STOCHASTIC INTEREST RATES, TRANSACTION COSTS, AND IMMUNIZING FOREIGN CURRENCY RISK

RAYMOND CHIANG
JOHN OKUNEV
MARK TIPPETT

INTRODUCTION

Since the advent of the deregulation of the foreign currency markets in 1973 the volatility of exchange rates has increased substantially. For example, the volatility of exchange rates between the U.S. and, the U.K., Germany, and Japan has generally increased by at least a factor of 6 from 1973 to 1990 compared with the volatility of exchange rates in the period, 1950–1973 (Levi, 1990). As a consequence of this increased volatility in exchange rates, it is important to develop hedging strategies to reduce the risk of foreign currency exposure. There are various ways to hedge this currency risk. A common approach is to provide a floor by purchasing a put option. This approach provides a safety net with upside potentials.

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■ *Raymond Chiang is a Professor of Finance at the Chinese University of Hong Kong.*

■ *John Okunev is Manager, Investment Technology, for Lend Lease Investment Management, Sydney.*

■ *Mark Tippett is a Professor of Accounting and Finance at the University of Wales, United Kingdom.*

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Exchange-traded foreign currency options include the British pound, French franc, German mark, Swiss franc, Canadian dollar, Japanese yen, and Australian dollar. If, however, an investor wishes to trade outside of these major currencies, the investor needs to purchase an option on the foreign currency from a major bank or financial institution. Since options are not traded freely, the premiums attached to these options may be more than the fair price.[1] Alternatively, the investor could purchase a cross-currency option. The difficulty with this strategy is that movements in the foreign exchange rate may not move in unison with any of the exchange rates trading in options.[2]

In this article an alternative approach for immunizing foreign currency exposure is proposed where foreign currency options traded on exchanges are not available. This is important because the nonmajor currency economies, such as those of Eastern Europe, Latin and South America, and the Pacific Rim economies, are becoming increasingly important. This is achieved by creating a synthetic put that replicates the payoffs of an actual put. This approach is not new. It is applied widely in stock portfolio insurance strategies. However, this approach differs from the traditional portfolio insurance strategies in that, it is self-financing. In other words, the investor does not have to reduce the original capital by the purchase of the put option. The strategy involves the creation of a synthetic put, where the hedge ratio specifies the appropriate combination of foreign and domestic bonds.

Because the hedge ratio is dependent on the option's time to maturity, the hedging strategy must involve continuous rebalancing; thus, transaction costs become a significant factor. Following the continuous-time framework of Leland (1985), a hedging strategy (of replicating a long put) that makes allowance for transaction costs under stochastic interest rates is developed. Not surprisingly, the accuracy of the hedging procedure hinges critically on both the revision period and the magnitude of the relevant transaction costs. Although Leland assumes constant interest rates, the proposed hedging strategy assumes stochastic interest rates. The incorporation of interest-rate risk is especially important for currency options because the domestic and the foreign interest rates are most prominent in the determination of exchange rates. Furthermore, interest rates in less developed financial markets may be more volatile than in well-developed financial markets.

[1]The bid–ask spread may be substantially higher than on those traded on an organized exchange. Discussions with financial institutions confirm that the premium on a nonmajor currency option varies a great deal among investment houses. They often cover themselves with wide bid–ask spreads.
[2]See Rumsey (1991) for a discussion on cross currency options.

The format of the article follows. The next section two describes a self-financing hedging strategy utilizing a synthetic put. Because the strategy is composed of the spot exchange rate, a domestic bond and a foreign bond, it can be implemented easily for the nonmajor currencies where there are no exchange-traded options. Then the Leland model is extended with a description of the adjustments necessary for the inclusion of transaction costs under stochastic interest rates. The immunization strategy can be extended into a discrete-time framework as well. The article provides some empirical results by way of simulations, and ends with a brief summary.

IMMUNIZATION STRATEGY

By way of example, consider the case where an investor wishes to invest in overseas bonds. Three strategies are open to the investor. The investor can invest in overseas bonds and at maturity convert the foreign capital at the spot exchange rate. The trouble with this strategy is that if the foreign currency weakens against the domestic currency, it is possible to lose a significant portion of the initial capital. On the other hand, if the foreign currency strengthens relative to the domestic currency, the investor obtains favorable gains. This strategy is therefore risky, because the returns are dependent upon the movements of the exchange rates. An alternative is to acquire a futures contract on the foreign currency. This strategy locks in the exchange rate at maturity of the bond and hence no gains or losses are experienced on the foreign currency.[3] This strategy is unsuitable because at interest-rate parity, there is no advantage in investing in overseas markets, as futures prices reflect the interest-rate differential. A third alternative is to purchase a put option on the foreign currency. The advantage of this approach is that if the exchange rate falls below the exercise price, then the put option can be exercised, and hence, the investor maintains a floor on the investment. If, on the other hand, the foreign currency strengthens against the domestic currency, the put option is worthless, but this is counterbalanced by the increase in value of the foreign capital converted at the favorable exchange rate.

If the currency option is traded on an exchange, there is a liquid and active market. Currently, only the major currencies have options traded on the exchanges. For the nonmajor currencies, such options must be purchased from major banks or financial institutions. Because there is less competition, less liquidity, and less depth than the exchange-traded

[3]Basis risk will exist if the hedging horizon is different from the delivery date of the futures contract. In addition, there may exist a degree of interest-rate risk from marking to market.

options, the trading costs and premiums will likely be higher. A vehicle for the investor to circumvent this is to create a synthetic put via the more active domestic and foreign bond markets. To facilitate the discussion the following terminology is adopted:

$S(t)$	the spot domestic currency price at time t of a unit of foreign exchange.
$\tau = T - t$	time to the option's expiration, T being the calendar date of the option's maturity.
$P_F(t)$	the foreign currency price at time t of a foreign pure discount bond maturing at time T.
$P_D(t)$	the domestic currency price at time t of a domestic pure discount bond maturing at time T.
K	the exercise price of the (put or call) option.
$C(S, P_D, P_F, K, \tau)$	the domestic currency price of a European call option with time τ to maturity.
$W(S, P_D, P_F, K, \tau)$	the domestic currency price of a European put option with time τ to maturity.

Suppose initially that an investor purchases foreign bonds and purchases a covering put on the foreign currency. The total cost of the portfolio is[4]

$$P_FS + W = P_FSN(d_1) + P_DKN(-d_2) \tag{1}$$

where $W = W(S, P_D, P_F, K, \tau)$ (the functional arguments are dropped for convenience). The principal problem with the above approach is that if the total capital amounts to P_FS, then it is necessary to borrow funds to purchase the put option. This problem can be overcome by selling a portion of the portfolio given by eq. (1) and investing the proceeds in domestic bonds. This has the effect of making the strategy self-financing, as it is not necessary to borrow funds to purchase the put option. The proportion of the portfolio retained is given by g. Because the package must be self-financing, it follows that

$$P_FS = g(P_FS + W) + DP_D \tag{2}$$

where D is the number of additional domestic bonds purchased. Assume the investor wants to realize a minimum the amount K at time T, then at maturity

[4]The value of the put is given by $W = KP_DN(-d_2) - SP_FN(-d_1)$, where $N(d_1)$ and $N(d_2)$ are cumulative probability density functions and are defined in eq. (8). The composition of these terms varies depending upon the option pricing model used.

$$gK + D = K \tag{3}$$

Solving eqs. (2) and (3) gives

$$g = \frac{P_F S - KP_D}{P_F S + W - KP_D} \tag{4}$$

If g is to be nonnegative, it is obvious from the above equation that $P_F S \geq KP_D$. When this condition is violated, the insurance strategy breaks down. That will imply that even if all funds are invested in domestic bonds, they will not accumulate to K at the completion of the insurance period.[5] Satisfying this condition requires that all proceeds be invested in domestic bonds as well as additional foreign bonds, which need to be sold short.

Following the put-call parity theorem, eq. (4) simplifies to $g = (C - W)/C$ Utilizing this result, eq. (2) can be rearranged as

$$P_F S = \left[N(d_1)\left(\frac{C - W}{C}\right)\right] P_F S + \left[1 - N(d_1)\left(\frac{C - W}{C}\right)\right] P_D \tag{5}$$

Equation (5) states the exact composition of the self-financing synthetic put using foreign and domestic bonds. As long as g is positive (or $C > W$), implementing the replication strategy is straightforward because there is no short selling involved. Because the replication involves only the purchase of domestic and foreign bonds, the strategy is easily doable for any nonmajor currencies.[6] Note that the proportion of investment devoted to the purchase of foreign bonds is

$$g\, N(d_1) = N(d_1)\left(\frac{C - W}{C}\right) \tag{6}$$

In the portfolio insurance context the traditional hedge ratio is given by $N(d_1)$. In the case of self-financing without having to finance the purchase of the put from incremental investments, the hedge ratio is given by eq. (6). Correspondingly, the investment in domestic bonds is given by

$$1 - N(d_1)\left(\frac{C - W}{C}\right) \tag{7}$$

[5]In essence, when $P_F S < KP_D$, the floor is set too high. To accomplish that objective, one needs to invest all the funds plus the funds from shorting the foreign bond in the domestic bond.

[6]In the nonmajor currencies, even if the put can be purchased from an institution, the premium is likely to be large because of the lack of an organized market. The proposed procedure provides a likely cheaper alternative, as bond markets tend to be more active with lower transaction costs.

TRANSACTION COSTS

It is apparent from the discussion in the previous section 2 that the immunization strategy is critically dependent upon the hedge ratio. This requires choosing a foreign currency option pricing model to model the hedge ratio. Option pricing models for European foreign currency options have been developed for the nonstochastic interest rate case by Biger and Hull (1983) and Garman and Kohlhagen (1983). Grabbe (1983), Hilliard, Madura, and Tucker (1991), and Chiang and Okunev (1993) have examined the case of stochastic interest rates. The Grabbe model assumes that the return-generating process of the foreign bond, the spot exchange rate, and the domestic bonds are driven by geometric Brownian motions. These assumptions can be criticized on the grounds that the bond return dynamics of a geometric Brownian motion does not preclude the possibility of negative interest rates. Also, the variance of the bond at maturity is not zero. Hilliard et al. (1991) models interest rates as an additive Brownian motion. Their model also does not preclude the possibility of negative interest rates. Chiang and Okunev (1993) model the bond dynamics of bonds as a Brownian bridge. Their model provides that the variance of the bond is zero when the bond is purchased and at maturity. However, that model still suffers from the possibility of permitting negative interest rates. Empirically, the assumption that the exchange-rate dynamics are geometric has been criticized by Tucker and Scott (1988), Tucker and Pond (1988), Jorion (1988), Giovannini and Jorion (1989), Hsieh (1989) and Melino and Turnbull (1991). Relaxing the assumption of lognormality and replacing the return-generating process with one that describes the behavior of exchange rates in a more realistic manner requires the use of numerical methods to value the option.[7] It is apparent that each of the models has its own difficulties. In this article the Grabbe model is selected because it provides closed-form solutions, and is preferable to the Biger and Hull (1983) and Garman and Kohlhagen (1983) models because it permits the possibility of stochastic interest rates. This is an important feature, as it is likely that the interest-rate regimes in less well-developed financial markets may be more volatile.

The value of a European put option in the Grabbe case is given by

$$W(S,P_D,P_F,\tau) = P_D K N(-d_2) - P_F S N(-d_1) \tag{8}$$

where

[7]Jump diffusion models of exchange rates have been examined by Jorion (1988) and Abraham and Taylor (1993).

$$d_1 = \frac{\log \dfrac{P_F S}{P_D K} + \dfrac{1}{2}(\sigma_F^2 + \sigma_S^2 + \sigma_D^2 - 2\rho_{SD}\sigma_S\sigma_D + 2\rho_{SF}\sigma_S\sigma_F - 2\rho_{FD}\sigma_F\sigma_D)\,\tau}{\sqrt{(\sigma_F^2 + \sigma_S^2 + \sigma_D^2 - 2\rho_{SD}\sigma_S\sigma_D + 2\rho_{SF}\sigma_S\sigma_F - 2\rho_{FD}\sigma_F\sigma_D)\,\tau}}$$

and

$$d_2 = d_1 - \sqrt{(\sigma_F^2 + \sigma_S^2 + \sigma_D^2 - 2\rho_{SD}\sigma_S\sigma_D + 2\rho_{SF}\sigma_S\sigma_F - 2\rho_{FD}\sigma_F\sigma_D)\,\tau}$$

where

σ_S^2 is the variance of the return of the exchange rate
σ_F^2 is the variance of the foreign bond return
σ_D^2 is the variance of the domestic bond return
ρ_{ij} are the respective covariance terms.

Adjustments can be made to the Grabbe model to accommodate interest rates being generated by either an additive Brownian (Merton, 1973) or Ornstein–Uhlenbeck (Vasicek, 1977) processes.[8] If interest rates can be described by an Additive brownian motion such as in Hillard et al. (1991), then $\sigma_B = \sigma_r\tau$, where σ_B is the standard deviation of the return on the bond and σ_r is the standard deviation of changes in interest rates. If, on the other hand, interest rates can be described by an Ornstein–Uhlenbeck process, then $\sigma_B^2 = \sigma_r^2[1 - e^{-2\beta\tau}]/2\beta$ (where β is the speed of adjustment). Empirical evidence suggests that β is not significantly different from zero. For small values of β the two expressions are essentially the same. Chan et al. (1992) have shown that the Vasicek and Merton models perform the worst of the seven interest models examined and can be rejected on the grounds of the assumption of constant volatility. The Grabbe model does not restrict the interest-rate process, but does encompass both the Vasicek and Merton models as well as other processes.

A not insignificant problem with the hedging strategy outlined previously 2 relates to the transactions costs associated with frequent rebalancing. Unless such costs are taken into account, there is the chance that even though the correct rebalancing strategy is followed, the terminal value of the firm's foreign currency portfolio will be eroded by transactions costs. Using a continuous-time framework, Leland (1985) developed a model that allows for discrete-time revision with the inclusion of transaction costs. In the Leland formulation adjustments are made

[8]Chan et al. (1992) have shown that the model that best describes movements in interest rates is characterized by the variance of changes in interest rates being dependent upon the level of the interest rate. Unfortunately, closed-form solutions of option pricing models do not exist to permit this characteristic.

through the variance term of the Black–Scholes formula to generate the upper and lower bound for the option. Boyle and Vorst (1992) embed the transaction costs problem in a true discrete-time framework with the Cox, Ross, and Rubinstein (1979) binomial model.[9] However, both models consider discrete-time revision only within the constant interest-rate framework. The Leland model is adopted and extended to include stochastic interest rates. As mentioned above, this is an important contribution, as interest rates tend to be more volatile in less-developed financial markets/countries.

Let k represent the round-trip transaction cost, measured as a fraction of the volume of transactions, and let Δt be the rebalancing interval. If the Leland analysis is amended to allow for stochastic interest rates, the variance and covariance terms for the synthetic long put position or the upper bound are defined as

$$\hat{\sigma}_S^2 = \sigma_S^2\left[1 + k\sqrt{\frac{2}{\pi(\sigma_S^2 + \sigma_F^2 + 2\rho_{SF}\sigma_S\sigma_F)\,\Delta t}}\right]$$

$$\hat{\sigma}_F^2 = \sigma_F^2\left[1 + k\sqrt{\frac{2}{\pi(\sigma_S^2 + \sigma_F^2 + 2\rho_{SF}\sigma_S\sigma_F)\,\Delta t}}\right]$$

$$\rho_{SF}\hat{\sigma}_S\sigma_F = \rho_{SF}\sigma_S\sigma_F\left[1 + k\sqrt{\frac{2}{\pi(\sigma_S^2 + \sigma_F^2 + 2\rho_{SF}\sigma_S\sigma_F)\,\Delta t}}\right]$$

$$\rho_{SD}\hat{\sigma}_S\sigma_D = \rho_{SD}\sigma_S\sigma_D\left[1 + \frac{k(\sqrt{1 - \rho_{GD}^2} + \rho_{GD}\sin^{-1}\rho_{GD})}{\rho_{GD}\pi}\right]$$

$$\rho_{FD}\hat{\sigma}_F\sigma_D = \rho_{FD}\sigma_F\sigma_D\left[1 + \frac{k(\sqrt{1 - \rho_{GD}^2} + \rho_{GD}\sin^{-1}\rho_{GD})}{\rho_{GD}\pi}\right] \tag{9}$$

The mathematical workings of these adjustments can be found in Appendix 1. To derive the lower bound (the synthetic short position), a slight adjustment is made to the variance and covariance terms in eq. (9). A long synthetic position is transacted at the upper bound and short positions at the lower bound.

It is of interest to note the difference between the Leland (1985) adjustment and the adjustments presented above. The first two adjustments of eq. (9) correspond with the Leland adjustments of adjusting the

[9] In a similar vein (but different tangent), Hull and White (1987) examine ways in which banks and other financial institutions can cover their risk of writing nonexchange-traded foreign currency options with the use of exchange-traded options of the same currency. Key factors affecting the delta hedge are also analyzed.

variance of the stock return. The last three terms of eq. (9) represent the adjustments for the covariance between the spot exchange rate and the foreign and domestic bonds, and the covariance between foreign and domestic bonds. These terms are absent from the Leland formulation, as there are no covariance terms in the Leland model.[10]

EMPIRICAL EVIDENCE

Testing the effectiveness of the hedging strategy requires daily prices of the foreign and domestic bonds together with the spot exchange rate over an extended period of time. Unfortunately, these data is difficult to obtain for emerging markets and are also difficult to obtain for well-established markets for an extended period. As an alternative, the hedging strategy is tested by way of simulations. The disadvantage of this is that the results of the simulations are based upon the statistical properties of the underlying distributions, and may not fully capture various shocks to foreign denominated bonds, especially in emerging markets.

To illustrate how the above procedures may be applied, consider the case where a U.S. investor is considering investing in foreign or domestic bonds. Suppose the investor wishes to invest \$100,000 for a period of 91 days. He could initially invest all of the funds in foreign T-bills and at maturity convert back to U.S. dollars. However, this strategy exposes the investor to foreign currency risk. If the U.S. dollar moves against the foreign currency (U.S. dollar strengthens), then at maturity the investor may have less than his original capital. Alternatively, the investor could invest all of the funds in U.S. T-bills. There is no risk involved in this strategy. However, by adopting this approach the possibility of a favorable exchange rate movement of the foreign currency against the U.S. dollar is foregone. By implementing the immunization strategy outlined above a floor is placed on the investment, but the benefits from favorable exchange rate movements are allowed.

The immunization strategy is demonstrated for investments in Australian and U.S. T-bills. Although options are traded on the U.S.–Australian exchange rate, this currency is selected because over the past 15 years Australian interest rates have been higher than U.S. interest rates, and hence, may have encouraged U.S. investors to invest in Australian bonds.[11] The data consist of daily U.S. and AUS T-bill prices and the spot exchange rates for a 91-day period for a period where Australian

[10]The constant interest rate case is embedded within this study's model and will be reflected in the variance–covariance terms, which will be small relative to the variance of the exchange rate.
[11]Daily bond prices on nonmajor countries could not be obtained.

interest rates were considerably higher than U.S. rates. It is assumed that a high interest-rate differential encouraged domestic investors to invest in Australian bonds as opposed to U.S. bonds. The data is for the period, July to August, 1986. To implement the strategy the variance–covariance matrix of eq. (8) needs to be estimated. This matrix contains the variance (σ_S^2) of the Australian dollar–U.S. dollar exchange rate return, the variance (σ_F^2) of the return on 91-day Australian government (pure discount) treasury bills, the variance (σ_D^2) of the return on 91-day U.S. government (pure discount) T-bills, and the associated covariances between each of these variables. Hence, ρ_{FD} is the correlation between the U.S. dollar return on U.S. T-bills and the Australian dollar return on Australian government T-bills. Similar meanings can be attached to ρ_{FS} and ρ_{SD}. Each parameter is estimated over a 3-month period. The estimated variance-covariance matrix of the annualized instantaneous returns are as follows:

$$\begin{bmatrix} \sigma_S^2 & \sigma_{SF} & \sigma_{SD} \\ \sigma_{SF} & \sigma_F^2 & \sigma_{FD} \\ \sigma_{SD} & \sigma_{FD} & \sigma_D^2 \end{bmatrix} = \begin{bmatrix} 0.05329 & 0.00000194 & -0.000000788 \\ 0.00000194 & 0.0000217 & 0.00000106 \\ -0.000000788 & 0.00000106 & 0.000000818 \end{bmatrix}$$

From the above matrix, it follows that

$$(\sigma_S^2 + \sigma_F^2 + \sigma_D^2 - 2\rho_{SD}\sigma_S\sigma_D + 2\rho_{SF}\sigma_S\sigma_F - 2\rho_{FD}\sigma_F\sigma_D) = 0.053315$$

In this case the variance of the exchange rate returns seem to be the dominant factor. However, this may not be the case for a country with volatile interest rates. At the beginning of the investment period, assume \$100,000 is available for investment in 91-day U.S. and Australian T-bills. Furthermore, assume that the exercise price is \$100,000. The yield on the U.S. T-bill is 6.1%, whereas the yield on the Australian T-bill is 13.1%. Table I outlines the computational procedures necessary to split the original principal between U.S. and Australian T-bills. Note that initially these proportions amount to 84.2% in U.S. bills and 15.8% in Australian bills. However, the passage of time and variations in the exchange rate will cause these proportions to vary when a floor of \$100,000 is set on the terminal value of the portfolio. In this case the U.S. dollar strengthened, and consequently, the Australian investment is shorted. The realized terminal value the bill portfolio is \$101,240.

If the \$100,000 is invested entirely in domestic bonds, then at the end of this period it will accumulate to 100,000/0.9852 = \$101,502.

TABLE I

Proportionate Investment Over Time

Days to Maturity	*Beginning Value*	*Spot Rate*	*AUS Bond Price*	*U.S. Bond Price*	*Proportion AUS*	*Proportion U.S.*	*Ending Value*[a]
91	100,000	0.655	0.9697	0.9852	0.158	0.842	99,791
84	99,883	0.622	0.9706	0.9863	0.0	1.0	99,903
77	100,015	0.642	0.9723	0.9879	0.0	1.0	100,045
70	100,116	0.633	0.9745	0.9889	0.0	1.0	100,136
63	100,228	0.608	0.9746	0.99	0.0	1.0	100,237
56	100,329	0.617	0.9749	0.991	0.0	1.0	100,349
49	100,440	0.61	0.9789	0.9921	0.0	1.0	100,460
42	100,551	0.622	0.9823	0.9932	0.0	1.0	100,572
35	100,673	0.606	0.9842	0.9944	0.0	1.0	100,673
28	100,774	0.608	0.9867	0.9954	0.0	1.0	100,795
21	100,886	0.61	0.9902	0.9965	0.0	1.0	100,906
14	101,007	0.621	0.9934	0.9977	0.0	1.0	101,027
7	101,116	0.631	0.9967	0.9988	0.0	1.0	101,139
0	101,220	0.625	1.0	1.0	0.0	1.0	101,240

[a]Rebalanced on a daily basis, but reported weekly.

Finally, if the $100,000 is invested entirely in Australian T-bills, it will accumulate to (100,000*0.622)/(0.655*0.9697) = $97,929 at the end of the investment period. The risk taken with this uninsured strategy, is that if the Australian dollar weakens against the U.S. dollar (as is the case here), there is no downside protection. As a consequence, the terminal value of the bill portfolio is below the desired $100,000. The proposed technique applies to any bond and foreign currency market. In the next section the immunization strategy is tested in varying market conditions by way of simulation.

Simulated returns are generated for foreign and domestic bonds and the exchange rate using the Box–Muller procedure. Specifically, let r_1 and r_2 be two independent rectangular random variables defined on the interval $[0,1]$; then the random variable X is normally distributed with mean zero and unit variance, where X is defined as:

$$X = \mathrm{Cos}\,(2\pi r_2)\,\sqrt{-2\log r_1} \qquad (10)$$

It therefore follows that $Z = \mu + X\sigma$ is normally distributed with mean μ and standard deviation σ. Finally, continuously compounded returns are created by the following transformation, $Y = e^Z - 1$. Estimates of μ and σ can be obtained from sample estimates by the following expression:

TABLE II

Results of Simulations with Transaction Costs set at 1% and with a Strike of 100,000

Strategy	*Mean*	*Minimum*	*Maximum*	*Standard Deviation*	*Skewness*	*Kurtosis*	*% of Floor Violations*
Insured	100,809	93,325	127,700	3,590	2.30	7.502	39.1%
Uninsured	96,108	72,112	132,538	9,387	0.26	0.086	61.5%

$$\mu = \log \frac{(1 + M)^2}{\sqrt{(1 + M)^2 + S^2}} \tag{11}$$

and

$$\sigma^2 = \log \frac{(1 + M)^2 + S^2}{(1 + M)^2} \tag{12}$$

where M is the sample mean and S^2 is the sample variance of returns in discrete time.

With the use of simulated prices for foreign and domestic bonds and exchange rates, the immunization strategy is simulated 2,000 times. The strike price is set at \$100,000, and rebalancing is done on a daily basis. Table II presents summary results of the ending value of the portfolio for the case with transaction costs set at 1%. Also displayed are the mean, standard deviation, maximum, minimum, skewness, kurtosis, and the percentage of times the strike is violated. The lower bound is violated because at certain points in the simulation the call value is worth less than the put value. This necessitates shorting the foreign position, and in some cases it is necessary to borrow additional foreign funds to invest in domestic bonds. As a matter of practicality, borrowing additional foreign funds is not permitted, and consequently the portfolio is no longer balanced. This situation usually occurs when the U.S. exchange rate strengthens considerably against the Australian dollar. Figures 1 and 2 display how the insured strategy is less volatile and possesses a higher lower bound when compared to the uninsured strategy.

Using eq. (9) to adjust the various variance-covariance terms has the effect of increasing the variance term in d_1 and d_2. This makes both the call and the put worth more, and it is difficult to say what effect this will have on the strategy without some empirical testing. As expected, the mean return is generally lower with the inclusion of transaction costs.

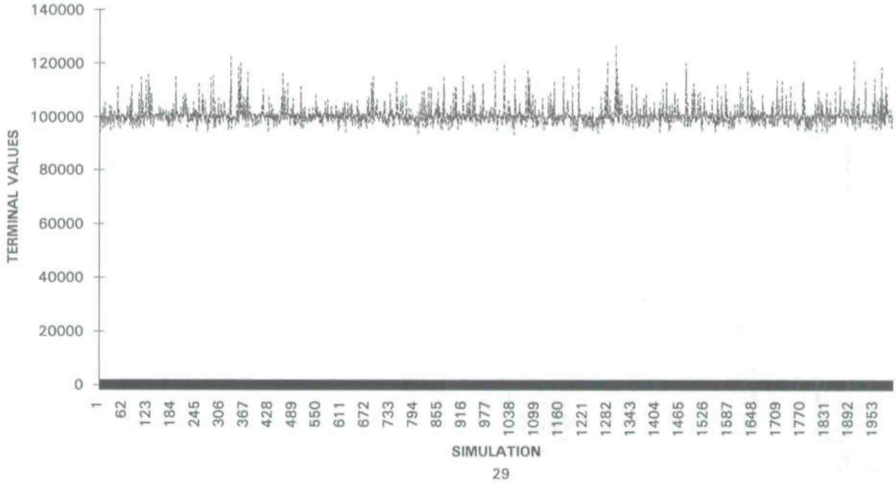

FIGURE 1
Terminal values of insured strategy with strike 100,000.

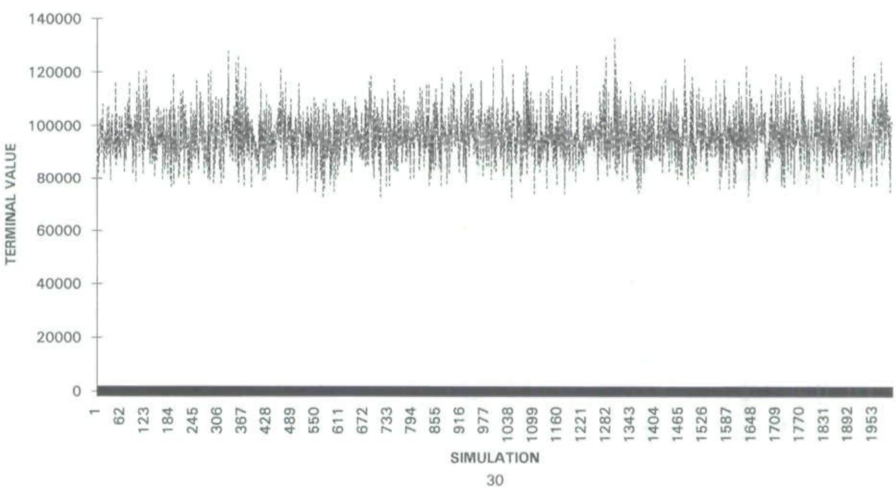

FIGURE 2
Terminal values of uninsured strategy.

However, this reduction does not appear to be substantial, as the mean terminal value with no transaction costs is \$100,939. The mean value with transaction costs is \$100,809, which is marginally below the terminal value of investing solely in U.S. T-bills. If one only invests in U.S. T-bills, one forgoes the opportunity of favorable exchange rate movements. The maximum realization is \$127,700, and the minimum terminal realization is \$93,325. The percentage of floor violations is 39.1%. How-

TABLE III

Results of Simulations with Transaction Costs Set at 1% and with Varying Strikes

Strike	*Mean*	*Minimum*	*Maximum*	*Standard Deviation*	*Skewness*	*Kurtosis*	*Percent of Floor Violations Set at Strike*	*Percent of violations below 95,000*
95,000	100,144	91,849	132,080	5,420	1.71	3.50	7.9%	7.9%
96,000	100,216	92,071	136,935	4,986	1.91	5.05	12.3%	4.9%
97,000	100,450	92,279	144.803	4,690	2.14	6.59	16.1%	3.0%
98,000	100,505	92,867	126,867	4,275	2.23	6.84	23.1%	1.0%
99,000	100,732	93,808	127,433	4,011	2.27	6.45	31.3%	1%
100,000	100,862	93,325	127,700	3,590	2.30	7.50	39.1%	0.6%
101,000	100,944	93,912	122,436	3,318	2.31	7.12	55.4%	0.3%
102,000	101,031	93,635	125,006	3,053	2.37	9.59	85.8%	0.1%
103,000	101,073	93,938	121,600	2,800	2.54	12.17	90.9%	0.1%
104,000	101,206	94,484	122,781	2,529	2.96	12.92	93.1%	0.01%
105,000	101,232	94,960	120,387	2,302	2.60	13.90	95.5%	0.01%

ever, if one is willing to accept a loss of, say, 5%, then the percentage of terminal realizations below $95,000 is only 0.6%. Table II also provides ending portfolio values when one invests solely in Australian T-bills. The average ending value for all simulations is $96,108. Note that the variance of the uninsured strategy simulations is greater than the variance of the insured strategy. The standard deviation of the uninsured strategy is $9,387, whereas the standard deviation of the insured strategy is $3,590, which means that the risk, while using the uninsured strategy, is 2.5 greater than when the insured strategy is used. The percentage of floor violations is 61.5%, with minimum and maximum values of $72,112 and $132,538. This clearly supports the view that the uninsured strategy is considerably more risky.

Table III presents the results of the simulations with varying strike prices. This examines the issue of whether it is beneficial to lower or raise the strike rather than setting the strike price at the initial investment. The general pattern is that lowering the strike results in a lowering of the mean with a corresponding increase in the standard deviation. Similarly, lowering of the strike results in a lowering of the minimum terminal value and an increasing maximum value. If one is more interested in obtaining large gains, but at the same time placing some restriction on lower returns, then it would be more appropriate to adopt a low exercise price strategy. Alternatively, if one does not want to accept a loss of more than 5%, then an exercise price of at least 100,000 should be set. This is because with a strike of at least 100,000, the terminal realizations below 95,000 are less than 1%. In fact, the level of the strike price can be regarded as an insurance level at which the hedger rims. Simulations are conducted with transaction costs varying from 0.25%, 0.5%, and 0.75%. The terminal values of these portfolios are a decreasing function of the terminal values with no transaction costs and transaction costs set at 1%. These results are not reported here but are available from the authors.

Clearly an exhaustive empirical study is required to evaluate the efficiency of the proposed techniques. Issues that need to be addressed include estimation problems relating to parameter estimates. Clearly these estimates will affect the hedge ratio and consequently the terminal value. Other considerations include the determination of an optimal balancing trigger, the desirability of locking in on accumulated gains, and the optimal period over which the immunization should occur. These decision variables should be evaluated in the context of their implementation costs, failure to satisfy the prescribed floor conditions, and path dependency.

CONCLUSION

This article proposes an alternative approach for immunizing foreign currency risk where options are not traded on the exchanges. A synthetic put is created that replicates the payoffs of an actual put. The approach differs from traditional portfolio insurance in that the technique is self-financing and includes transaction costs under stochastic interest rates.

Because the hedge involves revision, transactions costs become an important concern. In addition, interest-rate uncertainty is a prominent factor in currency risk as well. Leland's (1985) approach is extended to incorporate stochastic interest rates into the pricing of currency options with transactions costs. The bounds of the option price are derived by adjusting the variance and covariance terms of the Black–Scholes-type formula. The adjustments are slightly more involved than Leland's because of the interest-rate uncertainty. Furthermore, the approach is generic enough to the problem of immunizing a foreign equity portfolio. Future extensions of the study could examine an optimal balancing trigger and an optimal immunization period.

APPENDIX 1

Suppose E foreign pure discount bonds and N domestic pure discount bonds are purchased. Then, the value of this portfolio amounts to

$$J = EG + NP_D \tag{13}$$

where $G = P_FS$. Over the short interval $[t, t + \Delta t]$, the change in the value of the portfolio will be

$$\Delta J = E\,\Delta G + N\,\Delta P_D \tag{14}$$

$$\Delta J = EG\frac{\Delta G}{G} + NP_D\frac{\Delta P_D}{P_D} \tag{15}$$

If the change in option value over this interval is denoted as $\Delta C(G,P_D,t)$, then the difference, ΔH, between the change in the value of the portfolio and the change in the value of the option or the hedging error is given by

$$\Delta H = \Delta J - \Delta C \tag{16}$$

If one expands $C(G,P_D,t)$ as a Taylor series of order 2, noting that the option value satisfies Merton's (1973, p. 165) fundamental equation of value, and sets

$$E = \frac{\partial C}{\partial G} \quad \text{and} \quad N = \frac{\partial C}{\partial P_D}$$

one obtains [Apostol (1969, pp. 308–309)]:

$$\begin{aligned} \Delta H = {} & \frac{1}{2}\,G^2\,\frac{\partial^2 C}{\partial G^2}\left[\sigma_G^2\,\Delta T - \left(\frac{\Delta G}{G}\right)^2\right] + \frac{1}{2}\,P_D^2\,\frac{\partial^2 C}{\partial P_D^2} \\ & \left[\sigma_{P_D}^2\,\Delta t - \left(\frac{\Delta P_D}{P_D}\right)^2\right] + GP_D\,\frac{\partial^2 C}{\partial G\,\partial P_D} \\ & \left[\rho_{G,P_D}\sigma_G\sigma_{P_D}\,\Delta t - \left(\frac{\Delta G}{G}\,\frac{\Delta P_D}{P_D}\right)\right] + 0[(\Delta t)^{3/2}] \end{aligned} \tag{17}$$

where the σ and ρ measure the variance rate and the correlation, respectively, in the relevant variables. If one takes expectations across the above result and ignores the higher-order terms, one obtains

$$E(\Delta H) = 0[(\Delta T)^2] \tag{18}$$

so that in the absence of transactions costs, the expected hedging error is approximately 0.

To incorporate transactions costs the Leland (1985) analysis is amended to allow for stochastic interest rates. Hence, suppose the proportionate *round-trip* transactions costs (TC) amount to k. Then, on rebalancing, transactions costs will amount to

$$TC = \frac{1}{2} k|\Delta E(G + \Delta G)| \tag{19}$$

Using a first-order Taylor-series expansion for $\Delta\partial C/\partial G$ gives [Apostol (1969, pp. 308–309)]:

$$TC = \frac{1}{2} k \left| G^2 \frac{\partial^2 C}{\partial G^2}\left(\frac{\Delta G}{G}\right) + P_D G \frac{\partial^2 C}{\partial G\, \partial P_D}\left(\frac{\Delta G}{G}\frac{\Delta P_D}{P_D}\right)\right| + 0[(\Delta t)^{1/2}] \tag{20}$$

To proceed further, first note that

$$\frac{\partial^2 C}{\partial G^2} > 0 \quad \text{and} \quad \frac{\partial^2 C}{\partial G\, \partial P_D} < 0.$$

Using these results and the triangle in equality results in [Apostol (1967, p. 43)]

$$TC \geq \frac{1}{2} kG^2 \frac{\partial^2 G}{\partial G^2}\left|\frac{\Delta G}{G}\right| + \frac{1}{2} kP_D G \frac{\partial^2 C}{\partial G\, \partial P_D}\left|\frac{\Delta G}{G}\frac{\Delta P_D}{P_D}\right| + 0[(\Delta t)^{1/2}] \tag{21}$$

If one takes expectations across the above expression, one obtains [Patel and Read (1982, p. 310)]:

$$E(TC) \geq \frac{1}{2} kG^2 \frac{\partial^2 C}{\partial G^2}\sqrt{\frac{2\sigma_G^2\, \Delta t}{\pi}} + kP_D G \frac{\partial^2 C}{\partial G\, \partial P_D} \frac{\left[\sqrt{1-\rho_{G,P_D}^2} + \rho_{G,P_D} \sin^{-1} \rho_{G,P_D}\right]}{\pi} \sigma_G \sigma_{P_D}\, \Delta t + 0[(\Delta T)^{1/2}] \tag{22}$$

The hedging error is given by

$$\Delta H = \Delta J - \Delta C - \Delta TC \tag{23}$$

which gives

$$\Delta H = \Delta J - \Delta C - \frac{1}{2} kG^2 \frac{\partial^2 C}{\partial G^2} \sqrt{\frac{2\sigma_G^2 \,\Delta t}{\pi}} - kP_D G \frac{\partial^2 C}{\partial G \,\partial P_D}$$

$$\frac{[\sqrt{1 - \rho_{G,P_D}^2} + \rho_{G,P_D} \sin^{-1} \rho_{G,P_D}]}{\pi} \sigma_G \sigma_{P_D} \,\Delta t + 0[(\Delta T)^{1/2}] \tag{24}$$

If one substitutes for $\Delta J - \Delta H$ from Merton's (1973) fundamental equation (5) one obtains

$$\Delta H = \frac{1}{2} G^2 \frac{\partial^2 C}{\partial G^2} \left[\sigma_G^2 \,\Delta T - \left(\frac{\Delta G}{G}\right)^2\right] + \frac{1}{2} P_D^2 \frac{\partial^2 C}{\partial P_D^2}$$

$$\left[\sigma_{P_D}^2 \,\Delta t - \left(\frac{\Delta P_D}{P_D}\right)^2\right] + GP_D \frac{\partial^2 C}{\partial G \,\partial P_D} \tag{25}$$

$$\left[\rho_{G,P_D} \,\sigma_G \sigma_{P_D} \,\Delta t - \left[\frac{\Delta G}{G} \frac{\Delta P_D}{P_D}\right)\right] - \frac{1}{2} kG^2 \frac{\partial^2 G}{\partial G^2} \left|\frac{\Delta G}{G}\right|$$

$$- \frac{1}{2} kP_D G \frac{\partial^2 C}{\partial G \,\partial P_D} \left|\frac{\Delta G}{G} \frac{\Delta P_D}{P_D}\right| + 0[(\Delta t)^{1/2}]$$

Collecting like terms and taking expectations gives the expected hedging error inclusive of transaction costs, which gives

$$E(\Delta H) = -E(\mathrm{TC}) \tag{26}$$

so that

$$E(\Delta H) \le -\frac{1}{2} kG^2 \frac{\partial^2 C}{\partial G^2} \sqrt{\frac{2\sigma_G^2 \,\Delta t}{\pi}} + kP_D G \frac{\partial^2 C}{\partial G \,\partial P_D}$$

$$\frac{\left[\sqrt{1 - \rho_{G,P_D}^2} + \rho_{G,P_D} \sin^{-1} \rho_{G,P_D}\right]}{\pi} \sigma_G \sigma_{P_D} \,\Delta t + 0[(\Delta T)^{1/2}] \tag{27}$$

Following Leland (1985, p. 1289) the variance and covariance rates are redefined in the following terms:

$$\hat{\sigma}_S^2 = \sigma_S^2 \left[1 + k \sqrt{\frac{2}{\pi(\sigma_S^2 + \sigma_F^2 + 2\rho_{SF}\,\sigma_S\,\sigma_F)\,\Delta t}} \right]$$

$$\hat{\sigma}_F^2 = \sigma_F^2 \left[1 + k \sqrt{\frac{2}{\pi(\sigma_S^2 + \sigma_F^2 + 2\rho_{SF}\,\sigma_S\,\sigma_F)\,\Delta t}} \right]$$

$$\rho_{SF}\hat{\sigma}_S\sigma_F = \rho_{SF}\sigma_S\sigma_F \left[1 + k \sqrt{\frac{2}{\pi(\sigma_S^2 + \sigma_F^2 + 2\rho_{SF}\sigma_S\sigma_F)\,\Delta t}} \right]$$

$$\rho_{SD}\hat{\sigma}_S\sigma_D = \rho_{SD}\sigma_S\sigma_D \left[1 + \frac{k\,(\sqrt{1 - \rho_{GD}^2} + \rho_{GD}\sin^{-1}\rho_{GD})}{\rho_{GD}\,\pi} \right]$$

$$\rho_{FD}\hat{\Sigma}_F\sigma_D = \rho_{FD}\sigma_F\sigma_D \left[1 + \frac{k\,(\sqrt{1 - \rho_{GD}^2} + \rho_{GD}\sin^{-1}\rho_{GD})}{\rho_{GD}\,\pi} \right] \quad (28)$$

where

$$\rho_{GD} = \frac{\rho_{SD}\sigma_S + \rho_{FD}\sigma_F}{\sqrt{\sigma_F^2 + 2\rho_{SF}\sigma_S\sigma_F + \sigma_S^2}},$$

with σ_D^2 being unaffected. Given these definitions, following Leland (1985, p. 1290) one obtains

$$E(\Delta H) \leq 0[(\Delta t)^{1/2}] \quad (29)$$

inclusive of transactions costs. Note that as Δt tends to zero, the bound for the hedging error will also tend to zero.

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