Fractional Dynamics in International Commodity Prices

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INTRODUCTION

The analysis and forecasting of commodity prices continue to be important not only for utilizing commodity markets, but also for understanding their relationships to financial markets in a global context. Recent research in this area has been concerned with the underlying time series or structural character of prices. Can prices be better explained by classical integration order discovery or by fractal estimation which discloses underlying nonlinear dependence? Analytical approaches to answering such questions typically distinguish between long-term and short-term dependence. The long memory, or long-term dependence, property describes the correlation structure of a series at long lags. If a series exhibits long memory, there is persistent temporal dependence even between distant observations. Such series are characterized by distinct but nonperiodic cyclical patterns. The presence of long memory dynamics, which is a special form of nonlinear dynamics, gives nonlinear dependence in the

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first moment of the distribution and, hence, a potentially predictable component in the series dynamics. Fractionally integrated processes can give rise to long memory [Mandelbrot (1977a); Granger and Joyeux (1980); Hosking (1981)]. On the other hand, the short memory, or short-term dependence, property describes the low correlation structure of a series. For short memory series correlations among observations at long lags become negligible. Standard autoregressive moving average processes can exhibit short-term but not long-term dependence. The presence of fractal structure in asset prices raises issues regarding the statistical testing of pricing models based on standard statistical methods, theoretical and linear econometric modeling of asset prices, and pricing efficiency and rationality.

Much of the recent research on price behavior is directed to the analysis of long memory and fractional dynamics in common stock returns and other financial asset derivatives. A lesser amount of research is devoted to fractional order in commodity spot or futures prices. None of the latter studies, however, investigate fractional order across a wide range of spot commodity prices. The origins of fractional dynamics in commodity prices derive from nonlinear dynamic models, which are used often to provide an explanation of the complex, irregular movements found in spot prices, e.g., see Brock (1988); Chavas and Holt (1991); Jensen and Urban (1984); and Mackey (1989). Studies that extend cyclical analysis to embody fractional behavior include Burton (1993); Boldrin and Woodford (1990); and Yang and Brorsen (1991).

Such models are found useful because they recognize that demand and supply forces are more important for spot price determination. That is, dependence or reversals in spot prices can occur because of supply lags in annual planting periods or in longer perennial gestation periods. Business cycles, some of which have long memory, also can induce similar fluctuations in commodity demand and hence in spot prices. Early evidence of such price reversals appears in Houthakker (1961); Kendall (1953); and Labys and Granger (1970).

In fact, Houthakker's early work in this area stimulated Mandelbrot (1963a,b) to extend research on random walk in commodity prices to that of fractal structures. This permitted price behavior to be evaluated not only in the context of short-term random increments, but also in terms of fractional order associated with cyclic long-term memory. It would appear fruitful, therefore, to employ long memory tests to provide a better explanation of fluctuations in commodity spot prices, to provide insights into the kinds of nonlinear models that can be used for spot price explanations and forecasts, and also to lay the foundation for later more comprehensive tests of commodity futures prices. Among all financial markets, commodity futures represent the only assets linked to real physical markets.

The purpose of this study is to test for long memory across a variety of commodity spot prices. The data set consists of prices for 21 commodities traded on internationally important markets and a summary commodity price index. Also, in contrast to previous studies which employ rescaled-range (R/S) analysis to test for long memory in commodity prices, the approach taken here to evaluate long memory stems from developments in fractional order tests by Granger and Joyeaux (1980); Hosking (1981); Geweke and Porter-Hudak (1984); and Sowell (1990). The specific testing methodology employed is the semi-nonparametric procedure suggested by Geweke and Porter-Hudak (1984), which can potentially improve estimation efficiency over the nonparametric R/S approach. To provide robust evidence regarding the low-frequency properties of the sample commodity prices, the series are also subjected to two unit root tests that differ in their null hypothesis (unit root versus stationarity); these tests are the Phillips–Perron and Kwiatkowski, Phillips, Schmidt, and Shin unit-root tests. The obtained evidence suggests that characterization of the low-frequency properties for commodity prices based on integer integration tests can lead to erroneous inferences. A fractional order of integration is found for some commodity prices which appears to be robust. For other commodity prices additional flexible representations should be considered.

The plan of the article is as follows. It reviews the background to fractional price dynamics; presents the fractional order test; presents the data; discusses empirical results, and concludes with a summary of the evidence and suggestions for future research.

FRACTIONAL PRICE DYNAMICS

The origins of fractional dynamic explanations of commodity price behavior are diverse and can only be briefly summarized here. Some nonlinear dynamic models can generate deterministic chaos, while others can generate periodic behavior in the form of repeated sequences or reversals. Early explanations of commodity price behavior did not immediately address chaotic behavior, but were restricted to the linear analysis of autocorrelation or the lack of it. More recent work is concerned with the discovery of random walk in prices and its trend variants. Important studies in this regard are reviewed in DeCoster, Labys, and Mitchell (1992). The shift to nonlinear analysis can be attributed largely to Mandelbrot

(1963a,b) who replaced the Gaussian probability laws underlying random walk with those termed "stable Paretian." It is in this context that Frank and Stengos (1989) investigated the Martingale Hypothesis using an approach of Sims (1984) as well as a chaos-based approach. Although Frank and Stengos were not able to reject the Martingale Hypothesis in a series of standard econometric tests involving daily and weekly silver and gold prices, they did provide correlation dimension-based evidence of the presence of nonlinear structure.

Other nonlinear dynamic models are based on the search for autocorrelation or dependence in commodity prices, that is, they are more concerned with nonstationary trends and price reversals in the form of short- and long-term cycles. As recently interpreted by Tomek (1991) and Tomek and Myers (1993), some of this work can be identified with the structural analysis of commodity markets as compared to the above and other studies using reduced form analysis. Commodity cash or physical market prices are expected to exhibit price reversals or cycles, rather than to follow a random walk. This makes models emphasizing some form of autocorrelation in mean as well as in variance more useful for commodity market analysis. Tomek cites two reasons for this possibility. The first is that inventory holding is essential to commodity market adjustments and since stocks must be nonnegative, prices which are closely related to stock adjustments must also reflect this dependence, though nonlinearly. The second is that commodity prices are affected by many forms of information; and the costs of acquiring and acting on new information implies that prices are not likely to follow a random walk, e.g., see Grossman and Stiglitz (1980) and Sunder (1992). Persistence in price behavior can also occur in rational commodity markets, e.g., Labys, Badillo, and Lesourd (1995).

These two views of nonlinear commodity price behavior would seem to have converged in the form of investigations of fractional or fractal dynamics. The price behavior implied here would reflect variations in random walk behavior associated with short-term frequencies, but also the cyclic long-term dependence associated with long-term frequencies. Mandelbrot (1977a,b) earlier termed this behavior *fractal* and explained a fractal structure to be characterized by long-term dependence and nonperiodic cycles. More recently, several applications are made to commodity cash and futures prices, e.g., see Booth, Kaen, and Koveos (1982); Cheung and Lai (1993); Helms, Kaen, and Rosenman (1984); and Peterson, Ma, and Ritchey (1992). Most other fractal investigations are concerned with other forms of financial and futures prices. These include Aydogan and Booth (1988); Cheung (1993); Cheung, Lai, and Lai

(1994); Cheung and Lai (1995); Greene and Fielitz (1977); Fang, Lai, and Lai (1994); Booth, Kaen, and Koveos (1982b); Lo (1991); and Lee and Mathur (1992).

It is the purpose of this study to extend the latter experiments by investigating long memory with a much broader group of cash commodity price series in an attempt to try to understand the frequent and substantial instability in commodity price fluctuations. Contrary to previous studies which employed the classical or modified R/S analysis to analyze dynamic behavior in cash commodity prices, the fractional models considered are the ones investigated by Mandelbrot (1977a), Granger and Joyeux (1980), Hosking (1981), and Geweke and Porter-Hudak (1983). The long-term dependence in these models is captured by a single parameter, called the fractional differencing parameter. The testing method employed here is the GPH test suggested by Geweke and Porter-Hudak (1983). As confirmed with Monte Carlo evidence by Cheung (1993), the GPH testing methodology is robust to short-term dependence, nonnormal innovations, and variance nonstationarity. These robustness properties are of particular importance here since most of the sample series exhibit short-term dependence, their unconditional distribution is fat-tailed, or leptokurtic, and exhibit conditional heteroscedasticity effects. The GPH testing procedure can allow for a rich pattern of interactions between short-term and long-term dynamics and is, therefore, well-suited to capture long memory dynamics in the sample series. Additionally, the commodity price series are subjected to two integer order unit root tests, which differ in the way the null hypothesis is set up.

FRACTIONAL INTEGRATION TEST

A flexible and parsimonious way to model short-term and long-term behavior of time series is by means of an autoregressive fractionally integrated moving average model. A time series $\mathbf{y} = \{y_1, \ldots, y_T\}$ follows an autoregressive fractionally integrated moving average process of order (p,d,q), denoted by ARFIMA(p,d,q), if

$$\Phi(L)(1 - L)^d y_t = \Theta(L)\varepsilon_t, \qquad \varepsilon_t \sim iid(0, \sigma_\varepsilon^2) \tag{1}$$

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\Theta(L) = 1 + \vartheta_1 L + \ldots + \vartheta_q L^q$, and $(1 - L)^d$ is the fractional differencing operator defined by

$$(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k - d)L^k}{\Gamma(-d)\Gamma(k + 1)} \tag{2}$$

with $\Gamma(\cdot)$ denoting the gamma, or generalized factorial, function. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The ARIMA model is a special case of the ARFIMA model. The stochastic process $\mathbf{y}$ is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < 1/2$. The process is nonstationary for $d \geq 1/2$, because it possesses infinite variance, i.e., see Granger and Joyeux (1980).

ARFIMA series exhibit long-term persistence and short memory. The effect of the differencing parameter d on observations widely separated in time decays hyperbolically as the lag increases, thus describing the high-correlation structure of the series. At the same time, the temporal effects of the autoregressive and moving average parameters, $\Phi(L)$ and $\Theta(L)$, decay exponentially, describing the low-correlation structure of the series.[1] Assuming that $-1/2 < d < 1/2$ and $d \neq 0$, Hosking (1981) showed that the correlation function, $\rho(\cdot)$, of an ARFIMA process is proportional to j^{2d-1} as $j \to \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $j \to \infty$. This behavior is contrary to the faster, geometric decay of a stationary ARMA process. The ARFIMA process exhibits long memory for $0 < d < 1/2$, short memory for $d = 0$, and intermediate memory for $-1/2 < d < 0$.[2] This can be seen through the cumulative impulse response c of future values of the series to a unit innovation. For $d < 1$ the series is mean-reverting in that c at infinite horizon, denoted by c_∞, is zero even though it is not covariance stationary for $1/2 < d < 1$. For the unit-root case, $d = 1$, c_∞ is finite and nonzero; and for $d > 1$, c_∞ is infinite.

The existence of a fractional order of integration can be determined by testing for the statistical significance of the sample differencing parameter d, which is also interpreted as the long memory parameter. To estimate d and to perform hypothesis testing, the semi-nonparametric procedure suggested by Geweke and Porter-Hudak [GPH (1983)] is employed. The authors obtain an estimate of d based on the slope of the

[1]For ARFIMA processes the spectral density function, denoted by $f_y(\xi)$, is proportional to ξ^{-2d} as $\xi \to 0$, where ξ is the angular frequency. This contrasts with the behavior of the usual ARIMA $(p,1,q)$ model ($d = 1$) in which the spectral density has to behave like ξ^{-2} as $\xi \to 0$. Therefore, relaxing the assumption of an integer integration order gives rise to a richer range of spectral behavior near the zero frequency.

[2]A covariance stationary process is said to exhibit long memory if

$$\sum_{j=-n}^{n} |\rho(j)| \to \infty \qquad \text{as } n \to \infty$$

where $\rho(j)$ is the autocorrelation at lag j. The condition that the autocorrelations have an infinite sum suggests that autocorrelations at long lags are not negligible. If the autocorrelation sequence converges to a finite sum, the process is said to exhibit short memory or "antipersistence" [Mandelbrot (1977a, p. 232)]. It can be shown that stationary autoregressive moving average (ARMA) processes do not exhibit long memory.

spectral density function around the angular frequency $\xi = 0$. More specifically, let $I(\xi)$ be the periodogram of **y** at frequency ξ defined by

$$I(\xi) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\xi}(y_t - \bar{y})\right|^2 \tag{3}$$

Then the spectral regression is defined by

$$\ln\{I(\xi_\lambda)\} = \beta_0 + \beta_1 \ln\left\{\sin^2\left(\frac{\xi_\lambda}{2}\right)\right\} + \eta_\lambda, \qquad \lambda = I, \ldots, v \tag{4}$$

where $\xi_\lambda = 2\pi\lambda/T$ $(\lambda = 0, \ldots, T - l)$ denotes the harmonic ordinates of the sample, and T is the number of observations, $v = g(T) \ll T$.

Applying regularity conditions to the low-frequency ordinates used in the above spectral regression v, satisfied by, for example, $v = T^\mu$ with $0 < \mu < 1$, the negative of the OLS estimate of the slope coefficient gives a consistent and asymptotically normal estimate of d. With the proper choice of v, the asymptotic distribution of the estimate of d depends on neither the order of the ARMA component nor the distribution of the error term of the ARFIMA process. The number of low-frequency periodogram ordinates used in the spectral regression must be chosen. Improper inclusion of medium- or high-frequency periodogram ordinates will contaminate the estimate of d. At the same time, too small a regression sample will lead to imprecise estimates. Estimates of d based on $g(T) = T^{0.50}$, $T^{0.55}$, and $T^{0.60}$ are reported and the known theoretical asymptotic variance of the regression error, $\pi^2/6$, is used to compute the variance of the estimate of d. Granger and Joyeux (1980) and GPH (1983) have shown that ARFIMA models provide superior out-of-sample forecasts compared to more conventional procedures, especially over longer horizons.

Since the GPH test can be used as a unit root test, it is applied to the first differences of the series to ensure that stationarity and invertibility are achieved. The differencing parameter in the first-differenced data is denoted by $\tilde{d}$ in which case the fractional differencing parameter for the level series is $d = 1 + \tilde{d}$. The null unit-root hypothesis $d = 1$ $(\tilde{d} = 0)$ can be tested against fractional order alternatives.

DATA AND EMPIRICAL ESTIMATES

The price series of interest stem from 21 commodities traded on internationally important markets as well as the summary UNCTAD commodity price index (Ucam). All of the series stem from spot or physical

TABLE I

Summary Statistics

Commodity Price Series	*Mean*	*Standard Deviation*	*Median*	*Skewness*	*Kurtosis*
Ucam	0.2042E-2	0.0267	0.0000	0.862[a]	5.146[a]
Aluminum	0.2709E-2	0.0578	0.0005	−0.382[a]	3.349[a]
Cocoa	0.1362E-2	0.0687	−0.0036	0.473[a]	0.575[a]
Coffee	0.2305E-2	0.0633	0.0000	0.404[a]	3.815[a]
Copper	0.2487E-2	0.0692	0.0004	−0.237[c]	2.617[a]
Corn	0.2009E-2	0.0570	0.0000	0.036	4.115[a]
Cotton	0.1624E-2	0.0558	0.0025	−2.872[a]	31.206[a]
Gold	0.8422E-2	0.0688	0.0014	0.827[a]	6.565[a]
Jute	−0.0670E-2	0.0698	0.0000	−0.123	12.897[a]
Lead	0.1569E-2	0.0643	0.0016	0.274[b]	2.757[a]
Petroleum	0.7383E-2	0.1057	−0.0576	5.916[a]	65.993[a]
Rice	0.1573E-2	0.0460	0.0000	0.298[b]	2.519[a]
Rubber	−0.1563E-2	0.0518	−0.0031	0.415[a]	2.690[a]
Silver	0.4118E-2	0.0811	0.0000	0.669[a]	13.057[a]
Soybeans	0.2534E-2	0.0777	0.0014	−0.419[a]	12.166[a]
Sugar	0.2901E-2	0.1176	−0.0012	0.300[b]	0.723[a]
Tea	0.0496E-2	0.0930	−0.0027	0.178	19.110[a]
Tin	0.1963E-2	0.0463	0.0011	−1.070[a]	6.570[a]
Tungsten	0.0644E-2	0.0707	0.0000	0.720[a]	3.625[a]
Wheat	0.1918E-2	0.0472	0.0000	1.694[a]	16.805[a]
Wool	0.0775E-2	0.0697	0.0000	0.510[a]	20.950[a]
Zinc	0.2981E-2	0.0389	0.0000	0.751[a]	9.354[a]

Notes: Kurtosis refers to excess kurtosis. Commodity prices are in the form of returns.
[a,b,c]Indicate statistical significance at the 1, 5, and 10% levels, respectively.

markets and can be termed cash prices. These markets function competitively and none of the price series represent secondary or trade unit value prices except for jute. All of the price series are monthly; the majority begin in January, 1960 and end in June, 1994. Several of the series such as crude oil and gold do not begin until January, 1972. Further details on these series appear in the Appendix. Monthly prices, although not as frequent as daily prices, still represent a series in which both short-term and long-term memory can be observed.

Table I reports summary statistics for these prices transformed to returns series for the 22 series in the sample. Only for gold is the sample mean (positive) statistically, significantly different from zero. All returns series exhibit dependence in the fourth cumulants (kurtosis) and/or third cumulants (skewness).

Further evidence regarding temporal dependence in the first and second moments of the commodity returns series is reported in Table II. All returns series exhibit serial dependence in the conditional mean. To

TABLE II

Serial Dependence Tests

Commodity Price Series	*BP(6)*	*BP(12)*	*AR-BP(6)*	*AR-BP(12)*	*LM(6)*	*LM(12)*
Ucam	65.79	77.67	0.37	7.77	49.50	54.88
	(0.000)	(0.000)	(0.999)	(0.802)	(0.000)	(0.000)
Aluminum	29.05	35.44	11.27	18.27	37.59	53.96
	(0.000)	(0.000)	(0.080)	(0.107)	(0.000)	(0.000)
Cocoa	44.86	50.26	2.34	6.68	8.37	12.15
	(0.000)	(0.000)	(0.885)	(0.877)	(0.211)	(0.433)
Coffee	50.75	71.16	2.032	15.35	20.63	33.39
	(0.000)	(0.000)	(0.916)	(0.222)	(0.002)	(0.000)
Copper	44.94	56.52	2.28	7.40	26.07	28.16
	(0.000)	(0.000)	(0.891)	(0.829)	(0.000)	(0.000)
Corn	61.49	82.65	0.99	11.18	15.38	20.16
	(0.000)	(0.000)	(0.985)	(0.513)	(0.017)	(0.053)
Cotton	34.38	40.15	5.51	8.61	2.59	3.17
	(0.000)	(0.000)	(0.479)	(0.735)	(0.857)	(0.994)
Gold	5.74	25.32	3.77	18.41	23.93	23.97
	(0.452)	(0.013)	(0.707)	(0.103)	(0.000)	(0.020)
Jute	28.86	35.28	9.74	14.37	83.77	84.21
	(0.002)	(0.000)	(0.135)	(0.277)	(0.000)	(0.000)
Lead	21.95	26.43	4.07	6.97	70.20	70.85
	(0.001)	(0.009)	(0.666)	(0.859)	(0.000)	(0.000)
Petroleum	20.42	21.79	5.65	6.54	0.095	0.264
	(0.002)	(0.039)	(0.462)	(0.886)	(0.999)	(0.999)
Rice	132.65	137.29	10.52	12.70	3.17	11.74
	(0.000)	(0.000)	(0.104)	(0.390)	(0.787)	(0.466)
Rubber	56.64	60.57	0.85	6.10	41.85	58.96
	(0.000)	(0.000)	(0.990)	(0.910)	(0.000)	(0.000)
Silver	33.38	50.36	10.24	19.76	63.53	83.37
	(0.000)	(0.000)	(0.114)	(0.071)	(0.000)	(0.000)
Soybeans	15.74	27.08	7.72	13.99	17.29	21.63
	(0.015)	(0.007)	(0.258)	(0.301)	(0.008)	(0.041)
Sugar	42.85	47.03	18.51	31.41	41.37	42.70
	(0.000)	(0.000)	(0.005)	(0.001)	(0.000)	(0.000)
Tea	43.81	59.10	7.434	16.30	35.15	36.22
	(0.000)	(0.000)	(0.282)	(0.177)	(0.000)	(0.000)
Tin	63.81	67.13	5.76	9.54	42.64	43.62
	(0.000)	(0.000)	(0.449)	(0.656)	(0.000)	(0.000)
Tungsten	83.54	96.58	2.508	14.19	25.88	28.22
	(0.000)	(0.000)	(0.867)	(0.288)	(0.000)	(0.005)
Wheat	54.71	83.42	0.751	17.88	3.368	46.80
	(0.000)	(0.000)	(0.710)	(0.119)	(0.701)	(0.000)
Wool	11.73	16.04	0.36	4.84	37.44	41.82
	(0.068)	(0.18)	(0.999)	(0.963)	(0.000)	(0.000)
Zinc	76.01	84.69	2.32	10.11	2.35	8.005
	(0.000)	(0.000)	(0.887)	(0.605)	(0.883)	(0.784)

Notes: Commodity prices are transformed to returns. BP(*k*) is the Box–Pierce test statistic for autocorrelation of order *k*. AR–BP(*k*) is the Box–Pierce test statistic for autocorrelation of order *k* applied to the AR-filtered futures returns series. The AR representations for the commodity returns series are selected based on the Schwartz information criterion (the maximum lag order allowed is 12) and are of the following order for each series listed in order of their appearance in the table: 6, 1, 2, 1, 2, 3, 1, 0[1], 1, 1, 1, 1, 5, 2[3], 0[2], 1, 2[4], 1, 2[4], 1[2], 5, and 1 (the number in brackets is the actual AR order used in the estimation process for gold, silver, soybeans, tea, tungsten, and wheat as the residuals obtained for the SIC-based autoregression are not serially uncorrelated). LM(*k*) is Engle's (1982) LM test statistic for ARCH of order *k*. Marginal significance levels are given in parentheses.

capture the dynamics in the first moments, an autoregressive (AR) model is fit to each of the series with the lag length being chosen by the Schwartz information criterion (SIC). Serial dependence tests are then applied to the residuals obtained from passing each series through the AR filter to determine any remaining dependence in the conditional mean as well as in the second moments. The residual series are serially uncorrelated for all commodity returns series.[3] Significant variation in the second moments is detected by Engle's (1982) Lagrange multiplier test for autoregressive conditional heteroscedasticity (ARCH) for all commodity returns series except cocoa, cotton, petroleum, rice, and zinc.

The testing strategy for determining the low frequency properties of the sample series is as follows. First, the time series are subjected to two unit-root tests, both of which allow for integer orders of integration only, but they differ in how they establish the null hypothesis. Then, the fractional integration test, which allows the order of integration to take any value in the real line, is applied to the series. More specifically, the time series properties of the data are first investigated by means of the Phillips–Perron [Phillips (1987); Phillips and Perron (PP, 1988); and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS, 1992)] unit-root tests. The KPSS test is an alternative approach to the standard unit root tests, like the PP test, in which the unit-root null hypothesis is tested against the alternative of stationarity. In the KPSS test, stationarity (level or trend) is the null hypothesis to be tested against the alternative of a unit root. This test serves as a complement to the PP test. The combined use of the PP test and the KPSS test for a particular series produces the following alternatives:

i. Rejection by the PP test and failure to reject by the KPSS test provides evidence in favor of wide-sense stationarity; the series is $I(0)$.

ii. Failure to reject by the PP test and rejection by the KPSS test supports that the series is integrated of order one; the series is $I(1)$.

iii. Failure to reject by both PP and KPSS tests shows that the data are not sufficiently informative with respect to the low-frequency properties of the series; and

iv. Rejection by both PP and KPSS tests suggests that a series is not well represented as either $I(1)$ or $I(0)$ and alternative parameterizations need to be considered, such as, explosive roots, "flexible" trend alter-

[3]For gold, silver, soybeans, tea, tungsten, and wheat returns series, the reported diagnostic test statistics are not based on residuals obtained from fitting an SIC order-based autoregression because such a lag structure does not fully account for dependencies in the conditional mean of the series. The lag order for these returns series is increased until serial correlation up to twelfth order is fully accounted for in the corresponding residual vectors. This strategy results in autoregressions of order 1, 3, 2, 4, 4, and 2 for gold, silver, soybeans, tea, tungsten, and wheat returns series, respectively.

TABLE III

Phillips–Perron (PP) Unit-Root Test Statistics

	$l = 12$	
Commodity Price Series	$Z(\alpha)$	$Z(t_a)$
Ucam	−9.070	−2.152
Aluminum	−13.160	−2.486
Cocoa	−4.677	−1.349
Coffee	−4.100	−1.273
Copper	−21.000	−3.227[c]
Corn	−11.421	−2.351
Cotton	−10.845	−2.129
Gold	−4.555	−1.632
Jute	−21.358	−3.190[c]
Lead	−10.075	−2.010
Petroleum	−4.692	−1.675
Rice	−11.663	−2.336
Rubber	−14.103	−2.748
Silver	−5.941	−1.593
Soybeans	−15.419	−2.712
Sugar	−13.128	−2.548
Tea	−24.170[b]	−3.511[b]
Tin	−2.204	−0.761
Tungsten	−2.534	−0.821
Wheat	−9.707	−2.125
Wool	−13.515	−2.334
Zinc	−12.335	−2.241

Notes: The Phillips–Perron statistics test for a unit root in the univariate time-series representation for the log levels of each of the commodity price series against a trend-stationary alternative. *l* stands for the order of serial correlation allowed in constructing the test statistics. Inference is generally robust to the order of serial correlation allowed. The lag window suggested by Newey and West (1987) is used to ensure positive semidefiniteness. For more details on the tests see Phillips (1987), and Phillips and Perron (1986). The critical values are as follows [Fuller (1976); Dickey and Fuller (1981)] with rejections of the null hypothesis indicated by large absolute values of the statistics:

Critical Values	$Z(\alpha)$	$Z(t_a)$
10%	*−18.3*	*−3.12*
5%	*−21.8*	*−3.41*
1%	*−29.5*	*−3.96*

[a,b,c]Indicate statistical significance at the 1, 5, and 10% levels, respectively.

natives with exogenous or endogenous breaks, and stationarity around a nonlinear trend.

Table III reports the PP unit-root test results. The unit-root null hypothesis, which is tested against a trend-stationary alternative, is not rejected for any of the commodities in the sample except for tea. For tea, the unit-root null hypothesis is strongly rejected raising doubts about the first-differencing procedure to render the series stationary. Table IV reports the KPSS test results. Strong evidence in favor of the trend-sta-

TABLE IV

Kwiatkowski–Phillips–Schmidt–Shin (KPSS) Test for Trend Stationarity

	KPSS Test Statistic η_τ			
Commodity Price Series	$l = 6$	$l = 12$	$l = 18$	$l = 24$
Ucam	0.552[a]	0.313[a]	0.228[a]	0.184[b]
Aluminum	0.328[a]	0.199[b]	0.157[b]	0.139[c]
Cocoa	0.955[a]	0.535[a]	0.381[a]	0.302[a]
Coffee	1.015[a]	0.567[a]	0.404[a]	0.320[a]
Copper	0.252[a]	0.153[b]	0.119	0.104
Corn	0.601[a]	0.347[a]	0.255[a]	0.210[b]
Cotton	0.744[a]	0.434[a]	0.320[a]	0.260[a]
Gold	0.803[a]	0.453[a]	0.328[a]	0.265[a]
Jute	0.120[c]	0.079	0.070	0.072
Lead	0.598[a]	0.346[a]	0.256[a]	0.211[b]
Petroleum	0.826[a]	0.469[a]	0.337[a]	0.269[a]
Rice	0.572[a]	0.329[a]	0.244[a]	0.204[b]
Rubber	0.475[a]	0.276[a]	0.207[b]	0.172[b]
Silver	0.911[a]	0.512[a]	0.367[a]	0.293[a]
Soybeans	0.698[a]	0.415[a]	0.307[a]	0.249[a]
Sugar	0.445[a]	0.265[a]	0.190[b]	0.157[b]
Tea	0.433[a]	0.262[a]	0.196[b]	0.162[b]
Tin	1.031[a]	0.574[a]	0.405[a]	0.308[a]
Tungsten	1.145[a]	0.635[a]	0.448[a]	0.353[a]
Wheat	0.595[a]	0.339[a]	0.247[a]	0.201[b]
Wool	0.213[b]	0.125[c]	0.096	0.083
Zinc	0.309[a]	0.184[b]	0.139[c]	0.117

Notes: $\eta_\tau = \dfrac{T^{-2}\sum_{t=1}^{T} S_t^2}{s^2(l)}$ is the test statistic for the null hypothesis of trend stationarity, where $S_t = \sum_{i=1}^{T} e_i$, $t = 1, 2, \ldots, T$ (partial sum process of the residuals) with $\{e_t\}_1^T$ being the residuals from the regression of the series (log levels of commodity prices) on an intercept and a linear time trend, and $s^2(l)$ is a consistent estimate of the "long-run variance". The estimator used here is of the form

$$s^2(l) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2\,T^{-1}\sum_{s=1}^{l} w(s, l)\sum_{t=s+1}^{T} e_t\, e_{t-s}$$

where $w(s, l)$ is an optimal lag window and l is the order of serial correlation allowed. The triangular window $w(s, l) = 1 - s/l + 1$ [Newey and West (1987)], which guarantees the non-negativity of $s^2(l)$, is used here. The test is an upper-tail test and the critical values are 0.216, 0.146, and 0.119 at the 1, 5, and 10% levels, respectively [Kwiatkowski, Phillips, Schmidt, and Shin (1992)]

[a,b,c]Indicate statistical significance at the 1, 5, and 10% levels, respectively.

tionary null hypothesis is provided for jute and wool. Depending upon the autocorrelation lag structure allowed, evidence in favor of trend stationarity is obtained for copper and zinc as well.

If the PP and KPSS test results are now combined, the evidence can be interpreted as follows. For jute, wool, and probably for copper and zinc, the data are not informative enough to discriminate between the unit-root and stationarity hypotheses (failure to reject the null hypothesis in both PP and KPSS tests). The $I(1)$ or $I(0)$ representation appears to be inadequate for tea because both PP and KPSS tests reject their respective nulls. For the remaining commodities, the evidence appears to be robust and in favor of a unit root.

Possible fractional alternatives in the order representation of the time series are now investigated. To estimate the fractional order of integration, the GPH test is employed. The GPH test relaxes the arbitrary integer restriction in the order of integration imposed in the PP and KPSS tests by allowing the integration order to undertake any value in the real set of numbers (fractional integration). By doing so, the knife-edged $I(1)$ and $I(0)$ distinction in the PP and KPSS unit-root tests is avoided. Table V reports the empirical estimates for the fractional differencing parameter $\tilde{d} = 1 - d$ as well as the test results regarding its statistical significance based on the GPH test. To ensure that the stationarity and invertibility conditions are met, the GPH test is applied to the first differences of the (log) levels of the series, that is, the returns series. The $\tilde{d}$ estimates for $v = T^{0.50}, T^{0.55}$, and $T^{0.60}$ are reported to check the sensitivity of the results as to the choice of v. To test the statistical significance of the $\tilde{d}$ estimates, two-sided ($\tilde{d} = 0$ versus $\tilde{d} \neq 0$) as well as one-sided ($\tilde{d} = 0$ versus $\tilde{d} < 0$ and $\tilde{d} = 0$ versus $\tilde{d} > 0$) tests are performed. The known theoretical variance of the regression error $\pi^2/6$ is imposed in the construction of the t-statistic for $\tilde{d}$.[4]

Evidence in favor of a fractional order of integration is obtained for six series. In particular, the order of integration is significantly less than unity (unit-root case) for copper, soybeans, and tea.[5] For these commodities, the level series are not stationary (since $d = 1 + \tilde{d}$ exceeds 0.5) but they are mean reverting. They have a cumulative impulse response at infinite horizon, c_∞, equal to zero, suggesting that a shock to the series will eventually vanish but at a very slow rate. For Ucam, gold, and wool, the integration order of the level series exceeds unity in a statistically significant fashion.[6] The original series (levels) are neither kstationary nor mean reverting, with the corresponding returns series exhibiting long memory.

In addition, the previously obtained results of the PP and KPSS tests can be better understood by interpreting them on the basis of the GPH

[4]The results do not change materially if the t-statistics are based on the empirical error variance estimates.

[5]Note that for soybeans, the value of $\tilde{d}$ corresponding to $v = T^{0.60}$ is smaller (in absolute value) than that corresponding to $v = T^{0.50}$ and $v = T^{0.55}$. This may indicate possible "contamination" of the estimation result as more periodogram ordinates are included in the spectral regression.

[6]A similar remark to the one for soybeans in the previous footnote applies to gold.

TABLE V

Empirical Estimates for the Fractional-Differencing Parameter $\tilde{d}$

Commodity Price Series	$\tilde{d}$ (0.50)	$\tilde{d}$ (0.55)	$\tilde{d}$ (0.60)
Ucam	0.126	0.578	0.476
	(0.698)	(3.869)[a,d]	(3.798)[a,d]
Aluminum	−0.053	0.127	0.052
	(−0.255)	(1.020)	(0.370)
Cocoa	0.173	−0.004	−0.001
	(0.956)	(−0.031)	(−0.009)
Coffee	0.256	0.141	0.156
	(1.418)	(1.944)[f]	(1.248)
Copper	−0.405	−0.231	−0.251
	(−2.237)[b,h]	(−1.548)[i]	(−2.004)[b,h]
Corn	0.072	−0.058	−0.051
	(0.389)	(−0.379)	(−0.406)
Cotton	−0.265	−0.097	−0.082
	(−1.464)[i]	(−0.651)	(−0.654)
Gold	0.408	0.385	0.066
	(1.947)[c,h]	(2.262)[c,h]	(0.461)
Jute	−0.237	−0.190	0.033
	(−1.131)	(−1.115)	(0.233)
Lead	0.174	0.117	0.134
	(0.962)	(0.788)	(1.070)
Petroleum	0.028	0.009	−0.197
	(0.133)	(0.529)	(−1.379)[i]
Rice	0.090	0.096	0.158
	(0.499)	(0.647)	(1.266)
Rubber	0.095	0.215	0.111
	(0.525)	(1.412)[f]	(0.888)
Silver	0.044	0.170	0.057
	(0.246)	(1.141)	(0.460)
Soybeans	−0.298	−0.300	−0.141
	(−1.594)[i]	(−1.960)[b,h]	(−1.109)
Sugar	0.042	0.056	0.087
	(0.236)	(0.376)	(0.697)
Tea	−0.281	−0.173	−0.270
	(−1.555)[i]	(−1.157)	(−2.156)[b,h]
Tin	−0.021	−0.040	0.062
	(−0.116)	(−0.272)	(0.494)
Tungsten	0.227	0.198	0.111
	(1.256)	(1.326)[e]	(0.886)
Wheat	0.076	0.175	0.040
	(0.424)	(1.171)	(0.324)
Wool	0.201	0.337	0.278
	(1.114)	(2.257)[b,e]	(2.224)[b,e]
Zinc	−0.003	−0.020	0.073
	(−0.199)	(−0.134)	(0.582)

Notes: $\tilde{d}(0.50)$, $\tilde{d}(0.55)$, and $\tilde{d}(0.60)$ give the $\tilde{d}$ estimates corresponding to the GPH spectral regression of sample size $\nu = T^{0.50}$, $\nu = T^{0.55}$, and $\nu = T^{0.60}$, respectively. The *t*-statistics are given in parentheses and are constructed imposing the known theoretical error variance of $\pi^2/6$.

[a,b,c]Indicate statistical significance for the null hypothesis, $\tilde{d} = 0 (d = 1)$ against the alternative $\tilde{d} \neq 0 (d \neq 1)$ at the 1, 5, and 10% levels, respectively.

[d,e,f]Indicate statistical significance for the null hypothesis $\tilde{d} = 0 (d = 1)$ against the one-sided alternative $\tilde{d} > 0 (d > 1)$ at the 1, 5, and 10% levels, respectively.

[g,h]Indicate statistical significance for the null hypothesis $\tilde{d} = 0 (d = 1)$ against the one-sided alternative $\tilde{d} < 0 (d < 1)$ at the 1, 5, and 10% levels, respectively.

results. For copper, and especially for wool, the lack of power by the PP and KPSS tests to reject their respective nulls can be explained by the presence of a fractional order of integration. This is not true for zinc and jute. Actually, the GPH result confirms the PP result that both series are $I(1)$ and alternative time series models may have to be considered to adequately represent their low-frequency properties. For Ucam, gold, and soybeans, both the PP and KPSS tests lead one to conclude erroneously that the series are $I(1)$. However, the GPH test documents that the order of integration is, in fact, fractional. For tea, the contradiction between the PP and KPSS tests (both tests strongly reject their respective nulls) can be decided on the basis of the fractional alternative. These results suggest that characterization of the low-frequency properties of commodity prices on the basis of a single test could lead to erroneous inference. They also suggest that first-differencing the data may not always be the appropriate transformation to model commodity price behavior. Finally, the obtained evidence confirms that a more appropriate model is needed to explain price reversals than a pure martingale process for at least some of the commodities.

A Closer Look at the Fractionally Integrated Commodity Prices

In this subsection, the robustness of the fractal structure in the aforementioned six commodity price series to short-term dependencies, nonstationarity in the mean, and transient observations are investigated. Through extensive Monte Carlo simulations, Cheung (1993) showed that the GPH test is robust to moderate ARMA components, ARCH effects, and shifts in the variance. However, possible biases of the GPH test against the no long memory null hypothesis were discovered. In particular, the GPH test was found to be biased toward finding long memory ($\tilde{d} > 0$) in the presence of infrequent shifts in the mean of the process and large AR parameters (0.7 and higher). A similar point was observed by Agiakloglou, Newbold, and Wohar (1993) and Sowell (1992). Additionally, the GPH test is biased toward finding $\tilde{d} < 0$ in the presence of a large MA component (-0.9 and below). The presence of these bias-inducing features of the data in the six sample series for which a fractional integration order was found is now investigated.

To see if there is a shift in mean, the returns series for the three commodity price series with long memory are graphed. As Figures 1

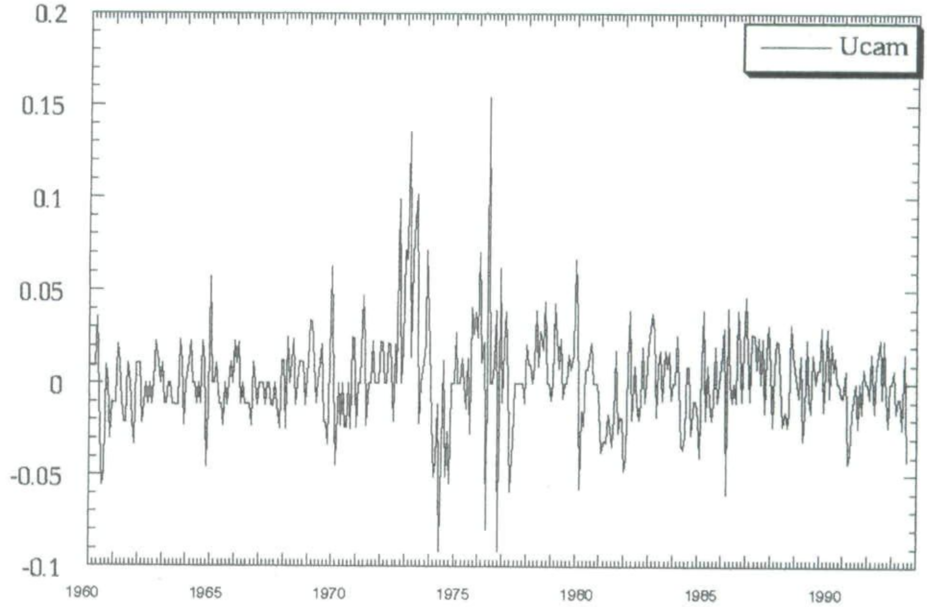

FIGURE 1
Ucam returns.

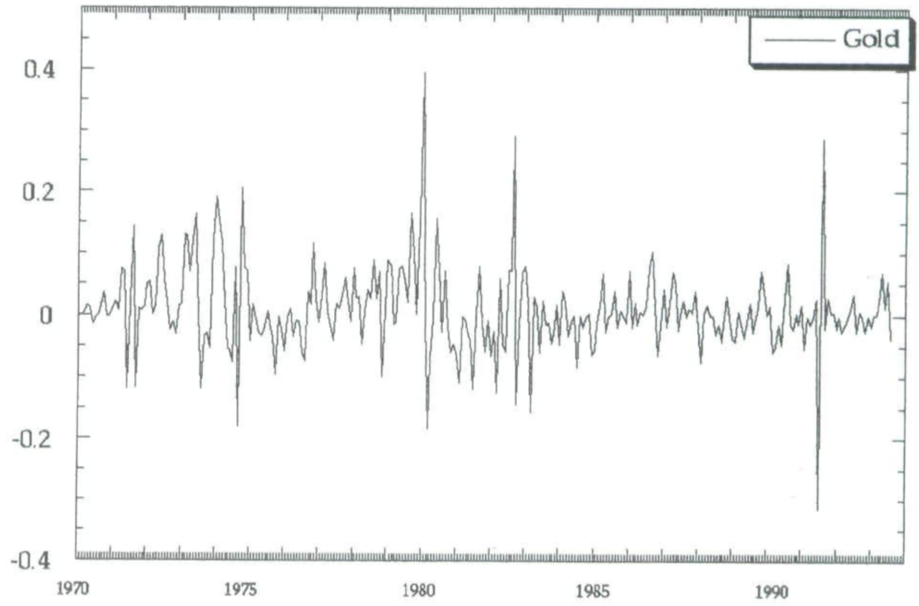

FIGURE 2
Gold returns.

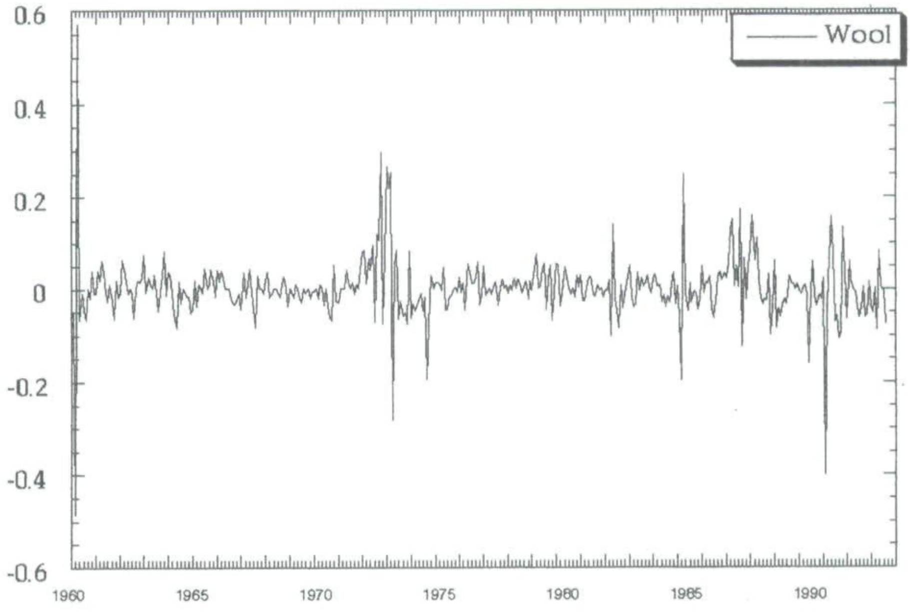

FIGURE 3
Wool returns.

through 3 indicate, there is no evidence that the data generating process for these three commodity price series experiences a shift in the mean. Therefore, the evidence of long memory for Ucam, gold, and wool cannot be a spurious artifact of changes in the mean of the series. To examine the possibility of spurious inference in favor of long-term persistence due to strong dependencies in the data, the previously estimated AR models for the three price series in question are examined. The largest AR parameter for Ucam, gold, and wool is 0.174, 0.097, and 0.122, respectively. In no case is the short-term dependence strong enough to possibly induce bias in the GPH test results. Neither shift in mean nor strong dependence are therefore responsible for finding long memory in the Ucam, gold, and wool price series.

The possibility that a large MA component may bias the GPH test toward finding $\tilde{d} < 0$ in the copper, soybeans, and tea price series is also investigated. To do so, a MA model is chosen for these three commodity series on the basis of SIC (a maximum order of 12 is allowed) and the value of the largest MA component is examined. The chosen MA models are of order 1, 2, and 4, respectively. The largest MA components are 0.401, -0.040, and -0.251 for the copper, soybeans, and tea price series, respectively. These values are well outside the bias-inducing MA

TABLE VI

Empirical Estimates for the Fractional-Differencing Parameter $\tilde{d}$ with Omitted Observations

Commodity Price Series	$\tilde{d}$ *(0.50)*	$\tilde{d}$ *(0.55)*	$\tilde{d}$ *(0.60)*
Copper	−0.090	0.036	−0.021
	(−0.706)	(0.327)	(−0.219)
Gold	0.759	0.532	0.133
	(3.603)[a,d]	(2.813)[a,d]	(0.721)
Soybeans	−0.366	−0.352	−0.330
	(1.814)[c,h]	(−2.433)[b,g]	(−2.985)[a,g]
Tea	−0.346	−0.268	−0.381
	(−2.107)[c,h]	(−2.107)[b,h]	(−2.770)[a,g]
Wool	0.164	0.190	0.264
	(0.855)	(1.329)[f]	(1.517)[f]

Observations corresponding to returns greater than 0.25 and less than −0.25 are omitted from estimation. The resulting sample sizes are 401 observations (out of 403) for copper; 279 (out of 283) for gold, 375 (out of 384) for soybeans, 394 (out of 403) for tea, and 387 (out of 403) for wool. There are no missing observations for Ucam. For explanation of rest of table see notes in Table V.

values and cannot therefore be held responsible for the finding of mean reversion in the aforementioned series.

Cheung and Lai (1993) found evidence of long memory in weekly gold returns series, but the persistence disappeared when only a few observations corresponding to major political events in the Middle East, together with the Hunt event, in late 1979 were omitted. This suggests the possibility that a few transient observations, which are unrepresentative of the underlying data generating process, can lead to misleading or incorrect statistical inference regarding long-term persistence in a series. To account for the effect of possible outliers, the following strategy is implemented for the six series for which evidence of a fractional integration order is found: observations corresponding to returns greater than 0.25 and lower than −0.25 are omitted and the GPH test is re-applied to the remaining observations for each of the series. The implicit and rather ad hoc assumption in doing so is that unusually high or low returns may represent potential outliers in the data and their effects on test results is indicative, to a certain extent, of the robustness properties of the evidence. There are zero such possible outliers for Ucam, 2 for copper, 4 for gold, 9 for soybeans and tea, and 16 for wool.

Table VI reports the fractional differencing parameter estimates for the five returns series for which there are omitted observations.[7] As can

[7]Similar evidence is obtained if the upper and lower bounds for the returns series are set at 0.30 and −0.30, respectively.

be seen, inference is robust to the presence of unusually high or low observations for gold, soybeans, and tea returns. The unstable evidence regarding long memory in gold returns found in Cheung and Lai (1993) is not confirmed here. It must be noted though that the data set used by Cheung and Lai differs from the one used here in both sample period and frequency of observation. The evidence in support of long memory somewhat weakens for wool returns. This might not be very surprising as quite a few observations are omitted in reestimating the fractional differencing parameter. The fractal structure in the copper series completely disappears. The two influential observations have magnitude -0.271 and -0.304 and occurred in 1966:9 and 1968:4, respectively. These changes represent the strong price corrections that were inevitable, given the extent to which copper prices had been driven upwards by low copper stock levels, demand pressures due to the war in Vietnam, and supply uncertainty in Zambia [Prain (1975)]. Even though these extreme observations may be realizations of the data generating process, the evidence of fractional dynamics in copper prices should be interpreted with caution. With the exception of copper, the evidence of fractal structure in several commodity prices appears to be robust to the presence of potential outliers.

Finally, the possibility of the need to have a time series representation more general than an ARFIMA model to completely describe the fractional behavior of the commodity price series is investigated. Figure 4 graphs the first 84 autocorrelation coefficients of the logarithms (of the levels) of the six commodity prices for which evidence of fractional integration was found. The correlograms exhibit a slow decline in the autocorrelation function. One interesting feature is the sinusoidal behavior of the correlogram for tea prices, which contrasts with the rather monotonic decline in the autocorrelation coefficients for the other commodity prices in question. A more complete description of cyclical components in the autocorrelation function for tea prices necessitates the use of a model that can allow for long memory harmonic behavior. One possible candidate is the Gegenbauer autoregressive moving average (GARMA) model considered by Gray, Zhang, and Woodward (1989), which makes use of the generating function of the Gegenbauer polynomials. The general form of the GARMA (p,u,λ,q) model, $\lambda \neq 0$, is given by

$$\Phi(L)(1 - 2uL + L^2)^{\lambda} y_t = \Theta(L)\varepsilon_t \tag{5}$$

which reduces to the ARFIMA model for $u = 1$ and $\lambda = d/2$. Provided that all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle, the process y is a long memory process if $0 < \lambda < 1/4$ when $u = \pm 1$ or $0 < \lambda < 1/2$

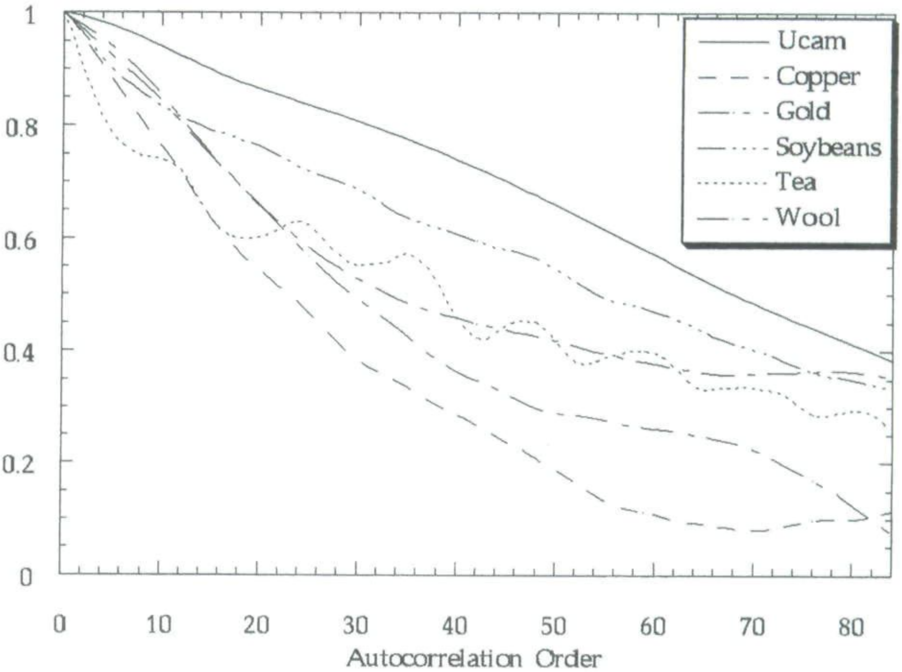

FIGURE 4
Correlogram for the Fractal commodity price series.

when $|u| < 1$. The autocorrelation function, $\rho(\cdot)$, of the GARMA process for $0 < \lambda < 1/2$ and $|u| < 1$ is proportional to $j^{2\lambda-1}\sin(\pi\lambda - j\omega_0)$ as $j \to \infty$, where ω_0 is the Gegenbauer frequency.

The significance of the extension to a GARMA model is the inclusion of periodic or quasi-periodic data in the long memory model.[8] It appears to be appropriate to estimate a GARMA model for the tea price series to account for the periodic component in its autocorrelation function. Since virtually no work has systematically considered the estimation of the parameters of the GARMA model, this possibility is not pursued here. It must also be noted that the GARMA model might prove to be useful for time series in which long memory is not apparent.

CONCLUSIONS

The low frequency properties of cash prices are investigated for several commodities with particular emphasis on the discovery of fractal structure. Several conclusions emerge from the analysis. First, at the empirical

[8]While the ARFIMA model has a peak in the spectrum at $f = 0$, the GARMA process can model long-term periodic behavior for any frequency $0 \leq f \leq 0.5$.

level, fractional orders of integration are confirmed for more than several commodity price series. Second, these orders sometimes vary among the series; these differences in memory lengths can help in constructing appropriate theoretical nonlinear models. Third, econometric time series models such as ARFIMA can be constructed to take advantage of the discovered fractional integers, where possible. And fourth, improved interpretation of long and short memory of futures prices can be made based on the new understanding of corresponding spot markets.

The results which suggest fractional orders of integration for a number of commodities confirm that commodity spot markets are likely to exhibit reversals generated by nonlinear processes. Fractional dynamics is an interesting form of dynamics characterized by irregular cyclic fluctuations with long-term dependence. By providing statistically significant evidence of fractal structure with long memory in several series highlighting the similarities and differences in dynamic behavior among the sample series, long memory is confirmed for a group of commodity price series. These results do not preclude the possibility of long memory presence for the rest of the price series. The fractional differencing method employed may have low power to detect long-term dependence, given the limited span of data available.

These findings further reveal that the discovered fractional orders also vary among different commodities. This is to be expected, since the processes generating the price movements vary considerably among commodities. That is, price fluctuations of agricultural annual crops are often of a shorter run nature, being driven by frequent changes in weather conditions. The price series of agricultural perennial crops often depend more on the gestation period of plantings, typically inducing longer term price reversals. And finally, price cycles of mineral commodities are not only affected by longer run lags in exploration and capital formation but also shorter run lags linked to business cycles. In the particular case here, the Ucam index, and prices for gold and wool, which is an industrial raw material, are more related to global business cycles of a medium or long-term nature. However, mean reversions in copper, soybean, and tea prices are more likely to represent short-term market turnarounds. Such strong differences in price cycle behavior among commodities have been found by Labys and Perrin (1976) and by Labys, Badillo, and Lesourd (1995).

Another conclusion is that the discovery of fractal dynamics for spot commodity prices calls into question linear models and invites the development of nonlinear pricing models at the theoretical level to account for fractional behavior. Mandelbrot (1971) observes that in the presence of long memory, the arrival of new market information cannot be fully arbitraged away and martingale models of asset prices cannot be obtained from arbitrage. The demand and supply conditions underlying commodity market behavior can be translated into a nonlinear paradigm capable of explaining market and price adjustments. Earlier possible theoretical bases for that paradigm are noted in works by Brock (1988); Chavas and Holt (1991); Jensen and Urban (1984); and Mackey (1989). They suggest how the traditional cobweb model can be nonlinearized to explain the chaotic dynamics and low frequency properties of commodity prices. The long memory property in the commodity returns series may also reflect the statistical property of fundamental factors underlying the series; they may be generated by an economic process that contains long-term dependence. Diebold and Rudebusch (1989); Porter-Hudak (1990); and Shea (1991) found that economic variables such as levels of output, money supply, and interest rates contain fractal structure. Also, statistical inferences concerning commodity pricing models based on standard testing procedures may not be appropriate in the presence of fractal dependence [Sowell (1990); Yajima (1985)].

The discovery of fractional orders of integration suggests possibilities for constructing nonlinear econometric models for improved price forecasting performance, especially over longer forecasting horizons. The nonlinear model construction suggested is that of an ARFIMA representation, which represents a flexible and parsimonious way to model both the short- and long-term dynamical properties of the series. Granger and Joyeux (1980) discussed the forecasting potential of such nonlinear models and Geweke and Porter-Hudak (1983) have confirmed this by showing that ARFIMA models provide more reliable out-of-sample forecasts than do more conventional procedures. However, it must be noted that estimation of an ARFIMA model, when appropriate, does not automatically imply improvements in forecasting accuracy in empirical samples. The impact of long memory is likely to be considerably further into the future and the adjustment to equilibrium will generally take a lot of time to complete. Therefore, any improvement in forecasting accuracy over the benchmark models may only be apparent in the very long run. For example, Cheung (1993) established the presence of long memory in four major spot currency rates, but the estimated ARFIMA models did not improve on the random walk in out-of-sample results. In general, the development of strategies to capitalize on fluctuating prices and nonperiodic cycles is a formidable task.

Finally, the present results can be employed to better interpret any existing price fluctuations as well as the efficiency of corresponding commodity futures markets. The conclusions of traditional tests of "efficient" markets hypotheses hang precariously on the presence or absence of long memory. Therefore, the finding of fractal structure reopens the debate on pricing efficiency and market rationality. As a consequence, the discovery of the long memory property in futures returns series may reflect the statistical property of fundamental factors underlying the spot prices; they may be generated by commodity markets processes that contain long-term dependence.

APPENDIX: DATA DESCRIPTION AND SOURCES

Unctad Commodity Price Index

This index (Ucam) represents a weighted average of some 50 primary commodity export prices of developing countries (United Nations, *Monthly Commodity Price Bulletin,* Geneva), 1960.01–1993.07.

Wheat

U.S. No. 2, Hard Red winter (ordinary) cash price, f.o.b. Gulf (International Wheat Council), 1960.01–1993.07.

Maize/Corn

U.S. Yellow, No. 3, Average Cash Price, Chicago (USDA), 1960.01–1993.07.

Rice

Thailand, White, 5% broken, end-of-month cash price, f.o.b. Bangkok, including export duty, Washington, D.C. (IMF Secretariat), 1960.01–1993.07.

Soybeans

U.S. Yellow No. 1, Average cash price, Chicago (USDA, Washington, D.C.), 1960.01–1993.07.

Coffee

Average of daily cash prices (Secretariat of the International Coffee Organization, London). Robustas, weighted average of ex-dock New York (60%), Angola Ambriz 2 BB, Uganda standard, 1960.01–1993.07.

Cocoa

Average of daily prices of the nearest three active future trading months on the London Terminal Market and on the New York Coffee, Sugar, and Cocoa Exchange at time of the London close. Article 26 of the International Cocoa Agreement, 1986 (International Cocoa Organization, London), 1960.01–1993.07. This is not strictly a cash price, but it is a useful surrogate.

Tea

London, auction cash prices, all tea (*Monthly Statistical Summary,* International Tea Committee, London), 1960.01–1993.07.

Cotton

Medium, U.S. Memphis Territory (medium staple) cash prices, Middling 1-3/32. Prior to July, 1981: S.M. 1-1/16 (USDA, Washington, D.C.), 1960.01–1993.07.

Wool

UK64's, dry-combed basis cash prices, (New Zealand Wool Marketing Corporation, Clacton-on-Sea, United Kingdom), 1960.01–1993.07.

Jute

Raw Bangladesh, B.W.D., f.o.b. Chittagong-Chalna, market cash prices (*The Public Ledger,* Watford, United Kingdom). Prior to March, 1980:-minimum export price (Bangladesh Ministry of Jute), 1960.01–1993.07. This also is not a cash price, but it is a useful surrogate.

Rubber

Singapore, f.o.b. in bales, No. 1 RSS. cash prices, closing quotations (*The Public Ledger,* Watford, United Kingdom), 1960.01–1993.07.

Aluminum

London Metal Exchange, high grade, cash prices. From February, 1970 to December, 1978: virgin ingot, 99.5% purity, c.i.f. Europe, Prior to January, 1970: virgin ingot, spot London (*Metal Bulletin,* London), 1970.01–1993.07.

Copper

London Metal Exchange, electrolytic wire bars, high grade, cash prices (*Metal Bulletin,* London), 1960.01–1993.07.

Lead

London Metal Exchange settlement and cash seller's price in warehouse excluding duty, main United Kingdom ports; Purity 99.97% Pb (*Lead and Zinc Statistics,* International Study Group, London), 1960.01–1993.07.

Zinc

London Metal Exchange, settlement and cash seller's price in warehouses excluding duty, main United Kingdom ports; virgin zinc, high grade (*Lead and Zinc Statistics,* International Study Group, London), 1960.01–1993.07.

Tin

Ex-works cash price, Kuala Lumpur market (ITC reference price since July 4, 1972). Tin trade was suspended between October 24, 1985 to end of January, 1986 (*Metals Week,* New York), 1960.01–1993.07.

Tungsten

Wolfram, c.i.f. European ports concentrates, cash prices, basis minimum 65% WO_3 (*Metal Bulletin,* London), 1960.01–1993.08.

Gold

United Kingdom, 99.5% fine, London afternoon fixing, average of daily cash prices (*Metal Bulletin,* London), 1970.01–1993.07.

Silver

Handy & Harman, 99.9% grade refined, average of daily cash price quotations, New York (*Metal Bulletin,* London), 1960.01–1993.07.

Crude Petroleum

Average of Dubai, United Kingdom Brent and Alaska N. Slope crude cash prices, reflecting relatively equal consumption of medium, light, and heavy crudes worldwide. Dubai: Fateh 32 API, spot, f.o.b. Dubai; United Kingdom: Brent Bland 38 API, spot, f.o.b. United Kingdom ports; United States: Alaskan N. Slope 27 API, spot, f.o.b. U.S. Gulf of Mexico ports, 1970.01–1993.07.

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