Linear Dependence, Nonlinear Dependence and Petroleum Futures Market Efficiency

ROGER A. FUJIHARA
MBODJA MOUGOUÉ

INTRODUCTION

The simple market efficiency hypothesis states that the futures price is an unbiased estimator of the future spot price as in (1):

$$E_t(\ln S_T) = \ln F_{t,T} \tag{1}$$

where S_T is the spot price at time T, $F_{t,T}$ is the futures price at time t with a contract maturity at T, and $E_t(\cdot)$ represents expectations formed at time t using all available information. Previous tests of (1) have been based on estimates of (2):

$$\ln S_T = \alpha_0 + \alpha_1 \ln F_{t,T} + v_T \tag{2}$$

where v_T is the news component which is orthogonal to the information incorporated in the futures price at time t. Assuming that market partic-

Send all correspondence to: Mbodja Mougoué, Department of Finance and Business Economics, School of Business Administration, Wayne State University, 5201 Cass Avenue, Detroit, Michigan 48202 USA. Tel: 1-313-577-4538; Fax: 1-313-577-0058; Internet: MMOUGOU@CMS.CC. WAYNE.EDU. We acknowledge James L. Hamilton for helpful comments and Andrew C. Szakmary for supplying the data used in this study. We also thank Shan Ming for able research assistance. The usual disclaimer applies.

■ *Roger A. Fujihara is an Account Manager with the Acacia Group in Fairfax, Virginia.*

■ *Mbodja Mougoué is an Assistant Professor of Finance at Wayne State University.*

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ipants are risk-neutral and expectations are rational, the hypothesis of simple efficiency involves the joint restriction that $\alpha_0 = 0$ and $\alpha_1 = 1$.

However, evidence that asset prices contain unit roots raises questions about the appropriateness of standard hypothesis tests applied to ordinary least squares (OLS) estimates of (2) [see, for example, Elam and Dixon (1988) and Shen and Wang (1990)]. In addition, nonstationary time series can yield spurious results [see, for example, Granger and Newbold (1974) and Engle and Granger (1987)] because the variables of interest may not be cointegrated, i.e., they may not be tied together in a long-run relationship.[1] Recognizing the econometric problems with specification (2), a standard procedure now consists of testing for unit roots in the spot and futures prices of an asset and then proceeding to conduct a cointegration analysis if a unit root is uncovered. Indeed, this latter procedure has led to the rapid growth in applications to various financial markets, with Chowdhury (1991), Lai and Lai (1991), Schroeder and Goodwin (1991), and Krehbiel and Adkins (1993) representing a small sample of such research. With respect to petroleum futures, Serletis and Banack (1990), for example, provide evidence that spot prices for crude oil, heating oil, and unleaded gasoline are cointegrated with their respective futures prices, but they do not explicitly test the joint restrictions implied by the unbiasedness hypothesis. Using monthly observations of 30-day crude oil futures, Crowder and Hamed (1993) test for cointegration using (2) and find evidence supporting the simple efficiency hypothesis. In contrast to Serletis and Banack, Crowder and Hamed's empirical evidence is based on a procedure developed by Johansen (1988, 1991) which is able to incorporate the joint restrictions.

A related test that futures prices are unbiased forecasts of future spot prices is based on Samuelson (1965), who demonstrates that a sequence of futures prices in an efficient market can be described by the following martingale process.[2]

$$E_t(\ln F_{t+1,T}) = \ln F_{t,T} \tag{3}$$

Then (3) can be made equivalent to (1) by assuming that arbitrage drives the futures price at maturity, $\ln F_{T,T}$, into equality with the spot price, $\ln S_T$.[3] When that arbitrage condition is combined with repeated appli-

[1]Nonstationary time series are said to be cointegrated if there exists a linear combination of the variables which is stationary. The resulting relationship implies that the variables will not drift apart over time. To analyze the long-run comovement of two or more time series, tests for cointegration can be applied following Engle and Granger (1987) and Johansen (1988, 1991).

[2]Fama (1970) also argued that asset prices should follow a martingale process.

[3]Strictly speaking, the time period, T, may not refer to the maturity of the futures contract. Energy

cation of the law of iterative expectations, the unbiasedness result follows.[4]

Under the null hypothesis of efficiency, as stated in (3), a typical test involves examining whether the change in futures prices is related to information at time $t - 1$, or any subset. Tests conducted by Hodrick and Srivastava (1987) found that the lagged basis, defined as the difference between the futures and contemporaneous spot price, was significant in predicting daily changes in foreign currency futures prices. McCurdy and Morgan (1987) found that daily foreign currency futures price changes were predictable using their own past values even after accounting for heteroscedasticity. Kaminsky and Kumar (1990) used various agricultural and raw materials commodities to examine the significance of lagged information variables by conducting weak- and semi-strong efficiency tests and concluded that it is not possible to make strong generalizations about the efficiency of commodity markets for short-term forecast horizons of one and three months. For longer forecast horizons of six and nine months, however, Kaminsky and Kumar rejected the null of simple efficiency for several markets.

The preceding discussion of market efficiency tests rely generally on the existence of linear predictability in the change in futures prices as evidence against efficiency. However, recent developments in the study of nonlinear relationships also provide an important contribution to the understanding of the behavior of asset prices. For example, Savit (1988, 1989) and Brock, Hsieh, and LeBaron (1991) argue that deterministic nonlinear dynamics can provide a useful description of movements in asset prices. In particular, the introduction of nonlinear feedback mechanisms can explain price fluctuations without relying on stochastic effects. These nonlinearities would imply that price series are forecastable, even though it appears that these series are not predictable using linear models.[5] Thus, evidence in support of or against the martingale behavior of asset prices should rely on both linear and nonlinear procedures.

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futures contracts are traded for delivery in all 12 months of the year. Trading for any contract terminates prior to the delivery month of that contract. Subsequently, there is a delivery period of three to four weeks. To avoid a riskless profit, the futures price should converge to the corresponding spot price on the day when delivery occurs. As Ma (1989) points out, since delivery can take place at any time during a four-week period, the futures price will converge to the cheapest-to-deliver spot price on the optimal delivery date for the holder of the delivery option. Ma (1989) indicates that the optimal delivery day is generally the first deliverable day for the heating oil and gasoline contracts. The crude oil futures price, however, predicts the spot price on the last trading day.

[4]If $\ln F_{t,T} = E_t(\ln F_{t+1,T})$ and $\ln F_{t+1,T} = E_{t+1}(\ln F_{t+2,T})$ then $\ln F_{t,T} = E_t(E_{t+1}(\ln F_{t+2,T})) = E_t(\ln F_{t+2,T})$. Successive applications would yield $E_t(\ln F_{T,T}) = \ln F_{t,T}$. Subsequently, arbitrage would produce $\ln F_{T,T} = \ln S_T$ and, thus, $E_t(\ln S_T) = \ln F_{t,T}$.

[5]Hsieh (1989a, 1991, 1993a) examines the issue of nonlinear dynamics for foreign exchange rates

This study extends the existing research on market efficiency in several important ways. First, it re-tests the martingale hypothesis based on linear dependence for a sample of three energy futures using relatively recent data. Second, this research provides a more detailed analysis of nonlinearities for petroleum futures than most of the previous research in this area.[6] More specifically, this study provides a more comprehensive analysis of market efficiency for futures markets by combining linear and nonlinear tests for the three energy futures. This analysis is useful because in the presence of any dynamics, linear or nonlinear, conditional densities can provide a better description of short-term price movements than can unconditional densities.[7] Third, while much of the previous research on commodities futures has analyzed market efficiency in terms of several weeks or months, this study employs daily data. The analysis of higher frequency data is of interest because evidence of asset prices displaying excess kurtosis appears to be more important at higher frequencies. Diebold (1986, 1988) indicates that, consistent with the generalized central limit theorem, changes in asset prices converge to a normal distribution under temporal aggregation. Finally, this research examines the possibility of time-varying variances for the three petroleum futures. This issue is worthy of analysis because energy futures are believed to be among the most volatile markets. Indeed, this research finds that the log changes of petroleum futures exhibit highly persistent time-varying volatility and attempts to explain this persistence in variance using the mixture of distribution hypothesis.

The findings of this article have important implications for practitioners or researchers who use volatility or variance models to price options or other derivative products. For example, the validity of the standard Black–Scholes options price formula hinges on the crucial assumption of random walk behavior for the price of the underlying asset. However, as demonstrated by Savit (1989), an option price computed for a nonlinear process may differ from that computed for the random process with the same unconditional distributions. An obvious consequence

and stock prices. Hsieh's results suggest that nonlinearities are largely attributable to changes in variance as opposed to changes in mean. Thus, conditional heteroscedasticity appears to be the main source for rejection of independent and identically distributed (iid) behavior of changes in exchange rates and stock prices. Indeed, the conditional heteroscedasticity can help explain the leptokurtocity and dependence in distribution for daily changes in agricultural and metals futures prices documented by Hall et al. (1989).

[6]This study is closely related to the article by Marshall and Stengos (1994) who examine the spot rate of return on gold.

[7]Standard methods of estimating a probability density use a smoothed histogram of past price changes. The result is the unconditional density. However, a conditional density can provide a more accurate assessment of prices changes because it uses more information.

of this disparity is that, in the presence of nonlinearities, hedging strategies based on the assumption of linear price movements will be inadequate.

The results of this research are also relevant for financial risk management, especially when highly leveraged instruments, such as futures contracts, are involved. For example, hedge ratios and the amount of capital needed to cover possible losses during the time a futures position is held depend critically on the probability distribution of changes in futures prices. However, when log price changes are not independent and identically distributed, their conditional density is more accurate than their unconditional density for describing short-term behavior. Hsieh (1993b), for example, shows that the conditional density yields different and more accurate capital requirements than the unconditional density.

Clearly, testing and accounting for nonlinearities is important since this may help improve short-term price predictability and may lead to the construction of more accurate hedge ratios and long-term improvements in investment strategies. The remainder of the article proceeds as follows. It discusses the data and provides some descriptive statistics and then establishes the methodology for analyzing the data generating process using various linear and nonlinear tests. The empirical results are presented and the article concludes by discussing some potential areas for future research.

DATA DESCRIPTION AND SUMMARY STATISTICS

The data consist of daily futures prices for crude oil, heating oil, and unleaded gasoline traded at the NYMEX from December 3, 1984 to September 30, 1993, a total of 2217 observations. These prices are transformed to daily returns, $r_t = 100 \times \ln(F_{t,T}/F_{t-1,T})$.[8] Since the maturity date changes over time, a single time series is constructed by using the nearby futures contract until the day prior to its last trading day at which point the data is rolled over to the next deferred contract.[9] Use of the

[8]A robust unit root test developed in Phillips (1987) and Phillips and Perron (1988) is implemented to provide further impetus for examining changes in the three futures prices. The choice of the Phillips–Perron method is appropriate since it is a general test which can be used even in the presence of autocorrelated innovation sequences. It is also well suited to handle departures from iid errors. The results of the unit root tests, not reported here but available from the authors upon request, firmly support the null hypothesis of a unit root in each of the three futures price time series.

[9]Ma, Mercer, and Walker (1992) discuss the implications of linking futures price series and the possible biases that may arise when selecting a particular method to rollover futures contracts. When rolling a contract over, the two most important decisions that must be made are the rollover date and the adjustment of futures prices on the rollover day. On the rollover day, the nearby contract

TABLE I

Summary Statistics for Futures Returns

	Crude Oil	*Heating Oil*	*Unleaded Gas*
Mean	.0144	.0171	.0408
SD	.0486	.0443	.0411
t-statistic	.2970	.3852	.9909
Skewness	−2.2774[a]	−2.0670[a]	−1.5691[a]
Kurtosis	39.8308[a]	37.8726[a]	27.3933[a]
Minimum	−38.4071	−35.0938	−29.8099
Maximum	12.3525	12.8019	10.6913

The data is for the period, December 3, 1984, to September 30, 1993. SD is the standard deviation of the mean. The *t*-statistic is for the null hypothesis that the mean equals zero.
[a]Indicates significance at the 1% level.

nearby contract avoids any serial correlation induced by price limits since such limits are removed in the month prior to delivery.

Various descriptive statistics are provided in Table I. The distributions of daily returns are negatively skewed and have excess kurtosis relative to the normal distribution. In view of the existing evidence [see, for example, Hall et al. (1989) and Akgiray et al. (1991)] that the excess kurtosis is due to a possible time-varying variance, a test for nonlinearity is conducted using the McLeod and Li (1983) method, which is based on the autocorrelation coefficients and the Box–Pierce Q-statistics for the squared data. These Q-statistics have been used in the literature to gauge the existence of conditional heteroscedasticity.[10] The Q-statistics reported in Table II are highly significant, indicating that the three daily returns time series exhibit possible conditional heteroscedasticity. Based on this evidence Table III reports the autocorrelation coefficients for the three petroleum returns, along with heteroscedasticity-consistent standard errors and Box–Pierce Q-statistics suggested by Diebold (1988). Several individual coefficients are significantly different from zero at the

price is generally different from that for the next-out contract and this may potentially introduce some artificial jumps in the constructed futures price time series. These jumps may, in turn, create extreme volatility and large price changes which may bias the parameter estimates of the true underlying distributions. The results of the current study are based on futures price series adjusted to remove the price jumps occurring at the rollover date. At each rollover date, the adjustment is accomplished by subtracting the difference in the price levels between the new contract and the old contract from all new prices, thereby eliminating price gaps resulting from contract rollover. To ascertain the robustness of those findings, unadjusted futures price series, including the jumps, are used. The results are qualitatively similar to those based on the adjusted data.

[10]See, for example, Hsieh (1989a, 1989b). Hsieh (1989a) points out that the McLeod and Li Q-statistic is related to Engle's (1982) test for heteroscedasticity since the former uses the autocorrelation coefficients of the squared data while the latter relies on the partial autocorrelation coefficients.

TABLE II

Autocorrelations Coefficients of Squared Returns

Lag	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
1	.0828	.0483	.1109
2	.0545	.0470	.0841
3	.1515	.1544	.1767
4	.0298	.0165	.0280
5	.0408	.0283	.0392
6	.0262	.0238	.0453
7	.0251	.0210	.0279
8	.0937	.1044	.1290
9	.0366	.0182	.0287
10	.0422	.0294	.0270
Box–Pierce			
$Q^2(30)$	181.269	118.905	221.808
	[0.000]	[0.000]	[0.000]
$Q^2(60)$	275.728	138.968	270.836
	[0.000]	[0.000]	[0.000]

The data is for the period, December 3, 1984, to September 30, 1993. $Q^2(p)$ is the Box–Pierce Q-statistic of the squared futures returns for lag p. Marginal significance levels in brackets.

10% level (or better) for all three series even though the only statistically significant Q-statistic is for unleaded gasoline for lag 60.

The serial correlation in the daily returns time series is evidence against the martingale hypothesis. However, market efficiency should not be ruled out without an extensive examination of the underlying linear dependence, as well as the evidence of nonlinearity provided by the McLeod and Li (1983) test. The next section addresses these issues.

METHODOLOGY

Weak-Form Efficiency Test

The weak-form test of market efficiency, applied to (3), assumes that current prices reflect the information contained in the historical sequence of prices. As a result, investors who rely on lagged changes in futures prices to predict changes in the current price should not expect to receive risk-adjusted excess returns, on average. The typical test equation for weak-form market efficiency can be stated as:

$$r_t = \beta_0 + \sum_{i=1}^{p} \beta_i r_{t-i} + v_t \tag{4}$$

TABLE III

Autocorrelation Coefficients for Futures Returns

Lag	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
1	.0205	.0566	.0765[a]
	(.0448)	(.0363)	(.0437)
2	−.0114	−.0042	.0480
	(.0384)	(.0360)	(.0395)
3	−.0717	−.0826	−.0608
	(.0575)	(.0567)	(.0527)
4	.0434	−.0095	.0045
	(.0318)	(.0273)	(.0286)
5	−.0650[a]	−.0650[b]	−.0180
	(.0349)	(.0310)	(.0311)
6	.0143	−.0263	−.0135
	(.0307)	(.0296)	(.0324)
7	.0241	.0243	.0189
	(.0304)	(.0288)	(.0286)
8	−.0635	−.0835[a]	−.0832[a]
	(.0471)	(.0482)	(.0464)
9	−.0250	.0090	.0150
	(.0338)	(.0279)	(.0288)
10	.0687[a]	.0717[b]	.0826[c]
	(.0353)	(.0313)	(.0284)
11	.0459	.0671[a]	.0632[a]
	(.0377)	(.0377)	(.0331)
12	.0298	.0028	.0418
	(.0326)	(.0260)	(.0262)
13	−.0109	.0228	.0331
	(.0303)	(.0250)	(.0260)
14	.0528[a]	.0276	.0232
	(.0299)	(.0286)	(.0268)
15	−.0177	−.0108	.0077
	(.0340)	(.0283)	(.0278)
Adjusted Box–Pierce			
Q(30)	24.453	27.567	35.358
	[0.751]	[0.593]	[0.230]
Q(60)	52.117	69.519	79.174
	[0.756]	[0.188]	[0.049]

The data is for the period, December 3, 1984, to September 30, 1993. Heteroscedasticity-consistent standard errors are in parentheses. Marginal significance levels are in brackets.
[a,b,c]Indicate significance at 10%, 5%, and 1% levels, respectively, for a two-tail test.

Under the null hypothesis of market efficiency, the regression coefficients β_i $(i = 0, \ldots, p)$ are all equal to zero and the error term, v_t, has mean zero and is not serially correlated. To ensure heteroscedasticity-consistent estimates of the standard errors, White's (1980) correction is used to estimate the covariance matrix.

Market efficiency is not testable except as a joint hypothesis with a precise pricing model that is hypothesized to generate equilibrium expected returns. Since pricing models that are consistent with a martingale process require strong assumptions, relaxing any one of these assumptions is likely to negate the correspondence between the martingale hypothesis and informationally inefficient markets. Therefore, a rejection of the martingale hypothesis implied by (4) does not necessarily mean that markets are informationally inefficient. For example, the existence of risk averse agents who possess rational expectations may drive a wedge between the current futures price and the expected futures price, leading to a risk premium in (3). That is, since an important role of the futures market is to diversify or transfer risk, investors can make use of all available information efficiently, but if participants are risk averse, they will require a premium to bear the risk of price fluctuations. In turn, if this risk premium is related to lagged values of returns or other information variables, then, conditional on the assumption of the average forecast error being zero, a rejection of the null hypothesis need not be associated with market inefficiency. Ma (1989) examines the existence of a risk premium for crude oil, heating oil, and leaded gasoline futures with maturities of 1, 2, 3, and 6-months. The results provide little evidence for a risk premium and there does not appear to be a systematic bias in the observed futures prices, supporting the simple efficiency hypothesis.

Existence of a risk premium is not the only reason for questioning any evidence against market efficiency provided by serially correlated excess returns. Kaminsky and Kumar (1990), for example, argue that such results need not imply market failure, because the econometrician is examining the ex post behavior of participants who may actually form expectations rationally ex ante. Similar to Frankel and Froot (1987), if investors are trying to uncover the true model of a process that changes over time, then, agents in the process of learning can make repeated mistakes. That is, even if market participants are using all available information efficiently, their information set is incomplete, due to their inability to readily discern shifts in the data generating process. However, Hsieh (1989a, 1991) points out that to the extent such shifts are infrequent, the nonstationary behavior arising from changes in the structure of the price process should have a minimal effect on high frequency data. As a result, if agents are presumed to be using information efficiently, then a rejection of the null hypothesis would tend to be associated with evidence of a risk premium.

Tests for Nonlinear Dependence

The linear specifications for market efficiency provided in the previous section may not fully account for the returns generating process underlying futures commodities. For one thing, the weak-form efficiency test is not designed to detect nonlinear behavior in the conditional mean. In addition, evidence of time-varying volatility would suggest that the distribution of the error term in the previous tests would not be independent and identically distributed (iid).

Brock, Dechert, and Scheinkman (1987) propose a test (BDS) for deviations from iid behavior. For a sequence of observations $\{x_t: t = 1, \ldots, N\}$ that are independent and identically distributed, an M-dimensional vector $X_t^M = (x_t, x_{t+1}, \ldots, x_{t+M-1})$ can be formed. The test computes a statistic based on the correlation integral defined by.[11]

$$C_M(\varepsilon, N) = \frac{2}{N_M(N_M - 1)} \sum_{t<s} I_\varepsilon(X_t^M, X_s^M) \tag{5}$$

where $N_M = N - M + 1$ and $I_\varepsilon(X_t^M, X_s^M)$ is an indicator function defined as,

$$\begin{aligned} I_\varepsilon(X_t^M, X_s^M) &= 1, \text{ if } \|X_t^M - X_s^M\| < \varepsilon \\ &= 0, \text{ otherwise} \end{aligned} \tag{6}$$

and $\|\ \|$ denotes the maximum-norm.

The test statistic is

$$\text{BDS}_M(\varepsilon, N) = \frac{\sqrt{N_M}\,[C_M(\varepsilon, N) - C_1(\varepsilon, N)^M]}{\sigma_M(\varepsilon, N)} \tag{7}$$

which has a standard normal limiting distribution. Under the null hypothesis that $\{x_t\}$ is iid, the term, $\sqrt{N_M}[C_M(\varepsilon, N) - C_1(\varepsilon, N)^M]$, has a normal limiting distribution with mean zero and standard deviation, $\sigma_M(\varepsilon, N)$. The null hypothesis of a random iid process is rejected if the probability of any two M-histories being close together exceeds the Mth power of the probability of any two points being close together.

In general, a rejection of the null hypothesis is consistent with some type of dependence in the returns that could result from a linear stochastic process, nonstationarity, a nonlinear stochastic process, or a nonlinear deterministic system. Following Hsieh (1991), linear dependence can be ruled out by filtering the changes in futures prices using an autoregression. In addition, nonstationarity is not likely to be an important issue given the use of daily data. To the extent that the nonstationarity

[11]The correlation integral measures the fraction of the pairs of points of $\{x_t\}$ that are within a distance of ε from each other. The value for ε is chosen relative to the standard deviation divided by the spread of the raw data.

can be associated largely with structural change, it is reasonable to assume that such changes occur infrequently. Thus, the use of high frequency data should minimize the effects of such nonstationary behavior. As a result, since the test utilized in this study analyzes the residual series from an autoregression of the raw data, the rejection of the null hypothesis can be associated with evidence for nonlinearity.

If nonlinearity is detected by the BDS test, the next step is to determine whether the source is the mean of the stochastic process or the variance, or possibly both the mean and the variance. For example, nonlinearity in the mean could arise from the additive nature of a time-varying risk premium component. On the other hand, nonlinearity in the variance can be regarded as being multiplicative in nature. Following Hsieh (1989a), these two types of nonlinearities in a series, v_t, which represents the residual from the filtered raw data series, x_t, may be expressed as follows:

1. Additive dependence

$$v_t = f[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] + w_t \tag{8}$$

2. Multiplicative dependence

$$v_t = g[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}]\, w_t \tag{9}$$

where w_t is an iid random variable with mean zero and independent of past v_t's and x_t's. The functions, $f(\cdot)$ and $g(\cdot)$, are some nonlinear functions of the v_t's and x_t's for finite p. Given the above formulations, additive dependence indicates that the nonlinearity enters through the mean of the stochastic process. Multiplicative dependence implies that the presence of the nonlinearity is transmitted through the variance, as suggested by the autoregressive conditional heteroscedasticity (ARCH) process developed by Engle (1982). Both additive and multiplicative dependence can influence the random variable, v_t, as in the case of the ARCH-in-mean (ARCH-M) model given by:

$$\begin{aligned} v_t &= f[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] \\ &\quad + g[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] w_t \end{aligned} \tag{10}$$

Hsieh (1989a, 1991) points out that $x(t)^2$ will be correlated with its own lags in both the additive and multiplicative cases, which is what the McLeod and Li (1983) test detects. The distinguishing feature, however, is that under additive dependence

$$E[v_t|v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] \neq 0, \quad (11)$$

while multiplicative dependence implies:

$$E[v_t|v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] = 0. \quad (12)$$

Hsieh provides a third-order moment test to detect the presence of additive dependence, given the null hypothesis of multiplicative dependence. The test statistic exploits the fact that, under multiplicative dependence, v_t is not correlated with terms such as $v_{t-i}v_{t-j}$, while additive dependence implies that the variable, v_t, is correlated with terms like $v_{t-i}v_{t-j}$. Thus, the null hypothesis of multiplicative dependence makes use of the fact that $E[v_tv_{t-i}v_{t-j}] = 0$, while the alternative hypothesis is $E[v_tv_{t-i}v_{t-j}] \neq 0$, implying additive dependence.[12] The test is designed to reject only in the presence of additive nonlinearity not multiplicative nonlinearity. Thus, the third-moment procedure is able to detect a specification such as the ARCH-M. Although the third-order moment test is used to discriminate between the source of nonlinear stochastic effects, the presence of deterministic nonlinear behavior cannot be ruled out even if the null hypothesis of zero third-order moments is not rejected. Hsieh (1991) notes that the test can fail to detect a chaotic process whose odd product moments are zero. However, this research does not explore alternative tests to determine if the conditional mean in (8) is zero.

Conditional Heteroscedasticity

While the third-moment test can suggest the source of the nonlinearity, it does not provide the researcher with a specific model to analyze the data generating process. The generalized ARCH (GARCH) model of Bollerslev (1986) provides a convenient framework for studying any nonlinearities suggested by the third-moment test. For instance, the GARCH model is able to characterize any multiplicative nonlinearity, while a GARCH-in-mean (GARCH-M) model is capable of capturing the process defined in (12).

The GARCH(1,1) model is described by:[13]

[12]Hsieh (1989a, 1991) points out that if the conditional mean is zero then the third-moment is zero but the converse is not true. That is, a process could have a zero third-moment and not have a zero conditional mean.

[13]The use of the GARCH(1,1) model is appropriate since it has been found to provide a parsimonious fit for many economic and financial time series [see, for example, Baillie and Bollerslev (1989), and McCurdy and Morgan (1987)].

$$r_t = \beta_0 + \sum_{i=1}^{p} \beta_i r_{t-i} + v_t \tag{13}$$

$$h_1 = \alpha_0 + \alpha_1 v_{t-1}^2 + \alpha_2 h_{t-1} \tag{14}$$

where h_t is the conditional variance of v_t which depends linearly on past squared values of the error term v_t, and the lagged conditional variance itself. The sum $\alpha_1 + \alpha_2$ is a measure of the persistence of a shock to the variance. As noted by Engle and Bollerslev (1986), if this sum equals one, the Integrated GARCH process (IGARCH) is said to be exhibited, implying that shocks to the conditional variance will persist over future horizons. With IGARCH-type models, the second and fourth unconditional moments are nonexistent even though the conditional distributions are still well defined.

The original work on GARCH modeling assumed a conditional normal distribution for the error term v_t. To the extent that any nonnormality is attributable mainly to excess kurtosis, a conditional student t density function may be more appropriate.[14] Indeed, many researchers have found that the GARCH model with the conditional normal distribution is unable to explain sufficiently the leptokurtocity in the standardized residuals [see, for example, Bollerslev (1987)].

ESTIMATION RESULTS

Weak-Form Efficiency Test

Table IV presents the weak-form tests for market efficiency. The statistics are distributed as a chi-square (χ^2) with (lag + 1) degrees of freedom. The insignificant χ^2 statistics for crude oil futures returns support the weak-form joint hypothesis that lagged returns do not have any informative impact on current returns and that the constant term is zero.[15]

[14]For the conditional normal process the appropriate log likelihood function is expressed as,

$$\text{Log}(L) = -\left(\frac{T}{2}\right)\log 2\pi - \sum_{t=1}^{T} \log h_t^{1/2} - \sum_{t=1}^{T}\left[\frac{v_L^2}{2h_t}\right]$$

The log likelihood function under the assumption that v_t follows a conditional student t density is given as,

$$\text{Log}(L) = \sum_{t=1}^{T}\left[\log\left(\Gamma\left(\frac{df+1}{2}\right)\right) - \log\left(\Gamma\left(\frac{df}{2}\right)\right) - \left(\frac{1}{2}\right)\log(\pi(df-2)\,h_t) - \left(\frac{1}{2}\right)(df+1)\log\left[1 + \left[\frac{v_t^2}{h_t(df-2)}\right]\right]\right]$$

where T is the sample size, $df > 2$ is the degrees of freedom, and $\Gamma(\ )$ is the gamma function.

[15]It should be noted, however, that none of the constant terms are significant even at the 10% level.

TABLE IV

Weak-Form Market Efficiency Tests for Futures Returns

lag	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
1	.2870	2.6655	3.8760
2	.3546	2.6858	6.8273[a]
3	2.6828	11.0170[b]	11.2878[b]
4	4.3910	11.3838[b]	11.2956[b]
5	8.5186	15.2909[b]	11.3261[a]
6	8.7931	15.6357[b]	11.3406
7	9.1367	16.0920[b]	12.1238
8	12.0770	23.3004[c]	16.5919[a]
9	12.1522	23.7945[c]	17.4343[a]
10	13.9852	25.9766[c]	25.4493[c]

The data is for the period, December 3, 1984, to September 30, 1993. The numbers in the table are the $\chi^2_{(p+1)}$ statistics for testing the null hypothesis of weak-form market efficiency and are based on the following regression,

$$r_t = \beta_0 + \sum_{i=1}^{p} \beta_i r_{t-i} + v_t$$

Under the null hypothesis of weak-form market efficiency, the regression coefficients β_i (i = 0, . . . , p) are all equal to zero and the error term, v_t, has mean zero and is not serially correlated.

[a,b,c]Indicate marginal significance levels of 10%, 5%, and 1%, respectively.

On the other hand, futures returns for heating oil and unleaded gas reject the weak-form market efficiency since the reported χ^2 statistics are highly significant for several lagged returns. These latter results are not surprising given the significant autocorrelation coefficients in Table III for heating oil and unleaded gas.

Nonlinear Dependence[16]

Since the presence of linear dependence can influence the outcome of tests for nonlinearity, a filter is used to remove the serial dependence in the raw data and the resulting residual series are re-tested.[17] An analysis of the residuals series in the weak-form test of market efficiency in the previous section reveals the presence of ARCH effects. The McLeod and Li test based on the Q-statistic also suggests the presence of nonlinearities.

[16]The tests are computed using an algorithm developed by Dechert and contained in Brock, Hsieh, and LeBaron (1991).

[17]The selection of the appropriate filter is based on the significance of the heteroscedasticity-consistent t-statistic through lag 20 being no greater than 5%. The resulting filter consists of an autocorrelation of lag 10 for heating oil and unleaded gas. However, for the sake of uniformity and ease of comparison of the empirical results, the same filter is used for crude oil even though none of the individual autocorrelations for crude oil is significant at the 5% level.

TABLE V

BDS Test Statistics of Residuals from Weak Form Regression

M	ε/σ	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
2	.5	13.523	11.143	13.017
3	.5	18.587	14.096	16.816
4	.5	24.222	17.021	20.527
5	.5	30.744	20.090	24.393
6	.5	38.778	23.553	29.547
2	1.0	15.441	13.012	14.613
3	1.0	19.613	15.985	18.041
4	1.0	23.487	18.687	20.960
5	1.0	27.202	21.269	23.531
6	1.0	31.168	23.897	26.442
2	1.5	16.451	14.932	16.702
3	1.5	19.867	18.073	19.819
4	1.5	22.446	20.316	22.176
5	1.5	24.283	21.994	23.826
6	1.5	26.047	23.547	25.532
2	2.0	15.667	16.051	18.981
3	2.0	18.233	19.402	21.561
4	2.0	20.499	21.366	23.280
5	2.0	21.679	22.495	24.248
6	2.0	22.743	23.465	25.259

The data is for the period, December 3, 1984, to September 30, 1993. The numbers in the tables are the BDS test statistics and are calculated as,

$$\mathrm{BDS}_M(\varepsilon, N) = \frac{\sqrt{N_M}\,[C_M(\varepsilon, N) - C_1(\varepsilon, N)^M]}{\sigma_M(\varepsilon, N)}$$

The BDS statistic has a standard normal limiting distribution. The null hypothesis of a random iid process is rejected if the probability of any two *M*-histories being close together exceeds the *M*th power of the probability of any two points being close together, where *M* is the vector dimension.
The 10%, 5%, and 1% critical levels are 1.645, 1.960, and 2.575, respectively.

Table V provides the results of the BDS test applied to the residual series from the linear filter. The values of the test statistics are all positive and significantly greater than zero, rejecting the null of iid behavior for the residuals.[18] The rejection of the iid behavior for the residual series implies that the conditional density differs from the unconditional density in describing short-term dynamics of petroleum futures prices. Furthermore, the presence of nonlinear dynamics implies that linear methods (e.g., Box–Jenkins) cannot be used to model the conditional density.

While the BDS test indicates the presence of nonlinearities, it does not indicate whether the stochastic process affects the mean or variance

[18]The results of the BDS test using the residual series are similar to the outcome of the test applied to the raw data.

TABLE VI

Third-Order Moment Test Statistics of Residuals from Weak-Form Regression

Lag				
i	*j*	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
1	1	−.0219	−.0270	−.0174
2	1	.0417	.0057	.0248
2	2	.0193	.0201	.0227
3	1	−.0298	−.0207	−.0311
3	2	.0050	.0082	.0172
3	3	−.0016	−.0092	.0115
4	1	.0211	.0491	.0403
4	2	.0085	.0005	.0022
4	3	.0315	.0244	−.0039
4	4	−.0079	−.0177	−.0165
5	1	−.0133	−.0064	−.0092
5	2	−.0186	−.0263	−.0100
5	3	.0156	.0473	.0247
5	4	.0132	−.0251	.0027
5	5	−.0158	.0051	.0151

The data is for the period, December 3, 1984, to September 30, 1993.
The 10%, 5%, and 1% critical levels are 1.645, 1.960, and 2.575, respectively.

(possibly both) of the series. To uncover the source of the nonlinear behavior, Hsieh's (1989a, 1991) third-order moment test statistics are calculated and the results are reported in Table VI for $i,j = 1,2,3,4,5$.[19] None of these third-order moment test statistics are significant, indicating a failure to reject the null hypothesis of multiplicative dependence. This result implies that nonlinearity arises solely from the variance of the process. It also implies that it may be more appropriate to use a GARCH model instead of a GARCH-M model.[20] Based on these results, there is little evidence of nonlinear predictability for the conditional mean of the petroleum returns which would rule out a nonlinear process for the risk premium.

GARCH Estimates[21]

Table VII shows the results of estimating a GARCH(1,1) model under the assumption that the innovations follow a conditional student-*t* den-

[19]Results for $i, j = 6, 7, 8, 8, 10$, not reported here but available on request, also fail to reject the null hypothesis.
[20]Hsieh's test has low power against the GARCH-M. Various other GARCH-M models are estimated with the contemporaneous and lagged variance, but none are found to be significant.
[21]The estimation procedure is the approximate maximum likelihood algorithm of Berndt, Hall, Hall, and Hausman (BHHH) (1974).

TABLE VII

GARCH(1,1) Estimates for Autoregression with Conditional-t Distribution

Coefficient	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
β_0	0.0629[b]	0.0474	0.0438
	(0.0279)	(0.0317)	(0.0293)
β_1	−0.0038	0.0269	0.0622[c]
	(0.0229)	(0.0230)	(0.0233)
β_2	−0.0080	−0.0083	0.0000
	(0.0211)	(0.0219)	(0.0218)
β_3	0.0094	−0.0153	−0.0092
	(0.0210)	(0.0220)	(0.0218)
β_4	0.0242	−0.0076	−0.0043
	(0.0217)	(0.0219)	(0.0222)
β_5	−0.0111	−0.0356	0.0034
	(0.0214)	(0.0220)	(0.0217)
β_6	0.0181	−0.0102	−0.0070
	(0.0215)	(0.0223)	(0.0223)
β_7	0.0185	0.0142	−0.0001
	(0.0212)	(0.0212)	(0.0219)
β_8	0.0029	−0.0188	−0.0169
	(0.0215)	(0.0220)	(0.0221)
β_9	−0.0086	−0.0114	0.0050
	(0.0214)	(0.0212)	(0.0215)
β_{10}	0.0501[b]	0.0466[b]	0.0537[b]
	(0.0207)	(0.0214)	(0.0214)
df	6.7026[c]	10.9136[c]	13.1674[c]
	(0.7662)	(1.8026)	(3.1116)
α_0	0.0447[c]	0.0633[c]	0.0595[c]
	(0.0135)	(0.0185)	(0.0159)
α_1	0.1163[c]	0.1063[c]	0.1161[c]
	(0.0160)	(0.0142)	(0.0147)
α_2	0.8785[c]	0.8781[c]	0.8665[c]
	(0.0149)	(0.0156)	(0.0162)
$\alpha_1 + \alpha_2$	0.9948	0.9844	0.9826
$\chi^2_1(\alpha_1 + \alpha_2 = 1)$	0.432	3.753	4.353
	[0.511]	[0.053]	[0.037]
$\chi^2_{11}\left(\sum_{i=0}^{10} \beta_i = 0\right)$	16.648	13.553	17.198
	[0.119]	[0.259]	[0.102]
SIC	8559.894	8674.690	8324.182
κ_1	5.596	4.505	3.717
κ_2	5.895	3.868	3.655
$Q^2(30)$	38.891	38.597	41.313
	[0.050]	[0.053]	[0.029]
$Q^2(60)$	72.312	53.876	70.408
	[0.070]	[0.556]	[0.093]

[a,b,c]Denote significance at the 10%, 5%, and 1%, respectively. SIC is the Schwarz information criterion for optimum lag selection. κ_1 and κ_2 are the sample kurtosis and implied estimates of the conditional kurtosis, respectively. $Q^2(p)$ is the McLeod–Li test for second-order dependence up to lag p. Standard errors in parentheses and marginal significance levels in brackets.

sity. GARCH effects are undeniably evident since the coefficients α_1 and α_2 are statistically significant for all three oil futures series. Additionally, the parameter estimates of the variance equation imply a very high persistence for shocks to the conditional variance. The response function of volatility to shocks decays at a slow daily rate measured by $\alpha_1 + \alpha_2$. These rates are 0.9948, 0.9844, and 0.9826 for crude oil, heating oil, and unleaded gasoline, respectively. For crude oil, for example, the proportion of shock remains at 0.9948^7 or 0.9642 after a week, and 0.9948^{30} or 0.8552 after a month. Even one year after the shock occurs, about 15% (0.9948^{365}) of the initial impact remains in effect.

Several diagnostic test statistics are also reported for the standardized residuals in Table VII. The $Q^2(30)$ and $Q^2(60)$ are the McLeod–Li test for second-order dependence. Based on these two statistics, the null hypothesis of uncorrelated squared residuals is rejected at better than the 5% level for the majority of the cases reported. A departure from normality is also suggested by the presence of excess kurtosis. The sample kurtosis figures, κ_1, are 5.596, 4.505, and 3.717 for crude oil, heating oil, and unleaded gasoline, respectively. Furthermore, the implied estimates of the conditional kurtosis, $\kappa_2 = 3(\text{df-2})(\text{df-4})^{-1}$, are 5.895, 3.868, and 3.655 for crude oil, heating oil, and unleaded gasoline, respectively. It is also interesting to note that these implied estimates of the conditional kurtosis are in close accordance with their sample analogues.

Overall, the results in Table VII provide some evidence that the three energy futures returns may be described by the GARCH(1,1) model. However, the data also suggest that an integrated GARCH (IGARCH) process [Engle and Bollerslev (1986)] may be an appropriate characterization for the crude oil series, since the null hypothesis that the sum, $\alpha_1 + \alpha_2$, is equal to unity cannot be rejected at any standard significance level (the marginal significance level for $\chi_1^2(\alpha_1 + \alpha_2 = 1$ is 0.511). For heating oil and unleaded gas, the results suggest that a near-IGARCH is more appropriate since the sum, $\alpha_2 + \alpha_2$, is greater than 0.98 but less than 1.0 for both series. Also, the null hypothesis that the sum, $\alpha_1 + \alpha_2$ equals one is rejected at the 5% level in nearly every case.

Lastly, the evidence provided in Table VII also supports the martingale hypothesis. In contrast to the results of the weak-form test contained in Table IV, the joint test that the constant term and all lagged terms are equal to zero cannot be rejected at the 5% level in every case. If the GARCH specification is appropriate, then it should result in more efficient estimators than OLS asymptotically. Alternatively, if the GARCH model is misspecified, the estimates of the β_i's could be inconsistent. This caveat is important since this study is not able to account fully for the second-order dependence in the standardized residuals for the condition t-distribution.

GARCH Estimates with Volume as a Mixing Variable

In this section, an attempt is made to uncover the source(s) of the GARCH effects documented in Table VII. Similar efforts have been made for common stock returns. Copeland (1976), Epps and Epps (1976), and Akgiray (1989), among others, have argued that the nonstationarity of variance in common stock returns is a function of the information arrival to the market. In a recent study, Lamoureux and Lastrapes (1990) (hereafter LL) empirically investigate the possibility that the daily stock returns are generated by a mixture of distributions in which the stochastic mixing variable is hypothesized to be the rate of information arrival. Using trading volume as a proxy for the rate of information arrival, LL study the GARCH effects for a sample of 20 common stocks and find that these effects vanish when volume is introduced as a proxy for the mixing variable.

This section takes an approach similar to that of Lamoureux and Lastrapes (1990) by investigating the extent to which trading volume explains the GARCH effects found in petroleum futures. To accomplish this goal, eq. (17) and (18) are re-estimated with volume included in the conditional variance equation.[22] As noted by Lamoureux and Lastrapes (1990), if volume is serially correlated, then α_1 and α_2 should become small and statistically insignificant when α_3, the coefficient on volume, is greater than zero. Moreover, the persistence of a shock as given by the sum, $(\alpha_1 + \alpha_2)$, should also become negligible if accounting for the flow of information explains the presence of GARCH effects in the data.

The results of the empirical estimations are given in Table VIII under the assumption of conditional-t distribution. The findings indicate that while α_3 (the coefficient on the volume variable) is significant, albeit small in size, both α_1 and α_2 remain highly significant as in Table VII. Thus, lagged squared residuals still appear to contribute significant information

[22]Volume is for the nearby contract until the day preceding its last day of trading when volume for the next-out contract is used. This method is in accordance with the way in which a single time series is constructed for futures prices. The volume measured is then scaled by dividing by 1000. Finally, a test for a unit root in the volume series using the Phillips–Perron method finds that volume is trend-stationary in level form. The volume series used in the GARCH model is accordingly detrended using fitted values from the following regression,

$$\mathrm{VOL}_t/1000 = \alpha_0 + \alpha_1 \times t + \zeta_t, t = 1, 2, \ldots, N.$$

Where N is the number of observations to which the data is fitted.

TABLE VIII

GARCH(1,1) Estimates for Autoregression with Conditional-*t* Distribution and Volume

Coefficient	*Crude Oil*	*Heating Oil*	*Unleaded Gas*
β_0	0.0669[b]	0.0500	0.0458
	(0.0278)	(0.0317)	(0.0293)
β_1	−0.0027	0.0275	0.0624[c]
	(0.0230)	(0.0231)	(0.0233)
β_2	−0.0080	−0.0082	−0.0002
	(0.0213)	(0.0220)	(0.0219)
β_3	0.0107	−0.0148	−0.0085
	(0.0211)	(0.0220)	(0.0218)
β_4	0.0254	−0.0075	−0.0040
	(0.0217)	(0.0220)	(0.0223)
β_5	−0.0108	−0.0360[a]	0.0039
	(0.0215)	(0.0219)	(0.0218)
β_6	0.0185	−0.0102	−0.0072
	(0.0214)	(0.0222)	(0.0223)
β_7	0.0191	0.0135	−0.0002
	(0.0210)	(0.0211)	(0.0218)
β_8	0.0038	−0.0186	−0.0176
	(0.0213)	(0.0219)	(0.0221)
β_9	−0.0078	−0.0105	0.0050
	(0.0212)	(0.0212)	(0.0214)
β_{10}	0.0519[b]	0.0464[b]	0.0541[b]
	(0.0204)	(0.0213)	(0.0213)
df	6.1650[c]	11.1939[c]	13.2858[c]
	(0.7955)	(1.9014)	(3.1493)
α_0	0.0140	0.0317	0.0498[c]
	(0.0144)	(0.0223)	(0.0167)
α_1	0.1210[c]	0.1092[c]	0.1192[c]
	(0.0168)	(0.0146)	(0.0150)
α_2	0.8652[c]	0.8719[c]	0.8605[c]
	(0.0161)	(0.0161)	(0.0166)
α_3	0.0008[c]	0.0019[b]	0.0012
	(0.0003)	(0.0010)	(0.0008)
$\alpha_1 + \alpha_2$	0.9862	0.9811	0.9797
$\chi^2_1 (\alpha_1 + \alpha_2 = 1)$	2.511	5.164	5.515
	[0.113]	[0.023]	[0.019]
$\chi^2_{11} \left(\sum_{i=0}^{10} \beta_i = 0 \right)$	18.988	13.811	17.708
	[0.061]	[0.244]	[0.089]
SIC	8557.833	8678.280	8329.640
κ_1	5.536	4.417	3.715
κ_2	5.771	3.834	3.646
$Q^2(30)$	41.577	40.139	41.672
	[0.020]	[0.028]	[0.020]
$Q^2(60)$	83.432	55.347	72.886
	[0.008]	[0.462]	[0.054]
	[0.070]	[0.556]	[0.093]

concerning the time-varying variance of petroleum futures returns even after accounting for the rate of information flow, as measured by volume. The results in Table VIII also show that the persistence in volatility, ($\alpha_1 + \alpha_2$), remains high and is not significantly distinguishable from the evidence in Table VII. Based on the Schwarz information criterion (SIC) criterion, the only improvement occurs for crude oil. This persistence in variance is similar to the results of Najand and Yung (1991) who examine the price-volume relationship for Treasury bond futures.

Taken together, these findings suggest that the introduction of volume as an explanatory variable in the conditional variance equation somewhat dampens the GARCH effects and helps reduce the marginal significance level for the null hypothesis of an IGARCH process for the three petroleum in futures prices. Volume is a significant explanatory variable in most cases, but does not replace the lagged variance or the lagged squared residuals in explaining the nonconstancy of petroleum futures return variances.[23] These results are at odds with those of Lamoureux and Lastrapes (1990) for individual stock returns.[24] This difference in results implies that, while GARCH effects for individual stocks may arise from time variation in information arrival, volume may be a poor proxy for analyzing this information flow in petroleum futures returns.

In a further attempt to uncover the source(s) of the GARCH effects shown in Tables VII, the basis, in place of volume, is used as an explanatory variable in the conditional variance, the results are not significant. Thus, it appears that variables other than volume or the basis play an important role in explaining the observed nonconstancy of petroleum futures return variances. Thus, further research should be directed toward uncovering these other possible sources of heteroscedasticity.

CONCLUSION

This article tests for linear and nonlinear dependence in three petroleum futures returns and finds evidence of both types of dependence in all cases. The evidence against market efficiency based on tests for linear dependence, however, is not supported by the data after the presence of

[23]A caveat raised by Lamoureux and Lastrapes (1990) in interpreting the results concerns the possibility of a simultaneity problem, if current volume is correlated with the disturbance term, v_t. Following Lamoureux and Lastrapes, lagged volume is used as an instrument for current volume. It is found that it has little explanatory power in the variance equation. Lagged volume is significant at the 5% level for only the crude oil series when estimated using the conditional normal distribution. The authors are currently conducting tests for causal relationships between price changes and volume separately.

[24]Lamoureux and Lastrapes (1990) do find that the GARCH effects remain for 4 of the 20 stocks in their sample, even in the presence of volume.

nonlinear dependence has been accounted for. This study finds that a GARCH(1,1) model provides support for the martingale hypothesis since it fails to reject the null hypothesis that the constant and lagged terms are jointly zero. However, the conditional distribution used in this study is unable to account fully for the second-order dependence in the return series. Although alternative conditional distributions are available, a more useful inquiry would involve explaining the documented GARCH effects. To that end, the GARCH(1,1) specification is augmented by using trading volume as a proxy for the rate of information arrival. It is found that while volume is a significant explanatory variable in most cases, it does not replace the lagged variance or the lagged squared residuals in explaining the nonconstancy of petroleum futures return conditional variances. This last finding runs counter to that of Lamoureux and Lastrapes (1990) who study the GARCH effects for a sample of 20 common stocks and find that these effects vanish when volume is introduced as a proxy for information arrival used as the mixing variable. Replacing volume with the basis in the conditional variance equation is also unsuccessful in accounting for the time varying volatility.

The third-order moment test suggests that the evidence for nonlinearity in the time series arises solely from the variance of the process. This provides some evidence against a nonlinear process for any risk premium. However, the standardized residuals from the GARCH estimates still exhibit substantial nonlinearities, a finding that is consistent with Hsieh's (1991) contention that the third-order moment test may not be robust to heavy-tailed data. Thus, the evidence against nonlinear factors influencing both the mean and variance cannot be rejected.

The overall results of this study have important implications for practitioners or researchers who use volatility or variance models to price options or other derivative products. For example, the validity of the standard Black–Scholes options price formula hinges on the crucial assumption of random walk behavior for the price of the underlying asset. Also, hedge ratios and the amount of capital needed to cover possible losses during the time a futures position is held depend critically on the probability distribution of changes in futures prices. Therefore, testing and accounting for nonlinearities is important since this may help improve short-term price predictability and may lead to the construction of more accurate hedge ratios and long-term improvements in investment strategies.

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