International Currency Relationship Information Revealed by Cross-Option Prices

ANDREW F. SIEGEL

INTRODUCTION

It is well known that options prices reveal current market sentiment regarding the current and future volatility of their underlying assets through the concept of implicit volatility introduced by Latané and Rendleman (1976). Information derived from current options prices is appealing because it can react quickly to market changes and does not have the statistical sampling error associated with classical measures based on samples of historical data.

More recently, it has been remarked that information about the *relationship* between two assets (e.g., the extent to which their price changes are associated or can be predicted one from the other) is similarly revealed by the prices of appropriate options. In particular, it was shown by Siegel (1995) that the systematic risk (beta) of a stock with respect to a market index would be revealed by implicit volatilities of three options: an option on the stock, an option on the market index, and an option to exchange stock for shares of the market index. At this time, such exchange options are not widely traded in the equities markets.

Recent innovations in the international exchange rate options markets have resulted in the availability of cross-options prices which permit estimation of market sentiment regarding relationships between particular pairs of currencies. The purpose of this study is to estimate the

The author is grateful for helpful comments provided by C. Bonser-Neal and K. Dewenter.

■ *Andrew F. Siegel is the Grant I. Butterbaugh Professor of Finance and Management Science at the University of Washington.*

The Journal of Futures Markets, Vol. 17, No. 4, 369–384 (1997)

CCC 0270-7314/97/040369-16

relationship between the German mark and the British pound, to estimate the relationship between the German mark and the Japanese yen, and to test these option-based measures of relationship against historically estimated values. For example, the prices of a pound/dollar option, a mark/dollar option, and a pound/mark option together imply a value for the correlation between the dollar/pound and the dollar/mark exchange rates. This implicit correlation in turn reveals market sentiment regarding how closely these currencies are expected to move together in the future.

Evidence is found that supports the use of cross-options data in inferring future relationships between foreign currencies: option-based measures are in many cases significantly better than historical measures, and are never significantly worse. Two groups of empirical tests are performed, each one comparing the options-based implicit measures to daily exchange-rate based measures of relationship. The first group of tests examines the actual risk of hedged positions constructed using recent information. Two of these cases favor the implicit measures while the other two show no significant difference. The second group of tests compares the time-varying pattern of information for each measure. In every case, the implicit measure contributes significant new information while the historical measure does not.

The theory and notation are outlined and the data set and estimation methodology are described including graphs of implicit and historical measures of volatilities and currency relationships. Test results, comparing the predictive abilities and information content of implicit measures to those of historical measures are presented and a summary and conclusions follow.

THEORY AND NOTATION

Given three currencies (the domestic currency D and two foreign currencies F and G) and three appropriate options (e.g., an option to exchange D for F, an option to exchange D for G, and a cross option to exchange F for G), the theory of option pricing leads not only to implicit volatilities for each exchange rate, but also to implicit measures of the relationship (correlation and regression) between changes in two exchange rates (see Figure 1). In this section the formula is derived for the implicit correlation between the two foreign exchange rates. A formula is derived also for the implicit beta hedge ratio of either foreign currency with respect to the other. Because any of the three countries could be labeled as "domestic" there are, in fact, three different correlations (and six different hedge ratios) that can be estimated in this way.

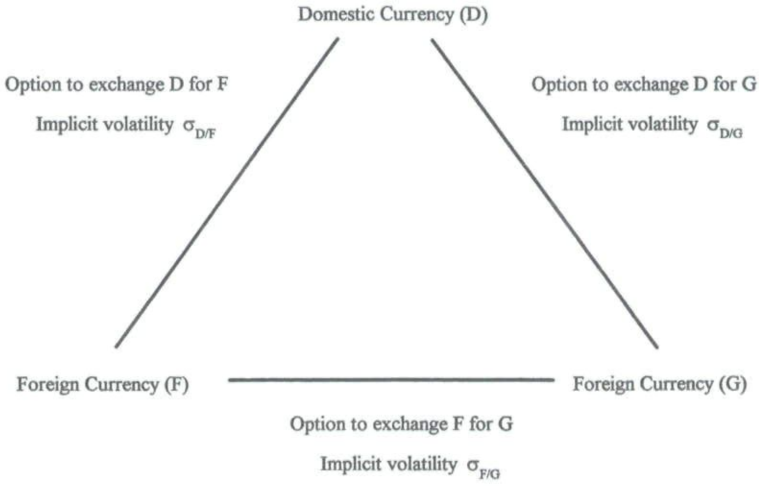

FIGURE 1

When options are traded on all three pairs of the currencies D, F, and G, the resulting implicit volatilities can be used to compute (1) the implicit correlation between changes in the exchange rates $X_{D/F}$ and $X_{D/G}$, and (2) the implicit beta ratio to hedge the risk (in D) of holding F by selling G.

The currency option pricing model of Garman and Kohlhagen (1983), which is based on Merton's (1973) proportional dividend extension of the Black and Scholes (1973) model, applies equally well to cross options and ordinary currency options, because a cross option may be viewed as an ordinary option with the exercise currency taken as the numeraire. This model gives the price of a European-style call option to exchange D for F as

$$\begin{aligned} C &= C(X_{D/F}, K, \tau, r_D, r_F, \sigma_{D/F}) \\ &= X_{D/F}e^{-r_F\tau}\, N(\zeta) - Ke^{-r_D\tau}\, N(\zeta - \sigma_{D/F}\sqrt{\tau}) \qquad (1) \\ \text{with } \zeta &= \frac{\ln[(X_{D/F}\, e^{-r_F\tau})/(Ke^{-r_D\tau})] + \sigma^2_{D/F}\,\tau/2}{\sigma_{D/F}\sqrt{\tau}} \end{aligned}$$

where the exchange rate, $X_{D/F}$, is the spot price of the foreign currency (i.e., the value in D of one unit of F), K is the exercise price (specified as an exchange rate), r_D is the domestic interest rate (to be compounded continuously), r_F is the foreign interest rate, τ is the time in years to maturity of the option, and $\sigma^2_{D/F}$ is the instantaneous variance of the rate of return of the exchange rate per unit time. If the option price, C, is observed, then the implicit volatility is defined as the value of σ for which $C(X_{D/F}, K, \tau, r_D, r_F, \sigma)$ equals this observed price. The exchange rate,

$X_{D/F}$, is assumed[1] to follow the following Itô process (where $Z_{D/F}$ is a standard Wiener process):

$$\frac{dX_{D/F}}{X_{D/F}} = (r_D - r_F)dt + \sigma_{D/F}dZ_{D/F} \tag{2}$$

Similar Itô processes hold for exchange rates, $X_{D/G}$ and $X_{F/G}$. Substitute the exchange-rate identity, $X_{F/G} = X_{D/G}/X_{D/F}$, into the Itô process

$$\frac{dX_{F/G}}{X_{F/G}} = (r_F - r_G)dt + \sigma_{F/G}dZ_{F/G} \tag{3}$$

for the cross-option and then simplify to find

$$\frac{dX_{F/G}}{X_{F/G}} = (r_F - r_G)dt + \sqrt{\sigma_{D/F}^2 + \sigma_{D/G}^2 - 2\sigma_{D/F}\sigma_{D/G}\rho_{D/F,D/G}}\,dZ_{F/G} \tag{4}$$

where $\rho_{D/F,D/G}$ is the instantaneous correlation between the processes, $Z_{D/F}$ and $Z_{D/G}$, also representing the correlation between movements of exchange rates, $X_{D/F}$ and $X_{D/G}$. Compare eqs. (3) and (4) to see that the volatility, $\sigma_{F/G}$, of the cross option is related to the volatilities of the ordinary options and the correlation between the exchange rate changes:

$$\sigma_{F/G} = \sqrt{\sigma_{D/F}^2 + \sigma_{D/G}^2 - 2\sigma_{D/F}\sigma_{D/G}\rho_{D/F,D/G}} \tag{5}$$

Solving this for the correlation shows how the three volatilities imply a value for the exchange-rate correlation:

$$\rho_{D/F,D/G} = \frac{\sigma_{D/F}^2 + \sigma_{D/G}^2 - \sigma_{F/G}^2}{2\sigma_{D/F}\sigma_{D/G}} \tag{6}$$

The ratio for hedging the risk (measured in units of D) of holding currency, F, by selling an offsetting position in currency, G, is also implied by the volatilities:

$$\beta_{(\text{hedge } F \text{ with } G)} = \rho_{D/F,D/G}\frac{\sigma_{D/F}}{\sigma_{D/G}} = \frac{\sigma_{D/F}^2 + \sigma_{D/G}^2 - \sigma_{F/G}^2}{2\sigma_{D/G}^2} \tag{7}$$

while the other hedge ratio (hedging the D value of G with an offsetting position in F) is

[1]These assumptions may be relaxed and the same pricing equations will still follow under more general conditions for the exchange-rate processes.

$$\beta_{(\text{hedge } G \text{ with } F)} = \rho_{D/F,D/G} \frac{\sigma_{D/G}}{\sigma_{D/F}} = \frac{\sigma_{D/F}^2 + \sigma_{D/G}^2 - \sigma_{F/G}^2}{2\sigma_{D/F}^2} \tag{8}$$

DATA AND ESTIMATION METHODOLOGY

Data for two sets of three countries are used; in each case the U.S. dollar is considered the domestic currency. One pair of foreign currencies is the British pound and the German mark; the other pair is the German mark and the Japanese yen. All trades in the data set for which implicit volatilities could reasonably be calculated are selected for further analysis. Because three options are needed for estimating currency relationships, and some classes of options are not traded every day, trading days are grouped by calendar month. Results in this section (implicit volatilities, correlations, and hedging betas) are presented primarily based on monthly averages of implicit volatilities of the selected options trades, and are compared to the corresponding classical historical measures which are based on daily exchange-rate changes (these are also computed for each calendar month).

Options data are obtained from the Philadelphia Stock Exchange, including the daily open, high, low, and close for each international currency option contract traded through August, 1993 with contemporaneous quotations of the spot exchange rate at the time of the trade. Contracts with different maturity dates or different strike prices are considered distinct contracts. The two cross options traded are the British pound/German mark cross option (data are available beginning September 23, 1992) and the German mark/Japanese yen cross option (data are available beginning January 3, 1992). Daily interest rates (1-month, 3-month, and 1-year maturities) and exchange rates are obtained from Datastream.

Trades are selected for further analysis if they are near the money (spot price within 5% of the strike price), have more than 10 but less than 100 days until maturity, and the price is such that the implicit volatility can be computed. To eliminate outliers, a trade is not selected if its volatility is more than 4 standard deviations away from the average volatility for the previous 25 trades (overall, 1.2% are rejected as outliers). Table I summarizes the characteristics of selected trades. Adjustment for early exercise is not necessary because only European-style option contracts are used. The put–call parity relationship is used to compute the implicit volatility for put options, where the theoretical price of a European-style put option is

TABLE I

Summary of Selected Trades for Each Class of Option: Time Period, Number of Trades, Percent of Selected Trades Representing Contracts Selected Only Once That Day, Percent Call Options, and Average Number of Trades Selected Per Month

	Pound Mark Cross	*Mark Yen Cross*	*Pound (Dollar)*	*Mark (Dollar)*	*Yen (Dollar)*
From	Sep-92	Jan-92	Jan-92	Jan-92	Jan-92
To	Aug-93	Aug-93	Aug-93	Aug-93	Aug-93
Number of trades selected	527	644	1028	3993	581
Percent unique	82.7%	80.4%	79.8%	74.9%	84.3%
Percent calls	62.2%	50.0%	57.1%	46.7%	58.0%
Average selected per month	43.9	32.2	51.4	199.7	29.1

$$P(X_{D/F}, K, \tau, r_D, r_F, \sigma_{D/F}) = C(X_{D/F}, K, \tau, r_D, r_F, \sigma_{D/F}) - X_{D/F}e^{-r_F\tau} + Ke^{-r_D\tau} \quad (9)$$

Implicit Volatilities

Implicit volatilities for these selected trades are averaged on a calendar-monthly basis for each of the five classes of options, with results as shown in Figure 2. These provide the basis from which implicit currency relationship measures (implicit correlation and implicit beta) are derived and tested. For comparison, the historical volatilities, based on the daily percent change in exchange rate from close to close, are computed each month and displayed in Figure 3. Comparing their general appearance, the implicit volatility graphs are smoother, in part because of the statistical sampling error of the historical volatilities.[2] While historical measures computed from a larger sample would have less sampling error, they would also lose sensitivity to changes over time because recent data would have less influence in the calculation.

Note how the overall pattern of volatility is somewhat similar, with Figures 2 and 3 both indicating a peak in the fall of 1992. This was a period of considerable uncertainty in the European Monetary System;

[2]Note that the number of implicit volatilities averaged per month from Table I is, for most option classes, only somewhat larger than the number of trading days per month used in the historical estimate, so that (with the exception of the mark option, which averaged about 200 selected trades per month) the two methods are roughly comparable in terms of sample size.

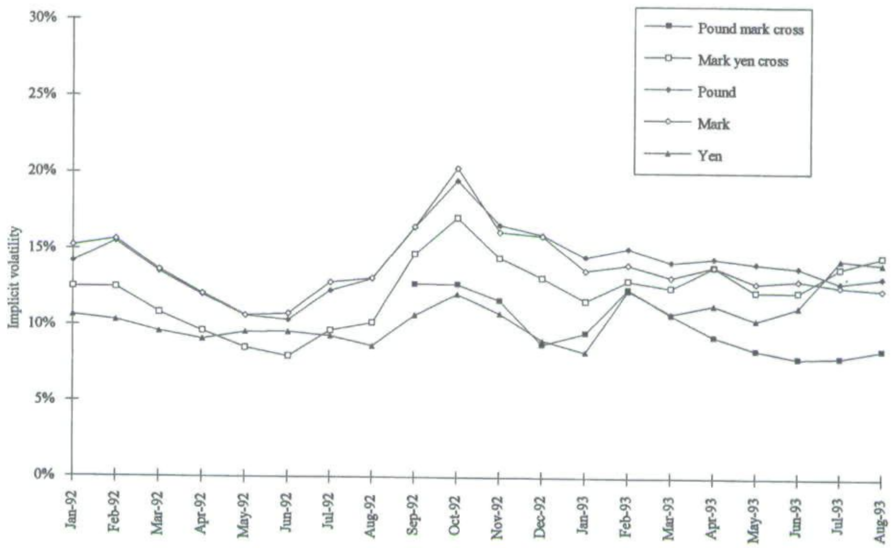

FIGURE 2
Monthly average implicit volatility.

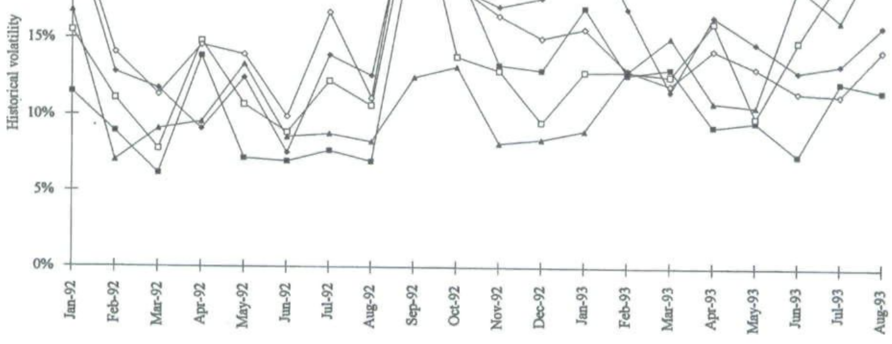

FIGURE 3
Historical exchange-rate volatilities: Standard deviations of daily percent changes, computed monthly.

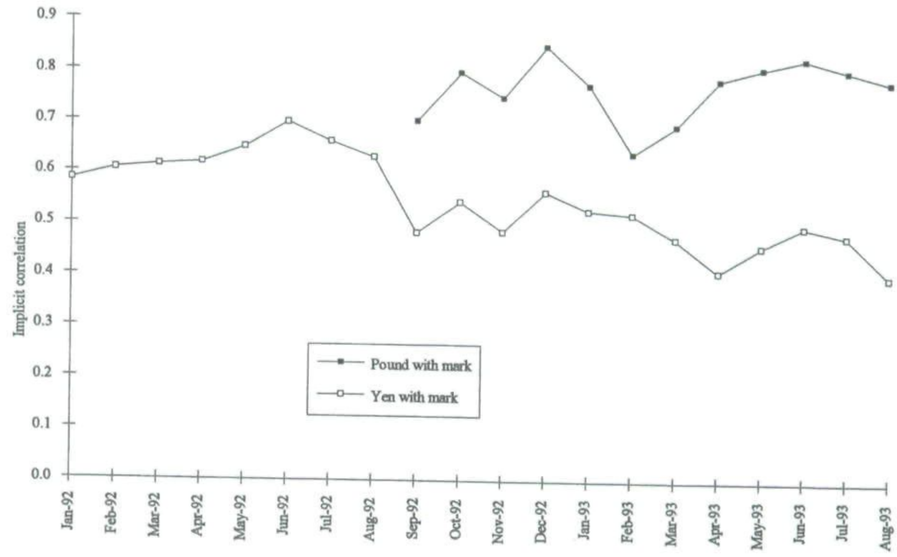

FIGURE 4
Implicit correlation of dollar-based exchange rate changes, computed from monthly average implicit volatilities.

the pound was removed from the Exchange Rate Mechanism on September 16.[3] Note also the peak in the pound/mark cross option volatility in early 1993. This was, again, a period of uncertainty in the European Monetary System, accompanied by interest rate cuts by Germany and Britain.[4]

Implicit Correlations

Correlations of dollar-based exchange rates are computed for each of the two cross-option combinations: pound with mark and mark with yen. A high correlation implies that the two currencies tend to move together, while a low correlation implies that they are expected to move more independently of one another. The implicit correlations displayed in Figure

[3]See, e.g., "Foreign Exchange: Volatile Currency Markets Resist Calming Efforts; Bundesbank Chief Denies Shift to Growth Policy," *The Wall Street Journal,* September 16, 1992, p. C1; "Foreign Exchange: Sterling Drops Against Mark and Dollar After Suspension from Monetary System," by C. Lee, *The Wall Street Journal,* September 17, 1992, p. C15; and "Currencies in Conflict—Conflicts of Interest: Currencies in Turmoil, Europe Faces Chaos at a Time of Unity; Germany's Monetary Policy Sinks the British Pound, and EC Goals are at Risk. Will French Vote to Unify?" *The Wall Street Journal,* September 17, 1992, p. A1.

[4]See, e.g., "European Currencies," *The Economist,* February 6, 1993, p. 76. The British base lending rates dropped one point to 6% from 7% on January 26 (see, e.g., "Foreign Exchange: U.K. Rates Cut Gives the Mark a Boost; Dollar Holds on, Ends Little Changed," G.L. White, *The Wall Street Journal,* January 27, 1993, p. C15).

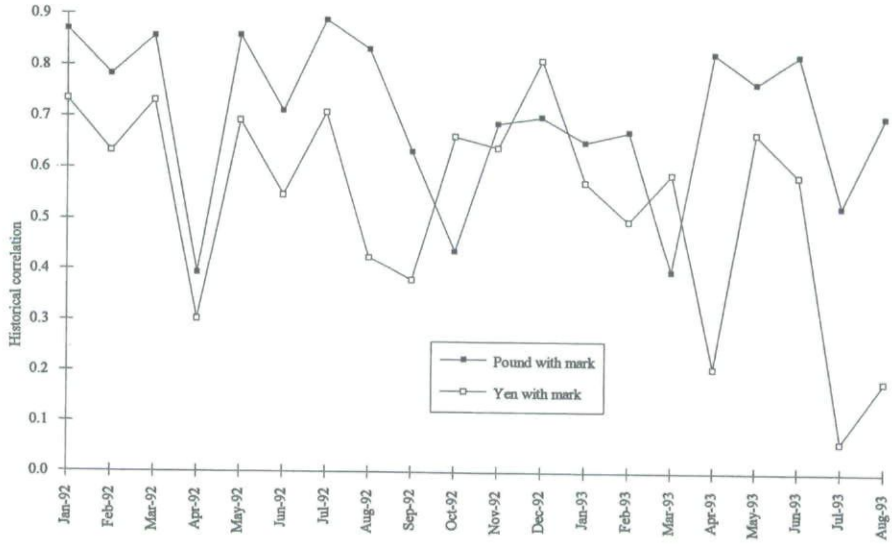

FIGURE 5
Historical correlation of daily dollar-based exchange rate changes, computed monthly.

4 are obtained by substituting appropriate monthly average implicit volatilities into eq. (6). For comparison, the historical correlation between daily exchange rate percent changes are computed each month and displayed in Figure 5. It is reasonable (due to geographical proximity) that the pound/mark correlation should be uniformly higher than the mark/yen correlation, as the implicit correlation shows in Figure 4. Note that this is not made clear by the noisier historical values in Figure 5.

The day-to-day variation of implicit correlation between the pound and the mark exchange rates is displayed in Figure 6 for the period, September, 1992 through August, 1993 (these are not grouped by month, but are computed using daily average implicit volatilities when available). The uncertainty noticed earlier in exchange rate and interest rate markets in early 1993 shows up clearly here (the lower correlation indicates greater uncertainty). The drop in the correlation appears to be related to differences between German and British interest rates (see Figure 7); the interest rate spread widened when British rates descended suddenly on January 26, while German rates descended more gradually over the following weeks.

Implicit Beta Hedge Ratios

Beta values for hedge ratios are computed for four situations involving hedging the dollar value of one foreign currency with another: (A) hedging

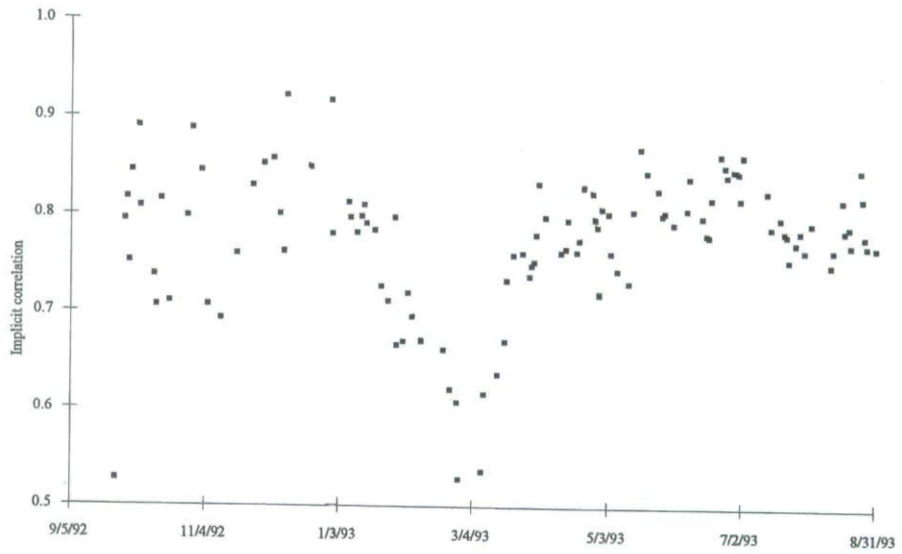

FIGURE 6
Implicit correlation of the (dollar-based) pound and mark exchange rates, daily.

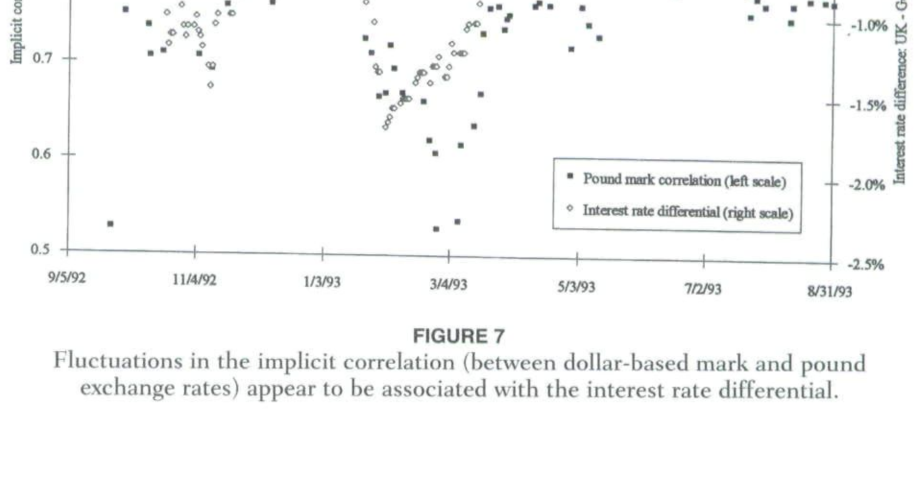

FIGURE 7
Fluctuations in the implicit correlation (between dollar-based mark and pound exchange rates) appear to be associated with the interest rate differential.

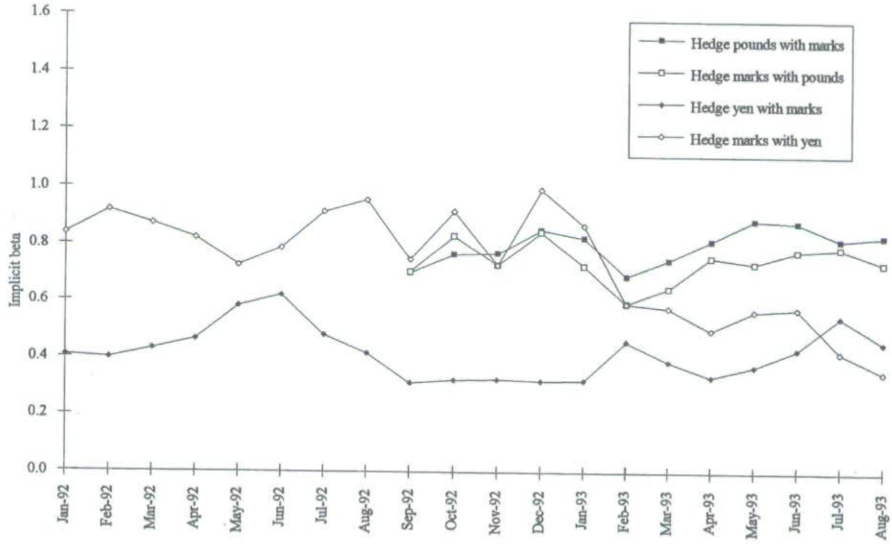

FIGURE 8
Implicit beta ratios for hedging the dollar value of one currency using another.

the pound using the mark, (B) hedging the mark using the pound, (C) hedging the yen using the mark, and (D) hedging the mark using the yen. These hedging situations are chosen for the purpose of comparing the information content of implicit and historical measures of relationship (see tests in the next section). The implicit beta values displayed in Figure 8 are obtained by substituting monthly average implicit volatilities into eqs. (7) and (8). For comparison, the historical regression coefficients to predict daily exchange rate percent changes for one foreign currency from another are computed each month and are displayed in Figure 9. Note that the implicit beta values are smoother, again due, in part, to the statistical sampling error of historical estimates. The implicit ratio for hedging yen with marks is the smallest hedge ratio (except for the last two months) in Figure 8 because the yen had low volatility during this time (see Figure 2). The largest changes in hedge ratio occur for hedging marks with yen.

TEST RESULTS

Tests are performed to compare the ability of each measure of relationship (implicit and historical) to explain relationships in future currency movements. Estimates for each method for each calendar month (as described in the preceding section) are used to explain daily currency movements

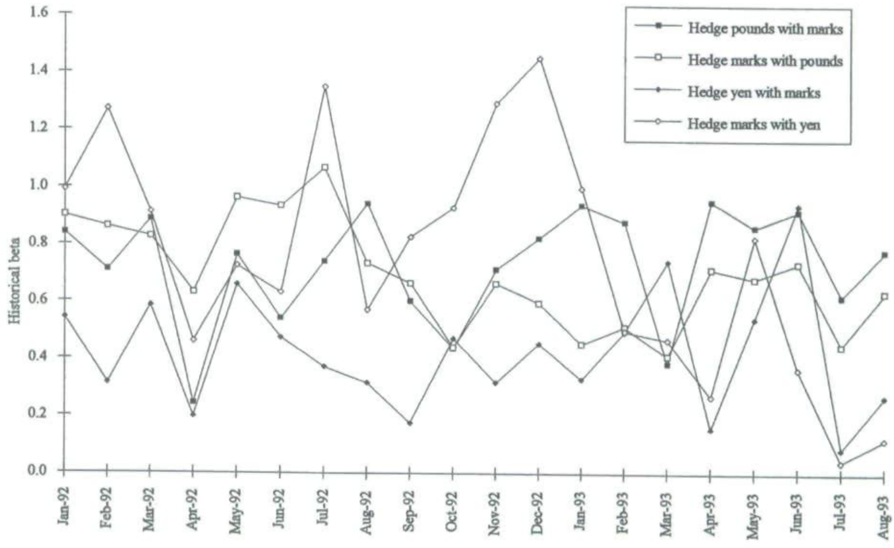

FIGURE 9

Historical beta ratios for hedging the dollar value of one currency using another.

over the course of the next month. All four hedging situations (as shown in Figures 8 and 9) are tested.

Two groups of tests are performed. The first group of tests evaluates hedges directly by looking at the volatility of the hedged position, asking "Do the implicit and historical measures lead to similar risk reduction, or does one method reduce risk significantly more than the other?" The second group of tests estimates a bias adjustment for each method and asks "Do the implicit and historical measures contain similar information regarding the time-varying pattern of hedge ratios, or does one method (or do both methods) contain significant useful hedging information that is missing from the other?"

Testing Hedging Efficiency

For the first group of tests, each beta value is considered with respect to its role as a regression coefficient and is evaluated according to its sum of squared residuals. Specifically, each beta value (implicit and historical) estimated from the previous calendar month (beginning the month before the test period) is used to construct, each day, a hedged portfolio consisting of a purchase of \$1 worth of the currency to be hedged and a short sale of beta dollars worth of the foreign currency used to construct the hedge. The dollar returns (changes in value representing the residual

TABLE II

Comparing the Hedging Effectivenes of Implicit Beta and Historical Beta. Risk is Reduced Significantly More by Implicit Beta in Two of the Four Hedging Situations, while Differences Are Not Significant in the Other Two Situations

Currency to Be Hedged	*Pound*	*Mark*	*Yen*	*Mark*
Currency Used to Construct Hedge	*Mark*	*Pound*	*Mark*	*Yen*
Test Period Begins	*10/1/92*	*10/1/92*	*2/3/92*	*2/3/92*
Test Period Ends	*9/30/93*	*9/30/93*	*9/30/93*	*9/30/93*
n	256	256	428	428
Risk of currency to be hedged	0.00855	0.00760	0.00652	0.00785
Risk of hedged position:				
using historical beta	0.00659	0.00582	0.00613	0.00700
using implicit beta	0.00643	0.00592	0.00593	0.00693
percent difference[a]	2.5%	−1.8%	3.3%	1.0%
Percent reduction in risk due to hedging:				
using historical beta	22.8%	23.5%	6.0%	10.8%
using implicit beta	24.8%	22.1%	9.1%	11.7%
p[b]	0.045[c]	0.352	0.023[c]	0.971

[a]This is the percentage by which the risk of the implicit beta hedged position is less than the risk of the historical beta hedged position.

[b]These p-values test the significance of the apparent difference in hedging effectiveness between the implicit and historical betas. Methodology is from Morgan (1939) as implemented by Kamara and Siegel (1987); this test for equality of variances recognizes the correlation between the samples.

[c]Significantly less risk using implicit beta instead of historical beta.

values) for these hedged portfolios are computed each day. The risk associated with each hedging technique is measured by the standard deviation of these daily returns over the test period.

Results are shown in Table II. The implicit beta hedge performs significantly better in some cases, but never significantly worse, than the historical beta hedge. Whenever the German mark is used to construct the hedge, the implicit beta hedge is significantly better, reducing risk by an additional 2.5% or 3.3% from the risk of the historical beta hedged position. Test results are inconclusive when the German mark is the currency being hedged; the observed differences between the implicit beta hedge and the historical hedge are not significant.

Testing Incremental Information Content

For the second group of tests, multiple regression analysis is used to assess the incremental information content of each hedge ratio (implicit and historical) with respect to the other, in terms of their abilities to explain exchange rate fluctuations. The dependent variable (to be predicted) consists of daily rates of return of the currency to be hedged. The

TABLE III

Multiple Regression Results Testing the Incremental Information Content of Each Hedge Ratio (Implicit and Historical) with Respect to the Other. In all Four Cases Implicit Beta Contains Significant Additional Information but Historical Beta Does Not

Currency to Be Hedged	*Pound*	*Mark*	*Yen*	*Mark*
Currency Used to Construct Hedge	*Mark*	*Pound*	*Mark*	*Yen*
Test Period Begins	*10/1/92*	*10/1/92*	*2/3/92*	*2/3/92*
Test Period Ends	*9/30/93*	*9/30/93*	*9/30/93*	*9/30/93*
n	256	256	428	428
R^2	43.8%	42.9%	18.1%	23.3%
Implicit beta				
Coeff	1.197	0.697	0.891	0.550
Std err	0.310	0.279	0.191	0.193
t	3.866	2.494	4.668	2.852
p	0.000141	0.0133	0.00000408	0.00456
Historical beta				
Coeff	−0.288	0.105	−0.064	0.311
Std err	0.335	0.356	0.187	0.186
t	−0.859	0.294	−0.343	1.669
p	0.391	0.769	0.732	0.0958

two independent (predictor) variables (one for the implicit beta and one for the historical beta), each represent daily rates of return on the hedge itself (i.e., beta times the daily rate of return on the currency used to construct the hedge). The t-test of each regression coefficient then indicates whether that variable provides significant incremental explanatory power over and above the information provided by the other variable.

Results are shown in Table III. In all cases the information content of the implicit beta hedge ratio is superior to historical beta. In all four hedging situations the t-test for the implicit beta regression coefficient is significantly different from 0, indicating that the time-varying patterns of implicit beta provide significant additional hedging information even after bias-adjustment of the historical beta. In no case is the t-test for a historical beta significantly different from 0, thus historical beta provides no significantly useful information beyond that already reflected in implicit beta.[5]

SUMMARY AND CONCLUSIONS

A new technology is proposed for estimating the relationship between changes in the exchange rates of two foreign currencies based upon op-

[5]Although multicollinearity is expected in a situation like this, it is not severe. Correlations between the two explanatory variables are 0.976, 0.979, 0.893, and 0.921 for the four situations, respectively.

tions prices. Prices of three currency options (e.g., an option to convert dollars to pounds, an option to convert dollars to marks, and a cross-option to convert marks to pounds) reveal current market sentiment regarding the association between future changes in any two of these currencies as measured with respect to the other (e.g., the association between dollar-based changes in the mark and the pound exchange rates). The beta hedge ratio of either foreign currency (e.g., mark or pound) with respect to the other is similarly revealed.

Implicit correlations and implicit beta hedge ratios are estimated for two situations. One situation uses the pound/mark cross option to study the relationship between the (dollar-based) pound and the mark exchange rates, while the other uses the mark/yen cross option. A visual comparison of implicit measures of association with historical measures shows that the historical measures are noisier, because of the statistical sampling error associated with their calculation, while the implicit measures are smoother. While historical measures computed from a larger sample would have less sampling error, they would then lose sensitivity to changes over time because recent data would have less influence in the calculation.

Empirical tests are performed in four cases involving hedging the dollar value of one foreign currency with another: (A) hedging the pound using the mark, (B) hedging the mark using the pound, (C) hedging the yen using the mark, and (D) hedging the mark using the yen. The implicit beta hedge ratios significantly reduce hedging risk, compared to historical hedge ratios estimated for the same time period, when the mark is used to construct a hedge either for the pound or for the yen. When the mark itself is being hedged, the two methods show no significant differences.

The information content of the implicit beta hedge ratio is found to be significantly higher than that of the historical hedge ratio estimated for the same time period. This result holds uniformly in all four hedging situations considered. Thus, the time-varying patterns of implicit beta provide significant additional hedging information even after bias adjustment of the historical beta. In no case does the historical beta provide any significantly useful incremental information beyond that already reflected in the implicit beta.

How well should implicit measures of currency relationships work in the real world? While results presented here show that they can represent a significant improvement compared to historical measures, they are not exact. Because the assumptions underlying option pricing models are not perfectly satisfied in the real world, implicit measures will not perfectly reflect the market. Nevertheless, just as the concept of implicit volatility has been found to be useful in many situations, its extension to implicit measures of relationship, while not perfect, appears to provide timely and improved estimates in currency markets.

BIBLIOGRAPHY

Black, F., and M. Scholes, (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 81:637–659.

Garman, M. B., and Kohlhagen, S. W. (1983): "Foreign Currency Option Values," *Journal of International Money and Finance,* 2:231–237.

Kamara, A., and Siegel, A. F. (1987): "Optimal Hedging in Futures Markets with Multiple Delivery Specifications," *Journal of Finance,* 42:1007–1021.

Latané, H. A., and Rendleman, R. J. (1976): "Standard Deviations of Stock Price Ratios Implied in Option Prices," *Journal of Finance,* 31:369–381.

Merton, R. (1973): "Theory of Rational Option Pricing," *Bell Journal of Economics,* 4:141–183.

Morgan, W. A. (1939): "A Test for the Significance of the Difference between the Two Variances in a Sample from a Normal Bivariate Population," *Biometrika,* 31:13–19.

Siegel, A. F. (1995): "Measuring Systematic Risk Using Implicit Beta," *Management Science,* 41:124–128.