PUT–CALL PARITY WITH FUTURES-STYLE MARGINING

STEPHEN A. EASTON

INTRODUCTION

The put–call parity relationship for futures options has been investigated by Ball and Torous (1986); Jordan and Seale (1986); and Followill and Helms (1990). Ball and Torous examined the parity relationship for American futures option, that is:

$$F_t e^{-r(\tau-t)} - X \leq C_t - P_t \leq F_t - Xe^{-r(\tau-t)}. \qquad (1)$$

where

F_t = futures price at time t,

X = exercise price;

C_t = call option price at time t,

P_t = put option price at time t,

r = risk-free rate of interest;

t = the time of the trade; and

τ = the expiry date for all contracts.

They used closing and settlement prices for options on three futures contracts traded on three United States futures exchanges for the period

The author would like to thank Rob Brown, Stephen Chambers, Peter Easton, Jack English, Robert Faff, Barry Goss, Paul Kofman, Paul Lalor, Martin Martens, Patricia McBride, Keith McLaren, Sean Pinder, two anonymous referees, and seminar participants at Macquarie University and Monash University for helpful comments. The study was funded by a grant from the Australian Research Council.

■ *Stephen A. Easton is an Associate Professor in the Department of Accounting and Finance at Monash University.*

The Journal of Futures Markets, Vol. 17, No. 2, 215–227 (1997)

CCC 0270-7314/97/020215-13

from October, 1982 to March, 1985. Close conformity was found between the closing and settlement prices and the parity relationship. However, as noted by Followill and Helms, such a result is not surprising as exchange settlement committees often specify settlement prices according to the models that are being tested.

Jordan and Seale and Followill and Helms examined the parity relationship for European futures options, that is:

$$(F_t - X)e^{-r(\tau - t)} - C_t + P_t = 0 \tag{2}$$

Jordan and Seale used transactions data to examine U.S. treasury bond futures options, and found that departures from the parity relationship were small, with profits generally being confined to floor traders. Followill and Helms used transactions data to examine U.S. gold futures options, and they, too, found evidence of some small departures from the parity relationship.

All of these studies related to markets where there was no futures-style margining of the option contracts. Lieu (1990) derived the put–call parity relationship for futures and futures option contracts where futures-style margining occurs on the option contracts. Specifically, he showed that under this condition the put–call parity relationship is:

$$F_t - X - C_t + P_t = 0 \tag{3}$$

Lieu (1990, p. 334) stated that it was not possible to test this theorem because there were no suitable markets that used futures-style margining for options. However, Chen and Scott (1993) showed that the put–call parity relationship that Lieu derived also applies to options on interest rate futures contracts. Therefore, the relationship could be examined using interest rate futures contracts traded on the London International Financial Futures Exchange.

The relationship derived by Lieu can be tested now by examining contracts traded on the Sydney Futures Exchange since all contracts traded on that exchange, including futures options, are subject to marking-to-market at the end of each day's trading. Further, since January, 1993 time-precise transactions data are maintained by the Exchange. These data are sourced directly from the verbal record of each trade as it occurs in the trading pits. Information provided by the Sydney Futures Exchange states that this data is time-precise to the nearest second.[1]

[1]Prior to 1993, the Sydney Futures Exchange only provided "CHIT" data. These data are sourced from the written records (or "chits") filled in by the traders on the trading floor. They are not time-precise.

The purpose of this article is to use all contracts traded on the Sydney Futures Exchange to examine the parity relationship in a market where futures-style margining is used. This is achieved by using all trades in all futures contracts, together with all trades in the related futures options contracts, from January, 1993 to December, 1994.[2]

A knowledge of the derivation of the put–call parity relationship is required to enable empirical tests of the relationship to be constructed. That relationship is derived, the data are described, the results are discussed, and a summary is provided.

PUT–CALL PARITY WITH FUTURES-STYLE MARGINING

Following Lieu (1990), eq. (3) may be proven as follows.[3] Suppose that this equation does not hold and it is observed in the market at time t that $F_t - X - C_t + P_t > 0$. In these circumstances an arbitrage opportunity is present. If the usual frictionless markets assumptions are made, including the assumption that it is possible to borrow and lend at the risk-free rate of interest, then the transactions needed to exploit this opportunity may be shown as follows.

A portfolio is established on day one by taking a long position in $e^{-r(\tau-t)}$ units of the call option (at price C_t) and short positions in $e^{-r(\tau-t)}$ units of the futures contract (at price F_t) and $e^{-r(\tau-t)}$ units of the put option (at price P_t). On the second day, a portfolio is established by taking a long position in $e^{-r(\tau-(t-1))}$ units of the call option (at price C_{t+1}) and short positions in units $e^{-r(\tau-(t-1))}$ of the futures contract (at price F_{t+1}) and $e^{-r(\tau-(t-1))}$ units of the put option (at price P_{t+1}). On the third day, a portfolio is established by taking a long position in $e^{-r(\tau-(t-2))}$ units of the call option (at price C_{t+2}) and short positions in $e^{-r(\tau-(t-2))}$ units of the futures contract (at price F_{t+2}) and $e^{-r(\tau-(t-2))}$ units of the put option (at price P_{t+2}). This procedure is repeated each day until the expiry of the futures contract.

At the end of the first day, the contracts are marked-to-market and the long position in the call option registers a gain (loss) of $e^{-r(\tau-t)}$ $[C_{t+1} - C_t]$. On the second day, the gain (loss) on the long position in the call option is $e^{-r(\tau-(t-1))}$ $[C_{t+2} - C_{t+1}]$, and on the third day, the gain (loss) is $e^{-r(\tau-(t-2))}$ $[C_{t+3} - C_{t+2}]$. If each of these daily gains (losses) is in-

[2]A wool futures contract and individual share futures contracts are traded also on the Sydney Futures Exchange, but futures options are not listed on these contracts.

[3]Lieu (1990) and Chen and Scott (1993) provide formal proofs of this relationship. Lieu assumed deterministic interest rates, while Chen and Scott showed that the relationship holds using stochastic interest rates.

vested (borrowed) at the risk-free rate of interest until the expiry of the futures contract, the terminal value of the long positions taken in the call option is:

$$\begin{aligned}
&e^{-r(\tau-t)}\,[C_{t+1} - C_t]\,e^{r(\tau-t)} + \\
&e^{-r(\tau-(t-1))}\,[C_{t+2} - C_{t+1}]\,e^{r(\tau-(t-1))} + \\
&e^{-r(\tau-(t-2))}\,[C_{t+3} - C_{t+2}]\,e^{r(\tau-(t-2))} + \\
&\ldots + e^{-r}[C_\tau - C_{\tau-1}]\,e^{r} \\
&= C_\tau - C_t
\end{aligned}$$

Given that $C_\tau = 0$ if $F_\tau < X$, and $C_\tau = F_\tau - X$ if $F_\tau \geq X$, the terminal value of the long positions taken in the call option is $-C_t$ if $F_\tau < X$, and $F_\tau - X - C_t$ if $F_\tau \geq X$.

It may be shown also that the terminal value of the short positions in the futures contract is $F_\tau - F_t$, and the terminal value of the short positions in the put options is $X - F_\tau - P_t$ if $F_\tau < X$ and $-P_t$ if $F_\tau \geq X$. The terminal value of the portfolio is therefore:

$$-C_t - [F_\tau - F_t] - [X - F_\tau - P_t] = -C_t + F_t - X + P_t \text{ if } F_\tau < X,$$

and

$$[F_\tau - X - C_t] - [F_\tau - F_t] - [-P_t] = -X - C_t + F_t + P_t \text{ if } F_\tau \geq X.$$

Therefore, for all F_τ, the terminal value of the portfolio is:

$$F_t - X - C_t + P_t$$

Given that $F_t - X - C_t + P_t$ is positive, an arbitrage profit is made.

Conversely, if it is observed in the market at time t that $F_t - X - C_t + P_t < 0$, then the same analysis is applied by taking short positions in the call option, and long positions in the futures contract and the put option. The terminal value of the portfolio is $-F_t + X + C_t - P_t$. Given that this value is positive, an arbitrage profit is made. Therefore, an arbitrage opportunity is present unless $F_t - X - C_t + P_t = 0$. It should be noted that the assumption of being able to borrow and lend at the risk-free rate of interest results in all interim net cash flows being zero.

DATA

To examine the put–call parity relationship, data in the four major contracts that are traded on the Sydney Futures Exchange are examined.

TABLE I

Number of Trades by Contract on the Sydney Futures Exchange from January, 1993 to December, 1994

	All-Ordinaries Share Price Index	*90-Day Bank Accepted Bills*	*3-Year Treasury Bonds*	*10-Year Treasury Bonds*
Futures contract	375,134	292,209	342,482	513,650
Call option contract	8,536	7,473	6,508	13,483
Call option contract ($F > X$)[a]	2,117	1,698	1,389	2,442
Call option contract ($F \leq X$)	6,419	5,775	5,119	11,041
Put option contract	6,855	8,812	5,809	12,270
Put option contract ($F > X$)	5,752	6,477	4,796	10,620
Put option contract ($F \leq X$)	1,103	2,335	1,013	1,650

[a]The futures price used to categorize options as either in- or out-of-the-money is obtained from the futures contract (with the same expiration month) that trades immediately prior to the option trade.

These are the All-Ordinaries Share Price Index Contract, the 90-Day Bank Accepted Bill Contract, and the 3-Year and 10-Year Treasury Bond Contracts. These data comprise all contracts for which futures options are traded.

The All-Ordinaries Share Price Index contract is based on the value of the Australian Stock Exchange All-Ordinaries Index multiplied by \$25, and is quoted to one full index point. The 90-Day Bank Accepted Bill contract is based on a 90-day bank accepted bill with a face value of \$100,000, and trades are quoted at 100 minus the annual percentage yield to two decimal places. The 3-Year (10-Year) Treasury Bond contract is based on a 3-Year (10-Year) government bond with a face value of \$100,000, offering a coupon rate of 12% per annum payable half yearly. Trades are quoted at 100 minus the annual percentage yield, with a minimum fluctuation of 0.005.

Since January 1993 the Sydney Futures Exchange maintains a trade-by-trade record of the time, price, and volume of every contract traded. These data are sourced directly from the verbal records of each trade as it occurs in the trading pits. The time of the trade is recorded accurately to the nearest second.

Table I shows the *number* of trades from January, 1993 to December, 1994 for each of four contracts. Table I shows that of the total number of trades in these contracts, more than 95% are in the futures contracts. The total number of trades in the call options is similar to the number of trades in the put options. Less than one quarter of the number of trades in call and put options are in-the-money.

TABLE II

Volume of Trades by Contract on the Sydney Futures Exchange from January, 1993 to December, 1994

	All-Ordinaries Share Price Index	*90-Day Bank Accepted Bills*	*3-Year Treasury Bonds*	*10-Year Treasury Bonds*
Futures contract	2,537,130	12,404,161	10,933,246	7,702,736
Call option contract	653,078	758,263	521,122	779,158
Call option contract ($F > X$)[a]	148,321	135,275	112,871	123,299
Call option contract ($F \leq X$)	504,757	622,988	408,251	655,859
Put option contract	571,469	843,418	480,358	706,795
Put option contract ($F > X$)	514,075	652,356	408,744	619,049
Put option contract ($F \leq X$)	57,394	191,062	71,614	87,746

[a]The futures price used to categorize options as either in- or out-of-the-money is obtained from the futures contract (with the same expiration month) that trades immediately prior to the option trade.

Table II is similar to Table I but shows the total *volume* of trades on each of the four contracts. For the All-Ordinaries Share Price Index Contract, 67% of the volume of trades is in the futures contract. For the 90-Day Bank Accepted Bill Contract, this figure is 89%, while for the 3-Year (10-Year) Treasury Bond Contract, this figure is 92 (84) %. The difference between Table I and Table II is due to the greater average volume of contracts per trade for futures options than for the futures contracts. Again, less than one quarter of the volume of trades in call and put options are in-the-money.[4]

Futures options traded on the Sydney Futures Exchange are American, that is, they can be exercised prior to the expiry date. However, Lieu (1990) and Chen and Scott (1993) showed that American futures options with futures-style margining should not be exercised prior to the expiry date. Given that all gains (losses) are registered daily via the marking-to-market of all positions, exercise prior to the expiry date is not rational. Consistent with their analyses, options on the Sydney Futures Exchange

[4]Very few options are priced below their intrinsic value. To examine the extent to which this occurs, an option is deemed to be priced below its intrinsic value if this is the case using both the matching futures price of the trade prior to the option trade, and the matching futures price of the trade following the option trade. For call options, the number of trades at prices less than intrinsic value for each of the four contracts are as follows: All-Ordinaries Share Price Index, 30; 90-Day Bank Accepted Bills, 37; 3-Year Treasury Bonds, 1; and 10-Year Treasury Bonds, 11. For put options, these numbers are 28, 108, 0, and 33, respectively. Across both put and call options, and across all four contracts, on only 20 occasions (out of a total of 69,746 trades) is the option price exceeded by its intrinsic value by more than $150. (Brochures provided by the Sydney Futures Exchange suggest that the transaction cost associated with buying and exercising an option, and then novating the futures position would exceed $150.)

are very rarely exercised prior to the expiry date. For example, during 1994, only 6 options on the All-Ordinaries Share Price Index Contract are exercised prior to the expiry date, 40 options on the 90-Day Bank Accepted Bill Contract are exercised prior to the expiry date, while 40 (300) options on the 3-Year (10-Year) Treasury Bond Contract are exercised prior to the expiry date.[5]

The data screening procedure that is used exploits the fact that approximately 95% of the total number of trades occur in the futures contract. Given the heavy trading in that contract, a *set* of futures prices may be used to examine the effects of nonsimultaneity between option prices and the prices of the underlying futures contract. If a particular pricing relationship is found using *each of the set* of futures prices matched to put and call options with respect to exercise price and expiration month, then it may be inferred that that relationship is robust to the effects of nonsimultaneity.

The following procedure is used to match put and call options with the same exercise price and expiration month, with a *set* of transactions in the underlying futures contract. Firstly, all prices for matching contracts (that is, contracts with the same exercise price and expiration month), are collected where:

$$F_1, P_2, F_3 \ldots\ldots F_{n-2}, C_{n-1}, F_n,$$

or

$$F_1, C_2, F_3 \ldots\ldots F_{n-2}, P_{n-1}, F_n, \tag{4}$$

where F_1 is the last trade in the futures contract that precedes the trades in the put and call options, P is the trade in the put option, C is the trade in the call option, $F_3 \ldots F_{n-2}$ are all the trades in the futures contract that occur between the trades in the options, F_n is the first trade in the futures contract that follows the trades in the options, and where the time period between the trading of F_1 and F_n is less than T. T takes on three values, namely 90, 120, and 150 seconds.[6] It should be noted that to be included in the sample, it is not necessary for there to be any trades in the futures contract *between* the trades in the options (that is, $F_3 \ldots F_{n-2}$ need not exist).

For each set of contracts, F_H is defined as the maximum futures price from F_1 to F_n, including $F_3 \ldots F_{n-2}$. Similarly, F_L is defined as the minimum futures price from F_1 to F_n, including $F_3 \ldots F_{n-2}$.

[5]This information is provided by Stephen Chambers from the Sydney Futures Exchange.

[6]The minimum value for T is set equal to 90 seconds to achieve a reliable sample size of matching contracts.

Given the heavy trading in the futures contract, this procedure is designed to allow for changes in conditions affecting equilibrium that may occur within the time period T. If, for example, a trading strategy involves taking a short position in the futures contract, and if that strategy is profitable using F_L, then it may infer that eq. (3) is not an accurate representation of the pricing relationship. Similarly, if a trading strategy involves taking a long position in the futures contract, and if that strategy is profitable using F_H, then it may, again, infer that eq. (3) is not an accurate representation of the pricing relationship.

As a second step in the screening procedure, when there is more than one trade in either the put or the call option, only those matching contracts that minimized the time period between the trading of the put and the call option are selected.

RESULTS

Table III presents an analysis of the extent to which eq. (3) provides an accurate representation of the pricing relationship. For this analysis, F_1 from eq. (4) is used as the futures price. (The analysis is repeated using F_n. The results are virtually identical and, therefore, are not reported.) For each contract, when all observations are considered, there is no evidence of any systematic violation of eq. (3). For example, when the time period between F_1 and F_n is less than 90 seconds, there are 93 observations for the All-Ordinaries Share Price Index contracts. Of these observations, 35 provide positive values for $F_1 - X - C + P$, while 44 provide negative values. Table III shows that for each contract and for each value of T, the median value of $F_1 - X - C + P$ is zero. The precise parity relationship is observed in over 15% of cases for the All-Ordinaries Share Price Index contract, and between one quarter and one third of cases for the other three contracts.

But systematic violations of the parity relationship are found when the observations are categorized as to whether the futures price is greater than, less than, or equal to, the exercise price. To test this relationship, a 2×2 chi-square statistic is calculated by comparing the expected and actual numbers of observations in four categories.[7] These categories are: where $F_1 > X$ and $F_1 - X - C + P > 0$; where $F_1 > X$ and $F_1 - X - C + P < 0$; where $F_1 \leq X$ and $F_1 - X - C + P > 0$; and where $F_1 \leq X$ and $F_1 - X - C + P < 0$. If put–call parity as shown in eq. (3) is an accurate representation of the pricing relationship, then no relationship

[7]To minimize the impact of outliers, only nonparametric statistics are used in this analysis.

TABLE III

Test of Put–Call Parity Relationship with Futures-Style Margining

T (*in Seconds*)		*All-Ordinaries Share Price Index*			*90-Day Bank Accepted Bill*			*3-Year Treasury Bond*			*10-Year Treasury Bond*		
		$F_1 > X$	$F_1 \leq X$	*All*	$F_1 > X$	$F_1 \leq X$	*All*	$F_1 > X$	$F_1 \leq X$	*All*	$F_1 > X$	$F_1 \leq X$	*All*
90	$F_1 - X - C + P > 0$[a]	19[b]	16	35	25	15	40	49	21	70	100	68	168
	$F_1 - X - C + P = 0$	8	6	14	17	22	39	31	40	71	64	63	127
	$F_1 - X - C + P < 0$	11	33	44	9	24	33	30	63	93	87	106	193
	Total	38	55	93	51	61	112	110	124	234	251	237	488
	χ^2[c]		7.10[d]			9.02[d]			22.78[d]			7.51[d]	
120	$F_1 - X - C + P > 0$	27	20	47	34	21	55	64	33	97	120	77	197
	$F_1 - X - C + P = 0$	12	8	20	22	32	54	36	51	87	77	73	150
	$F_1 - X - C + P < 0$	21	39	60	13	32	45	38	76	114	97	126	223
	Total	60	67	127	69	85	154	138	160	298	294	276	570
	χ^2		5.37[e]			10.77[d]			22.37[d]			12.70[d]	
150	$F_1 - X - C + P > 0$	34	21	55	42	32	74	70	37	107	140	86	226
	$F_1 - X - C + P = 0$	15	11	26	30	38	68	43	55	98	95	79	174
	$F_1 - X - C + P < 0$	26	49	75	18	47	65	40	84	124	111	136	247
	Total	75	81	156	90	117	207	153	176	329	346	301	647
	χ^2		9.41[d]			11.92[d]			25.32[d]			13.71[d]	

[a] F_1 is the last trade in the futures contract that precedes the trades in the put and call options. X = exercise price, C = call option price, and P = put option price.
[b] The number of observations where the conditions on $F_1 - X$ and $F_1 - X - C + P$ are met.
[c] χ^2 is a 2×2 chi-square statistic calculated by comparing observations where $F_1 - X > 0$ or $F_1 - X \leq 0$ with observations where $F_1 - X - C + P > 0$ or $F_1 - X - C + P < 0$.
[d,e] Indicates significance at the 0.01 (0.05) level.

between these variables should be found. Deviations of $F_1 - X - C + P$ from zero should be random. However, in all except one case the chi-square statistic is significantly different from zero at the 0.01 level.[8]

The analysis in Table III suggests that when $F_1 > X$, then, on average, $F_1 - X - C + P > 0$. Where $F_1 > X$, call options are in-the-money, and put options are out-of-the-money. Since in such cases, on average $F_1 - X - C + P > 0$, this suggests that in-the-money call options tend to be underpriced when compared with the parity relationship, while out-of-the-money put options tend to be overpriced when compared with the parity relationship. Table III also suggests that when $F_1 \leq X$, then, on average, $F_1 - X - C + P < 0$. Where $F_1 < X$, call options are out-of-the-money, and put options are in-the-money. Since in such cases, on average, $F_1 - X - C + P < 0$, this suggests that out-of-the-money call options tend to be overpriced when compared with the parity relationship, while in-the-money put options tend to be underpriced when compared with the parity relationship. In summary, therefore, in-the-money call and put options tend to be underpriced when compared with the parity relationship, while out-of-the-money call and put options tend to be overpriced when compared with the parity relationship.

The analysis in Table III suggests that when $F_1 > X$, then, on average, $F_1 - X - C + P < 0$. The arbitrage strategy to exploit such a relationship is as follows. Where $F_1 > X$, a long position is taken in the call option and short positions are taken in the futures contract and the put option. To allow for the effect of nonsimultaneity between the trades in the put option, the call option, and the futures contract, the short position in the futures contract is taken at the least favorable futures price, namely F_L. Where $F_1 \leq X$, a short position is taken in the call option and long positions are taken in the futures contract and the put option. In these cases, to allow for the effect of nonsimultaneity, the long position in the futures contract is taken at F_H. To the extent that nonsimultaneity might have an impact on the results, this procedure will result in a bias *against* finding a positive relationship between $F_1 - X$ and $F - X - C + P$. To further minimize the effects of nonsimultaneity, all observations are deleted where $F_H - F_L$ exceeds one tick size. The results from this analysis are provided in Table IV.

After allowing for the effect of nonsimultaneity, the extent of the apparent violations from the parity relationship are significantly reduced. But for the All-Ordinaries Share Price Index contract and the 3-Year Treasury Bond contract, there remains some evidence of a positive relation-

[8]Analyses are conducted also to examine whether systematic violations of the parity relationship are associated with the time to maturity of the options. No evidence of an association is found.

TABLE IV

Test of Put–Call Parity Relationship with Futures-Style Margining

T (*in seconds*)		*All-Ordinaries Share Price Index*			*90-Day Bank Accepted Bill*			*3-Year Treasury Bond*			*10-Year Treasury Bond*		
		$F_I > X$	$F_I \leq X$	*All*	$F_I > X$	$F_I \leq X$	*All*	$F_I > X$	$F_I \leq X$	*All*	$F_I > X$	$F_I \leq X$	*All*
90	$F - X - C + P > 0$[a]	16[b]	14	30	23	16	39	41	24	65	83	83	166
	$F - X - C + P = 0$	6	6	12	16	25	41	33	44	77	68	65	133
	$F - X - C + P < 0$	13	28	41	12	20	32	35	55	90	99	85	184
	Total	35	48	83	51	61	112	109	123	232	250	233	483
	χ^2[c]		3.35			3.24			8.84[d]			0.51	
120	$F - X - C + P > 0$	24	18	42	29	25	54	53	40	93	99	95	194
	$F - X - C + P = 0$	9	7	16	23	32	55	40	55	95	78	72	150
	$F - X - C + P < 0$	23	33	56	17	28	45	44	64	108	114	100	214
	Total	56	58	114	69	85	154	137	159	296	291	267	558
	χ^2		2.48			2.50			5.28[e]			0.21	
150	$F - X - C + P > 0$	30	20	50	36	37	73	58	44	102	113	105	218
	$F - X - C + P = 0$	12	8	20	29	39	68	46	59	105	96	78	174
	$F - X - C + P < 0$	25	38	63	24	40	64	48	72	120	128	107	235
	Total	67	66	133	89	116	205	152	175	327	337	290	627
	χ^2		4.61[e]			1.93			6.28[e]			0.32	

[a]X = exercise price, C = call option price, and P = put option price. F_1 is the last trade in the futures contract that precedes the trades in the put and call options. Where $F_1 > X$, F_L is used to calculate $F - X - C + P$. Where $F \leq X$, F_H is used to calculate $F - X - C + P$. Observations are only included where $F_H - F_L$ is less than or equal to one tick size.

[b]The number of observations where the conditions on $F_1 - X$ and $C + P$ are met.

[c]χ^2 is a 2 × 2 chi-square statistic calculated by comparing observations where $F_1 - X > 0$ or $F_1 - X \leq 0$ with observations where $F - X - C + P > 0$ or $F - X - C + P < 0$.

[d,e]Indicates significance at the 0.01 (0.05) level.

TABLE V

Test of Put–Call Parity Relationship with Futures-Style Margining (Quintiles Formed on the Basis of $F_1' - X$)[a]

All-Ordinaries Share Price Index (χ^2 = 10.51[f])[b]					
Median $F_1 - X$	\$1225	\$300	0	−\$300	−\$1263
$F_1 - X - C + P > 0$[c]	15	11	14	7	8
$F_1 - X - C + P = 0$	6	8	5	3	4
$F_1 - X - C + P < 0$	10	12	12	21	20
N	31	31	31	31	32
Median $F_1 - X - C + P$	0	0	0	−\$50	−\$45
90-Day Bank Accepted Bill (χ^2 = 10.39[f])					
Median $F_1 - X$	\$193	\$72	−\$24	−\$108	−\$239
$F_1 - X - C + P > 0$	18	21	12	11	12
$F_1 - X - C + P = 0$	12	14	16	16	10
$F_1 - X - C + P < 0$	11	7	13	15	19
N	41	42	41	42	41
Median $F_1 - X - C + P$	0	0	0	0	0
3-Year Treasury Bond (χ^2 = 34.12[e])					
Median $F_1 - X$	\$418	\$140	−\$56	−\$197	−\$608
$F_1 - X - C + P > 0$	26	34	19	20	8
$F_1 - X - C + P = 0$	22	20	15	20	21
$F_1 - X - C + P < 0$	17	12	32	26	37
N	65	66	66	66	66
Median $F_1 - X - C + P$	0	\$141	0	0	−\$142
10-Year Treasury Bond (χ^2 = 25.85[e])					
Median $F_1 - X$	\$1614	\$708	\$78	−\$476	−\$1125
$F_1 - X - C + P > 0$	47	57	51	45	26
$F_1 - X - C + P = 0$	36	36	37	33	32
$F_1 - X - C + P < 0$	46	37	41	52	71
N	129	130	129	130	129
Median $F_1 - X - C + P$	0	0	0	0	−\$73

[a]For the analysis presented in this table, T is set equal to 150 seconds.
[b]χ^2 is a 5 × 2 chi-square statistic calculated by comparing observations in the quintiles formed on the basis of $F_1 - X$ with observation where $F_1 - X - C + P > 0$ and $F_1 - X - C + P < 0$.
[c]F_1 is the last trade in the futures contract that precedes the trades in the put and call options. X = exercise price, C = call option price, and P = put option price.
[d]The number of observations where the conditions on $F_1 - X$ and $F - X - C + P$ are met.
[e,f]Indicates significance at the 0.01(0.05) level.

ship between $F_1 - X$ and $F - X - C + P$. This is especially true for the 3-Year Treasury Bond contract. The precise parity relationship is observed in over 13% of cases for the All-Ordinaries Share Price Index contract and between one quarter and one third of the cases for the other three contracts.

To examine the parity relationship further the samples for each contract are broken into quintiles based on the rank of $F_1 - X$.[9] The results from this analysis are provided in Table V. The results provided in Table V also show a strong relationship between $F_1 - X$ and $F_1 - X - C + P$. For all four contracts, a 5×2 chi-square statistic is calculated by comparing observations in the quintiles formed on $F_1 - X$ with observations where $F_1 - X - C + P > 0$ and $F_1 - X - C + P < 0$. In all cases, this statistic is significantly different from zero at the 0.05 level. While this apparent violation of the parity relationship is persistent, its magnitude is small. Within all of the quintiles across the four contracts, the median violation only ranges from zero to \$142. Brochures provided by the Sydney Futures Exchange suggest that the transaction costs associated with arbitrage trading would exceed \$150.

SUMMARY

In this article all contracts for which futures options are traded on the Sydney Futures Exchange are used to examine the put–call parity relationship where futures-style margining occurs for the option contracts. All trades that occur between January, 1993 and December, 1994 are studied. The precise parity relationship was observed in between 15% and one third of all cases, depending on the contract. The only systematic violation detected is that in-the-money put and call options are found to be underpriced by a small amount when compared with the parity relationship.

BIBLIOGRAPHY

Ball, C. A., and Torous, W. N. (1986): "Futures Options and the Volatility of Futures Prices," *Journal of Finance,* September, 41(4): 857–870.

Chen, R. R., and Scott, L. (1993): "Pricing Interest Rate Futures Options with Futures-Style Margining," *The Journal of Futures Markets,* 13(1): 15–22.

Followill, R. A., and Helms, B. P. (1990): "Put–Call–Futures Parity and Arbitrage Opportunity in the Market for Options on Gold Futures Contracts," *The Journal of Futures Markets,* 10(4): 339–352.

Jordan, J. V., and Seale, W. E. (1986): "Transactions Data Tests of Minimum Prices and Put–Call Parity for Treasury Bond Futures Options," *Advances in Futures and Options Research,* 1, Part A: 63–87.

Lieu, D. (1990): "Option Pricing with Futures-Style Margining," *The Journal of Futures Markets,* 10(4): 327–338.

[9]Tables III and IV provide an analysis of a two-way partitioning of the sample based on the rank of $F_1 - X$. Quintiles are selected for this analysis to provide a finer partitioning of the sample. Analyses using a three-way partitioning and quartiles yields virtually identical results and are, therefore, not reported.