of the underlying futures contract and the strength of the market's view about the appropriateness of the technical level. They can also use the market-view model to price and hedge a variety of path-dependent options with confidence because the market-view model can reproduce option prices with a fair degree of accuracy.

APPENDIX

Finite-Difference Approximation

The essential approximations used to numerically solve the governing partial differential equation are outlined below. The difference scheme is not robust; certain parameter sets give invalid results. Solutions should be checked by calculating the area under the probability density curve and also by calculating the expected futures price. If the results of these tests are not as expected, then the difference scheme has failed. One case known to cause problems is that when the current futures price is much larger than the technical level. Distributions in cases where the futures price is much greater than the technical level, that is, when the market feels that the current futures price is much too high, are often bimodal. However, calculated results always failed the probability distribution tests, sometimes terribly and sometimes just a little. A better numerical procedure must be developed to handle a wider range of parameters.

The method used to derive the differential equation is well established; see, for example Hull (1997). The governing equation is

$$\frac{\partial f}{\partial t} + \frac{B^2}{2} F^2[1/A + (1 - F/M)^2]^2 \frac{\partial^2 f}{\partial F^2} = rf \tag{A.1}$$

The equation is in backward form. It can be written in forward form by using the transformation $\tau = T - t$. When this is done the differential equation becomes

$$\frac{\partial f}{\partial \tau} = \frac{B^2}{2} F^2[1/A + (1 - F/M)^2]^2 \frac{\partial^2 f}{\partial F^2} - rf \tag{A.2}$$

The initial condition for a call is

$$call\ price = Max(F - K, 0), \qquad @\tau = 0 \tag{A.3}$$

A modified Crank–Nicolson scheme is used to discretize the differential equation. A uniform grid is used with time given by $j\ \Delta\tau$ and futures price given by $i\ \Delta F$. The approximation is centered at grid point $i\ \Delta F\ (j + \frac{1}{2})\ \Delta\tau$. The time derivative is approximated with the forward difference as

$$\frac{\partial f}{\partial \tau} = \frac{f_i^{j+1} - f_i^j}{\Delta\tau} \tag{A.4}$$

An averaged central difference is used for the first price derivative:

$$\frac{\partial f}{\partial F} = \frac{1}{2}\left[\frac{f_{i+1}^{j+1} - f_{i-1}^{j+1}}{2\ \Delta F} + \frac{f_{i+1}^{j} - f_{i-1}^{j}}{2\ \Delta F}\right] \tag{A.5}$$

An averaged central difference is also used for the second price derivative:

$$\frac{\partial^2 f}{\partial F^2} = \frac{1}{2}\left[\frac{f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}}{\Delta F^2} + \frac{f_{i+1}^{j} - 2f_i^{j} + f_{i-1}^{j}}{\Delta F^2}\right] \tag{A.6}$$

In the grid the futures price ranges from 0 to the maximum F_{infinity}; the range is divided into $N + 1$ increments. At $i = 1$ and $i = N$ the slope of the option price is approximated. At $F = \Delta F$ the slope of the call option curve (with respect to futures price) is approximately zero. While at $F = N\ \Delta F$ the slope is $\exp(-r\tau)$. The slope at these points is approximated by the averaged central difference [eq. (A5)]. This allows the value of the option at grid point $i = 0$ to be determined in terms of the values at grid point $i = 2$. Likewise, the value of the option at grid point $N + 1$ can be written in terms of the value at grid point $i = N - 1$. With these approximations a tridiagonal system of equations is produced at each time step, which can be solved easily by back substitution.

For i ranging from 1 to N the tridiagonal system of equations is

$$a_i f_{i-1}^{j+1} + b_i f_i^{j+1} + c_i f_{i+1}^{j+1} = A_i f_{i-1}^{j} + B_i f_i^{j} + C_i f_{i+1}^{j} \tag{A.7}$$

where

$$a_i = -X, \qquad b_i = \frac{1}{\Delta\tau} + 2X + \frac{r}{2}, \qquad c_i = -X,$$

$$X = \frac{B^2 i^2}{4}\left(1/A + \left(1 - \frac{i\,\Delta F}{M}\right)^2\right)^2,$$

$$A_i = X, \qquad B_i = \frac{1}{\Delta\tau} - 2X - \frac{r}{2}, \qquad \text{and } C_i = X \tag{A.8}$$

There are N equation and $N + 2$ option prices. The two additional equations come from the slopes at either end. At the bottom end ($F = \Delta F$)

$$f_0^{j+1} = f_2^{j+1} + f_2^j - f_0^j \tag{A.9}$$

At the top end ($F = N\,\Delta F$)

$$f_{N+1}^{j+1} = f_{N-1}^{j+1} - f_{N+1}^j + f_{N-1}^j + 4\,\Delta F \exp(-r(j + 0.5)\,\Delta\tau) \tag{A.10}$$

Excel Function

VBA code for an Excel function that implements the finite-difference scheme is given below. The program runs slowly in Excel; a typical calculation may take 2–3 s. It is much faster when the function is written as a DLL. The difference scheme given above does not always work when F is approximately 20% larger than M. Check all results by calculating the area under the probability curve and by calculating the expected futures price.

```
Function Euro_call(F_infinity As Double, F_max As Double, z As Double,_  F_price As
Double, t As Double, r As Double, sigma As Double, strike As Double)

Dim a(512) As Double, b(512) As Double, c(512) As Double
Dim u(0 To 512 + 1) As Double, uold(0 To 512 + 1) As Double
Dim gam(512) As Double, d(512) As Double
Dim, deltaN As Double, dt As Double, dx As Double, i As Integer
Dim bet As Double, slope As Double, j1 As Integer, j As Integer, x As Double
Dim ilow As Integer, iup As Integer, copt As Double, n As Integer, m As Integer

'F_infinity = Infinite futures price (for the finite difference scheme, approx = 2.5F)
'F_max = M technical level
'z = 1/A strength of view
'F_price = F futures price
```

```
't = T option term
'r = risk-free rate
'sigma = B volatility parameter

    n = 512: m = 30: dt = t/m: dx = F_infinity / (n + 1)
    For i = 0 To n + 1 Step 1: uold(i) = Application.Max(i * dx - strike, 0): Next i
    For j = 1 To m
    deltaN = Exp(-r * (j + 0.5) * dt)
    For i = 1 To n
    x = ((sigma * i) ^ 2) / 4 * (z + (1 - i * dx / F_max) ^2 ) ^ 2
    a(i) = -x: b(i) = 1 / dt + 2 * x + r / 2: c(i) = -x
    d(i) = -uold(i - 1) * a(i) + uold(i) * (2 / dt - b(i)) - uold(i + 1) * c(i)
    Next i
    c(1) = a(1) + c(1): d(1) = d(1) - a(1) * (uold(2) - uold(0)): a(n) = a(n) + c(n)
    d(n) = d(n) - c(n) * (4 * dx * deltaN - uold(n + 1) + uold(n - 1))
    bet = b(1): u(1) = d(1) / bet
    For j1 = 2 To n
    gam(j1) = c(j1 - 1) / bet: bet = b(j1) - a(j1) * gam(j1)
    u(j1) = (d(j1) - a(j1) * u(j1 - 1)) / bet
    Next j1
    For j1 = n - 1 To 1 Step - 1
    u(j1) = u(j1) - gam(j1 + 1) * u(j1 + 1)
    Next j1
    u(0) = u(2) + uold(2) - uold(0)
    u(n + 1) = 4 * dx * deltaN + u(n - 1) - uold(n + 1) + uold(n - 1)
    For i = 0 To n + 1: uold(i) = u(i): Next i
    Next j
    ilow = Int(F_price / dx): iup = ilow + 1: slope = (u(iup) - u(ilow)) / dx
    Euro_call = slope * (F_price - ilow * dx) + u(ilow)
    End Function
```

S&P Data Modeled

The S&P data examined in the article are given in Table AI.

TABLE AI

Daily Closing Prices of Call Options on the September 1996 S&P 500 Index Futures from June 26 to September 16, 1996. The Data Source is Bloomberg.

Date	8/6	8/5	8/2	7/31	7/30	7/29	7/26	7/25	7/24	7/23	7/22	7/19	7/18	7/18
Days	44	45	48	50	51	52	55	56	57	58	59	62	63	64
Futures	664.25	661.70	666.35	642.40	637.70	631.00	639.35	635.20	630.70	628.15	637.55	642.00	647.45	637.8
Strike	Call option prices (index points)													
510	154.25	151.70	156.35	132.55	127.85	121.35	129.35	125.30	121.05	118.70	127.70	132.00	137.45	127.90
530	134.25	131.70	136.35	112.75	108.25	101.90	109.85	105.75	101.65	99.35	108.15		117.55	108.25
550	114.35	111.80	116.45	93.55	89.05	82.75	90.30	86.65	82.80	80.50	88.95		98.05	89.15
580	85.10	82.60	87.15	65.30	61.25	55.25	62.10	58.95	55.85	53.65	61.30		69.40	61.30
600	66.10	63.60	68.00	47.40	43.85	38.40	44.50	41.80	39.30	37.20	44.00	46.20	51.30	44.20
610	56.65	54.40	58.60	38.85	35.70	30.70	36.20	33.75	31.60	29.75	36.00	36.00	42.65	36.15
620	47.60	45.45	49.50	30.95	28.10	23.60	28.55	26.45	24.65	23.00	28.40		34.60	28.85
625	43.20	41.05	45.05	27.15	24.60	20.35	24.95	23.10	21.50	19.90	24.95	27.60	30.80	25.40
630	38.90	36.85	40.75	23.75	21.40	17.40	21.45	19.95	18.55	17.10	21.70		27.15	22.20
635	34.70	32.70	36.50	20.35	18.25	14.50	18.25	17.00	15.75	14.40	18.65		23.65	19.10
640	30.60	28.75	32.25	17.15	15.25	12.00	15.30	14.25	13.25	12.00	15.75	17.80	20.35	16.30
645	26.65	24.95	28.30	14.25	12.65	9.65	12.60	11.75	10.95	9.85	13.20	15.00	17.35	13.75
650	22.95	21.35	24.50	11.60	10.30	7.70	10.20	9.50	8.85	7.95	10.90	12.40	14.60	11.45
655	19.45	18.00	20.95	9.30	8.20	6.00	8.10	7.60	7.15	6.40	8.90	9.40	12.05	9.35
660	16.20	14.90	17.60	7.35	6.45	4.60	6.30	5.95	5.55	4.90	7.10	8.00	9.75	7.60
665	13.15	12.00	14.55	5.65	4.90	3.40	4.80	4.60	4.30	3.80	5.60	5.80	7.80	6.10
670	10.55	9.70	11.70	4.25	3.70	2.50	3.60	3.50	3.25	2.90	4.30	4.80	6.15	4.80
675	8.30	7.40	9.30	3.00	2.60	1.75	2.60	2.60	2.40	2.15	3.25	4.40	4.70	3.80
680	6.35	5.65	7.25	2.00	1.85	1.25	1.85	1.80	1.70	1.55	2.40	2.80	3.50	2.90
685	4.75	4.20	5.50	1.35	1.35	0.90	1.30	1.25	1.20	1.15	1.80	1.80	2.60	2.25
690	3.45	3.10	4.05	0.95	1.00	0.65	0.90	0.90	0.90	0.90	1.25	1.50	2.00	1.75
695	2.45	2.20	2.90	0.60	0.65	0.45	0.60	0.60	0.65	0.80	0.85	1.00	1.30	1.25
700	1.65	1.50	2.05	0.40	0.45	0.30	0.40	0.40	0.45	0.45	0.60	0.75	0.80	0.80
705	1.10	1.05	1.45	0.20	0.25	0.20	0.25	0.25	0.30	0.30	0.40	0.60	0.60	0.60
710	0.70	0.70	1.00	0.10	0.10	0.10	0.15	0.15	0.20	0.20	0.30	0.45	0.45	0.45
715	0.50	0.50	0.70	0.05	0.05	0.05	0.10	0.10	0.15	0.15	0.20		0.35	0.35
720	0.35	0.35	0.45	0.05	0.05	0.05	0.05	0.05	0.10	0.10	0.15		0.25	0.25
725	0.20	0.20	0.30		0.10		0.05	0.05	0.10	0.10	0.10	0.15	0.20	0.20
Date	7/16	7/15	7/12	7/11	7/10	7/9	7/8	7/5	7/3	7/2	7/1	6/28	6/27	6/26
Days	65	66	69	70	71	72	73	76	78	79	80	83	84	85
Futures	632.10	630.25	648.05	648.35	661.70	659.15	656.30	661.10	676.10	678.80	680.80	676.80	673.05	668.65
Strike	Call option prices (index points)													
510	123.05	121.45	138.05	138.35	151.70	149.15	146.30	151.10	166.10	168.80	170.80	166.80	163.05	158.65
530	103.45	101.90	118.15	118.55	131.70	129.15	126.35	131.10	146.10	148.80	150.80	146.80	143.05	138.65
550	84.25	82.70	98.60	98.95	111.90	109.45	106.65	111.35	126.10	128.80	130.80		123.20	118.85
580	56.65	55.25	69.95	70.35	82.60	80.45	77.95	82.40	96.55	99.10	101.20	97.30	93.95	90.00
600	40.05	38.90	52.20	52.50	64.00	61.95	59.65	63.85	77.50	79.90	81.85	78.05	74.80	71.15
610	32.40	31.35	43.70	43.95	54.90	53.05	50.95	54.95	68.05	70.45	72.20	68.60	65.40	61.90
620	25.50	24.55	35.70	36.00	46.25	44.60	42.60	46.50	59.05	61.15	62.90	59.40	56.35	53.25
625	22.35	21.45	31.90	32.25	42.10	40.45	38.55	42.35	54.55	56.60	58.35	54.95	51.95	49.00
630	19.40	18.55	28.30	28.60	38.05	36.50	34.70	38.35	50.20	52.25	53.90	50.60	47.70	44.85
635	16.50	15.75	24.80	25.15	34.05	32.70	31.05	34.50	45.95	47.95	49.55	46.30	43.50	40.85
640	13.95	13.35	21.50	21.85	30.25	28.95	27.40	30.75	41.80	43.70	45.30	42.10	39.45	36.95
645	11.75	11.25	18.45	18.80	26.60	25.40	24.05	27.20	37.75	39.60	41.15	38.10	35.50	33.25
650	9.70	9.25	15.60	15.90	23.20	22.05	20.85	23.80	33.85	35.60	37.10	34.20	31.75	29.65
655	7.90	7.50	13.05	13.55	20.00	19.00	17.90	20.70	30.10	31.80	33.15	30.45	28.20	26.30
660	6.30	5.95	10.80	11.10	17.10	16.20	15.15	17.75	26.50	28.10	29.40	26.85	24.80	23.05
665	5.00	4.70	8.80	9.05	14.35	13.55	12.65	15.00	23.10	24.60	25.85	23.45	21.55	19.95
670	3.95	3.70	7.05	7.25	11.90	11.20	10.45	12.55	20.05	21.35	22.50	20.25	18.50	17.10
675	3.10	2.90	5.55	5.70	9.65	9.15	8.50	10.40	17.10	18.30	19.40	17.25	15.65	14.50
680	2.40	2.25	4.20	4.35	7.70	7.30	6.80	8.40	14.35	15.45	16.45	14.55	13.15	12.10
685	1.85	1.75	3.15	3.25	6.00	5.70	5.25	6.60	11.95	12.85	13.75	12.05	10.85	10.00
690	1.45	1.40	2.35	2.45	4.65	4.35	4.00	5.20	9.75	10.60	11.35	9.80	8.80	8.10
695	1.05	1.00	1.75	1.80	3.50	3.25	3.00	4.00	7.85	8.55	9.20	7.80	6.95	6.45
700	0.65	0.60	1.25	1.30	2.60	2.40	2.20	3.00	6.20	6.75	7.35	6.05	5.35	4.95
705	0.50	0.45	0.85	0.90	1.90	1.75	1.60	2.50	4.75	5.25	5.70	4.65	4.05	3.75
710	0.35	0.35	0.60	0.65	1.40	1.25	1.15	1.60	3.65	4.05	4.35	3.45	2.85	2.65
715	0.25	0.25	0.40	0.45	1.00	0.90	0.80	1.20	2.80	3.05	3.30	2.50	2.00	1.95
720	0.20	0.20	0.30	0.35	0.70	0.60	0.55	0.85	2.10	2.30	2.40	1.80	1.35	1.40
725	0.15	0.15	0.20	0.25	0.55	0.45	0.40	0.60	1.45	1.65	1.75	1.30	0.90	0.95

TABLE AI (Continued)

Daily Closing Prices of Call Options on the September 1996 S&P 500 Index Futures from June 26 to September 16, 1996. The Data Source is Bloomberg.

Date	9/16	9/13	9/12	9/11	9/10	9/9	9/6	9/5	9/4	9/3	8/30	8/29	8/28	8/27
Days	3	6	7	8	9	10	13	14	15	16	20	21	22	23
Futures	684.70	682.85	672.15	667.00	665.10	665.05	658.25	649.00	656.60	655.85	651.35	657.15	665.70	666.70
Strike	Call option prices (index points)													
510	174.70	172.85	162.15	157.00	155.10	155.05	148.25	139.00	146.60	145.85	141.35	147.15	155.70	156.70
530	154.70	152.85	142.15	137.00	135.10	135.05	128.25	119.00	126.60	125.85	121.35	127.15	135.70	136.70
550	134.70	132.85	122.15	117.00	115.10	115.05	108.25	99.05	106.60	105.85	101.40	107.15	115.70	116.70
580	104.70	102.85	92.15	87.00	85.10	85.10	78.30	69.60	76.90	76.15	71.75	77.35	85.80	86.80
600	84.70	82.85	72.15	67.05	65.20	65.15	58.50	50.35	57.30	56.55	52.35	57.80	66.05	67.00
610	74.70	72.85	62.20	57.10	55.25	55.25	48.70	40.85	47.65	46.95	42.85	48.10	56.25	57.25
620	64.75	62.85	52.25	47.15	45.35	45.40	39.05	31.80	38.20	37.50	33.50	38.60	46.55	47.55
625	59.75	57.90	47.30	42.20	40.45	40.50	34.30	27.35	33.55	32.90	29.05	33.95	41.75	42.75
630	54.75	52.90	42.35	37.30	35.60	35.65	29.55	23.05	29.05	28.05	24.80	29.40	37.10	38.10
635	49.80	47.95	37.40	32.40	30.75	30.80	24.90	19.00	24.65	24.10	20.65	25.05	32.45	33.45
640	44.85	43.00	32.50	27.55	25.95	26.05	20.45	15.20	20.40	19.90	16.70	20.85	27.90	28.90
645	39.85	38.05	27.60	22.80	21.30	21.45	16.25	11.70	16.40	15.95	13.10	16.85	23.60	24.55
650	34.90	33.10	22.75	18.20	16.90	17.05	12.40	8.70	12.80	12.45	9.95	13.25	19.45	20.45
655	29.95	28.15	18.15	13.80	12.70	12.95	9.00	6.20	9.60	9.30	7.20	10.00	15.60	16.55
660	25.00	23.25	13.70	9.80	8.95	9.25	6.10	4.15	6.90	6.65	4.95	7.20	12.10	12.90
665	20.10	18.40	9.50	6.30	5.75	6.10	3.85	2.65	4.70	4.55	3.25	4.90	8.95	9.75
670	15.25	13.70	6.00	3.65	3.40	3.75	2.30	1.60	3.00	2.90	1.95	3.10	6.30	7.00
675	10.60	9.35	3.35	1.85	1.80	2.10	1.30	0.95	1.85	1.80	1.15	1.85	4.25	4.80
680	6.40	5.55	1.60	0.80	0.85	1.00	0.70	0.60	1.10	1.10	0.70	1.05	2.70	3.15
685	3.10	2.80	0.70	0.30	0.40	0.45	0.40	0.35	0.65	0.65	0.40	0.60	1.65	1.95
690	1.20	1.20	0.35	0.15	0.20	0.25	0.25	0.25	0.40	0.40	0.25	0.35	1.00	1.20
695	0.35	0.35	0.15	0.10	0.10	0.15	0.15	0.15	0.25	0.25	0.15	0.20	0.60	0.70
700	0.15	0.15	0.10	0.05	0.05	0.10	0.10	0.10	0.15	0.15	0.10	0.10	0.35	0.45
705	0.10	0.05	0.05			0.05	0.05	0.05	0.10	0.10	0.05	0.05	0.20	0.25
710	0.05	0.05					0.05		0.05	0.05		0.05	0.10	0.15
715													0.05	0.10
720													0.05	0.05
725														0.05
Date	8/26	8/23	8/22	8/21	8/20	8/19	8/16	8/15	8/14	8/13	8/12	8/9	8/8	8/7
Days	24	27	28	29	30	31	34	35	36	37	38	41	42	43
Futures	664.20	668.80	671.85	666.00	667.95	668.20	668.10	663.60	663.60	661.80	667.75	662.45	666.20	666.20
Strike	Call option prices (index points)													
510	154.20	158.80	161.85	156.00	157.95	158.20	158.10	153.60	153.60	151.80	157.75	152.45	156.20	156.20
530	134.20	138.80	141.85	136.00	137.95	138.20	138.10	133.60	133.60	131.80	137.75	132.45	136.20	136.20
550	114.20	118.80	121.85	116.00	117.95	118.20	118.10	113.60	113.60	111.85	117.75		116.25	116.25
580	84.35	88.95	91.95	86.15	88.15	88.40	88.30	83.90	84.05	82.40	88.20	83.20	86.65	86.75
600	64.65	69.15	72.15	66.50	68.45	68.85	68.75	64.50	64.60	63.00	68.70	63.90	67.35	67.55
610	54.95	59.30	62.30	56.70	58.75	59.15	59.10	55.00	55.10	53.60	59.10	54.55	57.90	58.15
620	45.40	49.65	52.55	47.05	49.10	49.60	49.60	45.65	45.85	44.50	49.80	45.50	48.60	48.90
625	40.70	44.85	47.70	42.35	44.35	44.85	44.90	41.05	41.35	40.00	45.15	41.00	44.10	44.45
630	36.15	40.15	43.00	37.65	39.75	40.30	40.30	36.55	36.90	35.60	40.65	36.75	39.80	40.05
635	31.55	35.50	38.30	33.10	35.20	35.75	35.85	32.25	32.60	31.40	36.25	32.55	35.45	35.75
640	27.10	30.95	33.65	28.70	30.75	31.35	31.45	28.00	28.40	27.30	32.00	28.50	31.30	31.60
645	23.00	26.50	29.20	24.45	26.50	27.10	27.25	24.00	24.45	23.40	27.85	24.65	27.25	27.60
650	19.00	22.30	24.90	20.45	22.40	23.00	23.15	20.15	20.60	19.70	23.85	20.90	23.40	23.75
655	15.25	18.35	20.80	16.65	18.55	19.15	19.30	16.55	17.05	16.25	20.15	17.50	19.85	20.15
660	11.85	14.60	16.85	13.05	14.90	15.50	15.70	13.30	13.80	13.15	16.75	14.40	16.45	16.80
665	8.85	11.25	13.25	9.90	11.60	12.20	12.40	10.30	10.85	10.35	13.50	11.45	13.30	13.65
670	6.35	8.30	10.10	7.20	8.75	9.25	9.45	7.75	8.35	7.95	10.60	8.95	10.65	11.00
675	4.30	5.85	7.35	5.05	6.35	6.85	7.05	5.60	6.25	5.95	8.10	6.80	8.35	8.70
680	2.85	3.90	5.10	3.35	4.45	4.90	5.10	3.90	4.50	4.30	6.05	5.00	6.25	6.60
685	1.80	2.50	3.40	2.15	2.95	3.35	3.50	2.60	3.15	3.00	4.30	3.55	4.60	4.90
690	1.10	1.50	2.20	1.30	1.90	2.20	2.30	1.65	2.10	2.00	3.00	2.45	3.20	3.55
695	0.65	0.85	1.35	0.75	1.20	1.35	1.45	1.00	1.35	1.30	1.95	1.60	2.10	2.40
700	0.40	0.50	0.80	0.40	0.70	0.90	0.95	0.65	0.80	0.80	1.15	1.05	1.45	1.60
705	0.25	0.30	0.50	0.25	0.40	0.55	0.60	0.40	0.50	0.50	0.65	0.60	0.95	1.05
710	0.15	0.20	0.35	0.15	0.25	0.35	0.35	0.25	0.30	0.30	0.30	0.30	0.60	0.65
715	0.10	0.15	0.25	0.10	0.15	0.25	0.25	0.20	0.20	0.20	0.20	0.20	0.45	0.45
720	0.05	0.10	0.15	0.05	0.10	0.20	0.20	0.15	0.15	0.15	0.15	0.15	0.30	0.30
725	0.05	0.05	0.10		0.05	0.15	0.15	0.10	0.10	0.10	0.10	0.10	0.15	0.15

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