HEDGING RATIOS AND CASH/FUTURES MARKET LINKAGES

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INTRODUCTION

The use of futures instruments in hedging strategies has generated an extensive literature and such strategies have been widely adopted in a number of practical settings. If futures prices follow a martingale process, the optimal hedge ratio will be that which minimizes the risk (variance) of a portfolio comprising the futures contract and the underlying asset [see, for example, Anderson and Danthine (1981)]. The minimum variance hedge ratio can be conveniently estimated from simple univariate regressions in an analogous fashion to beta estimation within a market model context. Nonsynchronicities present within the indices can lead to biased hedging ratios for stock index futures (particularly in non-U.S. markets or when very short differencing intervals are used to generate large data sets over which relationships can be assumed to be stationary). Estimators that can handle such nonsynchronicities have been developed for estimating beta factors [e.g., Scholes and Williams (1977); Dimson (1979); Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983)] and these estimators may be used to generate unbiased stock index futures hedging ratios.

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This study demonstrates analytically the application of such estimators to stock index futures hedging ratios. It is shown, also, that in the presence of cash-futures markets linkages [that is, where one market leads the other market; see, for example, Chan (1992), Kawaller, Koch, and Koch (1987) Stoll and Whaley (1990)], the classical estimators that handle the nonsynchronicity problem will, themselves, provide biased estimators. Analytic expressions are derived for the biases in these estimators and estimates of the degrees of bias present in the traditional estimators are generated, together with an assessment of their performance in a hedging context.

The impacts of nonsynchronicities and cash-futures linkages are analytically developed in the next section. Expressions for the degree of bias present in the normal nonsynchronous trading estimators are derived and the means for empirically determining the proportionate degree of bias developed. In the third section simulations and empirical analyses (using U.K. futures and index data) of the biases in estimators such as Scholes–Williams (1977) in the presence of cash-futures linkages and the associated hedging performance are presented. The conclusions are contained in the final section of this article.

DEVELOPMENT OF ANALYTIC STRUCTURE

The articulation between linkages (i.e., leads and lags) in cash and futures markets and the normal econometric procedures for handling nonsynchronous trading (or observation) problems are developed in this section. It is well known that the minimum variance futures to cash hedge ratio, h, is given by

$$-h = \{\mathrm{cov}(R(C,t),\ R(F,t))\}\ \{\mathrm{var}(R(F,t))\}^{-1} \tag{1}$$

where $R(C,t)$ $((R(F,t))$ is "the return" on the cash (futures) instrument [see, for example, Figlewski (1984)], cov is the covariance operator and var, the variance operator. The hedge ratio can be estimated from a simple univariate regression of the return on the cash position versus the return on the futures contract.[1] However, where nonsynchronicities exist between observations on the cash and futures position, the simple univariate model will be subject to a classical errors in the variables problem. The nonsynchronicity problem will arise, in the main, due to trading fric-

[1]The minimum variance property follows directly from the Gauss–Markov theorem on least squares estimators. The returns definition used in this work is the natural logarithm of the price relative [Duffie (1989) provides an analytic discussion of the returns form].

tions in the constituent stocks in the index.[2] The measured returns on the index can be related to the "true" (i.e., perfectly co-synchronous with futures returns) returns as [see, for example, Cohen et al. (1983); Stoll and Whaley (1990); and Theobald and Price (1984)]

$$R(C,t,m) = \underline{w}'R + u(C,t) \tag{2}$$

where $\underline{w}$ is a vector with jth element, w_j, representing the proportion of the measured index return arising from period $t - j$; $\underline{R}$ a vector with kth element, $R(C,t - k)$, representing the "true" return on the cash market in period $t - k$; $u(C,t)$ a stochastic disturbance; a prime denotes a vector transpose; and $\underline{w}'\underline{i} = 1$, with $\underline{i}$, a vector of ones. The w_j's are assumed to be deterministic in the next section and will be treated as a random variable with a finite mean and variance in subsequent sections.

The analytic structure can be simplified by working within a "consecutive trades" framework as in Scholes and Williams (1977) and Miller, Muthuswamy, and Whaley (1994).[3] Within this framework, $\underline{w}' = [w_o\, w_1]$ and $\underline{R} = [R(C,t)\, R(C,t - 1)]$. Throughout the analysis, $R(C,t)$ and $R(F,t)$ are assumed to be strict sense stationary as is the cross-covariance structure[4]; $R(C,t)$ and $R(F,t)$ are assumed to have zero autocovariances at all lags.

Deterministic w_j's; No Cash-futures Linkages

Since $R(F,t)$ is assumed to be strictly stationary, the analysis of the hedge ratio can proceed by focusing only upon the numerator in eq. (1). Thus, the covariance between measured returns on the cash index, $R(C,t,m)$, and the futures contract, $R(F,t)$ is

$$\begin{aligned} \text{cov}[R(C,t,m), R(F,t)] &= \text{cov}[\underline{w}'\underline{R} + u(C,t), R(F,t)] \\ &= w_o \text{ cov}[R(C,t), R(F,t)] + w_1 \text{ cov}[R(C,t - 1), R(F,t)] \end{aligned} \tag{3}$$

assuming $\text{cov}[u(C,t), R(F,t)] = 0$. If there are no lead/lag linkages between the cash and futures markets, then $\text{cov}[R(C,t - 1), R(F,t)] = 0$ and with $w_0 < 1$ (as a result of the presence of nonsynchronicities) the measured covariance will be biased downwards due to the errors in the

[2]Even with "quote-driven" indices there will still be a nonsynchronicity problem as a result of stock quotes not being updated and being perfectly cosynchronous with the futures prices.
[3]The consecutive trades assumption underlying the analysis is reasonable for the stock indexes upon which futures contracts are created, comprising, as they do, the larger, more regularly traded stocks.
[4]The impacts of nonstationarity on foreign exchange hedge ratios are analysed by Kroner and Sultan (1993) within a GARCH type framework.

variables problem. If, as in Scholes and Williams (1977), the covariance between $R(C,t,m)$ and $R(F,t-1)$, is estimated, then

$$\text{cov}[R(C,t,m),R(F,t-1)] = w_1 \text{cov}[R(C,t-1),R(F,t-1)] \quad (4)$$

with no linkages between true returns. With stationarity of the cross-covariance structure, $\text{cov}[R(C,t-1), R(F,t-1)] = \text{cov}[R(C,t), R(F,t)]$. Summing eqs. (3) and (4),

$$\begin{aligned} &\text{cov}[R(C,t,m),R(F,t)] + \text{cov}[R(C,t,m),R(F,t-1)] \\ &\quad = \text{cov}[R(C,t),R(F,t)] \end{aligned} \quad (5)$$

since $w_0 + w_1 = 1$. Thus, the augmented covariance estimator on the left-hand side of eq. (5) provides an unbiased estimator of the true contemporaneous cross-covariance (as in the case of the Scholes–Williams estimator). An adjustment in the denominator of the hedge ratio for autocorrelation in the regressor is not necessary since $R(F,t)$ is assumed to have zero autocorrelations at all lags.

Stochastic w_j's; No Cash-Futures Linkages

The relatively simple analytic structure developed in the preceding section, becomes somewhat more complex when stochastic w_j's are introduced into the analysis, since covariances have to be determined between products of random variables. (The assumption that $w_o + w_1 = 1$ will modify to $E(w_o) + E(w_1) = 1$, in this section).[5] Generalizing the result in Mood, Graybill, and Boes (1974), Theorem 3, p. 180 for the variance of a product of random variables to the covariance involving product terms, one obtains:

$$\begin{aligned} &\text{cov}[R(C,t,m),R(F,t)] = E(w_o)\text{cov}\,[R(C,t),R(F,t)] \\ &\quad + E(R(C,t))\,\text{cov}[w_o,R(F,t)] + E[(w_o - E(w_o))(R(C,t) \\ &\quad - E(R(C,t)))(R(F,t) - E(R(F,t)))] + E(w_1)\text{cov}[R(C,t-1),R(F,t)] \\ &\quad + E(R(C,t-1))\,\text{cov}[w_1,R(F,t)] + E[(w_1 - E(w_1)) \\ &\quad (R(C,t-1) - E(R(C,t-1)))(R(F,t) - E(R(F,t)))] \end{aligned} \quad (6)$$

[5]The shift from the deterministic to stochastic (expectational) unity condition corresponds with the shift from the deterministic Dimson (1979) estimator to the stochastic Cohen et al. (1983) estimator. The interpretation of the w_j's as proportions of the measured/observed returns relating to the "true" returns in prior periods is still robust within this shift. For an equally weighted index, the unity condition ensures that all "true" returns are fully reflected within the measured index returns, while this holds approximately for a value weighted index.

With cov $[w_j, R(F,t)]$ terms equal to zero, eq. (6) simplifies somewhat. By using the result in Bohenstedt and Goldberger (1969) that if w_j, $R(C,t)$ and $R(F,t)$ are jointly multivariate normally distributed, then

$$E[(w_o - E(w_o))(R(C,t) - E(R(C,t)))(R(F,t) - E(R(F,t)))] = 0$$

and eq. (6) simplifies to

$$\begin{aligned} \operatorname{cov}[(C,t,m),R(F,t)] &= E(w_o)\operatorname{cov}[R(C,t),R(F,t)] \\ &\quad + E(w_1)\operatorname{cov}[R(C,t-1),R(F,t)] \end{aligned} \tag{7}$$

That is, eq. (3) with $E(w_j)$'s replacing w_j's. The analyses that are generated at eqs. (4) and (5) proceed directly as before with expectations of the stochastic index proportions replacing the deterministic proportions variable.[6]

Stochastic w_j's; and Cash-Futures Linkages

Linkages (in the sense of lead/lag relationships) between cash and futures markets have been detected in, for example, Chan (1992); Kawaller, Koch, and Koch (1987); and Stoll and Whaley (1990). In general, studies have found that the futures market leads the cash market. The covariance between the measured cash returns and the futures returns, from eq. (7) will be

$$\operatorname{cov}[R(C,t,m),R(F,t)] = E(w_o)\operatorname{cov}[R(C,t),R(F,t)] \tag{8}$$

since $\operatorname{cov}[R(C,t-1,m), R(F,t)] = 0$ as the cash market does not, in general, lead the futures. Summing the contemporaneous measured covariance with the covariance between the lagged futures and current cash returns yields

$$\begin{aligned} &\operatorname{cov}[R(C,t,m),R(F,t)] + \operatorname{cov}[R(C,t,m), R(F,t-1)] \\ &\quad = E(w_o)\operatorname{cov}[R(C,t),R(F,t)] + E(w_o)\operatorname{cov}[R(C,t),R(F,t-1)] \\ &\quad + E(w_1)\operatorname{cov}[R(C,t-1),R(F,t-1)] = \operatorname{cov}[R(C,t),R(F,t)] \\ &\quad + E(w_o)\operatorname{cov}[R(C,t),R(F,t-1)] \end{aligned} \tag{9}$$

[6]The assumption of normality for the weights, w_j, is somewhat stronger than is usually encountered in similar modeling scenarios [e.g., Cohen et al. (1983), Theobald and Price (1984)]. If the multivariate normality assumption is not made, the analysis holds as a good approximation, provided that the sum of the cross-products of the deviations from the means for the three random variables is small.

Thus, the Scholes–Williams type of adjustment for nonsynchroncities will lead to biased covariance estimators in the presence of linkages between the cash and futures markets.[7] The degree of bias will be higher, the stronger the futures lead effect and the more regularly traded (updated) the index; that is, $E(w_o) \rightarrow 1$ as the degree of nonsynchronicity declines. The intuition for this result is that as the degree of nonsynchronicity declines (i.e., as $E(w_o)$ approaches one) adding lagged autocovariance terms to the estimator will increasingly reflect linkage effects *only* with the result that the hedge ratio will be "over-estimated".

The degree of bias in the hedge ratio inherent at eq. (9) can be further analyzed using the cash-futures partial adjustment framework developed in Theobald and Yallup (1994). The $\text{cov}[R(C,t), R(F,t-1)]$ term at eq. (9) can be re-expressed as

$$\text{cov}[R(C,t), R(F,t-1)] = (1-g(c))\text{cov}[R(C,t), R(F,t)] \tag{10}$$

where $g(c)$ is a measure of the partial adjustment of the observed cash price to its intrinsic value. That is, the futures market will lead the cash market due to the cash market not fully adjusting to available information when $g(c) < 1$. Substituting (10) in (9) yields

$$\begin{aligned}&\text{cov}[R(C,t,m), R(F,t)] + \text{cov}[R(C,t,m), R(F,t-1)] \\ &\quad = \text{cov}[R(C,t), R(F,t)]\,(1 + E(w_o)(1-g(c)))\end{aligned} \tag{11}$$

The degree of bias can then be determined provided $E(w_o)$ and $(1-g(c))$ can be determined; the further $g(c)$ is from one, the greater the bias induced by cash-futures linkages.

Stoll–Whaley Estimator

Stoll and Whaley (1990) developed an alternative technique for handling the nonsynchronicity problem. They demonstrated that for a "true" process,

$$R(C,t) = \mu(C) + \vartheta(C,t) \tag{12}$$

where $\mu(C)$ is the process mean and $\vartheta(C,t)$, the process innovation, the measured return process can be represented via an ARMA process as

[7] If cash-futures linkages occur in both directions [Stoll and Whaley (1990) find some evidence for this], the expression at eq. (9) has an additional term; that is, $E(w_1)\ \text{cov}[R(C,t-1), R(F,t)]$. The impact of cash leads will be less than futures leads with $E(w_1) < E(w_o)$.

$$\phi(L)R(C,t,m) = W_o(\mu(C) + \vartheta(C,t)) + (1 - \phi(L))\, \varepsilon(C,t) \quad (13)$$

where $\phi(L)$ is a polynomial in the lag operator (which is a function of the weights, w) and $\varepsilon(C,t)$ incorporates spread and thin-trading error terms.

By using the residuals from a fitted ARMA (p,q) process, Stoll and Whaley (1990) demonstrated that a noisy, but unbiased, estimator of the true returns is generated. The numerator in the minimum variance hedge ratio that is obtained by using the ARMA (p,q) residuals is

$$\begin{aligned} &\text{cov}[w_o\vartheta(C,t) + (1 - \phi(L))\, \varepsilon(C,t), R(F,t)] \\ &= w_o \,\text{cov}[R(C,t), R(F,t)] \end{aligned} \quad (14)$$

assuming that the spread and thin trading error terms do not covary with $R(F,t)$.

Effectively, an identical degree of bias in the hedge ratio estimator will be generated as in the case at, for example, eq. (3). However, in this case, the ARMA (p,q) model estimates can be used to correct this estimator. Since, from Stoll and Whaley (1990), the polynomial lag operator, $\phi(L)$, is of the form,

$$\phi(L) = \{1 + (w_1/w_o)\, L + \ldots\ldots (w_{n-1}/w_o)\, L^{n-1}\}^{-1} \quad (15)$$

with L^p the "p" period lag operator, expanding $\phi(L)$ via a power series expansion, and with some rearrangement, the coefficient on the lag one measured return variable will be equal to w_1/w_o. Then, by making the "consecutive trades" assumption and using Slutsky's Theorem, a consistent estimator of w_o can be generated and, thereby, of the hedge ratio.

NUMERICAL AND EMPIRICAL ANALYSIS

The analytics developed in the previous section can be summarized in terms of hedge ratios as:

$$h_1 = E(w_o)h \quad (16)$$

where h_1 is the hedge ratio deriving from a regression of observed returns in the cash market versus contemporaneously observed returns in the futures market and h is the "true" hedge ratio; and

$$h_2 = h(1 + E(w_o)(1 - g(c))) \quad (17)$$

where h_2 is the hedge ratio estimated by incorporating the comovement between the return in the cash market and the lagged futures return in addition to the contemporaneous comovements between the two markets. The trade-off between not correcting for nonsynchronicities (i.e., using h_1) and potentially overestimating (i.e., using h_2) can then be assessed. For example, if only 95% of the index is co-synchronous with the futures return, then an underestimation of the true hedging ratio of 5% (i.e., using h_1) may be transformed into an overestimation of 9.5% by using h_2 if the cash market only adjusts by 90% of the futures market's adjustment.

Further insights into the impacts of cash/futures linkages upon hedging ratios are obtained via simulations. The simulations are conducted within the following equation system:

$$V(C,t) = V(C,t-1) + v(C,t) \tag{18}$$

with $V(C,t)$ the logarithmic intrinsic cash price, $v(C,t)$ the innovation in this price process;

$$P(C,t) = g(c)\,V(C,t) + (1 - g(c))\,P(C,t-1) + u(C,t) \tag{19}$$

with $P(C,t)$ the natural logarithm of the actual cash price, $u(C,t)$ a noise term;

$$r(C,t) = \mathrm{pr}(C,t-1) + R(C,t) \tag{20}$$

where $r(C,t)$ is the continuously compounded observed (i.e., with error due to nonsynchronicities/stale prices) return, p the autoregressive coefficient, and with $R(C,t)$ equaling $P(C,t) - P(C,t-1)$;

$$R(F,t) = (r-d) + u(C,t) \tag{21}$$

where $r - d$ is the differential between interest rates and dividend yields. The system is simulated by generating 1000 random normally distributed series of 100 observations for the $\{v(C,t),u(C,t)\}$ pair and using differing values for $\{g(c),p\}$. Hedge ratios are estimated using the observed (with error) series, $r(C,t)$ (i.e., generating h_1), and the actual (i.e., error purged) series, $R(C,t)$ (i.e., generating h). Corrections are made for stale prices by using a Scholes–Williams type adjustment (denoted by h_2) and by using the Stoll–Whaley-based procedures (denoted by h_3). The results of selected simulations are contained in Table I where the return series are generated for an intrinsic return volatility of 20% p.a.; a daily return interval is used.

It is immediately apparent that Scholes–Williams type adjustments do not perform particularly well, both in terms of the percentage bias and the mean square error criterion. This corresponds to the Scholes–Wil-

TABLE I

Percentage Biases of Simulated Estimators
[$\sigma\{V(C,t)\}$ = 0.2 p.a. for all simulations]

	Percentage Bias					Mean Squared Errors (X10^{-6})		
	h_1–h_2	h–h_1	h–h_2	h–h_3	h_1–h_3	h_1	h_2	h_3
$\sigma\{u(C,t)\} = 0$; $g(c) = 0.99$; $p = 0.05$	4.5%	0.4%	4.9%	0.8%	1.2%	.0053	.0247	.0111
$\sigma\{u(C,t)\} = 0$; $g(c) = 0.95$; $p = 0.05$	7.9%	0.4%	8.3%	1.0%	1.5%	.0107	.0642	.0165
$\sigma\{u(C,t)\} = 0$; $g(c) = 0.05$; $p = 0.10$	12.1%	0.8%	13.0%	1.0%	1.8%	.0270	0.0154	.01457
$\sigma\{u(C,t)\} = .01$; $g(c) = 0.95$; $p = 0.05$	8.7%	0.05%	8.8%	0.3%	0.3%	1.2951	.7956	1.1869
$\sigma\{u(C,t)\} = 0$; $g(c) = 0.90$; $p = 0.05$	12.1%	0.4%	12.6%	1.4%	1.8%	.0204	0.1301	.0222
$\sigma\{u(C,t)\} = 0$; $g(c) = 0.90$; $p = 0.10$	16.3%	0.8%	17.2%	1.4%	2.2%	.0396	0.2437	.0211
$\sigma\{u(C,t)\} = 0$; $g(c) = 0.90$; $p = 0.15$	20.5%	1.2%	21.9%	1.4%	2.5%	.0669	0.3964	.0202

h = actual hedge ratio.
h_1 = hedge ratio estimated from observed return series.
h_2 = hedge ratio corrected via Scholes–Williams adjustments.
h_3 = hedge ratio corrected via Stoll–Whaley adjustments.

liams estimator picking up the tardy response of the cash market and correcting it as a thin trading effect. The performance generally worsens as the partial adjustment coefficient declines. For relatively low degrees of nonsynchronicity (for example, where $p = 0.05$, corresponding to $w_o = 0.95$), the Stoll–Whaley adjusted estimator (h_3) does not outperform the unadjusted hedge ratio (h_1) on a mean-square error criterion. However, when there are higher degrees of nonsynchronicity/staleness (i.e., when $p = 0.1$, corresponding to $w_o = 0.91$), the Stoll–Whaley estimator does outperform the unadjusted estimator on this basis. The mean square error of both the Scholes–Williams and unadjusted hedge ratios increases relative to the Stoll–Whaley hedge ratio as the degree of thin trading (i.e., p) increases.

The simulation results are confirmed by estimating the three hedging ratios $\{h_1, h_2, h_3)$ using U.K. daily data on FTSE100 stock index futures contracts and FTSE100 (a value weighted index of the leading one hundred stocks) and FT-All Share (a broader based, value weighted index) indices for the period from 1985 to 1995.[8,9] As shown in Table II, the Scholes–Williams estimator (h_2) generally has the highest value for most partitions of the dataset and for both indices, consistent with its "over-estimating" characteristics previously indicated. The partial adjustment coefficient is, generally, close to, but below, one (i.e., full adjustment). For the overall dataset, the partial adjustment coefficient is slightly lower for the FT-All Share index than for the FTSE100, which is consistent with the broader content of the former. The percentage ranges for the FTSE100 hedge ratios are generally less than those for the FT-All Share index. This result is consistent with the wider constituency of the FT-All Share index (containing around 750 stocks) inducing both a slower adjustment towards intrinsic values, as manifested by the lower partial adjustment coefficients already referred to, and the greater number of stale prices. For example, for the whole dataset, the average proportion of prices corresponding to the contemporaneous period is estimated to be 94.1% for the FTSE100 as against 89.6% for the FT-All Share.

While the properties of the various hedging estimators have been empirically established, an equally important practical consideration is how well these estimators perform in minimizing the variance of the joint cash and futures position. In evaluating the performance of the differing hedge estimators, the variance of the "true" returns process should be used (that is, after adjusting for nonsynchronicities). If unadjusted returns are used, the traditional OLS estimator (h_1) will always generate the minimum variance position "in-sample" relative to the Stoll–Whaley-based (h_3) estimator, due to the Gauss–Markov theorem.[10] Similarly, "out-of-sample," h_1 is likely to be the best performer using unadjusted returns since h_1 would be anticipated to be the best predictor of subsequent h_1's (since h_1 will be biased downwards in the presence of nonsynchronicites).

[8]Dividend, interest rate and settlement effects were incorporated into the estimating procedures in Theobald and Yallup (1992). It was found that the resultant estimates were largely insensitive to such adjustments. When similar adjustments are incorporated into this article, results and conclusions are largely unchanged.

[9]Dickey–Fuller tests reject unit roots in all three return series for all partitions of the data. Lagrangian multiplier tests for ARCH effects find highly significant (at the 0.5% level) ARCH effects for the years 1985, 1987 and 1990 for all three return series, and 1984 for the FT-All Share Index, and 1991 and 1995 for the futures contract (significant effects at the 5% level are found for the other return series in these periods). The common incidence of such heteroskedastic effects in the regressors and regressands, will tend to "wash out" in the disturbances and, as a consequence, estimator efficiency is not impacted. In particular, for the FTSE100 index case, highly significant ARCH effects are found only in the crash subperiod.

[10]By an analogous argument, with true returns, the Stoll–Whaley estimator will be optimal "in-sample" relative to the OLS estimator due to the Gauss–Markov theorem. Accordingly, the analysis of hedging performance used here will concentrate on "out-of-sample" measures.

TABLE II

Estimator Results Using U.K. Index Data

		H1	*H2*	*H3*	*G(C)*	*H2-H1*	*H3-H1*	*E(WO)*	*E(W1)*
ALL	FT100	0.82079	0.90843	0.87254	0.956	10.68	6.31	94.1%	5.9%
	FTALL	0.73246	0.85429	0.81765	0.950	16.63	11.63	89.6%	10.4%
ALL[a]	FT100	0.81621	0.89350	0.87484	0.979	9.47	7.18	93.15	6.9%
	FTALL	0.72390	0.83410	0.80986	0.969	15.22	11.87	89.25	10.8%
1984	FT100	0.98272	1.00566	1.08933	1.107	2.33	10.85	88.4%	11.6%
	FTALL	0.84048	0.95347	0.99997	1.089	13.44	18.98	81.7%	18.3%
1985	FT100	0.91265	1.01261	0.95439	0.935	10.95	4.57	95.7%	4.3%
	FTALL	0.76816	0.93650	0.86348	0.899	21.91	12.41	89.4%	10.6%
1986	FT100	0.82421	0.92403	0.90878	0.986	12.11	10.26	90.3%	9.7%
	FTALL	0.69784	0.84994	0.81092	0.948	21.80	16.20	85.7%	14.3%
1987	FT100	0.82148	0.92118	0.86141	0.926	12.14	4.86	95.4%	4.6%
	FTALL	0.74753	0.88535	8.83554	0.932	18.44	11.77	89.6%	10.4%
1987[a]	FT100	0.76708	0.78749	0.81496	1.048	2.66	6.24	93.1%	6.9%
	FTALL	0.69315	0.75021	0.76827	1.046	8.23	10.84	88.6%	11.4%
1988	FT100	0.77665	0.82936	0.84256	1.028	6.79	8.49	91.2%	8.8%
	FTALL	0.72127	0.79875	0.81636	1.041	10.74	13.18	87.1%	12.9%
1989	FT100	0.73117	0.83370	0.81925	0.995	14.02	12.05	88.1%	11.9%
	FTALL	0.68956	0.78749	0.79242	1.026	14.20	14.92	85.6%	14.4%
1990	FT100	0.77504	0.85546	0.80791	0.937	10.38	4.24	96.0%	4.0%
	FTALL	0.70778	0.80513	0.75611	0.928	13.75	6.83	93.9%	6.1%
1991	FT100	0.84100	0.91882	0.82647	0.891	9.25	−1.73	101.6%	−1.6%
	FTALL	0.75146	0.84910	0.78087	0.907	12.99	3.91	96.5%	3.5%
1992	FT100	0.86046	0.94613	0.94233	0.998	9.96	9.51	91.1%	8.9%
	FTALL	0.79740	0.91728	0.88602	0.965	15.03	11.11	89.7%	10.3%
1993	FT100	0.83176	0.89773	0.91255	1.022	7.93	9.71	90.8%	9.2%
	FTALL	0.70266	0.79116	0.79025	1.002	12.59	12.47	88.7%	11.3%
1994	FT100	0.78774	0.89325	0.78626	0.864	13.39	−0.19	100.2%	−0.2%
	FTALL	0.65489	0.79274	0.68786	0.835	21.05	5.03	95.6%	4.4%
1995	FT100	0.81005	0.89589	0.79944	0.882	10.60	−1.31	101.2%	−1.2%
	FTALL	0.66081	0.76624	0.69826	0.893	15.95	5.67	95.0%	5.0%

[a]Denotes estimation results that are generated with the crash contract excluded.

The results reported in Table II confirm this. For instance, h_1, is the best predictor of h_1 for periods of one (two) years ahead in 11 out of 12 (9 out of 11) cases. Although h_2 is the best one year ahead predictor of h_2 (7 cases), h_1 is the best predictor of h_2 two periods ahead (h_1 is best in 5 cases, h_2 in 4). In the case of h_3 forecasts, h_1 is the best one year predictor (6 cases, as against 5 for h_3) and best two-year predictor (5 cases as against 3 for each of h_2 and h_3). The lesser degrees of freedom and/or increased noise in the $\{h_2h_3\}$ estimators is the likely cause of the poorer predictive power of these hedge ratios.

Table III contains the out-of-sample results for unadjusted returns. The hedge is formed by using one calendar year's data and the perfor-

TABLE III

Out-of-Sample Hedging Performance
Standard Deviation of Error (%)
(Unadjusted Returns)

	FTSE100			*FT ALLSH*		
	H1	*H2*	*H3*	*H1*	*H2*	*H3*
85	0.263972	0.268079	0.291775	0.262974	0.293577	0.312413
86	0.375352	0.406903	0.386047	0.361314	0.420627	0.388092
87	0.625151	0.656706	0.648152	0.637424	0.661108	0.641971
87X	0.421072	0.448660	0.442914	0.377654	0.412483	0.397616
88 (87)	0.330227	0.350993	0.335819	0.306028	0.336676	0.320415
88 (87X)	0.328863	0.328518	0.329679	0.307217	0.306172	0.307639
89	0.349827	0.361229	0.365296	0.295635	0.314796	0.321769
90	0.304875	0.308167	0.305021	0.295005	0.308021	0.309721
91	0.235320	0.227474	0.229090	0.235673	0.237819	0.232185
92	0.283413	0.289731	0.285074	0.369414	0.370256	0.366376
93	0.158364	0.177908	0.176615	0.181501	0.230074	0.215027
94	0.224783	0.247915	0.255354	0.295254	0.323560	0.323149
95	0.153382	0.164218	0.153514	0.158316	0.186199	0.159436

Note: 87X relates to 1987 data excluding the "crash" contract.

mance is evaluated over the subsequent calendar year.[11,12] As would be anticipated, h_1 generates the stronger performance for both indices (lower standard deviations in 11 out of 13 cases for the FTSE100 index and in 10 cases for the FT-All Share Index). The Stoll–Whaley estimator, h_3, outperforms the Scholes–Williams estimator for both indices (in 8 cases for the FTSE100 and 9 cases for the FT-All Share Index). Table IV contains the corresponding hedging performance results when the true returns are used in the hedging period,[13] with the $\{w_o\ w_1\}$ variables estimated from that same period.[14] The performance of h_1 relative to h_3 declines, although neither systematically outperforms the other in the majority of periods for either index. As would be anticipated from the simulation results, h_3's performance relative to h_1 improves with lower w_o's. For example, in 1993 when h_3 outperforms h_1, w_o is 91.1% in 1992 (the estimator period) for the FTSE100 index and for the performance evaluation period, 1993, w_o is 90.8%. Again h_3's performance is better

[11]The "crash" contract is excluded/included to control for the impact of that event.

[12]Similar performance tests are used by estimating the hedge ratio 2 years prior to the performance period. Broadly similar types of results are obtained.

[13]Using eq. (15), the true returns can be computed from the measured returns, $R(C,t,m)$ as $1/w_o$ $\{R(C,t,m) - w_1/w_o\ R(C,t-1,m)\}$ assuming that $(w_1)^2/(w_o)^2 \simeq 0$ and retaining the consecutive trades assumption.

[14]$\{w_o, w_1\}$ from the hedge estimating period is estimated also and broadly similar results are found.

TABLE IV

Out-of-Sample Hedging Performance
Standard Deviation of Error (%)
(Adjusted "True" Returns)

	FTSE100			*FT ALLSH*		
	H1	*H2*	*H3*	*H1*	*H2*	*H3*
85	0.266475	0.268413	0.284821	0.267190	0.274923	0.285853
86	0.400326	0.412202	0.402651	0.395720	0.411082	0.396760
87	0.650405	0.657579	0.652682	0.718641	0.666021	0.667255
87X	0.460767	0.474936	0.471256	0.444476	0.446329	0.440230
88 (87)	0.368888	0.373320	0.368041	0.359400	0.356874	0.352771
88 (87X)	0.375635	0.372361	0.369359	0.372001	0.358941	0.356260
89	0.399163	0.396956	0.397566	0.349839	0.342205	0.343051
90	0.319767	0.308968	0.307852	0.310404	0.303400	0.304123
91	0.231465	0.228049	0.226992	0.243137	0.234562	0.234405
92	0.320325	0.301420	0.326225	0.424649	0.400144	0.414507
93	0.173237	0.170834	0.170453	0.177308	0.200534	0.190895
94	0.225325	0.248901	0.256420	0.295218	0.313607	0.313287
95	0.153293	0.167884	0.153369	0.160330	0.171632	0.157039

than h_2 in the majority of cases reflecting the confounding impact of futures/cash linkages upon the Scholes–Williams estimator. The out-of-sample hedge performance tests indicate, then, that while Stoll–Whaley based estimators performed better than Scholes–Williams based estimators, performance is not unambiguously stronger than OLS based hedging strategies.

In the three cases for the FTSE100 index where w_o is less than 95% in both the estimating and performance periods, Stoll–Whaley procedures generate better hedge ratios in two cases (in the other case, the Scholes–Williams hedge ratio is better. Both periods, in this situation, are characterized by particularly low w_o's with partial adjustment factors of close to one). With the FT-All Share Index, of the four cases where w_o is less than 95% in both periods, OLS based procedures yield the best hedge ratio in two cases. This latter case, however, involves a cross-hedge which additionally impacts the performance.

CONCLUSIONS

The analytical section of this article demonstrates that thin trading adjustments in the presence of cash/futures linkages (that is, lags in the cash market response relative to the futures market) will lead to biased hedging ratios. Simulations confirm this and indicate that Stoll–Whaley-based adjustments generally outperform Scholes–Williams adjustments on a mean square error criterion. The results obtained using U.K. futures and index data are consistent with the analytic and simulation results. In particular, when the performance of the differing hedge ratios are evaluated using "out-of-sample" tests, the Stoll–Whaley-based estimator outperforms the traditional OLS based estimator only for higher degrees of nonsynchronicity between the cash and futures markets. That is, adjustments to OLS based hedging ratios are only beneficial when the proportion of the index traded consecutively with the futures is below around 95%.

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