Out-of-Sample Hedging Effectiveness of Currency Futures for Alternative Models and Hedging Strategies

ABE DE JONG
FRANS DE ROON
CHRIS VELD

INTRODUCTION

Since their introduction in the early 1970s, currency futures have become increasingly popular. Dubofski (1992, p. 314) points out that from October, 1989 to September, 1990 only, the Chicago Mercantile Exchange traded more than 7 million futures contracts on German marks and more than 9 million futures contracts on Japanese yen. These contracts are mainly used to hedge currency risk.

The authors are grateful to Frank Brouwer, Frank de Jong, Bruno Solnik, and two anonymous referees for providing many helpful comments. This article has also benefited from comments of the participants at the Inquire Conference in Barcelona (November 1995) and of seminar participants at Tilburg University and Erasmus University Rotterdam. The usual disclaimer applies. The research for this article was conducted while Frans de Roon was affiliated with the Department of Business Administration and CentER at Tilburg University.

- *Abe de Jong is a Ph.D. Student at the Department of Business Administration and CentER at Tilburg University, P.O. Box 90153, NL-5000 LE Tilburg, the Netherlands. Tel: +3113-4662020; fax: +3113 -4662875; e-mail: Abe.deJong@kub.nl*
- *Frans de Roon is an Assistant Professor at the Department of Finance at Erasmus University Rotterdam, the Netherlands.*
- *Chris Veld is an Associate Professor at the Department of Business Administration and CentER at Tilburg University.*

The Journal of Futures Markets, Vol. 17, No. 7, 817–837 (1997)

CCC 0270-7314/97/070817-21

The easiest way to use futures contracts for hedging purposes is by creating a naive hedge. In such a hedge a futures position is created with an equal magnitude and an opposite sign compared to the spot position. Unfortunately, the changes of spot and futures prices of currencies are not perfectly correlated. Naive hedging with currency futures transforms currency risk into basis risk.[1] A naive hedge reduces risk optimally only in the absence of basis risk. Therefore, many studies have tried to find the optimal proportion of futures to hedge with, that is, the hedge ratio. In these studies expressions for the optimal hedge ratio are derived for various utility functions. These hedges are referred to as model-based or optimal hedge ratio hedges. Different hedging strategies can be based on whether the hedge ratio is allowed to be time-varying or constant, and, if it is constant, on what level the hedge ratio is chosen. To test to what extent different hedging strategies improve the utility of a hedger, the hedging effectiveness can be measured. In this study different utility models are studied to compare the hedging effectiveness of different hedging strategies. These are the minimum-variance model of Ederington (1979), the α-t model of Fishburn (1977), and the Sharpe-ratio model of Howard and D'Antonio (1984, 1987).

The purpose of this article is twofold. The first purpose is to study the hedging effectiveness of futures for the different *utility functions* that are assumed in determining the hedge ratio and the hedging effectiveness. In the minimum-variance model and in the α-t model, hedging effectiveness is perceived as minimizing the risk of a portfolio. In the Sharpe-ratio model proposed by Howard and D'Antonio (1984), the trade-off between risk and return is optimized. Because effectiveness refers to attaining an objective and models of hedging effectiveness have various assumptions with regard to the perceived objective of the hedge, the specification of this objective is essential.

The second purpose of this article is to investigate the hedging effectiveness for *different hedging strategies in an out-of-sample setting*. The first strategy that is tested is the naive hedge, that is, a hedge ratio of 1. The second strategy uses the optimal hedge ratio. The third hedging strategy uses a constant hedge ratio, which is an average of the optimal hedge ratios. Both the theoretical and empirical literature generally assume an in-sample approach. In such an approach it is assumed that the optimal futures position and the hedging effectiveness can be determined in the same period. Because this is highly unrealistic in practice, practitioners are more interested in out-of-sample hedging performance. Therefore,

[1]The change of the basis is the change of the difference between the spot price of a currency and the futures price of the same currency.

this article assumes that a hedger first uses a specific period to calculate the model-based hedge ratio. After this period the amount of futures contracts, as calculated by this model-based hedge ratio, is used to hedge a spot position. Thus, this article uses an out-of-sample approach, whereas most previous empirical studies use an in-sample approach. Previous empirical studies that have used an out-of-sample approach include Malliaris and Urrutia (1991b), Benet (1992), and Geppert (1995). The approach taken in this article differs from these studies in two respects. First, this article does not use a moving-window approach. This makes it possible to avoid the overlapping-samples problem. Second, the previous studies have a different purpose. They concentrate on the effect of changes in the length of the estimation period on the effectiveness of the hedge. This study focuses on the effectiveness of currency futures for three different models, based on different utility functions. For this reason the minimum-variance model, the α-t model, and the Sharpe-ratio model are used and compared.

This study uses daily data for three different futures contracts traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME). These are the U.S. dollar futures on the British pound, German mark, and Japanese yen for the period from December, 1976 to October, 1993.

Studies that use an in-sample approach assume that the hedge ratio is constant over the sample period. In an out-of-sample setting, the estimated model-based hedges will be optimal only if the hedge ratio is constant. The out-of-sample hedging effectiveness, with the use of model-based hedges, therefore, depends on the stability of the hedge ratio. Therefore, following Malliaris and Urrutia (1991a), the stochastic behavior of the hedge ratio is studied. It is found that for the minimum-variance model and the α-t model, the hedge ratios appear to be constant and significantly different from one. If a time-varying model-based hedge ratio is not optimal in the out-of-sample setting because of stochastic behavior, the results may be improved with the use of a long-term average of the optimal hedge ratio. That is why a hedging strategy that applies a constant hedge ratio is tested also. This is the average optimal hedge ratio over the entire sample period.

The hedging effectiveness results of this study indicate that hedges are only effective for the minimum-variance model and the α-t model. Naive hedges are more effective than model-based hedges and are also more effective than constant hedge ratio hedges. For the Sharpe-ratio model, the hedging strategies considered here actually decrease the utility of a hedger, as the risk-return trade-off is worsened. For this model,

model-based, constant, and naive hedges are less effective than leaving the spot position unhedged.

The remainder of this article is organized as follows. Section 2 briefly describes previous theoretical and empirical research. Section 3 includes the methodology and data description. Section 4 presents and comments on the most important results. Finally, Section 5 gives a summary and some conclusions.

RELATED RESEARCH

The traditional method of determining the number of futures in a hedge is simply to measure the position in the underlying asset and to take an equal but opposite position in futures contracts. This method is known as the naive approach. Ederington (1979) was the first to suggest an alternative approach and to define a measure for the effectiveness of a hedge. Other approaches were suggested by Johnson and Walther (1984), who applied the α-t model of Fishburn (1977), and by Howard and D'Antonio (1984, 1987), who based the optimal hedge ratio on the Sharpe ratio. The hedge ratio, which is the ratio of the size of the futures position relative to the size of the spot position, is denoted by λ. In this study three hedge ratios are applied: $\lambda = 1$ for the naive hedge, $\lambda = \lambda^*$ for the model-based hedge, and $\lambda = \lambda^c$ for the hedge in which a constant hedge ratio, equal to the average of the model-based hedges, is used.

The Minimum-Variance Model

Ederington (1979) defines hedging effectiveness as the reduction in the variance of the value of a position hedged with futures. The objective of a hedge is to minimize the risk of a given position. This risk is represented by the variance of the returns. Ederington derives a closed-form solution for the optimal hedge ratio, the minimum-variance hedge, which is

$$\lambda^* = \frac{\sigma_{\mathrm{SF}}}{\sigma_{\mathrm{F}}^2}, \tag{1}$$

where

λ^* = the minimum-variance model-based hedge ratio
σ_{SF} = the covariance of the changes in the spot and futures prices
σ_{F}^2 = the variance of the change in the futures price

It is assumed that σ_{SF} and σ_{F}^2 are time invariant. The model-based hedge ratio minimizes the variance of the value of a portfolio, which includes a long (short) position in the spot currency and a short (long) position in the futures contract. The measure for hedging effectiveness can be defined as the percentage reduction of the variance. The minimum-variance measure for the hedged position is

$$\mathrm{HE}_{\mathrm{mv}} = 1 - \frac{\sigma_{r_p}^2}{\sigma_{r_s}^2}, \tag{2}$$

where

$\mathrm{HE}_{\mathrm{mv}}$ = the minimum-variance measure for the model-based hedged position

$\sigma_{r_p}^2$ = the variance of the portfolio returns, hedged with futures

$\sigma_{r_s}^2$ = the variance of the returns of the spot position

The minimum-variance model has been tested empirically for currency futures by Naidu and Shin (1981) and Hill and Schneeweis (1982a, 1982b). All these articles report substantial risk reduction in comparison with the unhedged spot position. Naidu and Shin (1981) conclude that with two out of three contracts it is possible to reduce variance by 70%–90%. Hill and Schneeweis (1982a) compare the hedging effectiveness of the minimum-variance hedge with the naively hedged position. They find that both techniques are highly effective. The largest reduction of variance is achieved with the minimum-variance hedge. Hill and Schneeweis (1982b) find that the hedging effectiveness increases with the investment horizon.

The theory of Ederington and its empirical tests have two drawbacks. The first is the assumption that hedgers have a mean-variance utility function with infinite risk aversion.[2] The second is that it is assumed that the hedge ratio and the hedging effectiveness can be determined in the same period. The three in-sample studies mentioned above optimize the hedging effectiveness with a hedge ratio with the use of Ederington's one-period model. More recently, Malliaris and Urrutia (1991b), Benet (1992), and Geppert (1995) test the relationship between the hedging effectiveness of currency futures contracts and the length of the hedge period. All these studies use the minimum-variance measure of hedging effectiveness in an out-of-sample setting. Although substantial risk reduction is reported, the results should be interpreted carefully. The results may be biased because a moving-windows approach is used.

[2]Alternatively, it can be assumed that hedgers have a mean-variance utility function and that the expected change in the futures price is zero.

The α-t Model

The minimum-variance model discussed above is based on the mean-variance portfolio theory of Markowitz (1952). In this theory risk is perceived as the variance of returns. Deviations below and above a specific return are assumed to be undesirable. Crum, Laughhunn, and Payne (1981) state that managers perceive risk as the failure to obtain a specific aspiration level, the target return. This is relevant for the treasurer of a firm who uses futures to prevent an unexpected loss caused by volatile currency prices or who tries to buy insurance against his risk exposure. Fishburn (1977) has formalized this perception of risk in his α-t model. Fishburn describes the expected disutility of an outcome under a target return, t, weighted by a measure for risk aversion, α. This measure for risk is defined as

$$G_{\alpha}(t) = \int_{-\infty}^{t} (t - Y(\lambda))^{\alpha}\, dF(Y(\lambda)), \tag{3}$$

where

$G_{\alpha}(t)$ = the expected utility of the loss
t = the target return
α = a measure for risk aversion for below-target returns
$Y(\lambda)$ = the return below the target return
$F(Y(\lambda))$ = the probability distribution of $Y(\lambda)$

An individual who has an α between 0 and 1 is risk seeking; if α is equal to 1, the individual is risk neutral; and, if α is larger than 1, the individual is risk averse. Laughhunn, Payne, and Crum (1980) have estimated α (assuming $t = 0$) for downside risk and found that 71% of the managers are risk seeking for below-target returns. Ahmadi, Sharp, and Walther (1986) estimated the risk measure in the α-t model of Fishburn (1977). The α-t model-based hedge ratio is found by minimizing risk. This problem is reflected in eq. (4), which is the sample equivalent of eq. (3)

$$\lambda^{*} = \operatorname{argmin}_{\lambda} G_{N\alpha}(t) = \frac{1}{N} \sum_{i=1}^{N} (t - Y_i(\lambda))^{\alpha} I(Y_i(\lambda) \leq t), \tag{4}$$

where

$G_{n\alpha}(t)$ = an estimate of the expected utility of a loss
I = the indicator function
N = the number of returns

The returns are continuously compounded or log returns. In this article the model-based hedge ratios are obtained by numerically optimizing eq. (4). Analogous to the minimum-variance measure of hedging effectiveness, the measure for the α-t model can be defined as the percentage reduction in risk

$$\mathrm{HE}_{\alpha t} = 1 - \frac{G_{n\alpha,h}(t)}{G_{n\alpha,s}(t)}. \tag{5}$$

Johnson and Walther (1984) apply the α-t model to the hedging effectiveness of forwards. The hypothesis tested is that model-based hedges are more effective than naive hedges. In their study the minimum-variance model is used to obtain the hedge ratio and the α-t model is used to calculate the effectiveness of this hedge. The results suggest that a naive hedge is more effective than a minimum-variance model-based hedge. Ahmadi et al. (1986) carry out similar research for currency futures and options. Hedging effectiveness is defined as the ratio of the expected utility of the below-target outcome of a position hedged with currency futures and a position hedged with currency options. Futures are found to be more effective than options. Two remarks have to be made with regard to these empirical studies. First, no model-based hedge ratio is calculated that optimizes utility according to the α-t model. The hedges in the empirical tests are therefore naive hedges [Ahmadi et al. (1986)] or minimum-variance model-based hedges [Johnson and Walther (1984)]. Therefore, the problem of maximizing the hedging effectiveness is avoided. Both methods imply that the measured hedging effectiveness can be improved by using a hedge ratio in accordance with the α-t model. Second, although Fishburn (1977) constructs a risk–return model, the measure for hedging effectiveness exclusively incorporates the risk element.

The Sharpe-Ratio Model

Howard and D'Antonio (1984) use the Sharpe ratio to construct a risk–return measure for hedging effectiveness.[3] They define this measure as the ratio of the Sharpe ratios for an unhedged spot position and a spot position hedged with futures. With this contribution the utility function of a hedger is extended from minimizing risk to optimizing risk and return.

[3]It can be argued whether it is appropriate to use a risk–return measure for the effectiveness of a hedge. If it is assumed that hedgers do not pursue risk reduction at all costs, a risk–return trade-off must be made.

The authors derive a closed-form solution for the optimal hedge ratio. This solution is given in eq. (6)

$$\lambda^* = \frac{(\varphi - \rho)}{\dfrac{P_F}{P_S} \cdot \dfrac{\sigma_S}{\sigma_F} \cdot (1 - \varphi\rho)}, \qquad \text{with } \varphi = \frac{E(r_F)/\sigma_F}{(E(r_S) - r^f)/\sigma_S}, \tag{6}$$

where

P_S = the spot price of a currency
P_F = the futures price of a currency
r_F = the change of the futures price of a currency
r_S = the change of the spot price of a currency
σ_S = the standard deviation of changes in the spot price of a currency
σ_F = the standard deviation of changes in the futures price of a currency
ρ = the correlation coefficient of the changes of the futures and the spot price
r^f = the risk-free interest rate

The returns of the spot and futures positions are again continuously compounded returns. In a reply to a comment of Chang and Shanker (1987) the original measure is amended by Howard and D'Antonio (1987). The measure they suggest gives the extra return that can be realized using futures in a position with the risk of the spot position without these futures. This risk-equivalent extra return is expressed per unit risk. The measure for hedging effectiveness is given in eq. (7)

$$\mathrm{HE}_{\mathrm{Sr}} = \frac{r^f + [(\overline{r_\lambda} - r^f)/\sigma_\lambda]\, \sigma_S - \overline{r_S}}{\sigma_S}, \tag{7}$$

where r_λ = the return of a spot position hedged with a fraction of λ futures contracts.

Chang and Shanker (1986) have applied the Sharpe-ratio model to test the hedging effectiveness of currency futures and of a synthetic option.[4]

METHODOLOGY AND DATA DESCRIPTION

The empirical studies discussed above generally assume that the model-based hedge ratio and hedging effectiveness are based on data from the

[4]A synthetic option can be created by buying a call option and selling a put option with the same exercise price and by lending or borrowing certain amounts. The synthetic instrument has the same characteristics as a futures contract.

same period. However, this is not a realistic assumption. Therefore, this study utilizes an out-of-sample test. Although Malliaris and Urrutia (1991b), Benet (1992), and Geppert (1995) also tested hedging effectiveness in an out-of-sample setting, this study's approach differs in two respects. First, because no moving-windows approach is used here, the overlapping-samples problem is avoided. Second, the hedging effectiveness of currency futures is tested for three different utility models, instead of exclusively for the minimum-variance model.

It is assumed that a hedger first uses an estimation period to calculate the model-based hedge ratio. After this period three hedges are simulated. First, the hedge ratio, as calculated by the model, is used to hedge a spot position. Second, a constant hedge ratio is used, which is equal to the long-term average of the model-based hedge ratios. Third, a naive hedge is used. The returns of the model-based hedged position, the constant hedge ratio position, and the naive hedge are compared with the returns of the unhedged spot position in the same period. In this study model-based hedge ratios and constant hedge ratios are calculated for each of the three previously discussed measures of hedging effectiveness. Previous studies have not calculated model-based hedge ratios for the α-t model. To make such a calculation, a numerical optimization procedure is used.

Daily data on spot and futures prices of the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME) are used. The data are obtained from the Futures Industry Institute. All prices are quoted in U.S. dollars. For the hedges on the British pound and the German mark, data are from December, 1976 to October, 1993. For the Japanese yen, data are only from April, 1977 to October, 1993. Currency futures for these three currencies include those with delivery dates in March, June, September, and December. The data set for the risk-free interest rate contains monthly data of U.S. Treasury Bills with a remaining maturity of 1 month. For each currency a data set is created by using the principle of rolling the hedge forward [see Hull (1993, pp. 39–40)]. For each contract the price 3 months before the delivery month and the price of the first day of the delivery month are used. On the first day of the delivery month the hedger closes futures contracts that will expire in the same month. Also, new contracts are opened on that same day. In other words, from the expiration date of the previous contract until the first day of the delivery month, the nearest-to-maturity contract is used. The data set obtained in this way is then split into periods of 90 days. Model-based hedge ratios are calculated by using daily data for the first 60 days of a period. Subsequently, a hedge is simulated for a hedge period

FIGURE 1
Sample with nonadjusted positions and model-based hedge ratios.

of 30 days. By calculating hedges in this way the overlapping-samples problem is avoided. This approach is illustrated in Figure 1.

The model-based hedge ratios are calculated by using eqs. (1), (4), and (6), respectively. Starting from December, 1976 for the British pound and the German mark and from April, 1977 for the Japanese yen, hedge ratios are calculated over periods of 60 days with 30-day intervals until October, 1993. The 30-day intervals are used to simulate hedges. The data set consists of 62 nonoverlapping observations for the British pound and the German mark and 42 nonoverlapping observations for the Japanese yen. The stochastic properties of the model-based hedge ratios are tested over the entire sample period. Tests are conducted to determine whether the model-based hedge ratios follow a random walk, and, if not, whether they are constant. Tests are conducted also to determine whether the constant hedge ratio is different from 1. The average hedge ratios are applied as the constant hedge ratios. This constant hedge is expected to be more optimal than a naive hedge. Also, if the standard deviation of the hedge ratio is high, it is expected that, due to this volatility, the effectiveness of the model-based hedge will be reduced in the out-of-sample test. This is followed by a simulation of the hedges. For each hedge the returns of four positions are calculated:

- Spot position without a hedge ($\lambda = 0$)
- Spot position with a naive hedge ($\lambda = 1$)
- Spot position with a model-based hedge ($\lambda = \lambda^*$)
- Spot position with a constant hedge ratio hedge ($\lambda = \lambda^c$)

The results are divided into three subperiods.

- Period I: December 2, 1976 to July 6, 1982 (20 hedges)
- Period II: July 7, 1982 to February 12, 1988 (21 hedges)
- Period III: February 16, 1988 to October 1, 1993 (21 hedges)

For each period the variance, the downside risk, and the Sharpe ratios are calculated over the 20 or 21 hedges. Finally, the measures of hedging effectiveness are applied to calculate the hedging effectiveness of currency futures.

RESULTS

The results are reported for the three models, three currencies, and three hedging strategies. For each model a comparison is made between a naive hedge, a hedge with the model-based hedge ratio, and a constant hedge ratio hedge that is equal to the long-run average of the model-based hedge ratio. Three issues are addressed. First, it is determined whether hedging improves the utility of a hedger in this setting. Second, it is determined whether it is more effective to hedge naively, to hedge with a time-varying model-based hedge ratio, or to hedge with a constant hedge ratio. Third, the stochastic behavior of the hedge ratio is revealed.

Statistical Properties of the Hedge Ratios

Tables I–III present summary statistics with respect to the time-series behavior of the model-based hedge ratios.

The average hedge ratios for the minimum-variance model and the α-t model are all statistically different from one at the 1% significance level. All averages are below 1 and show minor differences between currencies. That this result is obvious can be illustrated by using the minimum-variance model. Equation (1) for the optimal hedge ratio can be rewritten as

$$\lambda^* = 1 - \frac{\sigma_{\mathrm{B}}^2 + \sigma_{\mathrm{BS}}}{\sigma_{\mathrm{F}}^2}, \tag{8}$$

where B = the basis; in the case of currency futures this equals the interest differential between the home and the foreign country $(r_h^f - r_f^f)$.

The expression is less than one if $\sigma_{\mathrm{BS}} > 0$. This is the covariance between the interest differential and the spot price of the currency. In general this covariance is positive. Therefore, it is expected that the optimal hedge will tend to be below 1. This is apparent from the data if one considers the average hedge ratios. They are significantly below 1, and relatively constant over the three periods. For this reason hedges are simulated also with this constant hedge ratio. A strategy that uses these long-term averages of the optimal hedge ratios should yield higher effective-

TABLE I

Statistical Properties of the Optimal Hedge Ratios of the British Pound

	Minimum Variance	*α-t model* $\alpha = 0.50$	*α-t model* $\alpha = 1.00$	*α-t model* $\alpha = 2.50$	*Sharpe Ratio*
		Summary Statistics			
Minimum	0.21	−0.15	−0.16	−0.97	−36.06
Maximum	0.99	1.28	1.12	1.13	6.53
Average	0.78**[a]	0.75**	0.77**	0.77**	0.29
Std.dev.	0.13	0.30	0.22	0.28	5.23
		Autocorrelations			
ρ_1	0.23	0.24	0.49**[b]	0.35**	0.22
ρ_2	0.14	0.14	0.24	0.00	0.01
ρ_3	0.23	0.07	0.09	0.00	−0.08
		Unit-Root Tests			
DF test	−7.20**[c]	−6.40**	−5.20**	−5.64**	−6.21**
ADF(1) test	−5.14**	−5.49**	−6.20**	−10.83**	−4.94**
		Correlations			
	1.00	0.42	0.63	0.59	−0.05
		1.00	0.89	0.57	−0.07
			1.00	0.76	−0.02
				1.00	−0.07
					1.00

Note: The table entries present summary statistics for various optimal hedge ratios. All calculations are based on the sample period from December, 1976 to October, 1993, which gives a total of 62 nonoverlapping observations.
*Indicates a significance level of 5%.
**Indicates a significance level of 1%.
[a]The null hypothesis is that the average hedge ratio equals 1.
[b]The null hypothesis is that the first-order autocorrelation coefficient equals zero.
[c]The Dickey–Fuller (DF) and augmented Dickey–Fuller (ADF) tests are for the null hypothesis that the optimal hedge ratios follow a random walk.

ness compared to a naive strategy. The size of the standard deviations suggests that the optimal hedge ratios vary over time. Because optimal hedge ratios are used in an out-of-sample setting, variation in the optimal hedge ratios will decrease the hedging effectiveness. This suggests that, compared to an in-sample test, an out-of-sample test provides more reliable evidence of the effectiveness of a model-based hedge.[5]

The average hedge ratios for the Sharpe model differ from the average hedge ratios for the other models. Table III also shows that the average hedge ratios for the Sharpe model are much more volatile. This is not very surprising, because the optimal hedge ratios in the Sharpe-

[5]At first sight it may seem odd, in an out-of-sample approach, to first calculate an average hedge ratio over the entire sample period, and to use it from the beginning of the sample period. However, this can be justified by the assumption that the hedge ratio is constant over time (not only during the sample period).

TABLE II

Statistical Properties of the Optimal Hedge Ratios for the German Mark

	Minimum Variance	*α-t model α = 0.50*	*α-t model α = 1.00*	*α-t model α = 2.50*	*Sharpe Ratio*
		Summary Statistics			
Minimum	0.43	0.08	0.14	0.12	−43.09
Maximum	1.06	1.58	1.31	1.54	16.26
Average	0.81**[a]	0.85**	0.82**	0.82**	−0.79*
Std.dev.	0.11	0.31	0.20	0.24	6.86
		Autocorrelations			
ρ_1	0.15	0.22	0.12	0.18	−0.06
ρ_2	0.22	0.14	0.25	0.17	0.21
ρ_3	0.25	−0.03	0.06	0.22	−0.05
		Unit-Root Tests			
DF test	−6.51**[b]	−6.12**	−6.96**	−6.67**	−8.16**
ADF(1) test	−3.82**	−4.28**	−4.00**	−4.33**	−4.52**
		Correlations			
	1.00	0.37	0.53	0.50	−0.01
		1.00	0.85	0.21	−0.02
			1.00	0.37	0.02
				1.00	0.08
					1.00

Note: The table entries present summary statistics for various optimal hedge ratios. All calculations are based on the sample period from December, 1976 to October, 1993, which gives a total of 62 nonoverlapping observations.
*Indicates a significance level of 5%.
**Indicates a significance level of 1%.
[a]The null hypothesis is that the average hedge ratio equals 1.
[b]The Dickey–Fuller (DF) and augmented Dickey–Fuller (ADF) tests are for the null hypothesis that the optimal hedge ratios follow a random walk.

ratio model depend on expected returns in a similar way as mean-variance portfolio weights do, whereas the hedge ratios of the other models do not. Because it is well known that these portfolio weights are highly sensitive to expected returns, variation in expected returns directly translates to variation in the optimal hedge ratios.

As in Malliaris and Urrutia (1991a) an (augmented) Dickey–Fuller test is used to test the random walk hypothesis. They cannot reject the hypothesis that the hedge ratios follow a random walk. The results of the unit root tests in Tables I–III, however, show that the random walk hypothesis can be rejected for all cases. Because a unit root in the optimal hedge ratio would imply that shocks on the hedge ratio have a permanent rather than a transitory effect, the absence of a unit root also seems to be more plausible from an economic point of view.

Tables I–III also show that the null hypothesis that the first-order autocorrelation coefficient, ρ_1, is zero can be rejected in only three cases:

TABLE III

Statistical Properties of the Optimal Hedge Ratios for the Japanese Yen

	Minimum Variance	*α-t model α = 0.50*	*α-t model α = 1.00*	*α-t model α = 2.50*	*Sharpe Ratio*
		Summary Statistics			
Minimum	−0.03	0.00	−0.11	−0.79	−88.48
Maximum	1.00	1.20	1.13	1.44	41.41
Average	0.78**[a]	0.83**	0.79**	0.76**	0.94
Std.dev.	0.17	0.21	0.23	0.33	16.84
		Autocorrelations			
ρ_1	0.02	0.27*[b]	0.10	−0.02	0.01
ρ_2	−0.01	0.01	0.16	0.10	−0.01
ρ_3	0.01	−0.05	−0.21	−0.30	0.03
		Unit-Root Tests			
DF test	−6.07**[c]	−4.81**	−5.66**	−6.31**	−6.19**
ADF(1) test	−4.30**	−3.89**	−3.47*	−3.87**	−7.84**
		Correlations			
	1.00	0.30	0.76	0.90	0.04
		1.00	0.73	0.46	0.06
			1.00	0.90	0.08
				1.00	0.10
					1.00

Note: The table entries present summary statistics for various optimal hedge ratios. All calculations are based on the sample period from April, 1977 to October, 1993, which gives a total of 42 nonoverlapping observations.
*Indicates a significance level of 5%.
**Indicates a significance level of 1%.
[a]The null hypothesis is that the average hedge ratio equals 1.
[b]The null hypothesis is that the first-order autocorrelation coefficient equals zero.
[c]The Dickey–Fuller (DF) and augmented Dickey–Fuller (ADF) tests are for the null hypothesis that the optimal hedge ratios follow a random walk.

two times for the Fishburn hedges with respect to the British pound, and once for the Fishburn hedges with respect to Japanese yen. In general, however, this null hypothesis cannot be rejected. The fact that the hypothesis that $\rho_1 = 0$ cannot be rejected is consistent with the hypothesis that the optimal hedge ratio is constant.

The hypothesis that the average optimal hedge ratio is equal to 1 is always rejected. An exception is the case for the hedge ratios based on the Sharpe ratio, which show a large variability. The averages for the other hedge ratios are always between 0.75 and 0.85. It can be concluded that if the optimal hedge ratios are indeed constant over time, it appears that agents should not use a naive hedge ratio but should instead use a hedge ratio that is smaller than 1.

TABLE IV

Minimum Variance Hedging Effectiveness

Currency	*Period*	$HE_{mv,n}$	HE_{mv}	$HE_{mv,c}$
British pound	I	0.9662	0.9461	0.9530
	II	0.9887	0.9622	0.9523
	III	0.9892	0.9637	0.9537
German mark	I	0.9774	0.9223	0.9467
	II	0.9948	0.9666	0.9614
	III	0.9796	0.9627	0.9623
Japanese yen	I	n.a.	n.a.	n.a.
	II	0.9700	0.8816	0.9209
	III	0.9923	0.9539	0.9573

Note: $HE_{mv,n}$ is the minimum-variance hedging effectiveness with naive hedges; HE_{mv}, is the minimum-variance hedging effectiveness with model-based hedge ratios; $HE_{mv,c}$ is the minimum-variance hedging effectiveness with the long-term average hedge ratio (respectively, 0.78, 0.81, and 0.78 for all periods, see Tables I–III); n.a. = not available; that is, for this period not enough data are available to calculate the hedging effectiveness: Period I runs from December 2, 1976 to July 6, 1982 (20 hedges); Period II runs from July 7, 1982 to February 12, 1988 (21 hedges); Period III runs from February 16, 1988 to October 1, 1993 (21 hedges).

The Minimum-Variance Model

The results of the application of the minimum-variance model are summarized in Table IV. In this table the hedging effectiveness is given for each currency in the three periods.

In the column entitled $HE_{mv,n}$ naive hedges are used. In such a hedge the hedging effectiveness is calculated by using eq. (2). In the column entitled HE_{mv} the effectiveness of the minimum-variance hedge is presented. To calculate the numbers in this column, first the model-based hedge ratio is determined by using eq. (1). The hedging effectiveness is calculated by using eq. (2). In the column entitled $HE_{mv,c}$ the effectiveness of a constant hedge ratio is presented. To calculate the numbers in this column, first the model-based hedge ratios are determined for the entire sample period by using eq. (1). Then hedges are simulated by using the long-term averages of these hedge ratios. The hedging effectiveness is calculated by using eq. (2). For example, in Period I the hedging effectiveness of the British pound futures is, respectively, 96.62%, 94.61%, and 95.30%. In this example the hedger would have preferred the naive hedge, because the risk is reduced by 96.62%. With the use of a fixed minimum-variance hedge ratio, risk is reduced by not more than 94.61%. The constant hedge ratio reduces risk by 95.30%. In general, all three methods are effective in reducing currency risk. The results imply that for every period and every currency studied, naive hedging is more effective than hedging with the use of the model-based hedge. This conclusion

TABLE V

Target Return Hedging Effectiveness of the British Pound

	Period I			Period II			Period III		
α	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$
0.25	0.2754	0.4205	0.2295	0.2789	0.1967	0.5152	−0.3558	0.0625	0.1484
0.50	0.5696	0.5758	0.4776	0.5158	0.4243	0.7280	0.1284	0.3343	0.3559
0.75	0.7455	0.6997	0.6754	0.6734	0.6202	0.8038	0.4399	0.6208	0.5323
1.00	0.8496	0.7701	0.7905	0.7791	0.7414	0.8636	0.6405	0.7360	0.6604
1.50	0.9474	0.8956	0.9135	0.8981	0.8923	0.9382	0.8523	0.8928	0.8320
2.00	0.9816	0.9471	0.9585	0.9522	0.9515	0.9725	0.9395	0.9525	0.9145
2.50	0.9935	0.9701	0.9789	0.9770	0.9772	0.9880	0.9752	0.9770	0.9562
3.00	0.9977	0.9834	0.9891	0.9887	0.9886	0.9945	0.9899	0.9883	0.9701
3.50	0.9992	0.9908	0.9942	0.9943	0.9938	0.9973	0.9959	0.9937	0.9890
4.00	0.9997	0.9948	0.9970	0.9971	0.9962	0.9987	0.9983	0.9961	0.9946
4.50	0.9999	0.9970	0.9984	0.9985	0.9974	0.9993	0.9993	0.9971	0.9973
5.00	0.9999	0.9982	0.9991	0.9992	0.9980	0.9996	0.9997	0.9976	0.9987

Note: $He_{\alpha t,n}$ is the α-t model hedging effectiveness with naive hedges; $He_{\alpha t}$ is the α-t model hedging effectiveness with model-based hedge ratios [eq. (4)]; $HE_{\alpha t,c}$ is the α-t model hedging effectiveness with a constant hedge ratio (see row entitled "Average" in Table I). The effectiveness is calculated by using eq. (5); α is a measure for risk aversion: Period I runs from December 2, 1976 to July 6, 1982 (20 hedges); Period II runs from July 7, 1982 to February 12, 1988 (21 hedges); Period III runs from February 16, 1988 to October 1, 1993 (21 hedges).

differs from the empirical results of Hill and Schneeweis (1982a, 1982b). The results of the constant hedge ratio indicate that these hedges are less effective then the naive hedges. This is contrary to what might be expected, based upon the statistical properties of the hedge ratio.[6]

The α-t Model

The α-t model is tested for α's from 0.25 to 5.00 with intervals of 0.25. The target return is assumed to be zero. This seems to be a plausible target, because it corresponds to the case where treasurers try prevent the occurrence of a loss. The hedging effectiveness is given in Tables V–VII.

Equation (4) is used to obtain the hedge ratio for the model-based hedged position. The same equation is used to estimate the expected utility, $G_\alpha(t)$, of four positions: model-based hedged, hedged with a constant hedge ratio, naively hedged, and unhedged positions. Tables V–VII relate the three risk measures to the unhedged position, by using eq. (5). $HE_{\alpha t,n}$ is the hedging effectiveness of the naive hedge, $HE_{\alpha t}$ is the hedging

[6]The skewness of the distribution of hedge ratios is −1.57 (GBP), −0.88 (DEM), and −2.79 (JPY). The distributions are skewed to the left, which indicates that the average values are influenced by outliers (low hedge ratios).

TABLE VI

Target Return Hedging Effectiveness of the German Mark

	Period I			Period II			Period III		
α	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$
0.25	0.8156	0.3884	0.7033	0.7865	0.4265	0.5152	−0.2915	0.3063	0.1428
0.50	0.9089	0.6329	0.8263	0.8785	0.6231	0.7280	0.1767	0.5278	0.4443
0.75	0.9558	0.7444	0.8716	0.9303	0.6868	0.8038	0.4786	0.6929	0.6581
1.00	0.9789	0.8092	0.9091	0.9600	0.7490	0.8636	0.6701	0.8223	0.7772
1.50	0.9953	0.8738	0.9592	0.9871	0.8524	0.9382	0.8668	0.9103	0.8963
2.00	0.9990	0.8330	0.9829	0.9996	0.8905	0.9725	0.9452	0.9453	0.9492
2.50	0.9999	0.6394	0.9933	0.9987	0.9134	0.9880	0.9770	0.9653	0.9748
3.00	0.9999	0.2006	0.9973	0.9959	0.9287	0.9945	0.9902	0.9790	0.9869
3.50	0.9999	−0.6051	0.9989	0.9999	0.9392	0.9973	0.9957	0.9877	0.9929
4.00	0.9999	−1.9674	0.9995	0.9999	0.9460	0.9987	0.9981	0.9928	0.9961
4.50	0.9999	−4.1662	0.9998	0.9999	0.9494	0.9993	0.9992	0.9957	0.9978
5.00	0.9999	−7.6141	0.9999	0.9999	0.9497	0.9996	0.9996	0.9972	0.9987

Note: $He_{\alpha t,n}$ is the α-t model hedging effectiveness with naive hedges; $HE_{\alpha t}$ is the α-t model hedging effectiveness with model-based hedge ratios [eq. (4)]; $HE_{\alpha t,c}$ is the α-t model hedging effectiveness with a constant hedge ratio (see row entitled "Average" in Table II). The effectiveness is calculated by using eq. (5); α is a measure for risk aversion: Period I runs from December 2, 1976 to July 6, 1982 (20 hedges); Period II runs from July 7, 1982 to February 12, 1988 (21 hedges); Period III runs from February 16, 1988 to October 1, 1993 (21 hedges).

TABLE VII

Target Return Hedging Effectiveness of the Japanese Yen

	Period II			Period III		
α	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$	$HE_{\alpha t,n}$	$HE_{\alpha t}$	$HE_{\alpha t,c}$
0.25	0.7435	0.3627	0.6645	0.4268	0.3297	0.5290
0.50	0.7756	0.6100	0.7170	0.6815	0.6360	0.7159
0.75	0.7913	0.6697	0.7424	0.8278	0.7580	0.8013
1.00	0.7963	0.5071	0.7390	0.9088	0.8427	0.8313
1.50	0.7931	0.2524	0.7470	0.9753	0.8840	0.9114
2.00	0.7894	−0.1605	0.7442	0.9934	0.8962	0.9514
2.50	0.7915	−0.7896	0.7424	0.9983	0.9082	0.9734
3.00	0.7988	−2.0220	0.7430	0.9995	0.9140	0.9851
3.50	0.8094	−4.9407	0.7455	0.9999	0.9039	0.9917
4.00	0.8219	−10.9739	0.7497	0.9999	0.8775	0.9946
4.50	0.8351	−22.4345	0.7563	0.9999	0.8440	0.9968
5.00	0.8484	−43.8336	0.7548	0.9999	0.8038	0.9981

Note: $HE_{\alpha t,n}$ is the α-t model hedging effectiveness with naive hedges; $HE_{\alpha t}$ is the α-t model hedging effectiveness with model-based hedge ratios [eq. (4)]; $HE_{\alpha t,c}$ is the α-t model hedging effectiveness with a constant hedge ratio (see row entitled "Average" in Table III). The effectiveness is calculated by using eq. (5); α is a measure for risk aversion. Period II runs from July 7, 1982 to February 12, 1988 (21 hedges). Period III runs from February 16, 1988 to October 1, 1993 (21 hedges).

TABLE VIII

Comparison of Naive and Target Return Model-Based Hedging Effectiveness (8 cases)

α	0.25	0.5	0.75	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$HE_{\alpha t} > HE_{\alpha t,n}$	3	3	2	2	2	2	1	0	0	0	0	0
$HE_{\alpha t,c} > HE_{\alpha t,n}$	4	4	3	3	2	2	1	1	1	1	1	1
$HE_{\alpha t} > HE_{\alpha t,c}$	2	2	3	2	2	1	1	1	1	1	0	0

Note: The row $HE_{\alpha t} > HE_{\alpha t,n}$ expresses the number of observations for a specific α, where the model-based hedge results in a higher hedging effectiveness than the naive hedge. The row $HE_{\alpha t,c} > He_{\alpha t,n}$ naive expresses the number of observations for a specific α, where the constant hedge ratio hedge results in a higher hedging effectiveess than the naieve hedge. The row $HE_{\alpha t} > He_{\alpha t,c}$ expresses the number of observations for a specific α, where the model-based hedge results in a higher hedging effectiveness than the constant hedge ratio hedge; α is a measure for risk aversion.

effectiveness of the model-based hedge, and $HE_{\alpha t,c}$ is the hedging effectiveness of the hedges with a constant hedge ratio. For example, in Table VII the hedging effectiveness for the Japanese yen in Period II for an α of 0.25 is 0.7435 for the naive hedge, 0.3627 for the α-t model hedge, and 0.6645 for the constant hedge ratio hedge. This indicates that compared to the unhedged position, the disutility of a loss decreased by 74.35%, 36.27%, and 66.45%, respectively. As the parameter α increases from 1 to 5, hedgers become increasingly risk averse. This risk aversion reduces the effectiveness of the model-based hedges and results in negative effectiveness for high α's in the case of the German mark in the first period and the Japanese yen in the second period.

The results suggest that, in most cases, a substantial risk reduction can be realized. Especially, risk-averse hedgers are able to improve the hedging effectiveness by adding futures to their spot positions. In Table VIII the hedging effectiveness is compared for different α's by aggregating the periods and the currencies: three cases for the British pound and the German mark and two cases for the Japanese yen.

The hedging effectiveness of naively, model-based, and constant hedge ratio hedged positions are compared to find out which method yields the highest effectiveness. The first row, in which the model-based hedges are compared with the naive hedges, shows that in most cases the naive hedging strategy is more effective. This is especially true for risk-averse hedgers. The second row compares the hedge with a constant hedge ratio with a naive hedge. Contrary to what might be expected, it is concluded that the naive hedge is more effective for risk-averse hedges.[7]

[7]This result is thought to be caused by the skewness of the distributions of the hedge ratios. For the sample in this article the skewness of the distribution of hedge ratios appears to be negatively related to the hedging effectiveness.

TABLE IX

Sharpe-Ratio Hedging Effectiveness

Currency	*Period*	$HE_{Sr,n}$	HE_{Sr}	$HE_{\alpha t,c}$
British pound	I	−1.0012	−0.1113	−0.1138
	II	−1.9796	−0.1676	−0.1024
	III	−2.7537	−0.4157	−0.1013
German mark	I	−0.8841	−0.0456	0.0498
	II	−1.1209	0.0767	0.0316
	III	−1.6720	−0.3843	0.0793
Japanese yen	I	n.a.	n.a.	n.a.
	II	−0.6548	0.2143	−0.5861
	III	−1.4523	−0.0837	−1.4033

Note: $HE_{Sr,n}$ is the Sharpe-ratio hedging effectiveness with naive hedges [eq. (7) with $\lambda = 1$]. HE_{Sr} is the Sharpe-ratio hedging effectiveness with model-based hedge ratios [eqs. (6) and (7) with $\lambda = \lambda^*$]. $HE_{Sr,c}$ is the Sharpe-ratio hedging effectiveness with a constant hedge ratio [eq. (7) with $\lambda = \lambda$]. n.a. = not available; that is, for this period not enough data are available to calculate the hedging effectiveness. Period I runs from December 2, 1976 to July 6, 1982 (20 hedges); Period II runs from July 7, 1982 to February 12 (21 hedges), 1988; Period III runs from February 16, 1988 to October 1, 1993 (21 hedges).

For risk-seeking agents it is not clear whether they can improve their utility by using currency futures. The third row shows that the constant hedge ratio hedge is to be preferred over the model-based hedge.

The Sharpe-Ratio Model

In Table IX the hedging effectiveness is summarized for the cases where the Sharpe-ratio model is applied.

Equation (7) with $\lambda = 1$ expresses the hedging effectiveness of the naively hedged position. The hedging effectiveness of the model-based hedged position, HE_{Sr}, is calculated by using equation (7) with $\lambda = \lambda^*$. To find the model-based hedge ratio in this equation, eq. (6) is used. The hedging effectiveness of the positions hedged with the constant hedge ratio strategy, $HE_{Sr,c}$, is calculated by using eq. (7) with $\lambda = \lambda^c$. The long-term average hedge ratios, λ^c, are 0.29 (GBP), -0.79 (DEM), and 0.94 (JPY), respectively. For example, the hedging effectiveness of the British pound in the first period is negative for the model-based hedge, the constant hedge ratio hedge, and the naive hedge. This can be explained by considering the underlying Sharpe ratios. For example, the Sharpe ratio for the currency spot position is -0.4222. If this position is hedged, the ratio changes to -0.5336 for a model-based hedge and to -1.4233 for a naive hedge. In both cases the hedging effectiveness is negative because of a decrease in the Sharpe ratio. In Table IX only five cases exist where the effectiveness increases with the use of futures. Hedging effectiveness is negative in the other 19 cases. The use of futures apparently has a negative impact on the risk-return ratio.[8]

CONCLUSIONS

In this study three models for the hedging effectiveness of futures are applied to measure the effectiveness of currency futures. Each measure compares a hedged position with an unhedged (spot) position. Three types of hedges are studied: (a) the naive hedge; (b) a hedge using a model-based hedge ratio that varies over time, where the hedging effectiveness is maximized each period; and (c) a hedge in which the long-term average of the model-based hedge ratio is applied. Out-of-sample tests are performed to measure the hedging effectiveness for three utility functions. The first model is the minimum-variance model of Ederington (1979), which minimizes the variance of a portfolio. The second model is the α-t model of Fishburn (1977), which minimizes the disutility of a loss. The third model is the Sharpe-ratio model of Howard and D'Antonio (1984, 1987), which optimizes the risk–return trade-off. In the out-of-sample tests it is found that hedging is only effective when the minimum-variance model and the α-t model are used. When these two models are used the naively hedged position yields a higher effectiveness than the model-based hedges or the hedges based on a constant hedge ratio, which is equal to the long-term average of the model-based hedge ratios. If the Sharpe-ratio model is applied, hedging with currency futures does not improve the utility of a hedger.

BIBLIOGRAPHY

Ahmadi, H. Z., Sharp, P. A., and Walther, C. H. (1986): "The Effectiveness of Futures and Options in Hedging Currency Risk," *Advances in Futures and Options Research*, Pt. B, 1:171–191.

Benet, B. A. (1992): "Hedge Period Length and Ex-Ante Futures Hedging Effectiveness: The Case of Foreign-Exchange Risk Cross Hedges," *The Journal of Futures Markets*, **12**(2):163–175.

Chang, J. S. K., and Shanker, L. (1986): "Hedging Effectiveness of Currency Options and Currency Futures," *The Journal of Futures Markets*, 6(2):289–305.

[8]For each case the hedging effectiveness is higher when a model-based hedge is used. As in the previous model, the binomial test claims that this outcome has 0.4% probability in case both hedging techniques have equal probability. Therefore, naively hedged positions have a significantly lower effectiveness.

Chang, J. S. K., and Shanker, L. (1987): "A Risk-Return Measure of Hedging Effectiveness: A Comment," *Journal of Financial and Quantitative Analysis,* 22(3):373–376.

Crum, R. L., Laughhunn, D. L., and Payne, J. W. (Winter 1981): "Risk-Seeking Behavior and Its Implications for Financial Models," *Financial Management,* 20–27.

Dubofski, D. (1992): *Options and Financial Futures: Valuation and Uses,* New York: McGraw-Hill.

Ederington, L. H. (1979): "The Hedging Performance of the New Futures Markets," *The Journal of Finance,* 34(1):157–170.

Fishburn, P. C. (1977): "Mean-Risk Analysis with Risk Associated with Below-Target Returns," *The American Economic Review,* 67(1):116–126.

Geppert, J. M. (1995): "A Statistical Model for the Relationship between Futures Contract Hedging Effectiveness and Investment Horizon Length," *The Journal of Futures Markets,* 15(5):507–536.

Hill, J., and Schneeweis, Th. (Summer 1982a): "Forecasting and Hedging Effectiveness of Pound and Mark Forward and Futures Markets," *Management International Review,* 22:43–52.

Hill, J., and Schneeweis, Th. (Spring 1982b): "The Hedging Effectiveness of Foreign Currency Futures," *The Journal of Financial Research,* 5(1):95–104.

Howard, C. T., and D'Antonio, L. J. (1984): "A Risk-Return Measure of Hedging Effectiveness," *Journal of Financial and Quantitative Analysis,* 19(1):101–112.

Howard, C. T., and D'Antonio, L. J. (1987): "A Risk-Return Measure of Hedging Effectiveness: A Reply," *Journal of Financial and Quantitative Analysis,* 22(3):377–381.

Hull, J. C. (1993): *Options, Futures, and other Derivative Securities* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Johnson, L. J., and Walther, C. H. (1984): "New Evidence of the Effectiveness of Portfolio Hedges in Currency Forward Markets," *Management International Review,* 24(2):15–23.

Laughhunn, D. L., Payne, J. W., and Crum, R. L. (1980): "Managerial Risk Preferences for Below-Target Returns," *Management Science,* 26(12):1238–1249.

Malliaris, A. G., and Urrutia, J. (1991a): "Tests of Random Walk of Hedge Ratios and Measures of Hedging Effectiveness for Stock Indexes and Foreign Currencies," *The Journal of Futures Markets,* 11(1):55–68.

Malliaris, A. G., and Urrutia, J. (1991b): "The Impact of the Lengths of Estimation Periods and Hedging Horizons on the Effectiveness of a Hedge: Evidence from Foreign Currency Futures," *The Journal of Futures Markets,* 11(3):271–289.

Markowitz, H. M. (1952): "Portfolio Selection," *The Journal of Finance,* 7(1):77–91.

Naidu, G. N., and Shin, T. S. (1980): "Effectiveness of Currency Futures Market in Hedging Foreign Exchange Risk," *Management International Review,* 21(4):5–16.