Cross Hedging in Currency Forward Markets: A Note

UDO BROLL

In recent years, managers of international firms have become increasingly aware of how their organizations can be affected by risks beyond their control. In many cases, fluctuations in financial and economic variables, such as foreign exchange rates, interest rates, and goods prices have destabilizing effects on corporate strategies and performance. In principle, international firms can insulate themselves from exchange rate risks by using derivatives markets. The most important type of derivative is the forward contract. With a forward contract, the hedger promises to buy or sell an asset at a specified price on a specified date. Long positions enable users to protect themselves against price increases in the underlying asset; short positions protect hedgers against price decreases.

Hedging foreign exchange risk by offsetting a spot market position with an opposite one in currency forward contracts is important for international firms which are concerned with both the profitability and risk of their operations [Froot, Scharfstein, and Stein (1993); Wolf (1995); Zilcha and Eldor (1991)]. Forward markets are available in most major currencies of the world. However, not all currencies have forward markets [Buckley (1992)]. This is true for those currencies that are not fully convertible. In such cases an exporting firm may use forward contracts on other financial assets whose spot prices are highly correlated with the

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■ *Udo Broll is a Professor at the University of München, Germany.*

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exchange rate of the currency in which the export revenues are invoiced. Such risk management is called cross hedging [Eaker and Grant (1987); Grabbe (1991); Powers and Castelino (1991)].

There are two important ways to cross-hedge exchange rate risk. First, risk management by cross hedging is possible by using the regress-ability between the spot exchange rate and the spot price for a domestic financial asset correlated with the exchange rate. Second, an effective cross-hedge is possible by using forward foreign exchange contracts of two (or more) correlated currencies. Cross hedging is, of course, only successful when the two currencies behave similarly. Good examples of correlated currencies include: the German mark and the Dutch guilder, the British pound and the Australian dollar, the U.S. and the Canadian dollar, and for many of the European Union currencies.

In the first case of cross hedging a less common currency exposure, the firm has access to futures markets for a domestic financial asset whose spot price, $\tilde{G}$, is correlated to the foreign spot exchange rate, $\tilde{S}$ (tildes denote uncertainty). If S is regressable on G, the regressability assumption implies $\tilde{S} = a + b\tilde{G} + \tilde{\varepsilon}$, where ε is independent of G [Benninga, Eldor, and Zilcha (1984); Lence (1995); for another direction of regress-ability see Broll, Wahl, and Zilcha (1995); Broll (1997)]. In the second approach, the international firm can cross-hedge the exchange rate risk by using the forward market of a third country's currency correlated with the exchange rate in question.

The objective of this study is to make these arguments precise, to show the effectiveness of cross hedging, and to characterize the optimal hedge policy with the expected utility hypothesis. A competitive exporting firm is assumed and the optimal cross hedging strategy is obtained when a triangular parity condition holds between foreign currencies. Since the firm is interested in home currency income, the forward markets will always be used *jointly.* Contracting in just one forward market will necessarily result in an open position for at least one other foreign currency.

It is shown that even when cross currency forward markets are individually unbiased, the hedging problem for an exporting firm differs from that generally considered in the literature. This is so, because, with cross hedging, the international firm faces two tiers of uncertainty. First, by using currency forward markets jointly, the firm's export revenues are sold at an uncertain foreign exchange, and, then, the uncertain foreign proceeds are converted to domestic currency at an uncertain cross exchange rate. A full double-cross-hedge can reduce completely the two-tiered structure of exchange rate uncertainty. However, a full double-cross-hedge is optimal only if currency forward markets are "jointly unbiased."

DEFINING THE HEDGING PROBLEM

Consider a domestic firm which is exporting to country-1 a volume, PQ, of a given commodity at a future date at the then prevailing spot exchange rate, $\tilde{S}$. The producer need not hold the foreign exposure unhedged, although there is no direct currency forward market for $\tilde{S}$. However, there are currency forward markets for cross exchange rates, S_1 and S_2, whose random spot exchange rates are correlated to the exchange rate, $\tilde{S}$. More specifically, $\tilde{S} = \tilde{S}_1\tilde{S}_2$ with probability 1. Or, in other words, foreign exchange markets are arbitrage-free (assumption A.1). The existence of cross-currency forward markets for these two currencies allows the firm to use forward rates, F_1 and F_2, (at date, $t = 0$) for the spot exchange rate, $\tilde{S}_1$ and $\tilde{S}_2$, respectively. Thus, to hedge the revenue risk, the domestic exporting firm may sell or buy a forward country-2 currency contract, either in exchange for domestic currency against the forward rate, F_2, or in exchange for country-1 currency against the forward rate, F_1.

If the exporting firm buys forward a country-1 currency contract, Z_1, against the forward rate, F_1, (resulting in an open position for the currency of country-2) and sells a country-2 currency contract, Z_2, against the forward rate, F_2; then, the hedger will have final wealth, $\tilde{W}$, in domestic currency at time, 1, of

$$\tilde{W} = \tilde{S}PQ + Z_1(\tilde{S}_1 - F_1)\tilde{S}_2 + Z_2(F_2 - \tilde{S}_2) \qquad (1)$$

Note that wealth, $\tilde{W}$, is linear in $\tilde{S}$, $\tilde{S}_2$ and $\tilde{S}_1\tilde{S}_2$. This has implications for the optimal hedge policy (see next section). Now the firm's optimal cross hedging decisions are considered under the assumption that the firm maximizes the expected utility of its wealth.

DERIVING THE OPTIMAL CROSS-HEDGE

This section focuses on the firm's optimal cross-hedge. The firm enters into forward exchange contracts, Z_1 and Z_2, to maximize the expected utility of wealth. The decision maker's problem may, thus, be written

$$\max_{Z_1, Z_2} E[U(\tilde{W})]$$

where U is a strictly concave and twice continuously differentiable utility function and where wealth, $\tilde{W}$, is given by (1). The firm maximizes the expected utility of wealth, $E[U(\tilde{W})]$, with respect to currency forward commitments, Z_1 and Z_2. The necessary first-order conditions, which are also sufficient, for an optimal cross-hedge are

$$E[U'(\tilde{W}^*)(\tilde{S}_1 - F_1)\tilde{S}_2] = 0 \tag{2}$$

$$E[U'(\tilde{W}^*)(F_2 - \tilde{S}_2)] = 0 \tag{3}$$

where U' is the marginal utility of profit and an asterisk indicates an optimum level. From these conditions an optimal cross-hedge of a foreign exchange exposure is proved.

It should be noted that final wealth and the cross-currency forward contracts of the firm are linear. A linear forward contract implies: the gain when the value of the underlying asset moves in one direction is equal to the loss when the value of the asset moves by the same amount in the opposite direction. From (2) and (3) it can also be written: $E[U'(\tilde{W}^*)(F_1F_2 - \tilde{S})] = 0$. This condition shows that a cross-hedge defines a *synthetic* currency forward contract for the exchange rate, $\tilde{S}$, which has no direct currency forward market. A synthetic forward contract by cross transactions is possible because of the linear structure of wealth in the random spot exchange rates.

Combining (2) and (3) and the triangular arbitrage-free assumption, A.1, yields

$$E(\tilde{S}_1\tilde{S}_2) - F_1F_2 = -\frac{cov(U'(\tilde{W}^*), \tilde{S}_1\tilde{S}_2)}{E[U'(\tilde{W}^*)]} \tag{4}$$

$$E(\tilde{S}_2) - F_2 = -\frac{cov(U'(\tilde{W}^*), \tilde{S}_2)}{E[U'(\tilde{W}^*)]} \tag{5}$$

Now from (4) and (5) the main results can be stated and proved.

Proposition 1. *(Full double-cross-hedge). Suppose that the currency forward market for the spot exchange rate, $\tilde{S}_2$, is unbiased, i.e., $F_2 = E(\tilde{S}_2)$. The optimal risk management decision of the firm implies a full double-cross-hedge, i.e., $Z_1^* = -PQ < 0$ and $Z_2^* = F_1Z_1^* = F_1PQ > 0$, if and only if, currency forward markets are jointly unbiased, namely, $F_1F_2 = E(\tilde{S}_1\tilde{S}_2)$.*

Proof. First it is shown that

$$(PQ + Z_1^*)(E(\tilde{S}_1\tilde{S}_2) - F_1F_2) - (F_1Z_1^* + Z_2^*)(E(\tilde{S}_2) - F_2) \geq 0, \tag{6}$$

where the equality holds, if and only if, $\tilde{W}^*$ (the optimal level of wealth) is nonrandom. To prove (6), $cov(\tilde{W}, U'(\tilde{W})) \leq 0$ due to the strict concavity of U. This equality holds, if and only if, $\tilde{W}$ is not stochastic. Using eqs. (4) and (5) and the definition of wealth, $\tilde{W}$, it is found that:

$$cov(\tilde{W}^*, U'(\tilde{W}^*)) = -EU'(\tilde{W}^*)\{(PQ + Z_1^*)$$

$$(E(\tilde{S}_1\tilde{S}_2) - F_1F_2) - (F_1Z_1^* + Z_2^*)(E(\tilde{S}_2) - F_2)\} \leq 0 \qquad (7)$$

This proves (6), since the expected marginal utility is positive. Now, suppose $F_1F_2 = E(\tilde{S}_1\tilde{S}_2)$ and $F_2 = E(\tilde{S}_2)$. Then expression (7) holds with equality; therefore, $\tilde{W}^*$ must be nonstochastic. For wealth, $\tilde{W}^*$, to be nonstochastic, it must be the case that $Z_1^* = -PQ < 0$ (long position) and $Z_2^* = F_1Z_1^* = F_1PQ > 0$ (short position) because

$$\tilde{W}^* = \tilde{S}(Z_1^* + PQ) - \tilde{S}_2(Z_2^* + F_1Z_1^*) + F_2Z_2^* \qquad (8)$$

This proves the full double-cross-hedge theorem. □

It is inferred from Proposition 1 that a full double-cross-hedge completely insulates the wealth from any random variations of exchange rates. Thus, the firm sells forward all sales revenues in exchange for currency of country-2. At the same time, all receipts from this transaction are sold forward in exchange for currency of country-1, so that all revenue risks are fully hedged. The size of the risk premium in the forward market for foreign exchange, S_1, is immaterial to the firm's decision. This can be seen from wealth, $\tilde{W}$, in eq. (8). With a full double-cross-hedge the terminal wealth is nonrandom, which implies that the optimal portfolio strategy results in complete risk hedging. This full double-cross-hedge holds for any firm which has an objective function which is concave.

Proposition 1 shows that even when cross currency forward markets are individually unbiased, the hedging problem for an exporting firm differs from that generally considered in the literature. This is because with cross hedging the international firm faces two tiers of uncertainty. First, by using currency forward markets jointly, the firm's export revenues are sold at an uncertain foreign exchange, and, then, the uncertain foreign proceeds are converted to domestic currency at an uncertain cross exchange rate. Since the firm's profits are denominated in domestic currency, the firm is unable to use one forward market in isolation. Any returns from forward sales of country-1 currency against the forward rate, F_1, must be exchanged for domestic currency, so that the two forward markets will always be used jointly.

Remark. When the currency forward markets are "separately" unbiased so that there is no risk premium in either market [i.e., $F_1 = E(\tilde{S}_1)$ and $F_2 = E(\tilde{S}_2)$], the additional assumption that $cov(\tilde{S}_1,\tilde{S}_2) = 0$ is required for proposition 1 to be valid.

Under more general conditions, the following proposition concerning optimal cross hedging decisions is obtained. This is summarized as follows:

Proposition 2. *(a) Let the forward market for foreign exchange* $\tilde{S}_2$ *be unbiased, i.e.,* $F_2 = E(\tilde{S}_2)$. *Then,* $Z_1^* \geq -PQ$, *as* $F_1F_2 \geq E(\tilde{S}_1\tilde{S}_2)$ *and* $Z_1^* < -PQ$, *as* $F_1F_2 < E(\tilde{S}_1\tilde{S}_2)$.

(b) Let currency forward markets for $\tilde{S}_1$ *and* $\tilde{S}_2$ *be jointly unbiased, i.e.,* $F_1F_2 = E(\tilde{S}_1\tilde{S}_2)$. *Then,* $Z_2^* \geq F_1Z_1^*$, *as* $F_2 \geq E(\tilde{S}_2)$ *and* $Z_2^* < F_1Z_1^*$, *as* $F_2 < E(\tilde{S}_2)$.

The *proof* is straightforward from eq. (7). □

Note that Proposition 1 would follow by combining Proposition 2(a) and 2(b) when $F_1F_2 = E(\tilde{S}_1\tilde{S}_2)$ and $F_2 = E(\tilde{S}_2)$. Proposition 2(a) demonstrates that, with an unbiased forward market, the firm buys a greater or smaller amount of currency, S_1, than the value to be produced for exports, depending on whether the risk premium in the joint markets is negative or positive. Proposition 2(b) shows that, with jointly unbiased forward markets, the firm takes an overhedge or underhedge position in the forward market for currency, S_2, depending on the sign of the risk premium in this market.

Empirical Applicability

The discussion in this section deals with the empirical applicability of the model. The assumption that condition, $\tilde{S} = \tilde{S}_1\tilde{S}_2$, holds with a probability of one is very strong. Empirical studies [Duffie (1989); Eaker and Grant (1987); Graabe (1991); Jones and Jones (1987)] show that sister currencies usually are not perfectly correlated. In other words, nonperfect correlation between related currencies provides an imperfect hedging mechanism. However, the model of cross hedging as discussed is still applicable.

To model this, the assumption of the following regressability can be used: $\tilde{S} = \beta\tilde{S}_1\tilde{S}_2 + \tilde{e}$ where e is independent of the random product, $\tilde{S}_1\tilde{S}_2$, and zero-mean uncertainty, $E(\tilde{e}) = 0$, where $\beta > 0$. Under the conditions of Proposition 1, the optimal risk management is as follows: $Z_1^* = -\beta PQ < 0$ and $Z_2^* = F_1Z_1^* = F_1\beta PQ > 0$. This cross-hedge position has two distinctive properties. First, it leaves the hedger's expected wealth unchanged. This is due to the joint unbiasedness assumption. Second, it reduces all uncertainty about the hedger's wealth except that uncertainty which is by its nature unhedgeable and undiversifiable (the residual uncertainty e).

This example shows that the regression coefficient of spot exchange rate changes determines the optimal cross-hedge policy of the firm. The result may be useful and applicable to a large number of cross hedging cases. The strength of the result is derived from its generality: the optimal cross-hedge is free from assumptions about utility function and only a regression analysis is required to derive the optimal risk reducing policy. If the stochastic relationship can be estimated and remains stable over time, then the firm may achieve an acceptable reduction in the foreign exchange exposure of a less common currency.

CONCLUSIONS

It has been shown in the literature that an international firm facing exchange rate risk can eliminate this risk if it has access to a forward market on which the foreign currency is traded against the domestic currency. In the absence of a direct risk sharing market, international firms can reduce their income risk by engaging in cross hedging using the currency forward markets. Cross hedging is successful when the currencies behave similarly.

The article investigates conditions under which the optimal hedge is a fixed proportion of the cash position, independently of the risk-averse agent's utility function (full double-hedge property; see Proposition 1). In other words, the pure hedge ratio is equal to one, since $Z_1^* = -PQ$ and $Z_2^* = F_1Z_1^* = F_1PQ$ eliminates all exchange rate risk. Furthermore, optimal cross hedging policies in currency forward markets in the presence of normal backwardation and contango are studied.

To avoid transaction risks associated with open positions in foreign currencies, the exporting firm will have to use two forward currency markets in combination. It is shown that, in such a situation, individually unbiased currency forward markets do not imply a full hedge. If the forward market for the cross exchange rate, S_2, is unbiased, then the firm fully hedges, if and only, if the cross currency forward markets for exchange rate, S_1 and S_2, are jointly unbiased. The concept of joint unbiasedness used in the case of an optimal cross-hedge means that the expected value of the product, $\tilde{S}_1\tilde{S}_2$, is equal to the product, F_1F_2. Hence, it is the size of the risk premium in the joint markets, i.e., $E(\tilde{S}_1\tilde{S}_2) - F_1F_2$, that matters in the case of cross hedging.

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