CHARTING: CHAOS THEORY IN DISGUISE?

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INTRODUCTION

The primary hypothesis of this article is that technical modeling methods may represent crude but useful ways of exploring nonlinear qualities in data. More specifically, it is proposed that graphical technical analysis may be restated in terms of attractors and strange attractors. Further, it is suggested that technical analysis methods may allow prediction on systems of higher dimension than nonlinear methods do at this time. An objective algorithm that identifies technical patterns is applied to high-dimension nonlinear data, and provides support for the primary hypothesis. This article is distinguished from previous studies in that (a) a specific linkage/equivalence between technical analysis and nonlinear forecasting is proposed, and (b) statistically significant evidence in support of this specific linkage/equivalence is presented.

If this link is valid, it has important implications for the future study of technical and nonlinear analysis, which should be studied jointly, and also suggests that other disciplines applying nonlinear analysis might benefit from applying technical methods.

TECHNICAL ANALYSIS

Despite its inconsistency with the notion of market efficiency, it is clear that the use of technical analysis by practitioners is widespread and grow-

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ing.[1] Pioneering application and writing on technical analysis took place in the futures markets beginning more than 20 years ago [see, for instance, Hardy (1978), Kaufman (1978), Murphy (1986), Pring (1985), and Sklarew (1980)], but its use has grown dominant in the pits and spread to other markets over the past 20 years. Frankel and Froot (1990), for instance, report the transition of (the largely over-the-counter) foreign-exchange market forecasting firms from mostly fundamentalists to mostly chartists between 1978 and 1988. If this shift reflects demand, market participants appear to be embracing technical analysis. Carter and Van Auken (1990) surveyed chief investment analysts at insurance companies, investment banks, bank trust departments, and investment management firms. They found that 35% of all respondents use technical analysis and that as a group, on average, they judged technical analysis to be an important analysis technique. Taylor and Allen (1992) reported the results of a Bank of England survey of foreign-exchange chief dealers in which 90% of those surveyed said they used technical analysis in making decisions. Most respondents reported using both fundamental and technical analysis, with a bias toward using technical analysis for shorter horizons and fundamental analysis for longer horizons.

There is a growing body of empirical research that finds that in spite of market efficiency theory, technical analysis may be valuable for predicting future prices. Bohan (1981) and Brush (1986) each concluded that relative strength indicators may be of value in ranking securities. Pruitt and White (1988, 1989) tested equity and options markets with the use of a decision-making system based on cumulative volume, relative strength, and moving averages, and concluded that "The results of the study, which are consistent across several different return-generating models, provide impressive support for the success claims of technical analysts" (p. 58). In a follow-up study, Pruitt, Tse, and White (1992) showed that their earlier results hold up (even improve in many respects) in a subsequent period—a portion of which is subsequent to publication of their original findings. Neftci (1991) compared technical analysis methods to Wiener–Kolmogorov prediction theory (which, he claims, pro-

[1]Technical analysis refers to any of the many methodologies for forecasting future price activity on the basis of past price (and sometimes other) market data, without respect for economic theory. Technical analysis includes (a) graphical methods, which rely on identification of recurring price history formations such as trend channels, pennants, double bottoms, and head and shoulders (all named for the way they look on price charts), with the hope that future manifestations of these formations might be identified while they are in progress, allowing short-term prediction of prices based on the assumption that the formations will evolve as they have in the past; (b) mathematical methods in which any of several indexes, such as moving averages and relative strength indexes, are calculated and used in predictions based on assumptions that the calculated index will behave as it has in the past; and, (c) a growing number of other methodologies.

vides optimal linear forecasts.) He concluded that "Technical analysis might capture some information ignored by Wiener–Kolmogorov prediction theory," particularly when applied to nonlinear processes. LeBaron (1991) and Brock, Lakonishok, and LeBaron (1992) used moving averages and other technical methods to make predictions of foreign-exchange and stock prices, respectively. Both studies found technical analysis to be of value in making predictions, Brock et al. stated "Overall our results provide strong support for the technical strategies that we explored" (p. 1757). Taylor (1994) applied a channel rule to futures data, finding it to be "surprisingly profitable" (p. 215). Blume, Easley, and O'Hara (1994) showed that analysis of sequencing in volume and price data (only really available to those trading on exchanges) can be informative and advantageous to traders. Most recently, Chang and Osler (1995) developed and applied an objective algorithm to identify head-and-shoulders patterns, used bootstrap methods to test statistical significance of profits generated, and concluded that "aggregate profits would have been both statistically and economically meaningful regardless of transaction costs, interest differentials, or risk" (p. 37).

NONLINEARITY IN FUTURES AND OTHER FINANCIAL MARKETS

Separately, many researchers have been exploring nonlinear qualities and relationships in price data from various markets. Savit (1988), Brock (1991), and Hsieh (1991) all give useful introductions to the ideas involved in applying nonlinear methods to market prices. Using a variety of techniques, researchers have found evidence of nonlinear dependence in prices in a variety of markets. Many of these studies include the issue of whether nonlinearities are due to conditional heteroskedasticity (time-dependent variance) or deterministic chaos. The hypothesis of this article requires the presence of deterministic chaos, as discussed in the next section.

In studies involving futures markets and commodities prices, Lichtenberg and Ujihara (1988) applied a cobweb model with nonlinear terms to U.S. crude oil prices. DeCoster, Labys, and Mitchell (1992) applied the correlation dimension technique to daily sugar, silver, copper, and coffee futures prices. Both studies found evidence of nonlinear dynamic structure in the price series studied. Blank (1991) estimated correlation dimensions and Lyapunov exponents on soybean and S&P 500 futures prices, finding all results to be consistent with the presence of deterministic chaos. Vaidyanathan and Krehbiel (1992) studied mispricing in the S&P 500 contract, finding evidence of nonlinearity and low-order determinism. Yang and Brorsen (1993) found evidence of nonlinearity in several futures markets, which is consistent with deterministic chaos in about half of the cases. Hsieh (1993) found evidence of nonlinearity in four currency futures contracts, but found nonlinearity to be the result of predictable conditional variances.

Evidence of nonlinearity and chaos has been found in other financial markets as well. Hsieh (1989) used the BDS statistic [developed by Brock, Dechert, and Scheinkman (1987) as a more reliable test for nonlinearity] to provide evidence of nonlinear dependence in daily foreign exchange rates. Aczel and Josephy (1991) studied the correlation dimension of daily foreign exchange market data and its change over time, implying nonlinearity in the data. With the use of stock market data, Scheinkman and LeBaron (1989) found evidence of nonlinearity in weekly data by using the correlation dimension method (consistent with chaos), whereas Hsieh (1991) applied the BDS statistic to weekly stock returns to provide evidence of nonlinearity. Mayfield and Mizrach (1992) and Willey (1992) each found evidence of nonlinearity in S&P index cash prices, though neither study found evidence consistent with chaos.

Clearly, the review presented above suggests that there is ample evidence of the presence of nonlinearities in market prices, and some evidence of deterministic chaos. In addition to providing insight into market dynamics, this evidence of nonlinear dependence brings with it the possibility that nonlinear prediction techniques developed in the past 15 years might be applied to price data.

Traditional methods of forecasting in economics and finance involve theoretical specification of the equations of motion and interaction in the system under study, and then estimation of parameters in all equations with the use of linear regression techniques. The equations may then be used to forecast evolution of the system over time. Model specification has had a strong tendency toward linearity because of the importance of linear regression techniques in this methodology. There is, however, little theoretical justification for assuming that the equations governing a given system will all be linear. In fact, as nonlinear analysts often point out, concentration on linearity to the exclusion of all other functional specifications is like taking a bet that the unidentified animal in the next room is an elephant instead of one of any of the other species of animals. But until recently, the tools simply did not exist for exploring nonlinear specification of economic or other systems.

NONLINEAR PREDICTION METHODS

Fortunately, analytical methods developed over the past 15 years may now give researchers the tools needed to begin understanding economic and financial systems as nonlinear systems, and even to attempt prediction based on nonlinear methods. Nonlinear prediction methods do not require specification or knowledge of the equations governing the system under study. In fact, they do not require observation or even identification of the state variables of the system. Nonlinear prediction methods are based on two steps: (1) reconstruction of phase space, and (2) using the reconstructed phase space to make predictions. Each of these steps may be attempted by a variety of techniques.

Step One: Phase Space

A system's true phase space is constructed by plotting its evolution in its state variables over time [see Gleick (1987, pp. 49–51) for more detailed discussion of phase space]. Phase-space plotting is simply an alternative to representing data in the more common time-series format—one that can add important insight with respect to a system's long-term behavior. The motion of a clock pendulum, for instance, might be plotted in position versus time–space, in velocity versus time–space, or in position versus velocity–space; the third is called a phase–space portrait or plotting.

Simple linear and nonlinear systems will follow simple, repeating paths when plotted in their phase spaces, tracing out simple shapes called attractors. A simple attractor, like that of a clock pendulum, may easily be used to predict the future path of the system (as may either of the time-series plots for the pendulum discussed above.) More complex nonlinear systems that display chaotic behavior will display more complex behavior in phase space. In the extreme, chaotic systems trace out what are known as strange attractors, complex patterns in which near, but never exact repetition occurs (see Figure 1 for an example).[2]

Whatever the complexity of the system when represented in phase space, true phase-space construction requires identification and observation over time of all of the state variables of the system. The good news is that the equations governing the system are not needed, one simply needs to identify the state variables and measure them over time. The bad news is that in economics and finance as well as many other areas of study, the state variables of the system are not easily identified and/or they are not observable over time.

[2]Further description and discussion of the value of phase-space portraits can be found in Gleick (1987) and Savit (1988).

Phase-space reconstruction methods generally attempt to reconstruct or simulate true phase space, with the use of data for only one of the system's variables, one that is identifiable and measurable. Methods that are currently used for phase-space reconstruction include: (a) delay coordinates, developed by Takens (1981); (b) derivative coordinates, developed by Packard, Crutchfield, Farmer, and Shaw (1980); and (c) principal-value decomposition, developed by Broomhead and King (1987). Casdagli, Eubank, Farmer, and Gibson (1991) contains a useful review and comparison of these methods. This article deals exclusively with the delay coordinate or Takens method. It is the easiest to apply, most commonly used, and is the basis for the thesis that some forms of technical analysis are simply alternative but equivalent ways of doing nonlinear analysis.

The Takens method is based on the fact that the true phase space of a dynamical system with n state variables may generally be reconstructed by plotting an observable variable associated with the system against at least $2n$ of its own lagged values. The resulting plot has dimension of at least $2n + 1$, which is referred to as the embedding dimension. The key issue in using this method is the choice of lagging interval. In the words of Casdagli et al. (who refer to this interval as tau),

> If tau is too small each coordinate is almost the same, and the trajectories of the reconstructed space are squeezed along the identity line; this phenomenon is known as redundance. If tau is too large, in the presence of chaos and noise, the dynamics at one time become effectively causally disconnected from the dynamics at a later time, so that even simple geometric objects look extremely complicated; this phenomenon is known as irrelevance. (p. 58)

Another important issue in the use of this method results from the fact that the true dimension of the system being studied is often not known, and so the necessary embedding dimension cannot be easily discovered. Methods exist for estimating the dimension of the system, but as Ramsey, Sayers, and Rothman (1990) point out, serious problems may arise when applying these dimension estimation techniques in economics.

In any case, the goal of phase space reconstruction is to identify an attractor, simple or strange. If an attractor can be found, nonlinear prediction can be attempted.

Nonlinear Prediction

Assuming an attractor can be identified, nonlinear prediction may be attempted by a variety of techniques. Techniques explored in the litera-

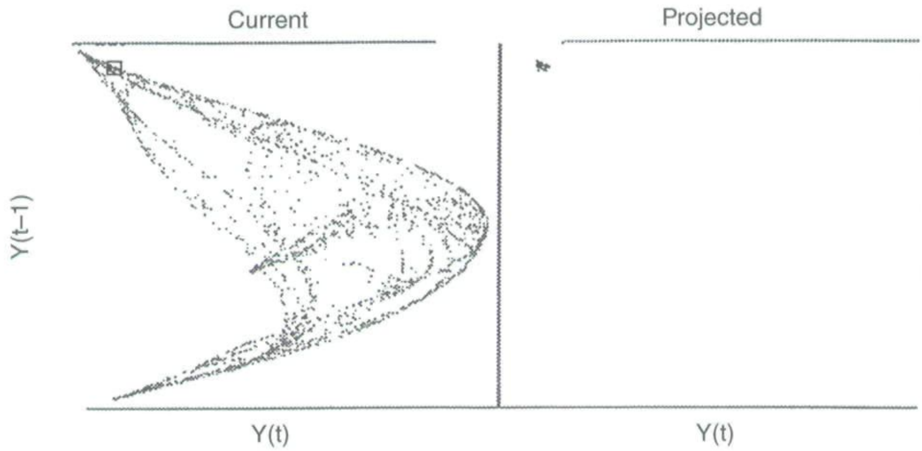

FIGURE 1
Evolution of points in the box after zero periods.

ture include (a) local polynomial mappings, developed by Farmer and Sidorowich (1987, 1988) among others, and probably the most commonly and easily used; (b) global prediction methods, developed by Farmer and Sidorowich (1987) among others; (c) radial basis functions, developed by Casdagli (1989); (d) neural-network-based methods developed by Lapedes and Farber (1987), among others. Useful descriptions and a comparison of these techniques can be found in Casdagli (1989) and in Schaffer and Tidd (1991). This article deals exclusively with local polynomial-mapping-based techniques.

To forecast with local polynomial mapping, one simply identifies the current position on the attractor, and then observes the evolution over time of points near the current point. So long as points near the current point evolve to points that are near each other on the attractor, prediction may be made with some confidence that the current point will evolve to the same region. As the forecasting horizon is stretched out, evolution of nearby points spreads out over the attractor, and predictive power is finally reduced to zero.

This process is illustrated in Figures 1–5. The attractor represented in these figures is that for the series of y_0's generated with the use of eqs. (1) and (2) (discussed in the next section), for $A = 3.75$ and $x_0 = y_0 = 0.10$. Figure 1 shows the attractor that has been identified on the left, with a box around the region surrounding the current point. The points in the box are shown alone in phase space on the right. These are the points whose evolution is to be studied. Figure 2 shows how the points in the box evolved after one period. At this point, they seem to be clus-

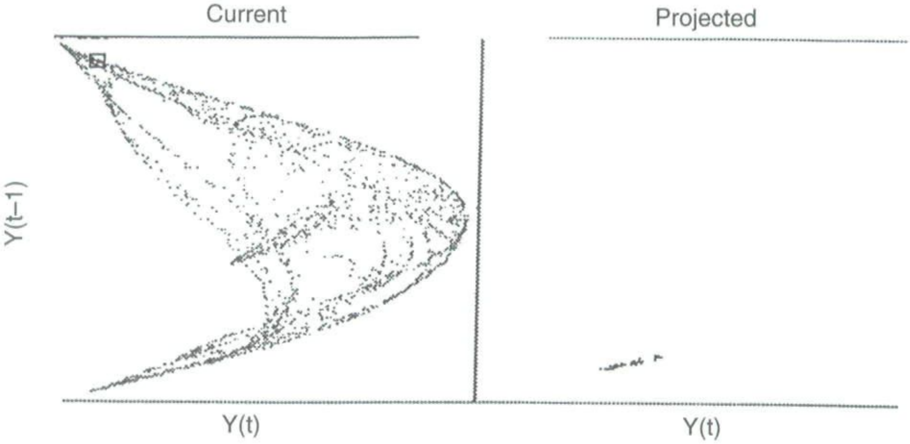

FIGURE 2
Evolution of points in the box after one period.

tered enough to allow meaningful and useful prediction. Figures 3–5 show the evolution of the points in the box after 2, 4, and 6 periods, respectively. Clearly, the predictive power afforded by the points in the box dissipates quickly in this case, and this result is generally true. It is a manifestation of sensitive dependence on initial conditions, which is one of the hallmarks of chaotic nonlinear processes.

TECHNICAL ANALYSIS AS NONLINEAR ANALYSIS

The thesis of this article is that some methods of technical forecasting are equivalent to some methods of nonlinear analysis. The following system of equations is used to demonstrate this idea:

$$x_t = A(x_{t-1} - x_{t-1}^2), \tag{1}$$

$$y_t = 4x_t\,(y_{t-1} - y_{t-1}^2), \tag{2}$$

where x_0 and y_0 must each be between zero and one. This system is made up of the logistic equation, probably the most commonly explored nonlinear system, with the output of the first logistic equation being used as input for the second logistic equation. Starting with any x_0 and y_0, the system is iterated to generate successive x_t's and y_t's for a given constant A. It is well known that for many (but not all) values of A between approximately 3.6 and 4.0, the series of x_ts generated will be chaotic [see Savit (1988) for a discussion of the evolution of this system as A increases

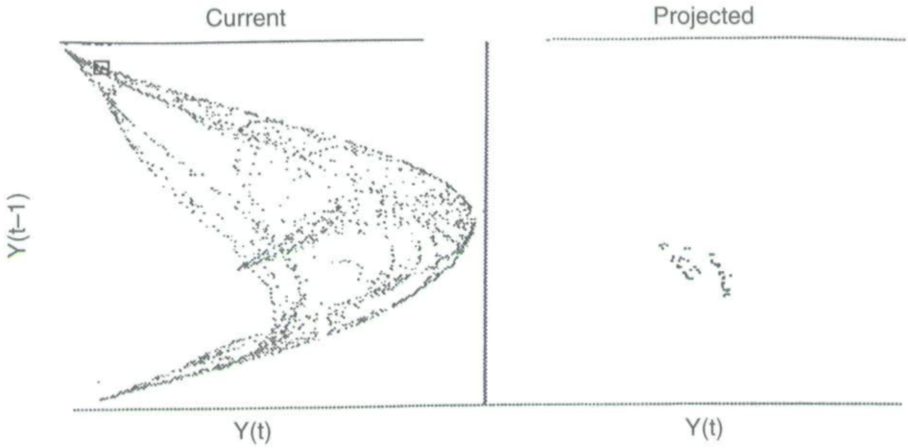

FIGURE 3
Evolution of points in the box after two periods.

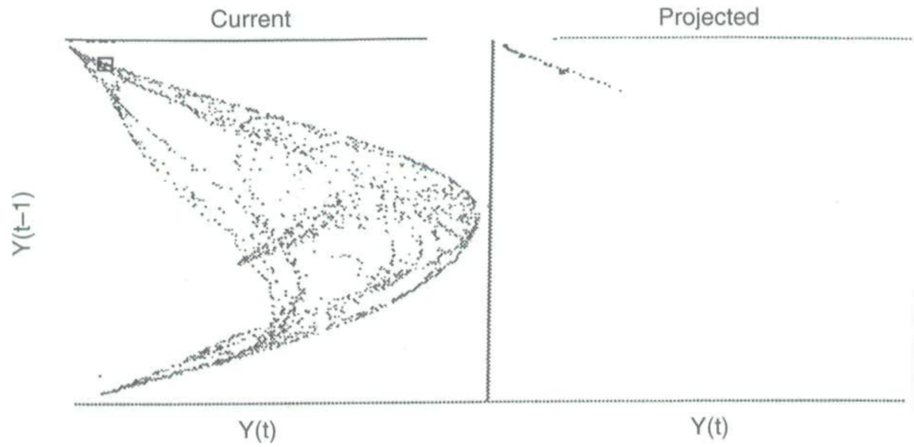

FIGURE 4
Evolution of points in the box after four periods.

from 0 to 4]. Output from this first logistic equation multiplied by 4, to give a parameter between 0 and 4, is then used in place of the constant A to generate a series of y_ts, which has higher dimension and greater complexity than the series of x_ts. Figures 6 and 7 show the behavior of the series, x_t and y_t, respectively, for $A = 3.7$ and $x_0 = y_0 = 0.1$. Each series is plotted as it evolves over a short period of time in the long graph at the bottom of each figure. The two boxes at the top of each figure show two different two-dimensional views of the three-dimensional mapping for the variable, x or y. This mapping is created in a way that is parallel

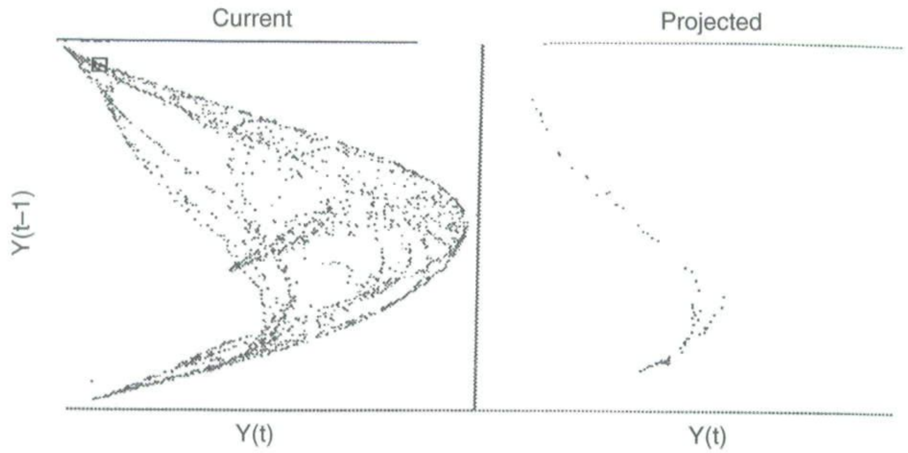

FIGURE 5
Evolution of points in the box after six periods.

to the Takens method of phase space reconstruction: every x_t is plotted versus its first and second lagged values, x_{x-1} and x_{t-2}, and the same is done with each y_t. Clearly, the y series displays more complexity. Yet it is unlikely that any financial or economic system is as simple as this. Still, the complexity of this series is sufficient to allow it to be used to demonstrate the idea that some technical and nonlinear methods may be equivalent.

Traditional graphical technical forecasting methods seek to catalogue recurring price history formations such as pennants, and head and shoulders. The plan of these methods is that future manifestations of these formations might be identified while they are in progress. Such identification might allow short-term prediction of the price series based on the assumption that the formation will evolve as it has in the past.

Essentially, these methods rely on identification and differentiation of price patterns for prediction. A slight difference in price patterns (whether support, resistance, or neck lines are broken, for instance) may lead to dramatically different future price behavior. Recall now the nonlinear method of prediction with the use of the Takens method of phase-space reconstruction along with forecasting with the use of local polynomial mapping. With the right choice of lagging interval, and the assumption that the prices being studied trace out an attractor (whatever the complexity), identification of a recurring price pattern may be equivalent to identification of a point on the attractor. For complex technical price patterns such as head and shoulders, the implication would be that the attractor involved might be of very high dimension. If they are indeed

A=	X
3.7	0.1
X(0)=	0.333
0.1	0.821811
	0.54182
	0.918529
	0.276884
	0.740811
	0.710437
	0.76115
	0.672662
	0.814695
	0.558579
	0.912303
	0.296022
	0.771054
	0.65316
	0.838206
	0.501783
	0.924988

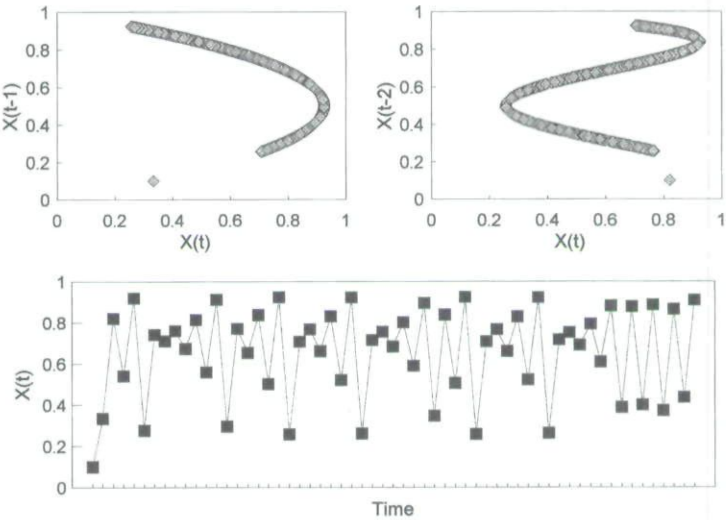

FIGURE 6
Evolution of X over time plotted as a time series (bottom) and in phase space from two perspectives (upper left and right).

A=	Y
3.7	0.1
X(0)=Y(0)=	0.11988
0.7	0.346833
	0.490975
	0.91823
	0.083158
	0.225926
	0.496975
	0.761122
	0.489201
	0.814314
	0.337843
	0.816348
	0.177523
	0.450323
	0.646712
	0.766038
	0.359726
	0.852185

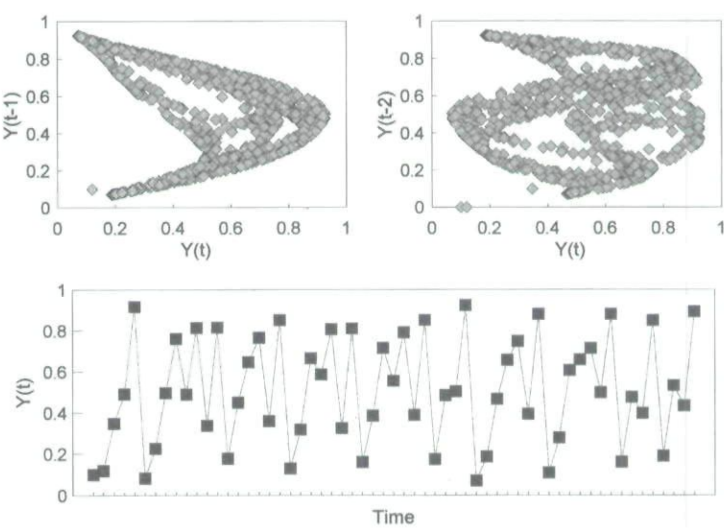

FIGURE 7
Evolution of Y over time plotted as a time series (bottom) and in phase space from two perspectives (upper left and right).

TABLE I

Data Sequences Selected for Study from the Generated Chaotic Series Y (All Groups have Time-Zero Observations between 0.88 and 0.89)

−2	0.747721	0.699425	0.685705	0.743225	0.686788
−1	0.392505	0.510319	0.518297	0.401047	0.516168
0	0.880807	0.882375	0.885893	0.886483	0.887155
1	0.109783	0.158987	0.14986	0.106328	0.147263
2	0.279277	0.467619	0.439825	0.273357	0.432126
3	0.607803	0.404887	0.43094	0.5937	0.436526
4	0.652739	0.860413	0.893073	0.6743	0.898969
5	0.724409	0.170238	0.115237	0.684079	0.106
6	0.474578	0.478307	0.317898	0.55117	0.289816
7	0.889798	0.479814	0.55162	0.846002	0.548314
8	0.139689	0.922119	0.847455	0.239107	0.815354
9	0.407842	0.074977	0.235039	0.668594	0.324523

equivalent, technical methods might allow analysis of higher-dimensional systems than current nonlinear methods allow.

The relatively low-dimension chaotic series, y_t, allows this point to be demonstrated. After iterating the x and y equations 1000 observations of y above 0.88 and below 0.89 (points found in the upper left portion of the attractor in Figure 7) are identified. These points, along with the points just preceding and just following them, are collected for study. Table 1 displays five of the data sequences thus collected: The time-zero observation in each case (in bold) shown is between 0.88 and 0.89, data above each point are the previous two observations, and data below each point are the nine observations that followed. Thus, the recent past and short-term future of the system are recorded for generated observations between 0.88 and 0.89.

Figure 8 is a plotting of each of these data groups. All of these plottings are tightly grouped at time zero. But succeeding points show that proximity at time zero is not sufficient to allow prediction for more than a couple of iterations. Neither do the points most closely clustered at time zero necessarily evolve with the most similarity: the time-zero points 0.880 and 0.886 share very similar futures, whereas points 0.882, 0.885, and 0.887 evolve similarly from time zero.

It is clear in looking at the plottings that similarity of evolution from time zero requires finding time-zero points that share most similar recent histories (most closely related $t - 1$ and $t - 2$ observations). In the words of technical analysis, prediction may be made by finding previous price patterns that are similar to the current price pattern.

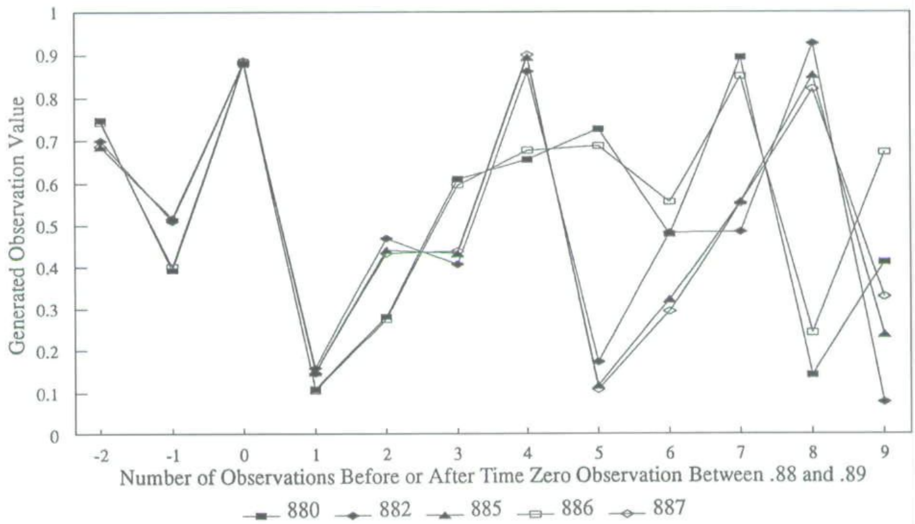

FIGURE 8
Evolution of zero points 0.880, 0.882, 0.885, 0.886, and 0.887 over time plotted as time series.

Figures 9 and 10 show that this is identical to saying that nearby points on an attractor may be used to predict evolution of the current point. These figures are laid out identically to Figures 6 and 7. The long plots at the bottoms are simple time-series plottings, and the two boxes at the top of each figure show two different two-dimensional views of the three-dimensional mapping for iterations of the series y. Figure 9 plots data related to time-zero points, 0.880 and 0.886, whereas Figure 10 plots data related to time-zero points, 0.882, 0.885, and 0.887. The first point in the each mapping is that for which y_t is the time-zero point, whereas the first point plotted in the time-series plots is for time $t - 2$, as indicated.

As in Figure 8, Figure 9 shows that time-zero points, 0.880 and 0.886 share nearly identical recent histories and very similar future evolutions for several iterations. The mappings demonstrate that these two points are close to each other in phase space, and that their future evolutions are also very similar when plotted in phase space. Figure 10 demonstrates the same ideas for time-zero points, 0.882, 0.885, and 0.887.

Finally, Figure 11 contrasts plottings of data related to zero points, 0.880 and 0.882. These points do not share recent histories or short-term futures that are nearly as similar as the other groupings, and they are not as close together, nor do they evolve as similarly in the mappings.

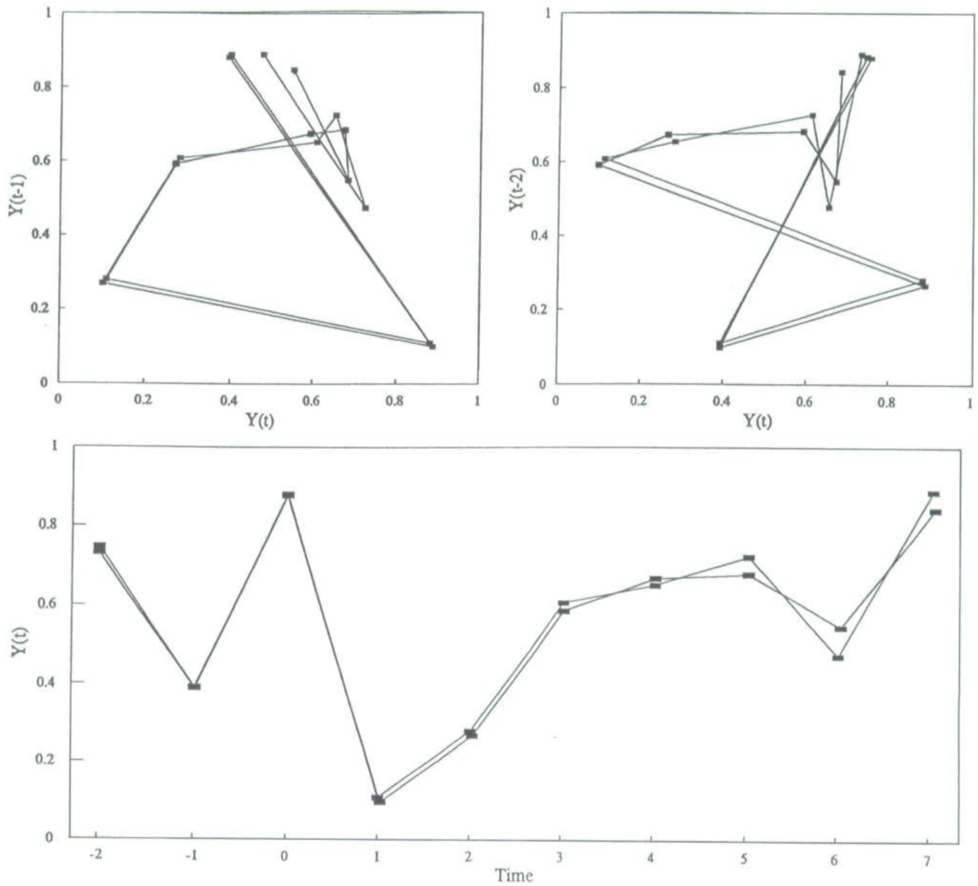

FIGURE 9
Evolution of zero points 0.880 and 0.886 over time plotted as a time series (bottom) and in phase space from two perspectives (upper left and right).

TECHNICAL ANALYSIS APPLIED TO A HIGH-DIMENSION NONLINEAR SYSTEM

The primary hypothesis of this article is that graphical technical analysis methods may be equivalent to nonlinear forecasting methods. This can be partially tested by applying technical analysis to known nonlinear systems to see if it has any predictive power. This is done by testing two secondary hypotheses:

1. Technical analysis has no more predictive power on nonlinear data than it does on random data.
2. When applied to nonlinear data, technical analysis earns no more hypothetical profits than those generated by a random buy/sell rule.

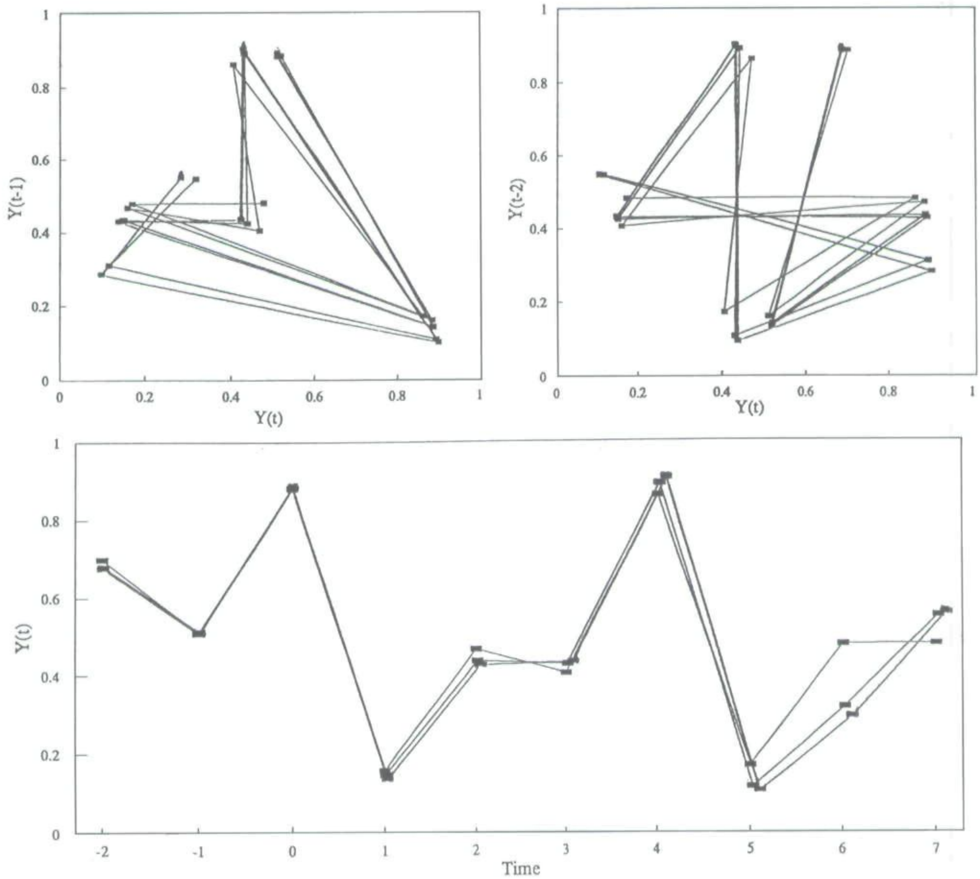

FIGURE 10
Evolution of zero points 0.882, 0.885 and 0.887 over time plotted as a time series (bottom) and in phase space from two perspectives (upper left and right).

These two hypotheses are tested and rejected in Sections D and E, respectively.

To begin with, high dimension nonlinear series based on the idea of successive dependent iterations used earlier are generated. The head-and-shoulders pattern identification algorithm (a schematic of a head-and-shoulders pattern is provided in Figure 12) developed by Chang and Osler (1995) is then applied to the nonlinear series. The goal is to see if (a) head-and-shoulders patterns can be identified in the series, and (b) any head-and-shoulders patterns identified in the nonlinear series will lead to good predictive ability. To do this, a single technical pattern-identifying program (that of Chang and Osler) is applied to a single nonlinear system distinguished solely by its combination of high dimension and ease of generation (but bearing no relationship to economic theory). It is found

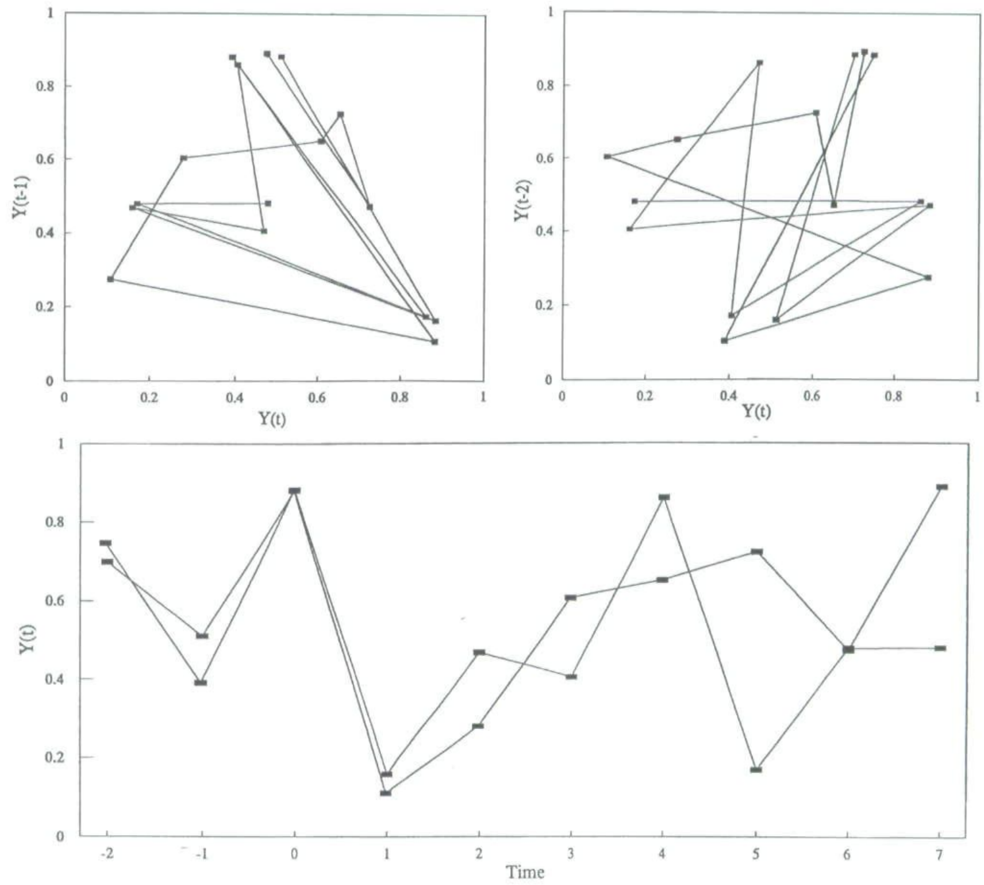

FIGURE 11
Evolution of zero points 0.880 and 0.882 over time plotted as a time series and in phase space from two perspectives (upper left and right).

that this arbitrarily chosen pattern can be applied profitably to the arbitrarily chosen nonlinear system.

A. Generation of the High Dimension Nonlinear Series

The high dimension nonlinear series is created with the use of this system of equations:

$$x_t = 4A_t\,(x_{t-1} - x_{t-1}^2), \tag{3}$$

$$y_t = x_t^{1/10}. \tag{4}$$

The entire A series is initially set at 0.925 (=3.7/4), x_0 is set to 0.10, and

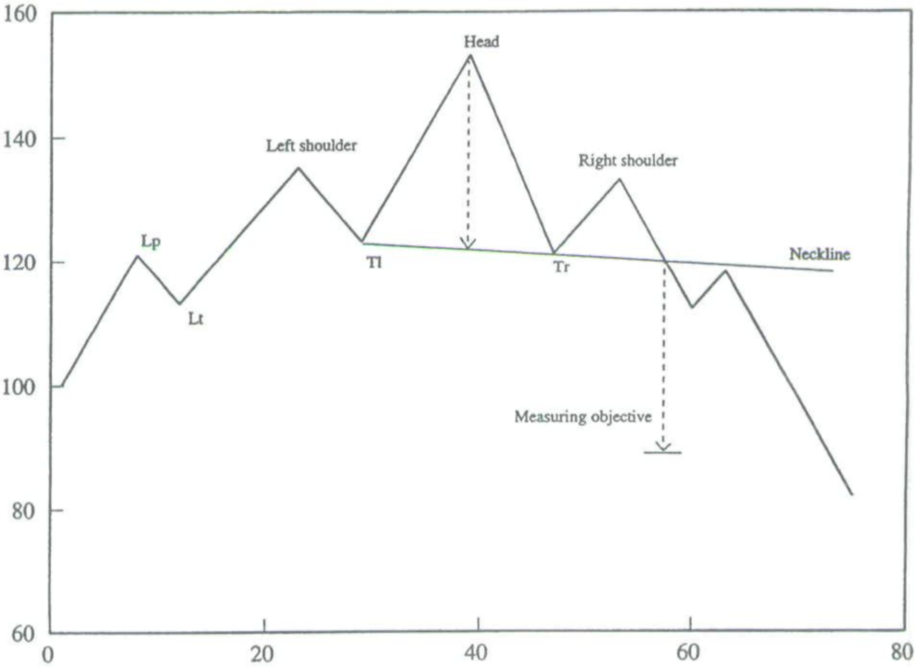

FIGURE 12
Schematic head-and-shoulders pattern.

eq. (3) is iterated 8000 times to produce a chaotic series, x. The tenth root of each value of x is then calculated to produce the series, y. Next, y_1–y_{8000} are plugged into Eq. (3) as values of A_1–A_{8000}, resulting in coefficient values of $4A_t$ that are between 0 and 4, but skewed toward 4 (by having taken the tenth root of x_t, which is between 0 and 1, to get A_t). X_0 is again set to 0.1, and Eq. (3) is iterated 8000 times again. This process is repeated 75 times.[3] The results from the 75th iteration of Eq. (3) form the basis for a simulated price series. The first 500 iterations are removed to exclude signs of initialization from the generating process. The remaining 7500 iterations are mean-shifted by subtracting the series mean from each value, and each mean-shifted value is then added (some values are negative) to a starting point of 40 (chosen so that the price does not become negative) to give the final nonlinear simulated price series. Eleven of these nonlinear simulated priced series are created for this study. The 11 are distinguished by the x_0's used to generate them, which are, 0.10 (as described above), 0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19, and 0.20.

[3]Data generated by repeating this process only 50 times are also tested and give very similar results.

B. Identifying Head-and-Shoulders Patterns

The head-and-shoulders (H&S) identification algorithm used begins by tracing a zigzag in the data. The zigzag is defined as a series of peaks and troughs, each of which differ by a minimum $x\%$ cutoff. The zigzag itself is then scanned for subseries that fit certain basic requirements. These requirements, which are taken directly from technical analysis manuals and practitioners, are[4]

1. *Basic head and shoulders:* The third peak (the head) must be higher than the second and fourth peaks (the left and right shoulders, respectively).
2. *Not too steep:* The second peak can be no lower than, and the second trough can be no higher than, the midpoint between the third trough and the fourth peak. Similarly, the fourth peak can be no lower than and the third trough can be no higher than, the midpoint between the second peak and second trough.
3. *Not too lopsided:* The distance between the second and third peaks can be no more than 2.5 times the distance between the third and fourth peaks. Likewise, the distance between the third and fourth peaks can be no more than 2.5 times the distance between the second and third peaks.
4. *Preliminary trend:* Because the H&S is supposed by technical analysts to presage a trend reversal, the requirement that there be an initial trend that could later be reversed is imposed. Specifically, the peak preceding the basic pattern must fall short of the left shoulder; likewise, the first trough must fall below the second trough.[5]

Head-and-shoulders patterns can also be inverted, in which case all the criteria listed above apply directly, with suitable substitutions of troughs for peaks and vice versa. The H&S patterns can also come in various sizes, for which reason zigzag patterns with different cutoffs are examined. Specifically, the multiples, 5.50, 5.25, 5.00, 4.75, 4.50, 4.25, 4.00, 3.75, 3.50, 3.25, and 3.00, times the square root of the series' mean-squared percent change are used.

C. Predicting and Trading with the Use of Identified Patterns

Once a pattern is identified, profits that might be earned by someone taking a speculative position are measured. Technical analysts agree that

[4]For a more complete description, see Chang and Osler (1995).

[5]This criterion is peripheral to the shape of the head-and-shoulders pattern itself. Tests show that the overall results are unaffected whether this criterion is imposed or not.

one should not enter until the price path crosses the neckline, defined as the line connecting the two troughs within the basic pattern. A market trade is hypothetically entered on that date, and exited 1–5 periods later.

Profits are measured as the percentage change in prices, signed so as to correspond to either a long or a short position. In choosing the sign of these positions the only departure from technical analysts' recommendations is taken. A long position is taken following a regular H&S and a short position following an inverse H&S. This rule, exactly the opposite of technical analysts' recommendations, is chosen because it is profitable for the high dimension nonlinear data used.[6]

D. Does Technical Analysis Work Better on Nonlinear Data than on Random Data?

Two approaches are taken to test this first hypothesis. First a crude but traditional measure of profitability, the hit ratio, or the fraction of total positions that are profitable is studied. One looks for these ratios to be above ½. As shown in Table 2, which reports these hit ratios for the 11 series, the ratios exceed ½ in almost all cases. If hit ratios are useful in evaluating predictive performance, it can be concluded that the H&S pattern used here has prediction value when applied to the nonlinear series studied here.

A more conservative approach is to test statistically whether measured profits exceed the values they might obtain if there were no intertemporal dependence in the data. To generate a distribution of profits with the null hypothesis that prices follow a random walk and thus have no intertemporal dependence, the bootstrap methodology following Efron (1979, 1982) is applied. The first step is to generate 10,000 simulated series by sampling with replacement from the period-by-period percent changes in the original nonlinear series. Though the distributions of these new series generally conform to those of the original nonlinear series in terms of average trend, skewness, kurtosis, et cetera, there is one important potential difference: by construction, any observed H&S patterns have no predictive power in the simulated data, and they might have predictive power in the original nonlinear data.

The H&S identification and profit-taking algorithm is run on each of the 10,000 simulated series. The distribution of these profits can be

[6]Because the nonlinear system under study has no theoretical foundation in economic reality, it should not be expected that the head-and-shoulders pattern could necessarily be applied here in the same way as it is in economic systems that generate prices. Nevertheless, the current study does provide evidence that graphical technical analysis can have predictive power when applied to nonlinear systems.

TABLE II

Hit Ratios
(Positions with Positive Profits Relative to Total Positions)

	Position Duration				
	1 Day	*2 Days*	*3 Days*	*4 Days*	*5 Days*
Series 1	0.77	0.62	0.85	0.77	0.46
Series 2	0.61	0.70	0.57	0.48	0.52
Series 3	0.67	0.73	0.67	0.67	0.87
Series 4	0.59	0.65	0.59	0.71	0.76
Series 5	0.69	0.63	0.56	0.50	0.63
Series 6	0.63	0.48	0.56	0.52	0.56
Series 7	0.58	0.74	0.58	0.58	0.53
Series 8	0.63	0.44	0.50	0.50	0.50
Series 9	0.58	0.71	0.63	0.54	0.54
Series 10	0.86	0.71	0.86	0.71	0.57
Series 11	0.65	0.60	0.70	0.75	0.55
Number above ½	11	9	11	10	10

TABLE III

Marginal Significance of Profits under Null Hypothesis That Prices Follow a Random Walk

	Position Duration						
	1 Day	*2 Days*	*3 Days*	*4 Days*	*5 Days*	*Number of Positions*	*p-Value for No. of Positions*
Series 1	0.04	0.20	0.24	0.28	0.31	13	0.85
Series 2	0.13	0.34	0.48	0.50	0.46	23	0.15
Series 3	0.00	0.00	0.05	0.09	0.20	15	0.76
Series 4	0.00	0.01	0.19	0.05	0.07	17	0.54
Series 5	0.00	0.00	0.00	0.01	0.07	16	0.68
Series 6	0.00	0.00	0.00	0.00	0.01	27	0.02
Series 7	0.10	0.21	0.58	0.77	0.74	19	0.44
Series 8	0.04	0.10	0.22	0.34	0.08	16	0.65
Series 9	0.01	0.01	0.07	0.01	0.04	24	0.12
Series 10	0.09	0.29	0.51	0.68	0.10	7	0.99
Series 11	0.01	0.07	0.09	0.33	0.66	20	0.35
Number ≤ 0.1	10	7	5	5	6	Average 18	0

used to test whether the profits from the original nonlinear data would likely have been observed under the null hypothesis. Table 3 presents the results. There are, on average, 18 positions taken based on identification of the H&S pattern in each of the 11 nonlinear data series generated, and that number of positions taken typically falls within the range of normalcy for the bootstrap simulated data, as shown by the P values in the right-hand column. In contrast, the profits on the positions are generally rather high compared with the range of normalcy, as evidenced by the marginal significance levels reported in the 1–5-day columns. Average percent profits over a position's first day are statistically significant at the 10% level in all series, and are significant at the 5% level in 8 of 11 cases. Profits over longer intervals are generally statistically significant at the 10% level or better, though the proportion generally declines as the horizons get longer. Though only about half of the nonlinear series' profits are statistically significant at the 10% level at the longer horizons, there seems to be some predictability at these horizons. Consistent with the earlier observations based on hit ratios, profits from applying the H&S pattern to the nonlinear series exceed the median of those from the bootstrap simulated data in almost all cases even at the 5-day horizon.

There is considerable evidence, then, that the first hypothesis should be rejected, and that technical analysis does work better on nonlinear data than on random data. This is consistent with the notion that graphical technical analysis is a way of doing nonlinear forecasting.

E. Does Technical Analysis Generate More Hypothetical Profits than Random Trading, Given Nonlinearities in the Price Series?

This hypothesis is tested with the use of the hit ratio discussed above. First, 100 nonlinear series are generated according to the process described in Section A by increasing x_0 by increments of 0.01 from an initial x_0 of 0.10. The head-and-shoulders algorithm described in Section B is then applied, and hit ratios are calculated for positions lasting 1–5 days for each of the 100 series. A random buy/sell strategy is then applied, and hit ratios are again calculated for positions lasting 1–5 days for each of the same 100 series. Table 4 reports the median hit ratios for the head-and-shoulders and random strategies, as well as the percentage of hit ratios that are above 50%. Median hit ratios for the random strategy are centered around 50%, as expected, but they are above 50% for 1–4-day positions taken with the use of the head-and-shoulders strategy.

The statistical significance of the difference in these hit ratio distri-

TABLE IV

Hit-Ratio Statistics Comparing the Random and Head-and-Shoulders Trading Strategies

	Position Duration				
	1 Day	*2 Days*	*3 Days*	*4 Days*	*5 Days*
Median hit ratio:					
• Random strategy	0.50	0.50	0.50	0.45	0.50
• Head-and-Shoulders strategy	0.67	0.57	0.54	0.54	0.50
Percent of hit ratios greater than 50%					
• Random strategy	51	60	51	44	53
• Head-and-Shoulders strategy	85	72	68	60	55

TABLE V

Marginal Significance of Maximum Differences between Cumulative Distributions of Hit Ratios under Random and Head-and-Shoulders Trading Strategies, Based on Kolmolgorov Tests

	Position Duration				
	1 Day	*2 Days*	*3 Days*	*4 Days*	*5 Days*
Maximum difference	0.031	0.543	0.335	0.315	0.543

butions is studied with the use of the Kolmolgorov test. In this test comparing unknown distributions, the maximum difference between two cumulative distribution functions is calculated and compared to a critical value, resulting in a marginal significance [see Press, Teukolsky, Vetterling, and Flannery (1992) and Birnbaum (1952)]. Table 5 reports the results of this study: The null hypothesis that the cumulative distributions of hit ratios are equivalent is rejected at about the 3% level for 1-day positions (but not at any reasonable significance level for 2–5-day positions).

Together, Tables 4 and 5 provide evidence, therefore, that the second hypothesis should be rejected: Technical analysis does seem to generate more hypothetical profits than random buying and selling when applied to a known nonlinear system.

CONCLUSION AND IMPLICATIONS

This article provides a theoretical foundation for technical analysis as a method for doing nonlinear forecasting on high dimension systems.

This article specifically shows how traditional graphical technical modeling methods may be viewed as equivalent to nonlinear prediction methods that use the Takens method of phase space reconstruction combined with local polynomial mapping techniques for nonlinear forecasting. Evidence in support of this hypothesis is presented in the form of an application of a head-and-shoulders formation identification algorithm to high-dimension nonlinear data, resulting in successful pattern identification and prediction.

Linkage of these methodologies, if it is valid, has a few implications. First, it may suggest that technical analysis methods are more broadly linked with nonlinear methods than shown here. Technical methods may generally be crude but useful methods of doing nonlinear analysis. Second, identification of such a link or links would aid the study and understanding of both technical and nonlinear methods. Just as the study of herbal remedies aids development in modern medicine, the study of technical methods (which, like herbal remedies, are empirically developed techniques with little theoretical foundation) might aid development of nonlinear techniques. This may be particularly true if technical methods point to ways of analyzing nonlinear systems of relatively high dimension. Third, if this link is valid, technical methods might be used in other disciplines applying nonlinear methods, such as physics, neurology, meteorology, et cetera. In fact, this seems an important direction for follow-up research. Identifying useful applications of technical analysis outside of finance would not only give new analytical tools to other disciplines, but would also help to validate the link between technical analysis and nonlinear modeling.

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