# ESTIMATING CASH SETTLEMENT PRICE: THE BOOTSTRAP AND OTHER ESTIMATORS

**JOHN CITA**
**DONALD LIEN**

## INTRODUCTION

Specifying a futures contract calls for the exchange to consider a host of design options and decisions. Certainly the most elemental problem is choosing the underlying asset. In the case of cash settlement contracts, this problem is essentially equivalent to specifying a settlement index.[1] Determining a settlement index is a multidimensional problem. It generally requires the exchange to specify a cash price sampling procedure and an estimator—as the means to both summarize and infer from the sample. Furthermore, each of these choices is likely to depend on the quality of the cash price information. Although closely related to its quality, it is likely that the availability of cash prices will also influence the decisions of contract designers.[2] As an example, if the exchange selects a settlement index that utilities cash prices that are less than public in

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[1]Many cash settlement contracts also use an index as the underlying cash commodity. In such cases, the underlying commodity doubles as the settlement index.

[2]Prices for assets traded over the counter (or in nonexchange-type settings) are certainly more private in nature and correspondingly more difficult to obtain. It is the relatively limited trading volume in assets of this type and the cost of continuously disseminating price information that restricts the availability of their prices.

■ *John Cita is the Chief of Economic Policy at the Kansas Corporation Commission.*

■ *Donald Lien is a Professor of Economics at the University of Kansas.*

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nature, as in the case of prices for assets traded over the counter (OTC), it immediately invites further design decisions. For one, the exchange must specify and implement a mechanism for soliciting or collecting the price data.[3] For another, construction of an accurate settlement index requires the price collection mechanism to be reliable.

This article explores the contract design problem of selecting an estimator for cash settlement contracts that rely upon a sampling of private cash price information at expiry.[4] In general, the exchange's selection of an estimator has implications for the quality of inference, but may also have implications for the quality of information or sample data submitted to the exchange. For instance, the incentive to possibly misreport or intentionally bias price information provided to the exchange is affected by estimator selection.[5] Hence, although estimator selection has these joint implications for contracts that rely on sampling, the focus in this article is on only the quality of inference. The contract design features regarding the method of collecting price reports, sample size, and possibly the family of estimators are set parametrically.

Like contract design features in general, the specification of an estimation procedure is guided by the exchange's objectives. It is frequently assumed the exchange prefers the overall contract design with maximal hedging performance. If the contract's underlying asset(s) are taken as given, the contract's hedging performance is generally maximized when its delivery risk is minimized. For the case of cash settlement contracts, the more accurate and precise the settlement index, the lower the contract's delivery risk. Hence, it is assumed the exchange selects the estimator that maximizes the accuracy and precision of the settlement index.

As prospective solutions to the estimation problem, two estimators, the adaptive truncated mean (ATM) and the adaptive Winsorized mean (AWM), are presented and discussed. Both use the bootstrap procedure. Barnett and Lewis (1984) is an excellent reference. The ATM and AWM perform favorably relative to other estimators, including the symmetric truncated mean (STM). The Chicago Board of Trade (CBOT) and Chicago Mercantile Exchange (CME) offer contracts, the Municipal Bond

[3]The exchange must actively seek out cash price information that is less than public. For instance, the Chicago Board of Trade compensates municipal bond price reporting brokers in the neighborhood of $90T annually for their reporting services. A slight distinction should be made between collecting and sampling price data. The former refers to costly search activities and decisions that may precede actual sampling. Sampling is required for the usual reasons.

[4]Besides trading OTC, relatively private cash price information may be generated by private negotiation, as in the case of Eurodollar transactions.

[5]Although it may appear that the incentive to intentionally misreport is small, this appearance is some what deceiving. As shown in Cita and Lien (1992), in the context of the municipal bond contract, the expected benefit from misreporting bond prices may be significant.

and Eurodollar, respectively, which use the STM to estimate the commercial value of underlying assets.[6] It appears the hedging performance of both contracts would possibly improve if one of the bootstrap-based estimators were used in place of the STM.

For simplicity, discussion focuses on the municipal bond contract.[7] However, the results obtained have implications for any cash settlement contract that requires a sampling of cash prices that are not directly assessable.[8] Section 2 provides a description of the municipal bond contract's basic settlement provisions. Section 3 provides a detailed description of the exchange's optimization problem. An explanation of the basic bootstrap procedure is contained in Section 4. Section 4 also contains a more detailed discussion of the bootstrap-based estimators, the ATM and AWM. The relative precision and accuracy (more specifically, the mean-square error) of these and other estimators (namely, the STM) are demonstrated in Section 5 via the results of a Monte Carlo study.[9] The final section contains conclusions.

## SOLICITING AND SUMMARIZING CASH PRICE DATA: THE CURRENT PROCEDURE

The municipal bond contract is supported solely on assets traded OTC. Because of the relative nontransparency of OTC prices, the CBOT must actively solicit and collect a sample cash price information. Under its current procedure, the CBOT solicits a sample of cash price information from brokers who are called price reporting brokers.

[6]The municipal bond contract is based on the Bond Buyer Index, which serves as the contract's settlement index. The Eurodollar contract is not based on an index. For the Eurodollar contract the use of an index is strictly for settlement purposes. What the municipal bond and Eurodollar contracts have in common is a reliance on cash prices that are relatively private and obtained via sampling, otherwise the two settlement procedures are substantially different.

[7]Because of the apparent problems in municipal bond (spot) market, the municipal bond contract may be worthy of added attention. Not only have relatively recent market conditions contributed to a downturn in overall activity (manifest by a sizable decline in the value of new bond issues and an outflow of municipal bond mutual funds), but the Securities and Exchange Commission is evidently ready to impose on the market a greater amount of oversight. The SEC is focusing on the need for "disclosure of more information about issuer's finances and market prices." Although this article focuses strictly on the municipal bond futures contract, improving the contract's hedging performance has implications for both the cash and futures markets. Another issue addressed in this article is the reliability of broker (more generally, agent) price reporting and its possible influence on contract hedging performance. In light of apparent problems in the cash market, such a concern may be well justified. Quotations taken from: "Municipal-Bond Industry Faces Dual Challenges: Falling Debt and Rising SEC Scrutiny," *Wall Street Journal,* February 6, 1995, pp. C1 and C19.

[8]Other contracts that call for some amount of cash price sampling are the Kansas City Board of Trade Value Line Index contract, which relies, in part, on assets traded OTC and the CBOT's GNMA contract, which takes a sampling of modified pass-through certificate prices.

[9]In the text, discussions focus on the municipal bond contract. The analysis of the Eurodollar contract and the Monte Carlo study results based upon that contract are provided in various footnotes.

### Soliciting Cash Prices: Price Reporting Brokers

The municipal bond contract is "based on The Bond Buyer Index of 40 actively traded, general obligation and revenue bonds." However, to simplify the following discussion, assume it is based on the issue(s) of a single municipal bond.[10] On a daily basis, the CBOT solicits six dealer-to-dealer municipal bond brokers for their assessment of the price at which \$100,000 face value of the bond will trade.[11] (Cash price reports are solicited from 1:45 to 2:00 p.m.) Collecting price reports from brokerage firms is generally perceived as being an objective means of collection. The basis for this view stems from the fact that brokerage firms "do not take positions in either the cash or futures markets, but merely act as agents who broker bonds among dealers." Nonetheless, when a reported price appears out of line the exchange seeks verification of its accuracy.[12]

### Summarizing Cash Price Reports: The Truncated Mean

In spite of the apparent objectivity of broker-reported prices, the CBOT routinely trims or truncates the sample of prices solicited from the six reporting brokers by discarding the extreme observations. The simple mean of the remaining four price observations yields an estimate of the bond's commercial value in the cash market. (At contract maturity the value of the contract, in accordance with settlement provisions, is set equal to the realized value of the index.)

## THE OPTIMAL CASH PRICE ESTIMATOR

Selection of an estimator, of course, requires a performance criteria. It is frequently suggested that the exchange selects contract provisions to minimize delivery risk or, equivalently, to maximize hedging performance. In the context of cash settlement contracts, such an objective requires the exchange to select an estimator that is as accurate and precise as possible.

### Specifying an Estimator: The Exchange's Objective Function

In more formal terms the accuracy of an estimator usually refers to its bias, where its precision is typically defined as the reciprocal of its vari-

[10]See Chicago Board of Trade (1987). All quotes concerning the MBI futures contract are taken from this pamphlet.

[11]Garbade and Silber (1984) provide a good description of the various types of price information and the relative advantages associated with each. The CBOT collects six price reports. For purposes of settling its Eurodollar contract, the CME randomly selects 12 banks, out of a panel of 20, from which to obtain prices.

[12]According to Dale Michaels, a representative of the CBOT, the exchange does inspect the price reports they receive and will double-check reports that appear out of line.

ance. It is not apparent whether the exchange is more concerned about the bias or variance of the estimator. However, if the exchange is indifferent about the problems created by the two, then the mean-squared error (MSE) of the estimator probably offers it an adequately balanced measure of the estimator's overall exactness. If the exchange is indifferent between precision and accuracy, then it is reasonable to assume that the exchange selects the estimator with the minimal MSE. Among estimators that are unbiased or nearly so, the exchange's problem is simplified, requiring it to opt for that estimator with minimum variance. Yet, when unbiasedness is lacking, the MSE of an estimator may be of prime interest to the exchange. It is assumed that the exchange selects an estimator from the family of truncated means to maximize the accuracy and precision of the settlement index; equivalently, the exchange seeks from the given family an estimator with minimum MSE.[13]

A more basic formulation of this problem is that the exchange seeks the proportion of truncation that minimizes the MSE. For this problem there are a number of ancillary considerations. First, the exchange may consider whether the proportion of truncation should be held constant or be allowed to vary over time. Under the current procedure the proportion is fixed, even when a contract which allows flexibility in this regard might be desirable. Second, the exchange may analyze the relative advantage of using truncation vis-à-vis Winsorization. If the latter is preferable, the optimal proportion of Winsorization can be established. The bootstrap procedure is especially helpful for exploring these as well as other questions that pertain to the problem of selecting an optimal estimator.

With the use of a Monte Carlo study it is possible to explicitly frame the exchange's problem and explore possible solutions. Several estimators that are reasonable candidates for the exchange's optimum are described.

## Some Possible Solutions

Having constrained the exchange's choice of an estimator to the class of trimmed means still leaves the exchange with considerable discretion. For instance, this class contains the simple mean—the trimmed mean with minimal (zero) trimming, and the median mean—the (symmetrically)

[13]The selection of an estimator has been isolated to the family of trimmed means for two reasons. The primary reason is that that is the family from which exchanges currently select. It is not clear why exchanges employ trimmed means. However, it seems reasonable to suggest that trimmed means are used because of a concern about intentionally biased price reporting and that intentionally biased reports are most likely to be extreme observations. Second, it narrows the search for an optimal estimator to a search for the optimal amount of trimming.

trimmed mean with maximal trimming that retains only the median observation. For purposes of simplification, it is assumed that the exchange considers only symmetric, as opposed to asymmetric, truncation.[14] Furthermore, because all of the basic distributions investigated in the Monte Carlo experiment are symmetric, only symmetrically trimmed means are used. Alternatively, asymmetric distributions imply it may be more appropriate to explore asymmetric trimming [Barnett and Lewis (1984)].

Let $X$ denote a random sample of size $n$ from a distribution $F$: $X = (X_1, X_2, \ldots, X_n)$. Also let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ represent the order statistics. There are a number of different ways to define a trimmed mean. The trimmed mean based on sample $X$ can be defined by

$$T(\alpha,\alpha;X) = \frac{X_{(1+\alpha m)} + \cdots + X_{(n-\alpha m)}}{n(1 - 2\alpha)}$$

where $\alpha \in \Lambda = \{i/n : i = 0, 1, \ldots, (n - 1)/2\}$. This is an example of a symmetrically trimmed mean. For $\alpha$-trimmed means the proportion of trimming is given by $2\alpha$, which leaves a censored sample size of $n(1 - 2\alpha)$. For the municipal bond contract, $\alpha = \frac{1}{6}$. With the use of the above notation then, that contract specifies using $T(\frac{1}{6}, \frac{1}{6}; X)$ for settlement.[15] Because this is the status quo estimator, it is denoted SQE.

An $\alpha$-Winsorized mean can be defined by the expression:

$$W(\alpha,\alpha;X) = \frac{rX_{(r+1)} + X_{(r+1)} + \cdots + X_{(n-r)} + rX_{(n-r)}}{n}$$

where $r = \alpha n$. Like the truncated mean, the Winsorized mean omits the extreme observations; the $r$ largest and smallest observations are dropped. However, the $r$ largest values are each replaced by the value of the nearest observation to be retained, that is, $X_{(n-r)}$. Similarly, the $r$ smallest values are each replaced by $X_{(r+1)}$. Replacement of the trimmed observations, that is, Winsorization, is motivated by the belief that the direction of residuals for the extreme observations carries useful information that should be retained in some form. Truncated and Winsorized means are based on trimming without and with replacement, respectively.

The simple mean, $T(0,0;X)$ is denoted as SIM and the median, $T(\frac{1}{2}, \frac{1}{2}; X)$, readily available when $n$ is an odd number, is denoted as MED.[16]

[14]This appears reasonable, because the exchange generally will not be in a position to anticipate the direction of intentional bias, and brokers are as likely to inject a positive bias as to inject a negative bias.

[15]The Eurodollar contract also uses $T(\frac{1}{6}, \frac{1}{6}; X)$ for purposes of settlement.

[16]In general, maximal trimming would eliminate all observations; thus there must be a suited limiting argument so that at least one observation is preserved.

In summary, the family of trimmed means contains two subfamilies—trimming without replacement and trimming with replacement, that is, Winsorization. Each may be further divided depending on whether the proportion of trimming is held fixed or allowed to vary.

## ADAPTIVE MEANS: THE BOOTSTRAP AND OTHER PROCEDURES

In general, adaptive means depend or rely in part on (and as such adapt to) the actual sample. Descriptions follow of two methods—Jaeckel's and the bootstrap—that can be used to select the proportion of trimming so as to minimize the sample variance of the truncated mean. Each method is capable of determining the optimal $\alpha$, denoted $\alpha^*$, in the $\alpha$-trimmed mean for estimating the location parameter of a symmetric distribution. They are optimal in the sense that over an allowable range $(\alpha_0, \alpha_2)$, $\alpha$ is selected to minimize the sample variance, $s^2(\alpha)$, of $T(\alpha,\alpha;X)$. The optimally trimmed mean, $T(\alpha^*,\alpha^*;X)$, is referred to in this study as the adaptive trimmed mean, ATM. Both methods also can be used to establish the optimally Winsorized mean, $W(\alpha^*,\alpha^*;X)$, which is referred to as the adaptive Winsorized mean, AWM. These descriptions are offered as simplified derivations of the adaptive means.

To avoid possible confusion, both adaptive means are optimal not because they necessarily solve the exchange's optimization problem, but because they minimize the sample variance of the trimmed or Winsorized estimator. Clearly, both appear worthy of consideration as possible solutions to the exchange's optimization problem—especially if the adaptive mean is unbiased. The question of whether they are optimal in terms of the exchange's contract design problem is explored by way of the Monte Carlo experiment.

Before proceeding it is important to note that procedures for determining an optimally trimmed mean are not designed to recognize and subsequently cast out spurious observations. Rather, the intent of optimal trimming is to use the information embedded in the sample data as well as possible, given that some sort of contamination may exist [Jaeckel (1971)].

### Jaeckel's Approach

Jaeckel (1971) considers two procedures for estimating the center of a symmetric distribution. Both procedures start with a family of possible estimators—trimmed means with different (but symmetric) trimming proportions and L estimators. His problem is straightforward: "determine the estimator in the family with smallest estimated asymptotic variance." Jaeckel's basic method (of which there are extensions) involves essentially three steps. First, the sample observations are used to estimate the asymptotic variance of each member of the family. Second, the estimator in the family with the smallest asymptotic variance is determined. Last, the estimator with minimum sample variance is taken as the best location estimator. The optimal estimator under this procedure, $T_J(\alpha^*,\alpha^*;X)$, maximizes the precision (being the inverse of the sample estimator's variance) of the location estimate over the family of trimmed means. Jaeckel shows that the estimator established by his approach converges to the true best estimator in the family; hence, using the optimally trimmed mean "is asymptotically as good as knowing beforehand which estimator in the family is best for a given distribution."

There are, however, some disadvantages to Jaeckel's method. As stated by Mehrotra, Jackson, and Schick (1991), "explicit expressions for the asymptotic variances may not be available, and (or) it will be difficult to obtain an estimate of the variance." They also suggest "the asymptotic variance may not be a true representative of the actual variance of a trimmed mean, especially in small samples." It seems likely this could be a problem for an application like the municipal bond contract. Furthermore, Jaeckel's estimator of the asymptotic variance of the trimmed means becomes unstable for values close to $\frac{1}{2}$. Along the same lines, Jaeckel rules out the possibility that $i = 0$, being the simple mean.

Because of the drawbacks with Jaeckel's technique, Mehrotra et al. suggest applying the bootstrap approach in which sample variances are estimated with the use of the bootstrap method, with the trimming proportion that minimizes the bootstrap variance estimate being optimal.

## The Bootstrap Approach

There are a number of different applications of the bootstrap method. It is used here to calculate the bootstrap estimate of the variance of particular trimmed means. That is, for a given sample size, $n^0$, and proportion of trimming, $2\alpha^0$, the application yields the bootstrap variance of $T(\alpha^0,\alpha^0;X)$. Repeating this over a family of trimmed means, that is, over a permissible range of $\alpha$, enables selection of the trimming proportion that minimizes the bootstrap variance estimate. A truncated mean that utilizes the optimal proportion of trimming, as determined by the bootstrap method, is sometimes referred to as a bootstrap adaptive trimmed mean. [Barnett and Lewis (1984)]. The bootstrap procedure is rather simple yet computationally cumbersome.

Again, it is presumed that the distribution function from which (in this context, cash price) observations are drawn, $F$, is symmetric at zero, but is otherwise unknown. The symmetry assumption is unnecessary, but the instant application is both reasonable and reduces the bootstrap calculations by half.[17] Let $F^0$ denote the empirical probability distribution function, a function of only the sample observations. The bootstrap variance of a particular $\alpha$-trimmed mean estimator is simply its variance, calculated in the usual way, evaluated at $F = F^0$. Because $F^0$ is the nonparametric maximum-likelihood estimator (MLE) of $F$, the bootstrap estimate of the variance is the nonparametric MLE of the variance of $F$.

Fisher and Hall (1991) describe algorithms for the exact computation of bootstrap estimators and show that such computations are practical for small samples. For example, in the case of the municipal bond contract, which relies on a price sample size of six, the entire bootstrap distribution has only 462 atoms. Although exact computation is competitive with Monte Carlo simulation, the simulation method offered by Efron (1977, 1982) is used to explore for the optimal estimator for the municipal bond contract.[18] (To be sure, the adaptive means offered here as potential solutions to the exchange's problem, ATM and AWM, are bootstrap adaptive means.) The bootstrap ATM is asymptotically normal and has an asymptotic variance equal to the smallest among all possible trimmed means; hence, the ATM works asymptotically as well as the best trimmed mean [Leger and Romano (1990)]. With the same method it is possible to determine the optimally Winsorized mean.

## A MONTE CARLO EXPERIMENT AND RESULTS

For each simulation, $N = 1000$ bootstrap replications are used for each trial. A trial is simply a sample set drawn from the original data, $X_1$, $X_2, \ldots, X_n$. For the municipal bond contract, $n = 6$. For each simulation, 2000 trials are drawn. With the use of the bootstrap method, the ATM and AWM are determined for each trial. Also, the SIM, MED, and estimates of each of the remaining possible trimmed and Winsorized means (with fixed proportions of trimming), including the SQE, are calculated

[17]With asymmetric distributions, the bootstrap method determines the optimal amount of trimming from the right and left sides, thereby determining the optimal amount of asymmetric trimming.
[18]See the Appendix for an outline of the bootstrap procedure based on Efron's Monte Carlo algorithm.

**TABLE I**

Monte Carlo Results (Scenario: Normal Distribution, No Intentionally Biased Reporting)

| *Estimator* | *MSE* | *Bias* | *Variance* |
|---|---|---|---|
| $T(0, 0, 6)$ | 0.1656 | 0.0003 | 0.1656 |
| SQE | 0.1841 | 0.0003 | 0.1841 |
| $T(\frac{1}{3}, \frac{1}{3}, 6)$ | 0.2163 | 0.0023 | 0.2163 |
| $T(\alpha^*, \alpha^*, 6)$ | 0.1674 | 0.0004 | 0.1674 |
| $DB(\alpha^*, \alpha^*, 6)$ | 0.1674 | 0.0003 | 0.1674 |
| $CM(\alpha^*, \alpha^*, 6)$ | 0.4103 | 0.4250 | 0.2297 |
| $W(0, 0, 6)$ | 0.1656 | 0.0003 | 0.1656 |
| $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.1799 | 0.0010 | 0.1799 |
| $W(\frac{1}{3}, \frac{1}{3}, 6)$ | 0.2163 | 0.0023 | 0.2163 |
| $W(\alpha^*, \alpha^*, 6)$ | 0.1674 | 0.0004 | 0.1674 |

Notes: The SQE is the same as $T(\frac{1}{6}, \frac{1}{6}, 6)$. $DB(\alpha^*, \alpha^*, 6)$ denotes the double bootstrapping estimator and $CM(\alpha^*, \alpha^*, 6)$, the augmented bootstrap estimator with centering at the median.

for each trial. The bias, variance, and mean-squared error of each estimator over the sample set of 2000 trials are ascertained.[19]

Because the distribution of cash prices is uncertain, a number of different cases for F are considered: normal, uniform, double exponential, logistic, and triangular.

It is assumed that the exchange is uncertain whether all or none of the submitted price reports are actually intentionally biased, and if so by what amount. This study models and analyzes a number of different contamination, or intentionally biased, reporting scenarios. Contamination is presumed to influence only the estimate of the location parameter. Within the Monte Carlo study price observations are intentionally biased as follows: A predetermined number of actual price reports are distorted by adding to them an amount equal to a predetermined number of (population) standard deviations.[20] By increasing the overall amount of contamination, in terms of both the number of distortions and the amount of the distortions, it is possible to assess the various estimators' robustness against intended contamination. (In this way the exchange can explore which estimator is best for its purposes.)

Table I shows the complete results for the normal distribution when zero contamination is introduced. The results for the AWM are identical

[19]A brief discussion of the simulation results for the Eurodollar contract ($n = 12$, $\alpha_t = \frac{1}{6}$) can be found in Footnote 25.

[20]The Monte Carlo results will be the same regardless of the nature (or the source) of biased reports. That is, the same approach and conclusions apply to the case of unintentionally biased reporting. Hereby a reporting broker draws an observation from a wrong distribution but reports his observation truthfully.

to those of the ATM, and are not reported.[21] With either the MSE or variance used as performance criteria, both the ATM and AWM are superior to the status quo estimator (SQE). Both are also less biased. Although the SIM is better than the adaptive means, this is explained in part by two factors: the relatively small sample size ($n = 6$) and a finite number of bootstrap replications ($N = 1000$). (However, when additional contamination is introduced the performance of the SIM is diminished. For example, when one price report is biased by one standard deviation and another by two standard deviations, then under the normal distribution it is found that the SIM performs worse than the SQE and ATM. Both the ATM and SIM have smaller variances but larger biases than the SQE.) Again, the bootstrap estimators are expected to perform asymptotically as well as optimally trimmed means. In this particular case, as $N$ nears infinity, the performance (measured by sample variance of the estimator) of the bootstrap estimators will converge to that of the SIM. Under the normal distribution, the adaptive means have equivalent performance. The performance of the double bootstrap procedure is essentially equivalent to that of the single bootstrap.[22] The bootstrap procedure based on augmented bootstrap samples centered at the median is outperformed by all other estimators and, except for double bootstrapping, has the greatest computational expense.[23]

A comparison of the estimators over the various cash price distributions is shown in Table II. Again, the bootstrap adaptive means perform quite well. For most distributions the ATM performs nearly as well as the SIM, and frequently better than the SQE.[24]

[21] In the case of the Eurodollar contract, AWM is frequently better than ATM.

[22] The double bootstrap, as the name implies, performs two bootstrap procedures on the sample observations. The first determines the bootstrap estimator, $T(\alpha^*)$. A set of expanded bootstrap samples is then derived, based on the original observations, $X$, but now augmented by information contained in $T(\alpha^*)$. A second bootstrap is then conducted with the augmented bootstrap sample. See Mehrotra et al. (1991).

[23] This procedure is identical to double bootstrapping, except the expanded bootstrap sample is augmented by information contained in the sample median.

[24] This is based upon a comparison of MSEs. Except for the logistic distribution, the difference between the MSE for the ATM and the MSE for the SQE is significant at the 1% level. The same conclusions apply to the difference between the MSEs for the SIM and the SQE. The statistical tests are constructed as follows. Let MSE1 and MSE2 denote the mean-squared errors of two estimators, say, ATM and SQE. The variance of (MSE1 − MSE2) is

$$\mathrm{Var}\left(\frac{\sum (u_i)^2}{n} - \frac{\sum (v_j)^2}{n}\right) = \mathrm{Var}\,\frac{(u^2 - v^2)}{n} = \frac{\{E(u^2 + v^4 - 2u^2v^2) - E^2(u^2) + 2E(u^2)E(v^2)\}}{n},$$

where $u_i$ and $v_i$ are, respectively, ATM and SQE estimates from the $i$th trial, $i = 1, 2, \ldots, 2000$; similarly, $u$ and $v$ denote the representative ATM and SQE, respectively. If the expectations are replaced by their sample analogs, a variance estimator is available. For distributions considered in the Monte Carlo study, the standard errors (i.e., the squared roots of the variances) are very small,

**TABLE II**

Monte Carlo Results (Scenario: Normal Distributions, No Intentionally Biased Reporting)

| *Distribution* | *Estimator* | *MSE* | *Bias* | *Variance* |
|---|---|---|---|---|
| | SIM | 0.16766 | −0.00321 | 0.16765 |
| | SQE | 0.18908 | −0.00248 | 0.18908 |
| Normal | MED | 0.21726 | −0.00472 | 0.21724 |
| | ATM | 0.18818 | −0.00174 | 0.18818 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.18892 | −0.00247 | 0.18892 |
| | SIM | 0.17010 | 0.01914 | 0.16973 |
| | SQE | 0.24975 | 0.02144 | 0.24929 |
| Uniform | MED | 0.33569 | 0.02250 | 0.33519 |
| | ATM | 0.17371 | 0.01993 | 0.17331 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.23287 | 0.02109 | 0.23243 |
| | SIM | 0.16516 | 0.01761 | 0.16485 |
| | SQE | 0.13476 | 0.01628 | 0.13449 |
| Double Exponential | MED | 0.13559 | 0.01456 | 0.13538 |
| | ATM | 0.16044 | 0.02369 | 0.15988 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.14183 | 0.01685 | 0.14155 |
| | SIM | 0.16580 | 0.01810 | 0.16547 |
| | SQE | 0.16452 | 0.01779 | 0.16420 |
| Logistic | MED | 0.18627 | 0.01678 | 0.18590 |
| | ATM | 0.16482 | 0.02046 | 0.16441 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.16518 | 0.01813 | 0.16485 |
| | SIM | 0.16775 | 0.01982 | 0.16736 |
| | SQE | 0.20710 | 0.02015 | 0.20670 |
| Triangular | MED | 0.24920 | 0.02005 | 0.24879 |
| | ATM | 0.16890 | 0.02057 | 0.16847 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.20241 | 0.02019 | 0.20200 |

Note: All distributions are standardized to have their variances equal to 1.

For each distribution and when contamination is introduced, the precision of the SIM stays constant. Likewise, although not constant, the precision of the bootstrap estimators remains essentially static. This is some indication of their robustness. Alternatively, for most of the other estimators precision diminishes as contamination is increased—as shown by a comparison of Tables III and IV.

With the exception of the double exponential distribution, the adaptive means are nearly always superior to the SQE, given the contamination scenarios considered in Tables III and IV. Overall, under rather di-

ranging from 0.0022 to 0.0031. The usual $z$ test (i.e., dividing the actual MSE difference by the standard error) is applied to test the equality between MSE1 and MSE2.

**TABLE III**

Monte Carlo Results (Scenario: Various Distributions, One Intentionally Biased Report with One Standard Deviation)

| *Distribution* | *Estimator* | *MSE* | *Bias* | *Variance* |
|---|---|---|---|---|
| | SIM | 0.19437 | 0.16346 | 0.16765 |
| | SQE | 0.21365 | 0.15084 | 0.19090 |
| Normal | MED | 0.24542 | 0.14469 | 0.22448 |
| | ATM | 0.19528 | 0.16398 | 0.16839 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.21329 | 0.15288 | 0.18991 |
| | SIM | 0.20426 | 0.18581 | 0.16973 |
| | SQE | 0.26605 | 0.17276 | 0.23621 |
| Uniform | MED | 0.35082 | 0.17745 | 0.31933 |
| | ATM | 0.20586 | 0.18740 | 0.17074 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.25021 | 0.17119 | 0.22090 |
| | SIM | 0.19881 | 0.18428 | 0.16485 |
| | SQE | 0.17402 | 0.16468 | 0.14690 |
| Double Exponential | MED | 0.17707 | 0.14650 | 0.15560 |
| | ATM | 0.19546 | 0.18694 | 0.16052 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.18224 | 0.17074 | 0.15309 |
| | SIM | 0.19970 | 0.18500 | 0.16547 |
| | SQE | 0.20118 | 0.16945 | 0.17247 |
| Logistic | MED | 0.22429 | 0.15850 | 0.19916 |
| | ATM | 0.19990 | 0.18712 | 0.16489 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.20314 | 0.17310 | 0.17317 |
| | SIM | 0.17508 | 0.08786 | 0.16736 |
| | SQE | 0.21468 | 0.08799 | 0.20694 |
| Triangular | MED | 0.25830 | 0.08559 | 0.25098 |
| | ATM | 0.17612 | 0.08921 | 0.16816 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.20976 | 0.08878 | 0.20188 |

Note: All distributions are standardized to have their variances equal to 1.

verse if not realistic conditions, the bootstrap estimators perform quite well compared to the SQE.[25]

As a final comment, with a RATS program (which contains a bootstrap command) on an IBM486 PC, it takes less than 2 minutes to derive the daily settlement index for the municipal bond contract. Thus, the computational cost of the proposed estimators should not be a limiting

[25]The results for the Eurodollar contract are somewhat similar to the case of the municipal bond contract. A comparison of the SQE to the ATM or the ATW shows the latter generates both a lower variance and MSE for all distributions but the double exponential. However, the bootstrap estimators are usually more biased than the SQE. For the uniform distribution with no contamination, the ATM is unambiguously better than the SQE with smaller variance, bias, and MSE. Although it is assumed that the exchange is primarily interested in the sample MSE of an estimator, the exchange may be concerned also with either the bias or variance and, therefore, may consider the trade-offs between the two.

**TABLE IV**

Monte Carlo Results (Scenario: Various Distributions, Two Intentionally Biased Reports; Each with One Standard Deviation)

| *Distribution* | *Estimator* | *MSE* | *Bias* | *Variance* |
|---|---|---|---|---|
| | SIM | 0.27663 | 0.33013 | 0.16765 |
| | SQE | 0.29472 | 0.32165 | 0.19126 |
| Normal | MED | 0.32704 | 0.31670 | 0.22675 |
| | ATM | 0.27965 | 0.33163 | 0.16968 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.29470 | 0.32330 | 0.19018 |
| | SIM | 0.29398 | 0.35248 | 0.16973 |
| | SQE | 0.34480 | 0.34469 | 0.22599 |
| Uniform | MED | 0.41251 | 0.34382 | 0.29430 |
| | ATM | 0.29829 | 0.35506 | 0.17222 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.33517 | 0.34498 | 0.21616 |
| | SIM | 0.28801 | 0.35094 | 0.16485 |
| | SQE | 0.26239 | 0.33843 | 0.14786 |
| Double Exponential | MED | 0.26885 | 0.31660 | 0.16861 |
| | ATM | 0.28641 | 0.35506 | 0.16034 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.27020 | 0.34571 | 0.15068 |
| | SIM | 0.28930 | 0.35189 | 0.16547 |
| | SQE | 0.28836 | 0.34218 | 0.17127 |
| Logistic | MED | 0.31030 | 0.32847 | 0.20240 |
| | ATM | 0.29110 | 0.35518 | 0.16494 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.29164 | 0.34675 | 0.17140 |
| | SIM | 0.19167 | 0.15591 | 0.16736 |
| | SQE | 0.23039 | 0.15736 | 0.20563 |
| Triangular | MED | 0.27223 | 0.15385 | 0.24856 |
| | ATM | 0.19388 | 0.15711 | 0.16919 |
| | $W(\frac{1}{6}, \frac{1}{6}, 6)$ | 0.22613 | 0.15853 | 0.20099 |

Note: All distributions are standardized to have their variances equal to 1.

factor. Nevertheless, as traders currently lack full understanding and appreciation of the new method, they may have less confidence in the settlement procedure. Consequently, the trading volume may decline. As usual, the efforts exerted by the exchange(s) in educating traders, and the general public, will greatly benefit the implementation of the new method.

## CONCLUSION

Based on the results of the Monte Carlo study, it does appear that the overall accuracy of the settlement provisions (or index) of the municipal bond contract can be improved upon. In some sense the bootstrap approach (viz., the ATM estimator) is simply an extension of the current procedure in that it allows the exchange to continue the practice of trimming the extreme observations. But bootstrapping enables the amount of trimming to be variable and, most importantly, optimally determined. The SIM also appears, somewhat surprisingly, as a viable alternative to the SQE. Excessive contamination, for example, an amount of drift equal to two standard deviations or more, might render the SIM less accurate than the adaptive means. With the safeguards currently employed, it may be unreasonable to suggest the possibility of exorbitant distortions in reality. But in the absence of rigorously verifying cash price reports, dishonest observations most likely go undetected if they are not also extreme observations. For the contamination scenarios investigated, the contaminated observations do not always end up on the extreme. It appears the adaptive means better accommodate—respond to—contamination of interior observations. Under fairly general conditions the bootstrap estimators, based on trimming or Winsorization, appear to be more accurate estimators than the SQE estimator.

## APPENDIX

This outline of the bootstrap procedure is based on Efron's Monte Carlo algorithm.

1. Fit the nonparametric MLE of $F$,

$$F\text{: mass } (1/n) \text{ at } X_i, \qquad I = 1,2, \ldots ,n$$

(The bootstrap distribution puts mass $1/n$ on each of the fixed data points.)

2. Draw $N$ independent samples:

$$Y_1 = (Y_{1,1}, Y_{1,2}, \ldots , Y_{1,n}), \ldots , Y_N = (Y_{N,1}, Y_{N,2}, \ldots , Y_{N,n})$$

of size $n$ from $X$, equivalently $F^0$. Each of the $N$ independent samples is referred to as a bootstrap sample and each is taken with replacement. The size of each bootstrap sample, $n$, is equal to the size of the sample drawn from the original data. Swanepoel (1986) argues the optimal bootstrap sample size $m$ (in terms of minimizing the MSE of the bootstrap estimator) may be smaller than $n$. The selection of n follows from institutional considerations.

3. For each $\alpha \in \Lambda$ calculate the trimmed means $T\,(\alpha,\alpha,;Y_i)$ for each $i$, $0 \leq i \leq N$;

$$\bar{V}(\alpha) = \frac{1}{(N-1)} \sum_{i=1}^{N} (T(\alpha,\alpha,;Y_i) - \bar{T}(\alpha))^2$$

This result is the bootstrap variance of the trimmed mean whose proportion of trimming is $\alpha$. The average of the trimmed mean over the bootstrap sample is denoted $\bar{T}(\alpha)$.

4. The proportion of trimming associated with the minimal $V(\alpha)$, which is denoted as $2\alpha^*$, indicates the optimal proportion of trimming. (The bootstrap method could be used to select that member of the family of trimmed means with minimum MSE or bias. However, this determination would require knowledge of the population mean, which of course is assumed unknown.) The estimate for the center of symmetry is given by $T^B(\alpha^*,\alpha^*,X)$, referred to as the bootstrap adaptive trimmed mean.

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