CONTINUOUSLY TRADED OPTIONS ON DISCRETELY TRADED COMMODITY FUTURES CONTRACTS

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INTRODUCTION

Japanese and U.S. commodity futures markets differ substantially in their market structures. These structural differences include the auction system and the frequency of trading. The *open outcry auction* system used on most commodity futures markets in the U.S. results in a nearly continuous series of transaction futures prices during the trading day, whereas the *Itayose-hoh* (or single fixed-price) auction method used on many Japanese commodity futures markets results in a single transaction futures price for each contract month during a trading session. Marsh and Webb (1983) have shown that these structural differences affect the behavior of transaction futures prices.

Perhaps surprisingly, some of these structural differences between Japanese (*periodic call*) and U.S. (*continuous*) commodity futures markets

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do not extend to the related commodity futures option markets. In particular, some Japanese commodity futures option contracts are traded *continuously* for large parts of the trading day, even though the underlying futures contract is traded *discretely* using the Itayose-hoh trading method. This gives rise to what Leong (1992) has called *hedge slippage costs,* or hedging errors, even when option positions are delta hedged. This study examines the nature and extent of hedge slippage costs arising from holding Tokyo Grain Exchange (TGE) soybean futures option positions. In addition, it provides an explanation for the differential success of soybean futures options traded on the TGE and on the Chicago Board of Trade.

THE STRUCTURE OF JAPANESE COMMODITY FUTURES AND OPTION MARKETS

The influence of market structure on the behavior of speculative prices has received much attention in the financial economics literature since Garman's (1976) ground-breaking article on speculative prices in auction and dealer markets. Garbade and Silber (1979) have argued that trading frequency is an important structural characteristic of financial markets.

The Tokyo Grain Exchange, the largest grain futures market in Japan, uses the Itayose-hoh trading method for its commodity futures contracts. On the TGE, futures contract months are auctioned *in seriatim* during each of the trading sessions held during the trading day. The number of trading sessions varies from four to six, depending upon the commodity. After transaction futures prices have been determined for each of the six contract months, trading ceases for that commodity futures until the next trading session begins.

Webb (1991) argues that the Itayose-hoh trading method closely resembles a (time-consuming) Walrasian tâtonnement for a single commodity. The auction begins when an initial *provisional* price is posted. Traders respond by placing orders to buy or sell. The provisional price is increased (decreased) if there is *hana* or excess demand (supply). This iterative process continues until a market clearing price for a given contract month is established. All transactions occur at the market clearing price. As with a Walrasian tâtonnement, traders are allowed to recontract prior to the determination of the equilibrium price.

Not all Japanese commodity futures markets can be characterized as periodic call markets, however. In contrast to the Itayose-hoh trading method with its discrete trading, some Japanese commodity futures exchanges (e.g., the Tokyo Commodity Exchange) trade commodity futures continuously. The continuous-trading method is called *Zaraba.*

Although the TGE trades commodity futures contracts with the use of the Itayose-hoh method, the TGE trades commodity futures options with a combination of continuous and discrete (*Ita-awase-hoh*) trading. Under the Ita-awase-hoh trading method, participants submit orders to buy or sell at various prices. The price with the greatest quantity of *matching* buy and sell orders is the transaction price at which all trades are executed. Put differently, the transaction price is simply the *mode* of matching buy and sell orders. Like the Ita yose-hoh trading method, the Ita-awase-hoh trading method is a single fixed-price auction. However, unlike the Ita yose-hoh auction method, there are no provisional prices in the Ita-awase-hoh trading method. As a result, transaction prices in the Ita-awase-hoh auctions are potentially noisier than those in Ita yose-hoh auctions. TGE futures options are traded continuously during much of the trading day. Orders submitted during continuous trading are ranked by both price and time. The Ita-awase-hoh trading method differs from the Zaraba method in that the *temporal* priority of orders is not considered. Thus, if two orders to buy call options for a given contract month and exercise price arrived at different times they would be treated equally under the Ita-awase-hoh trading method.

There is a growing literature on the single fixed-price auction system. Marsh and Webb (1983) compare the behavior of *transaction* TGE and Chicago Board of Trade (CBT) soybean futures prices to ascertain whether differences in market structure influence the behavior of the futures prices of a similar commodity. Using the number of iterations before an equilibrium price is reached as a proxy for "speed of convergence," Webb (1991) concludes that *provisional* TGE soybean futures prices converge rapidly to a final transaction futures price. In an extensive examination of provisional TGE soybean futures prices, Iwata et al. (1994) estimate an excess demand function for individual trading firms. Low and Muthuswamy (1994) examine the behavior of commodity futures prices traded on the Manila International Futures Exchange, which also employs the single-fixed price auction trading system. Webb (1995) examines both provisional and transaction TGE azuki (red bean) futures prices to evaluate the importance of noise or noninformational factors in Itayose-hoh auction markets.

COMMODITY FUTURES OPTION TRADING IN THE U.S. AND JAPAN

Exchange-traded commodity futures options have enjoyed a long and colorful history in the United States. As Mehl (1934) points out, their history can be traced to transactions in *privileges* on the CBT during the 1860s. Options trading flourished despite periodic attempts to restrict it by both exchanges and state governments until it was banned by the U.S. government. This prohibition against exchange-traded commodity futures options ended in the early 1980s.

Since the prohibition on exchange traded commodity futures options ended in the United States, numerous commodity futures option contracts have been introduced. Many of these option contracts (such as options on CBT soybean and corn futures) have met with great success. The rapid acceptance of these contracts is shown in Figures 1–3, which depict the ratio of CBT soybean futures call and put option volumes to CBT soybean futures volume during the first 3 years after the contract was introduced (i.e., October 31, 1984–October 30, 1987). As is readily apparent, there was a secular increase in the ratio of the volume of soybean futures options trading to soybean futures trading during this period.

In comparison, the Japanese experience with options on commodity futures is more recent. Options on commodity futures were introduced by the TGE in June 1991 for the imported soybean futures contract. Unlike the experience of the CBT with options on its soybean futures contracts, the TGE's experience with options on its imported soybeans futures contract has not been as successful on a relative basis. This is shown in Figures 4–6, which depict the ratio of TGE soybean futures call and put option volume to TGE soybean futures volume during the first three years after the contract was introduced (i.e., June 1991 to June 1994). For instance, a comparison of Figures 3 and 6 indicates that the ratio of total TGE soybean option volume to TGE soybean futures volume 3 years after the introduction of soybean futures options trading, was significantly below the ratio of total CBT soybean option volume to CBT futures trading 3 years after the (re)introduction of soybean futures options.

To gain a better perspective on TGE soybean futures options trading it is useful to consider some statistics on options volume in more detail. Figure 7 shows the monthly volumes of TGE call and put option trading from June 1991 to December 1995. As is readily apparent, the volume of TGE option trading was relatively low until July 1994. (For example, only 60,418 option contracts were traded in 1993.) The low volume of option trading led the TGE to introduce a system of designated option market makers in July 1994.[1] In response to this change in market structure, the volume of option trading increased almost fourfold during July

[1]The TGE increased the size of the soybean futures contract starting with the April 1994 contract.

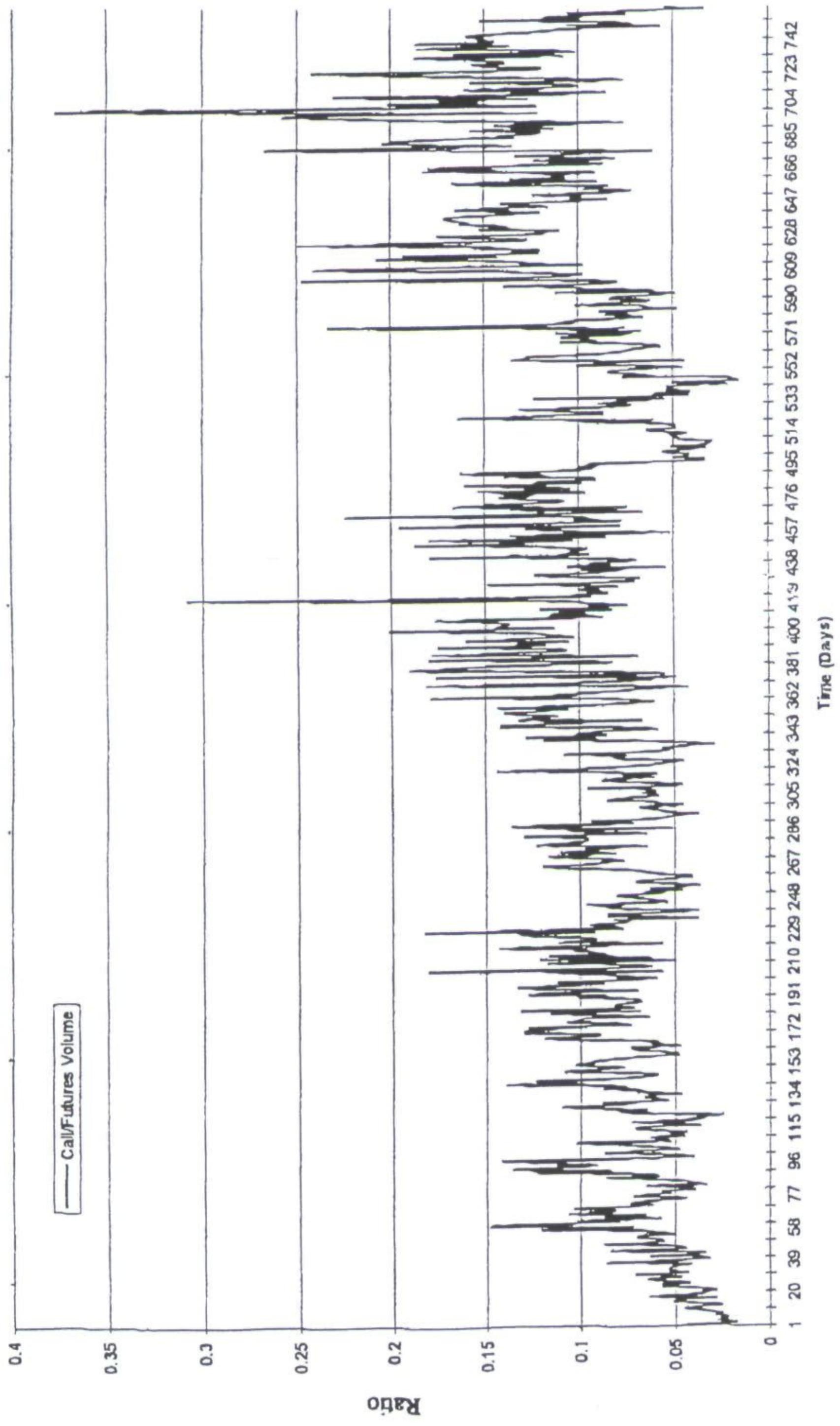

FIGURE 1
Ratio of the daily volume of CBT call options/soybean futures (10/84–10/87).

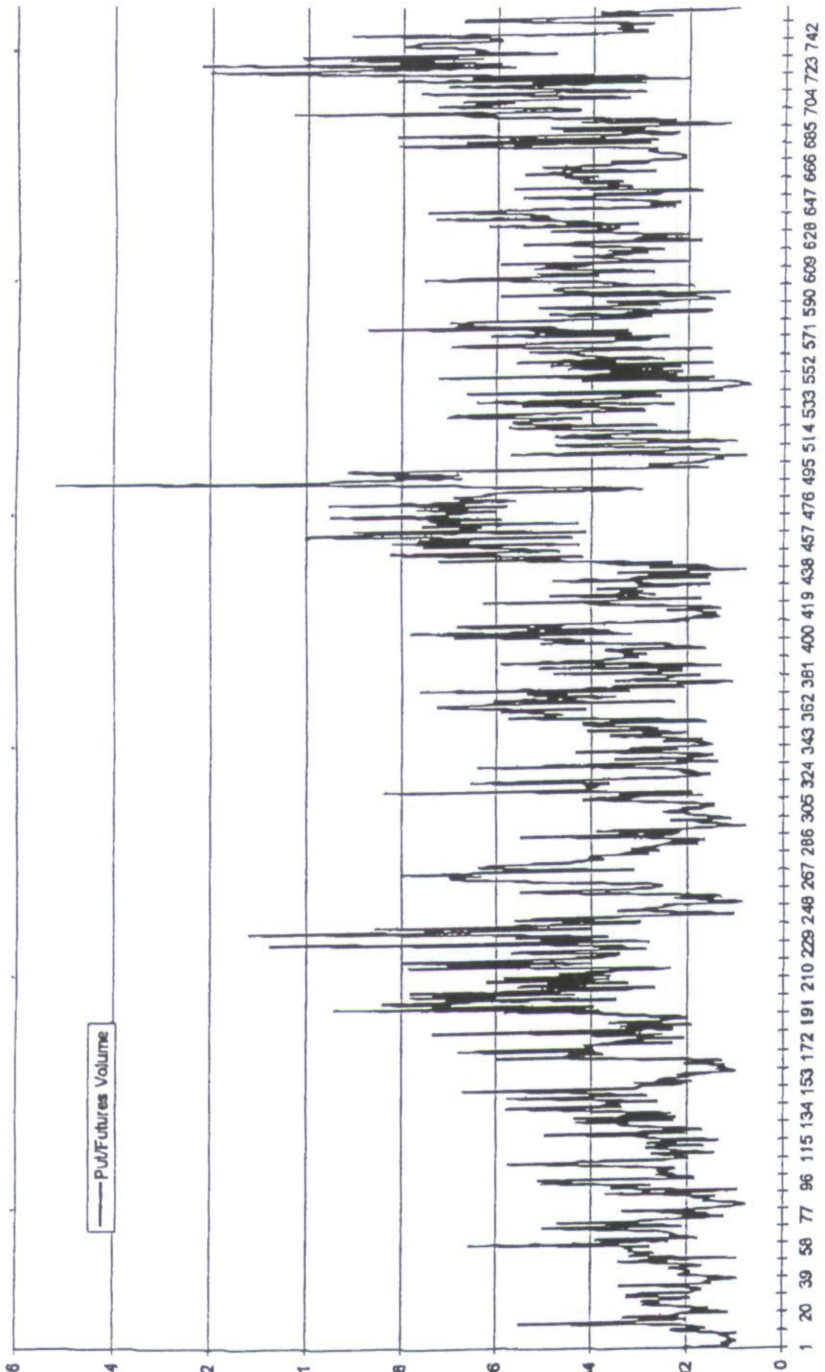

FIGURE 2
Ratio of the daily volume of CBT put options/soybean futures (10/84–10/87).

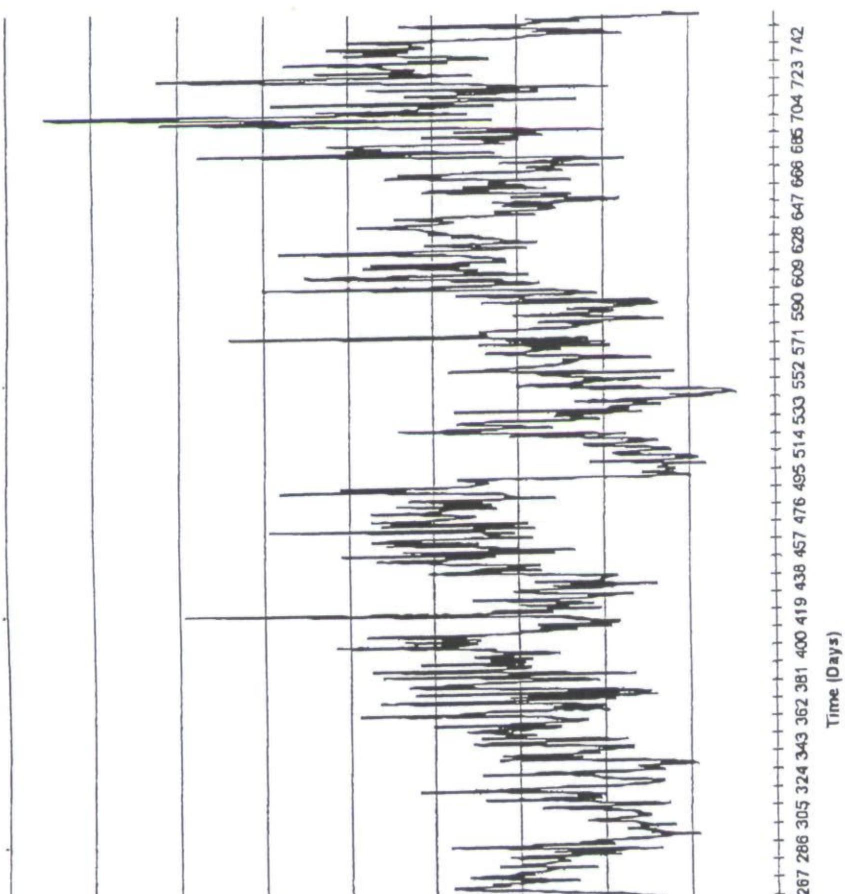

FIGURE 3
Ratio of the daily volume of CBT put and call options/CBT soybean futures (10/84–10/87).

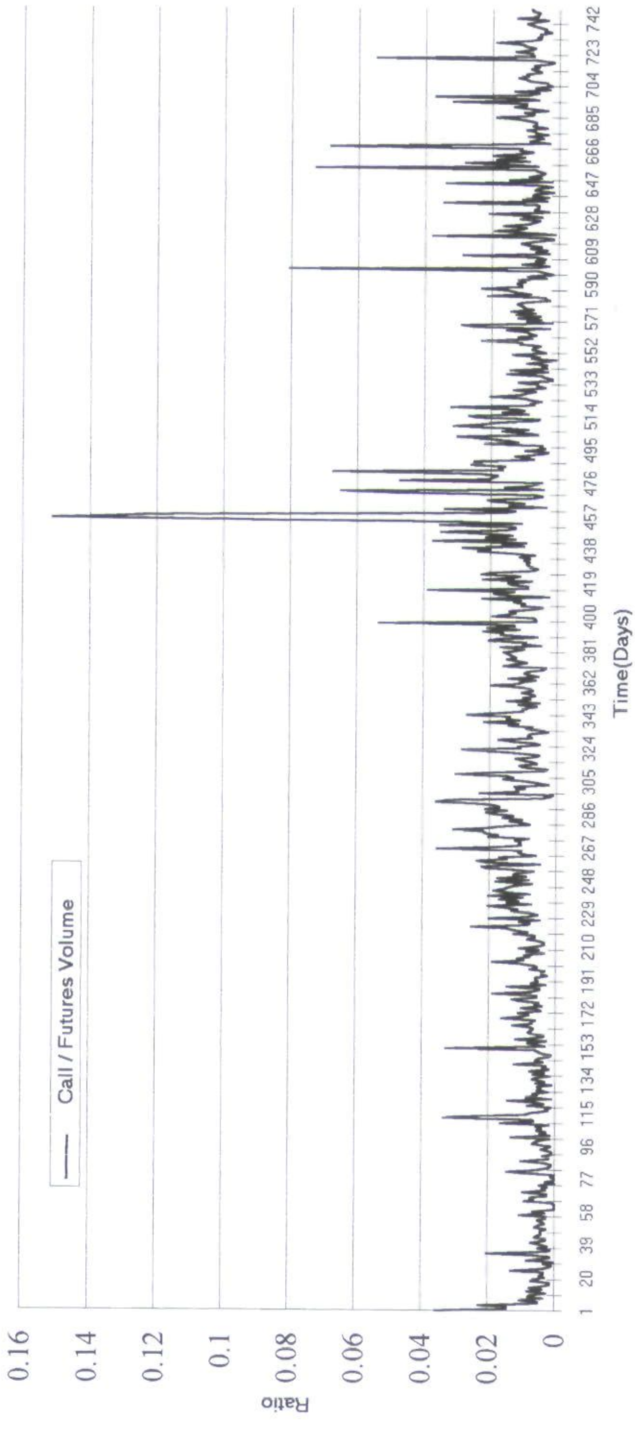

FIGURE 4
Ratio of the daily volume of TGE call options/soybean futures (6/91–6/94).

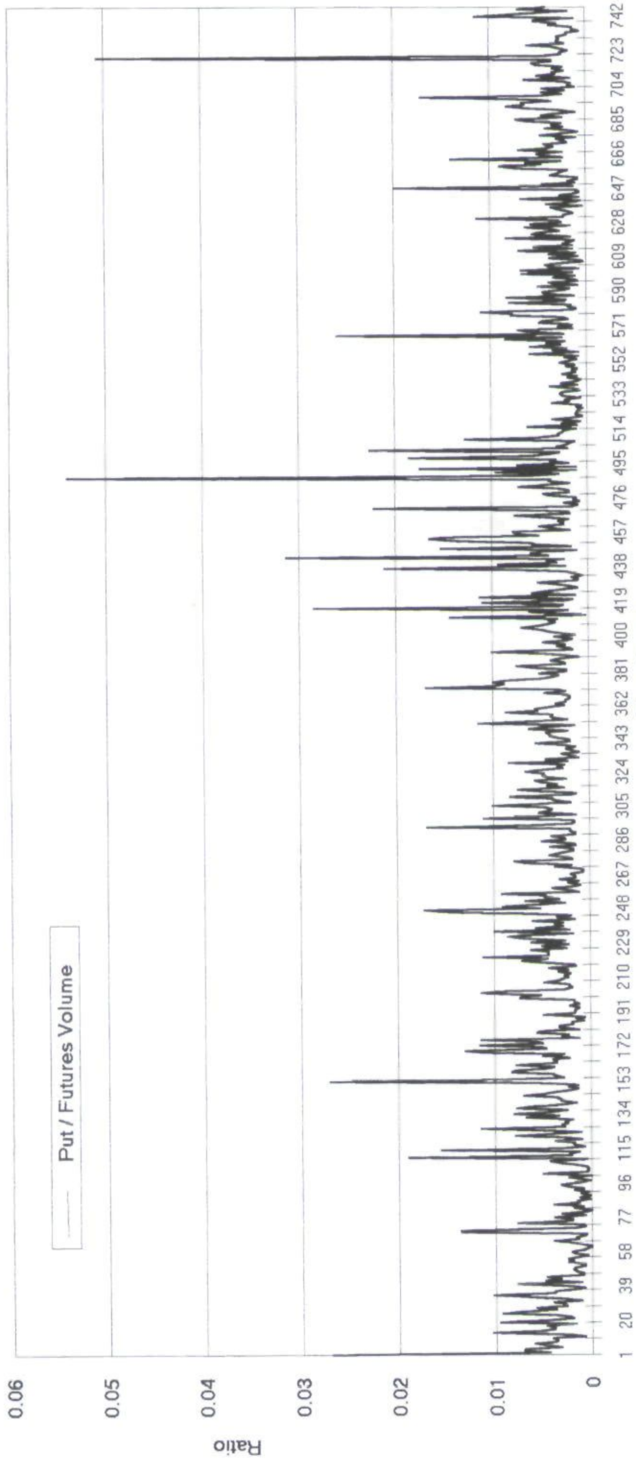

FIGURE 5
Ratio of the daily volume of TGE put options/soybean futures (6/91–6/94).

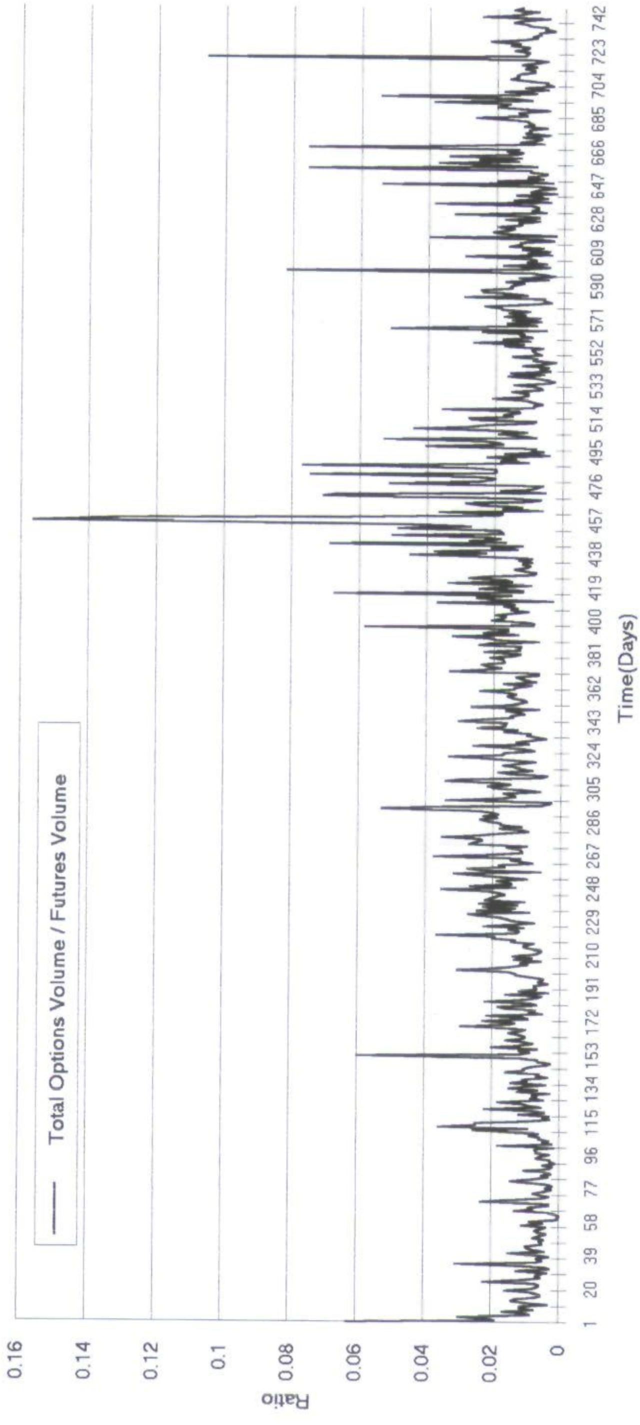

FIGURE 6
Ratio of the daily volume of TGE soybean total options/futures (6/91–6/94).

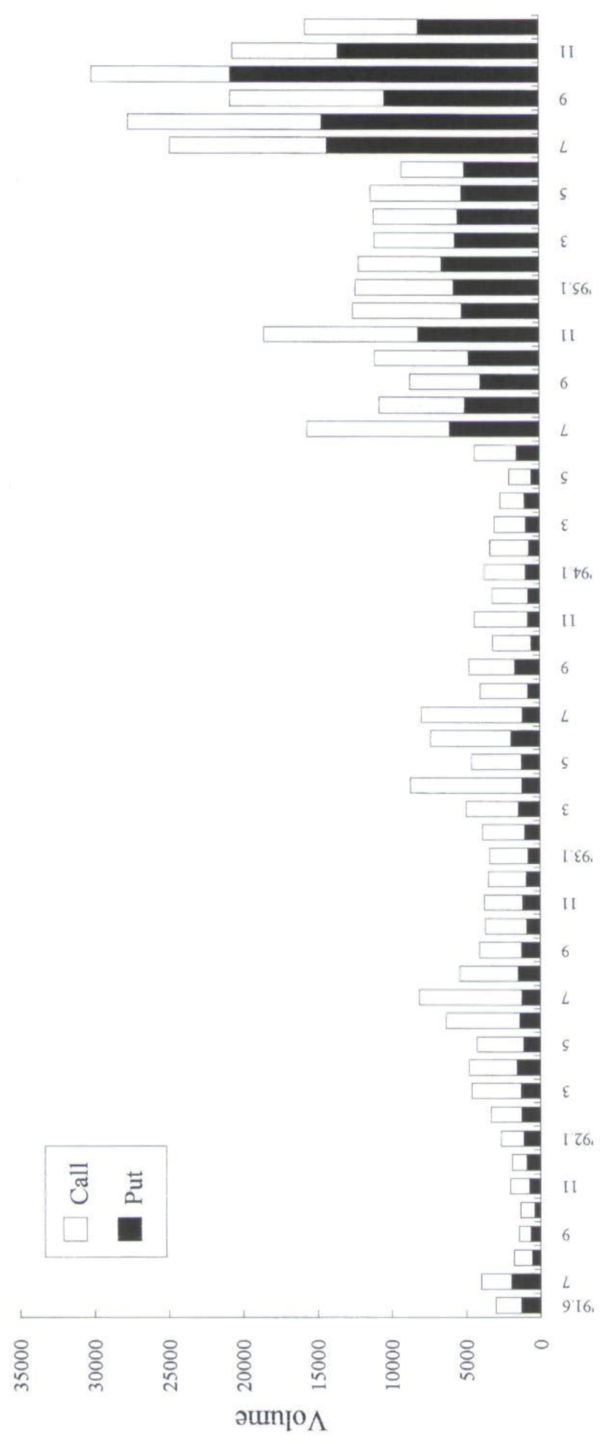

FIGURE 7
Monthly volume of soybean futures option in TGE (July 1991–December 1995).

1994, and the volume of trading in subsequent months remains higher than it was before the change in market making. Figure 7 also indicates that the proportion of put trading increased significantly following the change in market making.

As noted above, both the Ita-awase-hoh and Zaraba trading methods are used in trading commodity futures options on the TGE. Trading in soybean futures options opens at 10:00 (Tokyo time) coincident with the start of trading in the underlying soybean futures contract. During the period, 10:00 to 10:20, all orders are accumulated for each option. At 10:20 a single market clearing price is established for each contract month and strike price. This price is established by taking the mode of matching buy and sell orders. This is known as the Ita-awase period. Option contracts are traded continuously between 10:20 and 11:30. Trading ceases for lunch between 11:30 and 13:00. A second Ita-awase period occurs coincident with the start of the first afternoon futures trading session at 13:00 and lasts until 13:15. Continuous trading in the futures option resumes at 13:15 and lasts until 14:30, when a third Ita-awase period begins. The final Ita-awase period ends at 14:45.[2]

The question naturally arises as to whether there is a discernible intraday pattern in trading volume. Figure 8 shows the distribution of volume in TGE soybean futures options for various 5-minute intervals during the trading day. Keep in mind that the volume of option trading reported during the time intervals ending at 10:30, 13:15, and 14:45 represent trransactions under the Ita-awase-hoh trading method. As is readily apparent, TGE soybean futures option volume exhibits a U-shaped pattern during the trading day.

Also of interest is the *composition* of options trading. In the U.S., soybean futures option trading volume tends to be greatest in the near-the-money options for the first three contract months. Table I shows the volume of TGE options traded for all contract months during August 1994. The columns in Table I represent various exercise prices, and the

[2]The exchange has made a number of changes in the time of the Ita-awase and Zaraba periods since the introduction of TGE soybean futures option trading on June 3, 1991. Prior to July 1, 1995, the first Ita-awase period began at 10:00 and ended at 10:30. The TGE allowed continuous trading in options between 10:30 and 14:15 from the inception of option trading until September 30, 1993. This was followed by a closing Ita-awase-hoh period which began at 14:15 and ended at 14:30. On October 1, 1993, the TGE expanded the continuous trading period by 10 minutes by changing the end of the continuous trading session to 14:25 from the previous 14:15. As a result, the closing session was changed to start 10 minutes later and ran from 14:25 to 14:40. On June 29, 1994, the TGE created two continuous trading periods to allow traders time to break for lunch. As a result, the first Zaraba session began at 10:30 and ended at 11:45. Trading resumed at 13:00 with an Ita-awase period that ended at 13:15. The second continuous trading period began at 13:15 and ended at 14:30. This was immediately followed by an Ita-awase-hoh period that ended at 14:45. This arrangement lasted until July 1, 1995, when the schedule described above was adopted.

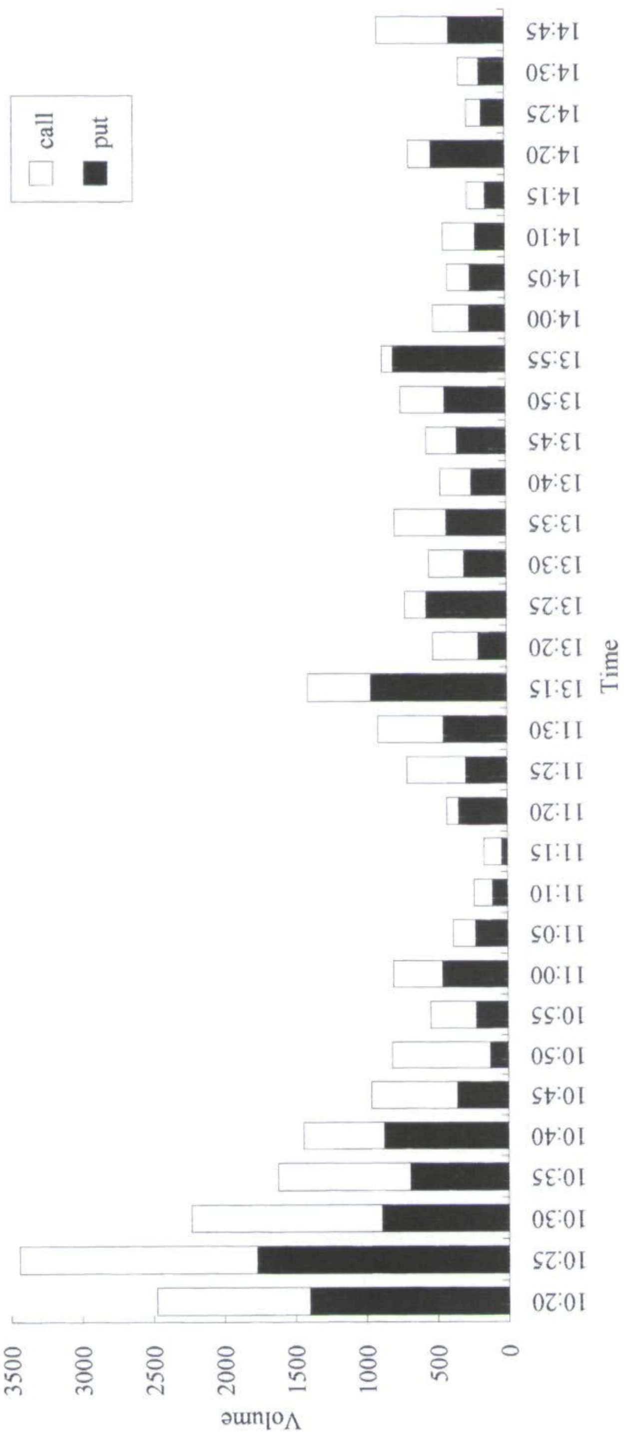

FIGURE 8
Time pattern of volume of soybean futures option in TGE during August 1995.

TABLE I

The Distribution of TGE Soybean Futures Option Volume during August, 1994

	22,000	*24,000*	*26,000*	*30,000*	*32,000*	*34,000*	*36,000*	*Sum*
Call								
October 1994	0	0	38	23	0	0	0	61
			(0.62)	(0.38)				
December 1994	3	282	677	578	38	10	0	1588
	(0.00)	(0.18)	(0.43)	(0.36)	(0.02)	(0.01)		
February 1995	0	430	836	836	6	0	15	2123
		(0.20)	(0.39)	(0.39)	(0.00)		(0.01)	
April 1995	1	230	434	295	0	10	0	970
	(0.00)	(0.24)	(0.45)	(0.30)		(0.01)		
June 1995	0	0	0	0	0	0	0	0
August 1995	0	0	0	0	0	0	0	0
Put								
October 1994	0	0	0	0	0	0	0	0
December 1994	0	732	537	237	0	0	0	1506
		(0.49)	(0.36)	(0.16)				
February 1995	0	586	790	310	0	0	0	1686
		(0.35)	(0.47)	(0.18)				
April 1995	0	601	337	151	0	0	0	1089
		(0.55)	(0.31)	(0.14)				
June 1995	0	0	0	0	0	0	0	0
August 1995	0	0	0	0	0	0	0	0

rows correspond to various futures contract months. The relative amounts of call (put) trading for each contract month and exercise price are reported in parentheses.

Table I indicates that during August 1994 options on the nearby futures contract (i.e., October 1994) were not very actively traded, and options on the two most deferred contract months (i.e., June 1995 and April 1995) were completely inactive. The most actively traded options were (with two exceptions) on the third contract month out (i.e., February 1995).[3] Although this reflects the demand preferences of individual market participants, it is also a reflection of the new market-maker system the TGE introduced in July 1994.

Clearly, TGE soybean futures options have not enjoyed the same success as CBT soybean futures options have. One possible explanation

[3]Under the TGE option market-maker system, at least two market makers must indicate bid and offer prices for all listed options. Further, market makers are required to trade at least 100 option contracts each day. However, option market makers are required to trade only three specified contract months; namely, the second, third, and fourth contract months. The two most deferred contract months and the nearby contract month are specifically excluded. Thus, in August 1994, TGE option market makers had to trade options on the December 1994, February 1995, and April 1995 soybean futures contracts.

for the limited success of the TGE soybean futures options may be that potential users are unfamiliar with the uses of options. Another possible explanation is that market making in TGE options is discouraged by concern over potentially high hedge slippage costs.

HEDGE SLIPPAGE COSTS

In their seminal article, Black and Scholes (1973) advance the first general equilibrium model of option pricing. The fundamental insight that enabled Black and Scholes to solve the problem of option pricing was exceedingly simple; namely, that a perfectly hedged security position should earn the risk-free interest rate, otherwise arbitrage profit opportunities would arise. Application of this insight allowed Black and Scholes to remove the uncertainty surrounding future values of the underlying security from the problem of pricing an option on that security. It also allowed them to determine the price of an option without regard to the risk preferences of market participants.

Maintaining a riskless hedge entails dynamically adjusting the hedge as the price of the underlying security changes. The assumptions underlying the Black and Scholes option pricing model of infinite divisibility (of the underlying security or option) and zero taxes and transaction costs make it possible to create and dynamically maintain perfectly hedged (*delta neutral*) security positions in theory.[4] In practice, however, such frequent adjustments may be neither practical nor possible. Discrete hedging means, as Boyle (1977) points out, that even a delta hedge may not be risk free. A similar problem arises if changes in the expected volatility of the underlying security are stochastic.[5]

The above-mentioned hedging errors caused by discrete (rather than continuous) hedge adjustments or stochastic volatility are called hedge slippage costs by Leong (1992).[6] Hedge slippage costs arise from both the inability of option sellers to create and dynamically maintain riskless hedges and from the convex relationship of option prices to the price of

[4]*Delta* is the first derivative of the option price function with respect to the price of the underlying security. *Delta neutrality* is the notion that one can *continuously* hedge a security position such that the value of the position is invariant to changes in the price of the underlying security position.

[5]*Kappa, zeta, sigma, lambda,* and *vega* are all terms used to denote the change in the value of an option with respect to a change in the volatility of the underlying security.

[6]Leong argues that option writers expect to incur losses associated with their inability to perfectly hedge option positions. Indeed, Leong argues the price of an option may be viewed simply as the present value of expected future slippage costs. This is somewhat misleading, however, because it ignores the option premium that option writers would demand even if continuous hedging were possible.

the underlying security.[7] This introduces another risk to market makers. If the risk of losses due to hedge slippage cannot be easily diversified away, option market makers will demand a higher risk premium for writing options.

There are two components to hedge slippage costs: the potential loss associated with not being able to hedge immediately after acquiring risk exposure from a security position and the potential loss associated with implementing an *imperfect* hedge. The second component of hedge slippage cost associated with discretely adjusted delta hedges has been examined by Boyle and Emanuel (1983) and Galai (1983), who call such hedging errors "*residual return*." Their approach is followed here.

One of the assumptions of the Black–Scholes option pricing model is that changes in the underlying security price, F, follow a stochastic differential equation such as

$$dF = \alpha F\,dt + vF\,dW \tag{1}$$

where α is a constant; volatility, v, is both positive and constant; and W is a standard Brownian motion process. Using Ito's lemma, the call premium, C, obeys the following stochastic differential equation:

$$dC = C_1\,dF + (1/2\,C_{11}v^2F^2 + C_2)\,dt, \tag{2}$$

where $C_1 = \partial C/\partial F$, $C_{11} = \partial^2 C/\partial F^2$, and $C_2 = \partial C/\partial t$. Imposing the no-arbitrage-profits condition leads to the equality $(1/2C_{11}v^2F^2 + C_2)\,dt = rC\,dt$ and the following partial differential equation:

$$1/2C_{11}v^2F^2 + C_2 = rC. \tag{3}$$

The solution of eq. (3), which satisfies the boundary condition, $C = \max(F\text{-}K,0)$, at expiration date, t^*, is the Black-Scholes formula:

$$C = e^{-\mathrm{rT}}[F\Phi(\mathrm{d}_1) - K\Phi(\mathrm{d}_2)]. \tag{4}$$

Here: K is the exercise price; T is time until expiration; $d_1 = \ln(F/K)/(vT^{1/2}) + vT^{1/2}/2$, $d_2 = d_1 - vT^{1/2}$, and $\Phi(\cdot)$ is the cumulative probability function for a standardized normal variable.

[7] *Convexity* (or gamma) is the second derivative of the option price function with respect to a change in the price of the underlying security and measures the change in delta as the price of the underlying security changes. As Hull and White (1992) point out, gamma indicates delta's "sensitivity to large [price] jumps" and "can be used as an indicator of the frequency with which a delta-neutral portfolio should be rebalanced" (p. 73). Engle and Rosenberg (1995) argue that modeling stochastic volatility as a GARCH process produces better estimates of gamma.

Thus, the Black–Scholes option pricing model assumes that for an infinitesimal interval, dt,

$$dC - C_1\, dF - rC\, dt = 0 \tag{5}$$

holds. It should be emphasized that eq. (5) holds only if the hedge position is continuously adjusted as the price of the underlying security changes.

For example, an agent who has hedged a long call position faces the risk of losses if he can only discretely adjust his hedge position during a small (but not infinitesimal) time interval. As a discrete analogue of eq. (5), the error of not being able to continuously adjust an existing hedge position is expressed as

$$X = C(t + \Delta t) - C(t) - \delta[F(t + \Delta t) - F(t)] - rC(t)\,\Delta t. \tag{6}$$

Here $\delta = C_1(t)$ is the partial derivative of eq. (4) with respect to F. Because the error, X, comes from displacing the infinitesimal, dt, with a discrete interval, Δt, in (5), X tends to zero as $\Delta t \to 0$.

The distribution property of the error, X, can be derived by following Boyle and Emanuel (1983). Using Taylor's expansion, one obtains

$$\begin{aligned} C(F + \Delta F, t + \Delta t) &= C(F,t) + C_1\,\Delta F + C_2\,\Delta t + 1/2\; C_{11}(\Delta F)^2 \\ &\quad + 1/2 C_{22}(\Delta t)^2 + C_{12}\,\Delta F\,\Delta t + \cdots. \end{aligned} \tag{7}$$

Here the partial derivatives are the values at time t_0. If one uses this and denotes the term of order $(\Delta t)^{3/2}$ by $O[(\Delta t)^{3/2}]$, eq. (6) may be expressed as

$$X = C_2\,\Delta t + 1/2\; C_{11}(\Delta F)^2 - rC\,\Delta t + O[(\Delta t)^{3/2}]. \tag{8}$$

From the Black-Scholes formula [eq. (4)]:

$$C_2 = rC - e^{-rT}F(2T^{1/2})^{-1}\, v\phi\,(d_1) \tag{9}$$

and

$$C_{11} = e^{-rT}\,(FvT^{1/2})^{-1}\,\phi(d_1) \tag{10}$$

are derived. Here $\phi(\cdot)$ denotes a standard normal distribution density. From eq. (1) and given $F(t_0)$, $F(t)$ takes the value

$$F(t) = F(t_0)\exp[\alpha(t - t_0) - 1/2\, v^2(t - t_0) + v(t - t_0)^{1/2}z], \tag{11}$$

where z follows a standard normal distribution, $N(0,1)$. Setting $t - t_0 = \Delta t$ and expanding the exponential function part, one obtains:

$$F(t) = F(t_0)\,[1 + \alpha\,\Delta t - 1/2\, v^2\,\Delta t + v(\Delta t)^{1/2} z + 1/2[\alpha\,\Delta t - 1/2\, v^2\,\Delta t + v(\Delta t)^{1/2} z]^2 + \cdots]. \tag{12}$$

From this eq. (13) is obtained:

$$\Delta F = F(t) - F(t_0) = F(t_0)[\alpha\,\Delta t + vz(\Delta t)^{1/2} + 1/2\, v^2(z^2 - 1)\,\Delta t + O((\Delta t)^{3/2})]. \tag{13}$$

Therefore:

$$(\Delta F)^2 = F(t_0)^2 v^2 z^2\,\Delta t + O((\Delta t)^{3/2}). \tag{14}$$

Substituting Eqs. (9), (10), and (14) into (8), one obtains:

$$X = e^{-rT} vF(2T^{1/2})^{-1}\,\phi(d_1)(z^2 - 1)\,\Delta t + O((\Delta t)^{3/2}). \tag{15}$$

Because z^2 obeys the chi-squared distribution with one degree of freedom its mean and variance are 1 and 2, respectively. If $\lambda = e^{-rT} vF(2T^{1/2})^{-1}$ $\phi(d_1)$, then

$$X = \lambda(z^2 - 1)\,\Delta t + O((\Delta t)^{3/2}). \tag{16}$$

Here λ is a given value, because it is determined at time t_0. It can be shown from eq. (16) that if one neglects the higher-order terms of Δt and given λ, the discrete hedge error, X, has mean 0 and variance $2\lambda^2(\Delta t)^2$, and obeys a leftward-skewed distribution with a long tail to the right. From the property of the standard normal variate, z, the probability that X takes negative values is 0.6826. As can be easily shown, the same conclusion applies also to the put option case.

It should be stressed that the hedge slippage error defined above has an expected value of zero. Therefore, the risk of hedge slippage should be evaluated with the use of a measure of dispersion such as the standard deviation.

In addition to the hedging error arising from discrete adjustments to hedged positions discussed above, there is another, perhaps more important, component of hedge slippage costs, namely, the loss from being completely unhedged initially. Most option market makers do not wish to have a directional position on at any time. Their inability to immediately hedge an option position acquired as a result of a customer-initiated transaction means that they will have an unwanted directional bet on. This component of hedge slippage costs can be thought of as the loss (i.e., change in the price of an option) an option market maker incurs when the price of the underlying futures contract changes adversely before the market maker can hedge his position. This loss is a function of the delta of the option and size of the change in futures price.

HEDGE SLIPPAGE COSTS ON THE TGE

Unlike the CBT, which trades both commodity futures and the related futures options continuously during the trading day, the TGE trades soybean futures periodically and soybean futures options continuously during parts of the trading day. This difference in trading frequency of options and futures between the CBT and the TGE allows one to assess the influence of market structure on hedge slippage costs.

To understand how hedge slippage costs arise, consider the problems that an option market maker on the TGE faces. *First,* because futures are traded periodically under the single-fixed price auction system, there is no assurance that the market maker will be able to lay off his risk in the futures market simultaneously with or immediately after selling an option. *Second,* there is considerable uncertainty over the futures price at which the option market maker can transact even if the option is sold coincident with the determination of the futures price. *Third,* if the option is sold between trading sessions, the option market maker must wait until the next trading session to hedge his position in the underlying futures market. Moreover, the discrete nature of the auction system means that there is more time for new information to accumulate. The greater the accumulation of new information, the greater the probability that prices will change by more than the minimum price change of 10 yen.

Note that even if a riskless hedge is constructed when the option is sold, it must be adjusted as the price of the underlying futures changes. The TGE auction process constrains the price of the futures to change only during the trading session. In theory, the futures price changes with the arrival of new information. Thus, dynamic adjustments to the price of the hedged position are not possible, even if desired.[8]

[8]Essentially, the market maker is concerned with the price change from the last *transaction* price to the opening *provisional* price as well as the price change from the opening *provisional* price to the final *transaction* price. Normally, the former is substantially greater than the latter. However, the latter is considerably greater than the 10 yen minimum price move as well. There is also an observed tendency for the second component of the price change to be negatively correlated with the first component [cf. Webb (1991) pp. 666–667]. This may be interpreted as an overreaction on the part of the market to the price move between trading sessions.

Put differently, the essential problem that TGE option market makers face is that the price of the underlying futures will follow a jump process rather than a diffusion process. In a sense, one may regard a jump in the futures price as equivalent to a momentary increase in volatility. Periodic futures auctions mean that TGE option market makers will be unable to hedge these price jumps away. Similarly, when changes in speculative prices are random or exhibit mean reversion hedge slippage costs are potentially greater than when changes in speculative prices are serially correlated.

The uncertainty over which price an option market maker can transact at to lay off his risks introduces significant hedge slippage costs. Other things equal, the more difficult it is for an option market maker to lay off his option-related risks in the futures market, the less he is willing to make a market in options. Simply put, the greater the hedge slippage costs, the less attractive is option market making.

For instance, suppose an option market maker sells an at-the-money ¥24,000 strike price call option on a soybean futures contract. If the underlying futures contract price rises by ¥10, the option premium would rise by about ¥5 (assuming a delta equal to 0.5) per tonne before the market maker can hedge. The loss the option market maker would face is approximately ¥150 (= ¥5 × 30 tonnes) per contract. The loss would be correspondingly lower or higher as delta falls or rises. Similarly, the loss would change with the size of the jump in futures prices. For instance, suppose soybean futures prices jump ¥200 between trading sessions. The writer of an at-the-money call option (who is unable to hedge immediately) would lose about ¥3,000 (= ¥100 × 30) per option contract. This is the first component of hedge slippage costs.

As for the second component of hedge slippage cost (i.e., the hedging errors arising from discrete adjustments to the hedge), suppose a market maker delta hedges a short call position from time, t_1 to t_{1+h}. At the same time when he sells the call, suppose he buys δ units of the underlying futures contract, where $\delta = \partial C/\partial F$. Let n represent the number of times the position is rebalanced (i.e., ranging from the initial acquisition of a short futures position to the final closing of both the option and futures positions). Assume that the ith rebalancing takes place at $t_i (i = 1, \ldots, n)$. So $t_1 + h = t_n$. The cost of the hedging error, X_i, at the ith rebalancing is

$$X_i = C_{i-1} - C_i - \delta_i(F_{i-1} - F_i) - rC_i(t_{i-1} - t_i)$$
$$(i = 1, \ldots, n - 1), \tag{17}$$

where C_i and F_i denote the call premium and futures price at the ith adjustment, respectively.

Let $G(n)$ denote the present value at time t_1 of the total hedging errors during the interval. Defining $B_i \equiv \exp(-r(t_i - t_1))$. $G(n)$ is expressed as

$$G(n) = \sum_{i=1}^{n-1} B_i X_i = \sum_{i=1}^{n-1} B_i(C_{i+1} - C_i)$$
$$- \sum_{i=1}^{n-1} B_i \delta_i(F_{i+1} - F_i) - r\sum_{i=1}^{n-1} B_i C_i(t_{i+1} - t_i). \qquad (18)$$

Let $t_{i+1} - t_i$ represent the time interval between two consecutive trading sessions and h denote the length of a trading day. The standard deviation of $G(n)$ can be calculated by conducting a simulation, assuming that changes in futures prices follow a geometric Brownian motion process.

SIMULATION OF HEDGE SLIPPAGE COST

The two components of the hedge slippage cost associated with holding TGE soybean futures option positions are simulated under the assumptions of discrete and continuous trading and constant and stochastic volatility, respectively. For purposes of the simulation, futures trading is assumed to occur four times per day (10:00; 11:00, 13:00, and 14:00) under discrete trading and 240 times or every minute between 10:00 and 14:00 under (almost) continuous trading. Similarly, futures options are assumed to be traded continuously from 10:00 to 14:00. This is slightly different from real option trading on the TGE, but it is assumed for simplicity.

In the first case, a trader is assumed to sell a call option and buy delta units of a futures contract simultaneously. Traders are assumed to close their positions at the end of the trading day. This allows one to measure the second component of hedge slippage cost (i.e., the hedging errors). The assumption of simultaneous hedging means that only the second component of hedge slippage costs exists. In the second case, a trader is assumed to sell a call option at 10:00 and is unable to hedge by buying delta units of the futures contract until 11:00 (i.e., asynchronous hedging). This allows one to measure the first component of hedge slippage costs (i.e., the potential loss from not being able to immediately hedge the risk exposure of an option position.) as well as the second.

TABLE II

Constant Volatility, Case 1 (Synchronous Hedging)

	Discrete Trading	*Continuous Trading*
Component	Second component	Second component
Average	0.108179	0.077172
S.D.	0.841351	0.147774

As shown in Table I, the most actively traded commodity option contracts are based on the second, third, and fourth deferred delivery months for the futures contract. The hedge slippage costs of option contracts calling for December 1995 delivery are simulated. Specifically, the hedge slippage cost is simulated for July 3, 1995, the first trading day in July 1995. This day is selected because the December delivery month futures contract is actively traded in July.

$G(n)$, defined in eq. (18) in the previous section, is simulated. Because B_i will be nearly 1 in such a small interval as four hours, $B_i = 1$ in this simulation. Thus

$$G(n) = \sum_{i=1}^{n-1} (C_{i+1} - C_i) - \sum_{i=1}^{n-1} \delta_i(F_{i+1} - F_i) - r(t_{i+1} - t_i)C_i. \tag{19}$$

The underlying futures price is assumed to follow a geometric Brownian motion process such as eq. (1). One thousand simulations are conducted for each case.

When constant volatility is assumed, v is set equal to 0.25 (which is near the value of implied volatility on July 3, 1995), α is set equal to 0, and the underlying futures contract is assumed to be traded four times per day. The opening futures price and option premium are assumed to be ¥24,210 and ¥1,565.83, respectively. The basic statistics associated with the simulation of hedge slippage costs under the assumptions of constant volatility and simultaneous hedging are reported in Table II. The assumption of simultaneous hedging means that only the second component of hedge slippage costs exists in this case. The frequency distributions of the second component of hedge slippage costs under the assumptions of constant volatility and discrete or continuous trading are shown in Figures 9 and 10, respectively.

When stochastic volatility is simulated, volatility is assumed to change every minute according to the following formula:

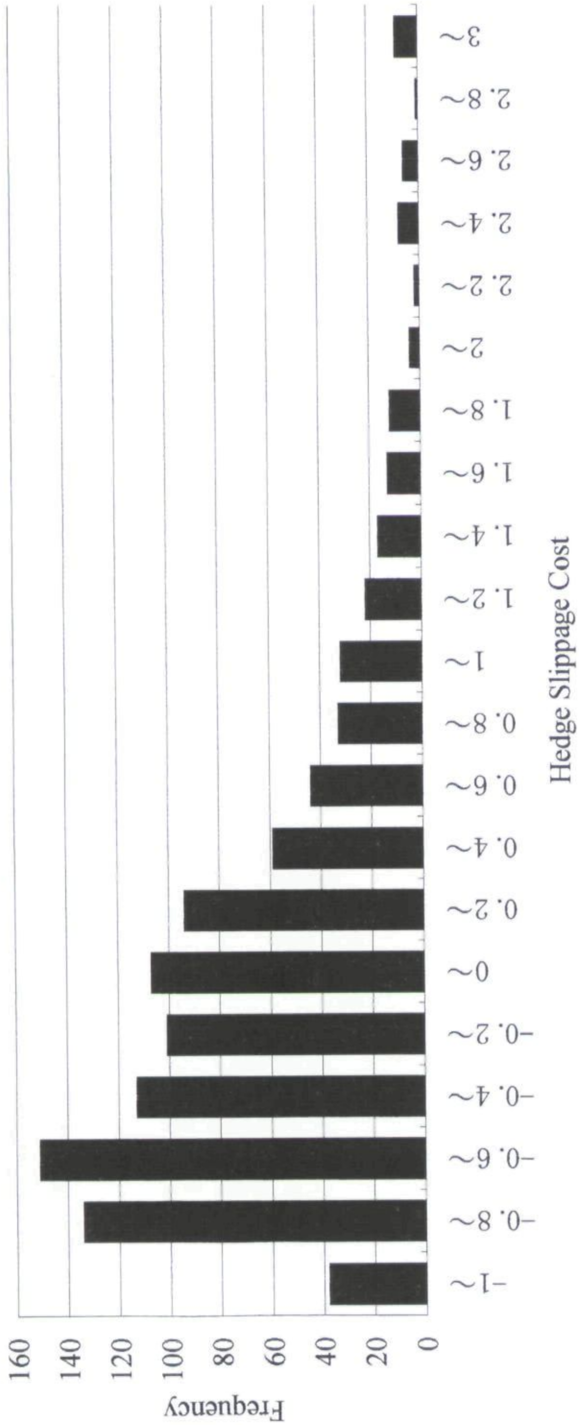

FIGURE 9
Frequency distribution of hedge slippage cost: discrete trading and constant volatility.

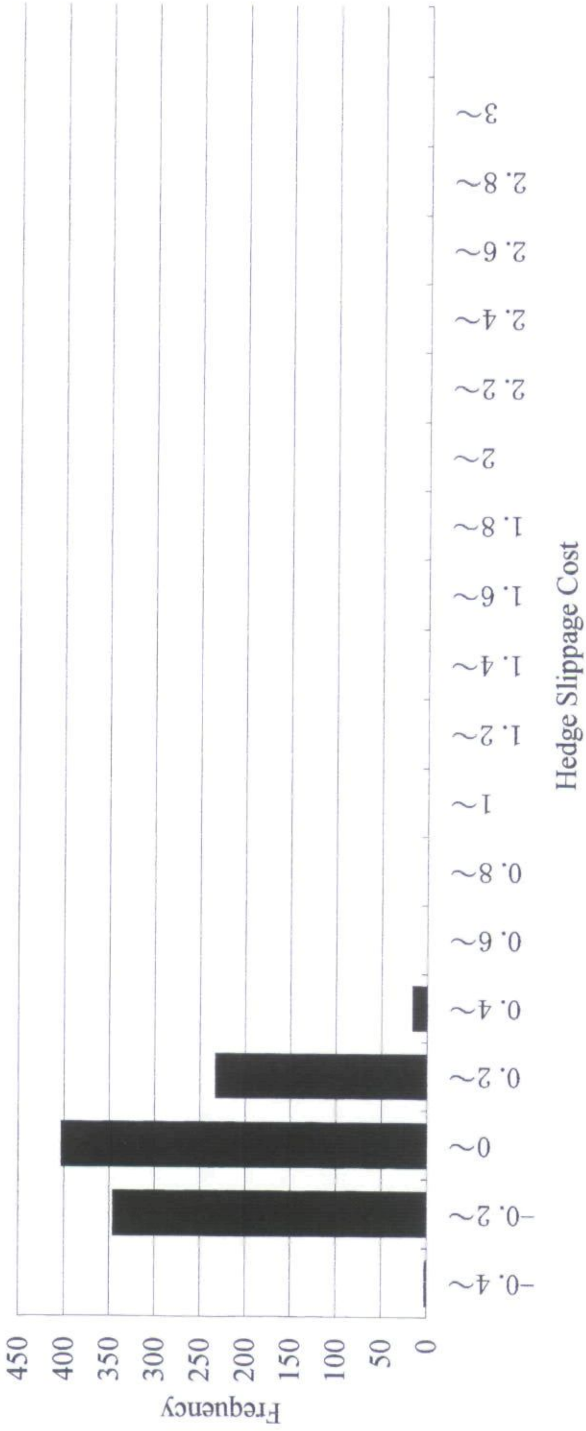

FIGURE 10
Frequency distribution of hedge slippage cost: continuous trading and constant volatility.

TABLE III

Stochastic Volatility, Case 1 (Synchronous Hedging)

	Discrete Trading	*Continuous Trading*
Component	Second component	Second component
Average	0.298771	0.099546
S.D.	19.332954	11.698803

TABLE IV

Constant Volatility, Case 2 (Asynchronous Hedging)

Component	First component	Second component
Average	−0.844220	0.085824
S.D.	34.570182	0.788670

TABLE V

Stochastic Volatility, Case 2 (Asynchronous Hedging)

Component	First component	Second component
Average	1.467070	0.234361
S.D.	35.811306	13.68766

$$dv = \mu v \, dt + \xi v \, dW_2, \tag{20}$$

where W_2 is a standard Wiener process and ξ is the volatility of v. ξ can be calculated theoretically as follows:

$$\xi = (\text{S.D. of } (\Delta v/v)^* \sqrt{365}. \tag{21}$$

That is, ξ provides an estimate of the annualized sample standard deviation of the daily rate of change in the implied volatility. Actual TGE data on implied volatility are used and ξ is set equal to 0.37759. Following Hull and White (1987) the correlation between dW_1 and dW_2 is assumed to be zero and $\mu = 0$. The basic statistics of simulated hedge slippage cost under the assumption of stochastic volatility and simultaneous hedging are reported in Table III.

In Case 2, asynchronous hedging is assumed. The simulation results are shown in Tables IV and V. In this case, both components of hedge slippage cost exist. The frequency distributions of the second component of hedge slippage cost under the assumptions of stochastic volatility and

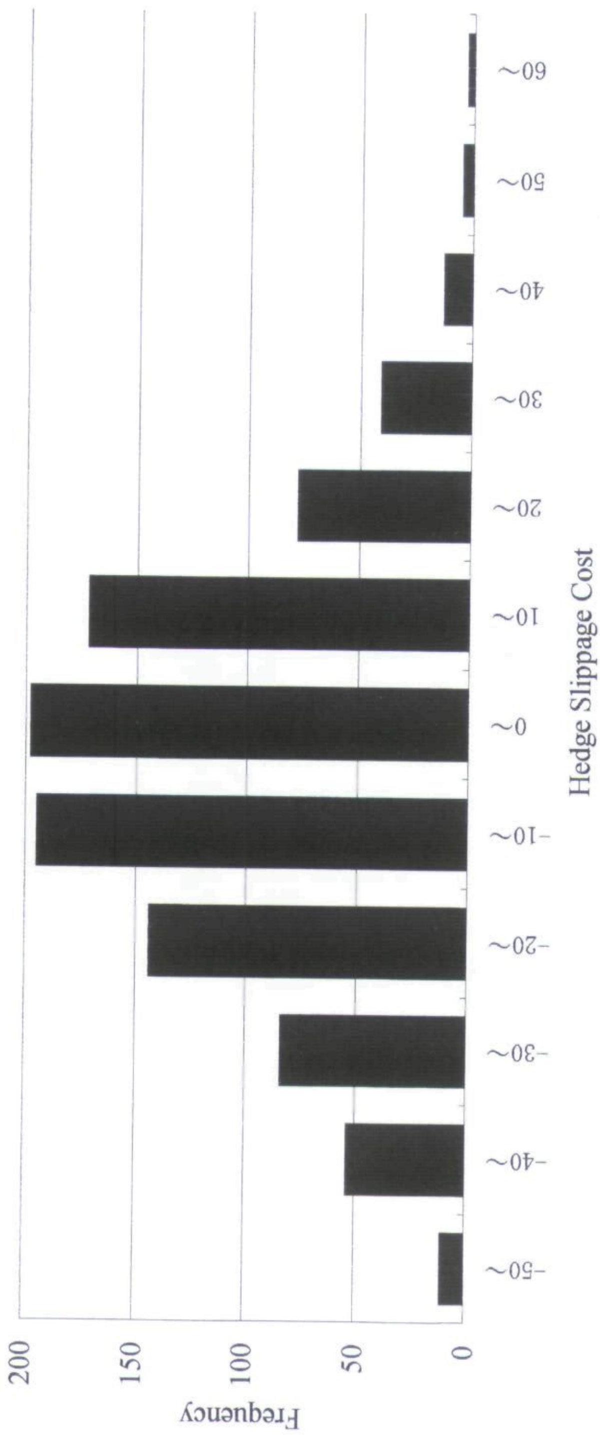

Figure 11
Frequency distribution of hedge slippage cost: discrete trading and stochastic volatility (second component).

discrete or continuous trading are shown in Figures 11 and 12, respectively.

It is readily apparent from Tables II–V that the standard deviation of the second component of hedge slippage cost is larger under the assumption of discrete trading than it is under the assumption of continuous trading. Under the assumptions of constant volatility and discrete trading, the standard deviation of the second component of hedge slippage cost during the 4-hour trading day (10:00 to 14:00) is 0.841351, which, when multiplied by the leverage factor of 30, is only 25.24 yen per contract. This compares with a leveraged estimate of ¥4.43322 for the second component of hedge slippage cost under the assumption of nearly continuous trading. These simulation results suggest that when volatility is constant, the size of the second component of hedge slippage costs arising from an imperfectly delta hedged option position is not affected greatly by whether trading is discrete or continuous.

Tables II and III also suggest that the second component of hedge slippage costs is much larger under the assumption of stochastic volatility than under the assumption of constant volatility. For instance, in the case of discrete trading under conditions of stochastic volatility, the standard deviation of the second component of hedge slippage cost is approximately ¥579.98 (that is, 30×19.332954). This compares with a standard deviation of approximately ¥350.96 (that is, 30×11.698803) in the case of continuous trading under the assumption of stochastic volatility.

The results change, however, when asynchronous hedging is assumed because the first component of hedge slippage costs must also be considered. It is immediately apparent from Tables IV and V that the absolute value of the first component of hedge slippage cost is much larger than the second. For instance, the standard deviation of the sum of both components in Table IV is 34.5790 yen [i.e., $(34.570182^2 + 0.778670^2)^{1/2}$] and when this value is multiplied by the leverage factor of 30 the hedge slippage cost is approximately ¥1,060.76 per contract. Not surprisingly, both components of hedge slippage costs are even higher under the assumptions of asynchronous trading and stochastic volatility. The standard deviation of the sum of both components in Table V is 41.1542 yen [i.e., $(38.811306^2 + 13.68776^2)^{1/2}$] and the hedge slippage cost is approximately ¥1,234.62 (that is, 41.1542×30) per contract.

The above simulation results suggest that the second component of hedge slippage costs increases as the frequency of trading decreases and when volatility is stochastic. The simulations also suggest that the first component of hedge slippage costs substantially exceeds the second com-

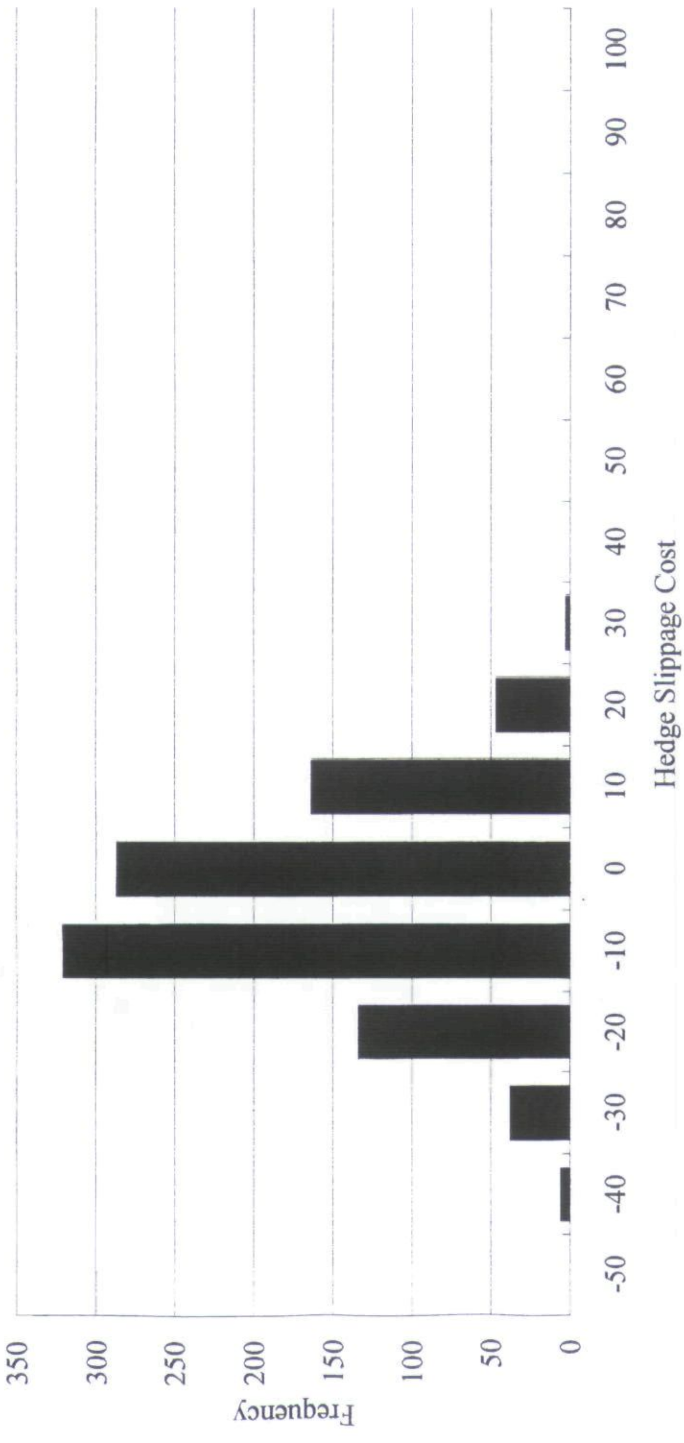

FIGURE 12
Frequency distribution of hedge slippage cost: continuous trading and stochastic volatility (second component).

ponent. This means that a market maker in TGE futures options faces significant risk when he cannot immediately hedge his exposure in the underlying futures market. Not surprisingly, the risk a market maker faces increases when volatility is stochastic (i.e., kappa risk).

HOLDING LENGTH OF OPTIONS BY TGE MEMBERS

Hedge slippage costs are a function of many factors, including the length of time the option position is held. The simulations reported in the previous section of hedge slippage cost are conducted under the assumption that a TGE option market maker closes his position daily. However, in practice, a marketmaker rarely closes his TGE option position daily, because of market illiquidity. The question naturally arises as to how long TGE market makers *actually* hold option positions. Data on TGE member soybean futures option positions during 1995 are examined to answer this question.

Let t_0 denote the start and t^* denote the end of trading of a given maturity month option. Suppose a member trades or exercises (or is exercised on) a specified option at time points, $t_1, t_2, \ldots, t_n$. Let $t_{n+1} = t^*$. Denote the quantity of options bought or received from exercise at t_i by $q(t_i) > 0$, and the quantity of options sold or exercised at t_i by $q(t_i) < 0$. Let $Q(t_i)$ denote the quantity of options held at t_i, with $Q(t_i) > 0$ representing a long position and $Q(t_i) < 0$ representing a short position. Then

$$Q(t_i) = Q(t_{i-1}) + q(t_i), \qquad i = 1, \ldots, n + 1. \tag{22}$$

Here $Q(t_0) = Q(t^*) = 0$.

The average holding period H is defined as

$$H = \sum_{i=1}^{n} (t_{i+1} - t_i)J_i \Big/ \left(\tfrac{1}{2} \sum_{i=1}^{n} |qt_{i+1}| \right), \tag{23}$$

where $J_i = |Q(t_i)|$.

If a member trades only once for his own account during a trading day, it is assumed that trading is done at the start of trading (i.e., 10:30). If a member trades m times for his own account during a trading day it is assumed that these trades are equally distributed across the trading day. Thus, the time of the jth trade is set on that day, $t_{i+j-1} = t_i + (j - 1)A/(m - 1), j = 1, \ldots, m$. Here $A = 4.25/24$ day, where 4.25 means the hour length between 10:30 and 14:45 (the closing time of the last Ita-awase-hoh session), and t_i is the time at 10:30.

TABLE VI

An Example of the Calculation of the Holding Length of TGE Options

1	Year	Month	Day	t_i	Premium	Sell	Buy	Exercise	Exercised	q	Q
1	95	6	29	179.438	940	10	0	0	0	−10	−10
2	95	7	12	192.438	720	0	1	0	0	1	−9
3	95	7	27	207.438	940	0	9	0	0	9	0
4	95	7	31	211.438	850	5	0	0	0	−5	−5
5	95	8	16	227.438	810	10	0	0	0	−10	−15
6	95	9	18	260.615	—	0	0	0	1	1	−14
7	95	9	19	261.615	—	0	0	0	6	6	−8
8	95	10	3	275.615	—	0	0	0	1	1	−7
9	95	10	23	295.615	—	0	0	0	2	2	−5
10	95	11	13	316.615	—	0	0	0	2	2	−3
11	95	11	15	318.615	—	0	0	0	3	3	0

TABLE VII

Average Holding Length of Options by Selected TGE Members (in days)

Member	Mean	Standard Error	Standard Deviation	Sample Size
AC	93.2098	(18.8147)	53.2161	8
DF	59.9209	(20.3382)	53.8098	7
FF	4.6782	(1.7267)	3.8609	5
HK	33.8307	(9.5865)	30.3153	10
JA	37.4260	(10.3335)	37.2578	13
KS	56.0812	(10.5365)	40.8078	15
KT	31.7682	(11.3069)	42.3064	14
KY	36.6951	(13.4744)	44.6894	11
MB	40.7164	(9.9971)	44.7085	20
TG	25.4601	(5.0171)	20.6858	17
UT	33.1861	(8.7106)	36.9561	18
YT	52.6793	(15.1224)	62.3513	17
POOLED	41.5679	(3.5620)	44.3464	155

As an example, consider the case of a December 1995 call option with an exercise price of ¥26,000 yen held by TGE member Daiwa Futures Inc. (DF) as its own position in Table VI. If the member acquires the option position on the first day of trading and holds it until the last day of trading (November 15, 1995), then $t^* - t_1 = 318.725$ and $Q(t^*) = 0$.

Table VI reports that the average holding period, H, for this option position by TGE members is 48.7863 days; $\Sigma_{i=1}^{n}(t_{i+1} - t_i)\ |Q(t_i)| = 1219.66$; and $1/2\ \Sigma_{i=1}^{n}\ |q_i| = 25$ contracts. The mean holding period of December TGE soybean futures options and the associated standard deviation for each of the 12 principal TGE member firms are shown in Table VII. The table indicates that the total mean over the 12 TGE member firms is 41.5679 days, and the standard deviation is 44.3464 days. These results suggest that TGE option market makers hold their option positions for a relatively long time period.

IMPLICATIONS AND CONCLUSIONS

The previous simulation experiment shows that the standard deviation of the delta hedging error under the assumption of discrete hedge adjustments is ¥25.24 when multiplied by a leverage factor of 30 (i.e., 0.8414 $\times$ 30). On the other hand, the standard deviation of the delta hedging error under the assumption of nearly continuous trading is ¥4.432 (i.e., 0.1477 $\times$ 30).

To check the results of the discrete delta hedging case, the standard deviation of X_i is calculated according to the Boyle and Emanuel approximation given in eq. (6). As explained above, the Boyle and Emanuel approximation neglects $O(\Delta t^{3/2})$ terms in X_i. For simplicity of exposition, set $\Delta t = t_i - t_{i-1} = 1/24/365 = 0.0001142$ year or 1 hour, the typical length of time between consecutive Itayose-hoh trading sessions of the underlying futures contract. According to the conditions taken in the simulation, the time before expiration T is set $= 136/365$ (i.e., 0.3726 year). Assuming that the exercise price, K, is ¥24,000; the futures price, F, is ¥24,210; the short-term interest rate, r, is 0.0126; and volatility, v, is 0.25, then $d_1 = 0.1334$.[9] Thus, X has mean ¥0 yen and standard deviation of ¥0.3151 (i.e., $2^{1/2} \cdot \lambda \, \Delta t = 1.4142 \cdot 1951.15 \cdot 0.0001142$).

According to the simulation assumption rebalancing occurs at 10:00, 11:00, 13:00, and 14:00. If one sets $B_i = 1$, and assumes that λ remains at nearly the same value, the standard deviation of $G(\mathrm{n})$ (i.e., the present value of the total hedging errors) will be ¥0.6302 (that is, $[0.3151 \cdot 4]^{1/2}$). If one multiplies this by the leverage factor of 30, the standard deviation of the discrete hedging error per option contract becomes ¥18.91. This figure is a little smaller than the estimate of ¥25.24 from the simulation. If one considers that the theoretically estimated value neglects the term of order, $(\Delta t)^{3/2}$, one should say that these two figures are roughly the same.

[9]That is, $\ln(F/K)/(vT^{1/2}) + 1/2vT^{1/2} = \ln(24210/24000)/(0.25 \cdot (0.3726)^{1/2}) + (1/2)(0.25 \cdot (0.3726)^{1/2}))$ and $\lambda = 1951.15$ [i.e., $e^{-rT}vF(2T^{1/2})^{-1}\,\phi(d_1) = \exp(-(0.0126 \cdot 0.3726) \cdot 0.25 \cdot 24210 \cdot (2 \cdot (0.3726)^{1/2}]^{-1} \cdot (1/(2\pi)^{1/2})\exp(-(0.1334)^2/2)$].

If stochastic volatility is assumed, the simulation suggests that the standard deviation of the discrete delta hedging error is ¥19.3330. Multiplying this by the leverage factor of 30, one obtains ¥579.99. If one compares this figure with that of the constant volatility case (i.e., ¥25.24), the stochastic volatility makes delta hedging more risky.

The survey of the previous section indicates that the average holding length of option positions by TGE members for their own accounts is almost 42 days. This suggests that most market makers choose not to close their positions daily, presumably because of a lack of market liquidity. It also means that TGE market makers are exposed to overnight risk in their option positions. If this overnight position is delta hedged, the standard deviation of the delta hedge error would be ¥6.2999 [that is, $2^{1/2}\lambda\,\Delta t = 1.4142 \cdot 1951.15 \cdot 20/(24 \cdot 365)$ or ¥6.2999]. This assumes that the overnight interval, Δt, is approximately 20/24 of a day. In the case of option positions held over a weekend, Δt will be $(48 + 20)/24$ days. Then $2^{1/2}\lambda\,\Delta t =$ ¥21.4196 [that is, $1.4142 \cdot 1951.15 \cdot (48 + 20)/(24 \cdot 365)$ or ¥21.4196]. Assume for simplicity that the 42-day average holding period for option positions by TGE members contains 31 trading days, 24 overnights and 6 weekends. Then, if these hedge errors are mutually independent, the standard deviation of delta hedging errors will be 61.1915 yen per tonne [that is, $1.1227^2 \cdot 31 + 6.2999^2 \cdot 24 + 21.4196^2 \cdot 6)^{1/2} = (3744.3994)^{1/2}$]. Taking into account the leverage factor causes the standard deviation to rise 30-fold to ¥1, 835.74.

In addition to the delta hedge error, one must consider the first component of hedge slippage error (i.e., the risk that prices may move adversely before a hedge can be put on). For instance, suppose a TGE market maker who sells a December '95 call with a ¥24,000 strike price on July 3, 1995 is unable to hedge his option position for at least 1 hour. Assuming $v = 0.25$, $F =$ ¥24,210 and the delta $= e^{-\mathrm{rt}}\Phi(d_1) = \exp[-(0.0126 \cdot 0.3726] \cdot \Phi(0.1334) = 0.9953 \cdot 0.5531 = 0.5504$. The standard deviation of the call premium variation for an hour would be approximately calculated as ¥35.59 per tonne [i.e., $0.550424210 \cdot (0.25^2/(24 \times 365))^{1/2} = 0.5504 \cdot 64.667$ or 27.16]. The corresponding simulation result of 62.75 yen reported in Table II is considerably greater. This difference may be caused from the approximation error in the former. The standard deviation adjusted for the leverage factor is ¥1067.70.

If the two components of hedge slippage costs are mutually independent, the leveraged standard deviation of the sum of both components will be $(1067.70^2 + 1835.74^2)^{1/2}$ or 2123.66 yen for a 1-hour unhedged position. In addition to this, if volatility is perceived to be stochastic, market makers will fear even higher hedge slippage costs. This may represents a significant proportion of an option's premium. For instance, the opening premium of the December '95 ¥24,000 strike price call was ¥1610 per tonne on July 3, 1995 or ¥48,300 per option contract.

It is important to keep in mind that even if the underlying futures are traded continuously, the delta hedging error due to the overnight and over-weekend holding of hedged positions would remain. However, if this is the case, the holding length of the option might be reduced because the market liquidity of each option series could increase.

Soybean futures options are traded on both the Chicago Board of Trade and the Tokyo Grain Exchange. Yet the volume of soybean futures option contracts traded in proportion to trading in the underlying futures contract is significantly smaller for the TGE than the CBT. This is true even if one compares the proportion of options traded to futures volume at the two exchanges, because soybeans futures options were first introduced. This study examines one potential explanation for that difference, namely, the existence of significant hedge slippage costs on the TGE.

Hedge slippage costs arise because option market makers are unable to create and dynamically maintain delta-neutral hedges due to institutional factors and convexity in option prices. TGE soybean futures options are traded continuously during large parts of the trading day, whereas TGE soybean futures contracts are traded discretely. This poses a burden on option market makers because the frequency of hedge adjustment in a Itayose-hoh auction is limited to the frequency of trading of the underlying futures contract. The potential for large price changes between trading sessions on the TGE entails large hedge slippage costs before the hedge can be adjusted. High hedge slippage costs raise the price of, and reduce the demand for, TGE options. By increasing the risks of market making, high hedge slippage costs also reduce the desire of option market makers to make markets in such options.

The Itayose-hoh auction system is one of the strengths of futures trading on the Tokyo Grain Exchange. As Webb (1995) argued, the Itayose-hoh auction system tends to produce prices that are less noisy than futures prices in continuously traded markets. However, the very advantage of the Itayose-hoh auction trading system for futures becomes a marked disadvantage for trading options on TGE futures contracts because of the substantial hedge slippage costs associated with the potential for large changes in the price of soybean futures between trading sessions.

Significant hedge slippage costs on the TGE represent one possible explanation for the differential success of options on CBT and TGE soybean futures contracts. To be sure, other explanations may exist. Which explanation best explains the behavior of TGE soybean futures option trading activity is ultimately an empirical issue.

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