The Predictive Power of Implied Stochastic Variance from Currency Options

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INTRODUCTION

The purpose of this article is to test the hypothesis that the implied variance extracted from option prices is the market expectation of the future variance of the underlying assets, and thus should dominant time series variance forecasts. Recently, Day and Lewis (1992), Canina and Figlewski (1993), and Lamoureux and Lastrapes (1993) found that implied volatility is a biased forecast of future volatility, and that implied volatility does not outperform forecasts from either GARCH models or simple historical-based statistical models.[1] These findings, based on stock index and

[1]The empirical evidence on testing the implied volatility hypothesis is mixed. Latane and Rendleman (1976), Chiras and Manaster (1978), and Beckers (1981) found evidence favorable to the hypothesis: the weighted implied standard deviation explains more of the cross-sectional variation in the future standard deviations of individual security returns than do historical volatilities. Day and Lewis (1992) found that the implied volatilities from S&P 100 index options contain incremental information to the GARCH models. Canina and Figlewski (1993) found evidence against the hypothesis: the implied volatilities from S&P 100 index options have little predictive power for subsequently realized volatility than simple historical volatilities. Lamoureux and Lastrapes (1993) rejected the orthogonality restriction that the forecast from time-series models should not have predictive power in addition to implied volatility. Recently, Fleming (1993), Jorion (1995), and Xu and Taylor (1995) found that implied volatility is a high quality estimator in terms of *ex ante* forecasting power.

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individual stock options, are inconsistent with the hypothesis that implied volatility is an efficient predictor of future volatility. This contradiction motivates the present article, which explores the performance of implied volatility in the foreign exchange market. The article seeks to extend existing research by (a) reducing the measurement errors, (b) adopting a stochastic volatility option pricing model, and (c) improving the techniques of statistical inference.

Previous studies have focused on stock and stock index options. This study seeks to extend the debate to foreign currency options traded on the Philadelphia Stock Exchange (PHLX). There are at least three major reasons for this extension. First, the PHLX is the major organized exchange for trading foreign currency options. The PHLX commenced trading in currency options in 1982. Since then, the size of the market has grown very rapidly. By 1993, the Australian dollar, British pound, Canadian dollar, German mark, Japanese yen, French franc, Swiss franc, and the European currency unit were traded. Second, measurement errors usually encountered in the studies of stock and stock index options can be corrected or substantially mitigated in the study of PHLX foreign currency options. Harvey and Whaley (1991) suggested that measurement errors caused by "infrequent trading," "bid–ask spread bounce," and "nonsynchronous trading" can substantially distort inference in the studies of S&P 100 index options. The simultaneous recording of exchange rates and option prices in the PHLX database alleviates dramatically the "nonsynchronous trading" problem. Third, the stochastic nature of exchange rate volatility motivates the exploration of the implications of the stochastic volatility option pricing model on the estimation of implied variance.

The importance of the study depends upon whether the conclusion holds over a wide variety of currencies and sampling periods. In this study, prices of dollar/mark and dollar/yen options, from January, 1986 to February, 1993, are analyzed. These two currencies are highly volatile, and options on them are highly liquid compared with options on other currencies.

Another potential source of bias is the misspecification of the valuation model for currency options. The stochastic nature of exchange rate volatility suggests that the estimated implied volatility from the class of Black–Scholes (1973) constant volatility models is subject to model specification errors. Recently, various stochastic volatility option pricing models have been developed. Examples include Scott (1987), Hull and White (1987, 1988), Wiggins (1987), Melino and Turnbull (1990), Stein and Stein (1991), and Heston (1993). These models make it theoretically attractive to adopt a logically consistent stochastic volatility model, and to explore the implication of the stochastic volatility model in the study of implied variance.

The out-of-sample tests focus on examining the quality of the implied variance as an *ex ante* variance forecast over a fixed horizon. Statistical problems arise in testing of unbiasedness and orthogonality restrictions with highly persistent variables. The estimation and statistical inference are complicated by the presence of serial correlation and finite sample bias in regression coefficients. To overcome these problems, the Fully Modified least square estimator of Phillips and Hansen (1990) along with the Newey and West (1987) variance–covariance estimator is employed.

The out-of-sample tests suggest that the implied variance from currency options is an efficient, but biased forecast of the future variance of the exchange rate, and that the MA(60) and GARCH variance forecasts do not contain significant incremental information. These results are consistent with the findings of Fleming (1993) and Jorion (1995) that implied volatility is a dominant, but biased estimator in terms of *ex ante* forecasting power. Furthermore, the forecasting performance of implied variances extracted from the model of Hull and White (1987) confirms the usefulness of the stochastic volatility model in the study of implied variance.

The remainder of this article is organized as follows: it introduces the data, the option pricing model, and develops the procedures to estimate the implied variance, then discusses time series formulations of the GARCH model and variance measures. This is followed by tests of unbiasedness and orthogonality restrictions.

IMPLIED VARIANCE ESTIMATION

This section discusses the data source, option pricing model, and implied variance estimation procedure. Two conditions are crucial for minimizing errors in the estimation of implied variance: (a) the correct measurement of option prices and other exogenous economic inputs, and (b) the validity of the option pricing model.

Data Description

Trading records for foreign currency options are taken from the "Foreign Currency Options Pricing History" database of the Philadelphia Stock Exchange (PHLX). One of the advantages of using PHLX currency options is that the simultaneous recording of exchange rates and option prices alleviates dramatically, if not completely, the "nonsynchronous trading problem" that arises in studies of stock index options. The database contains prices for options on Deutsche mark, Japanese yen, British pound, Swiss franc, Australian dollar, Canadian dollar, and European currency unit. The record for each option trading contains the following: the date of trade, currency and option identification, maturity symbol, strike price, time of trade, and the option premium per contract. The trade-by-trade option prices cover the period from January, 1983 to February, 1993. Most of the options are American-style with maturity of less than one year. The maturity date is the third Wednesday of the contract months: March, June, September, and December. Contracts maturing in the nearest other two months are also traded in the PHLX.

Transaction prices for American-style options on the dollar/mark and dollar/yen exchange rates, with time to maturity ranging from 15 to 90 calendar days, are selected. These two currency options are chosen for two reasons: they are highly volatile; and they are the most frequently traded. Because trading records before 1986 are poorly documented, the selected sample starts in January, 1986.

The interest rates used are the daily closing prices of Euro Deposits in terms of the U.S. dollar, German deutsche mark, and Japanese yen. The maturities are of 30, 60, and 90 days. The proxies used for intermediate rates are the rates whose maturities are the closest to the option expiration date.

The Pricing of Foreign Currency Options

The stochastic nature of exchange rate volatility suggests that the estimated implied variance from the constant volatility Black–Scholes (1973) model is subject to specification error. Recently, various stochastic volatility models have been developed, i.e., Scott (1987), Hull and White (1987, 1988), Wiggins (1987), Melino and Turnbull (1990), and Heston (1993). These models make it theoretically attractive to recover the market expectation of future variance from a stochastic volatility model. However, no published article has extracted implied variance from a stochastic volatility model.[2] To meet this challenge, this article seeks to extend the

[2]Jorion (1995) claims that this is because of the computational costs usually involved. The main difficulties in extracting implied variance from a stochastic volatility model are that the closed form pricing solution is unknown, and that the parameter estimation is nontrival. For example, Wiggins (1987) proposed finite difference "hopscotch" numerical methods with logarithmically transformed state variable. Melino and Turnbull (1990) used GMM to estimate the volatility process and Monte Carlo simulation to compute the option price. Although Heston (1993) developed an analytic approach based on the Fourier inversion method, the estimation of parameters is still burdensome.

existing empirical literature by adopting the well-known model of Hull and White (1987), and to explore the implication of the stochastic volatility model on the study of implied variance.

Hull and White (1987) Stochastic Volatility Model

It is assumed that:

a. The market is frictionless: trading takes place continuously in time and there are no transaction costs, taxes, or short sale restrictions.

b. The instantaneous risk-free interest rate, r_F, and domestic/foreign interest differential, $b = r_D - r_F$, are known and nonstochastic.[3]

c. The state variables are the underlying asset, $S(t)$, and the instantaneous variance, $V(t)$. The Hull and White (1987) model is based on the following stochastic processes:

$$dS(t) = \phi S(t)dt + \sigma S(t)\,dw \tag{1}$$

$$dV(t) = \mu V(t)dt + \xi V(t)dz \tag{2}$$

The variable, ϕ, is a parameter that may depend on $S(t)$, $\sigma(t)$, and t. The variables, μ and ϕ, are assumed to be constant. dz and dw are increments to standard Wiener processes with instantaneous correlation, ρ. Notice that the instantaneous variance, $V(t) = \sigma^2(t)$, follows a geometric Brownian Motion, so ϕ is the volatility of variance parameter. Making the variance random helps to capture the kurtosis of the underlying assets.

If the instantaneous variance is uncorrelated with aggregate consumption (i.e., the random variance has no systematic risk) and the instantaneous correlation, ρ, between the underlying asset and its variance process is zero, the Hull and White option price, C^{HW}, [their eq. (8)] is the integral of the Black–Scholes price, $C^{BS}(\bar{V})$, over the distribution of the mean variance, $\bar{V}$,

$$C^{HW}(S_t, \sigma_t^2) = \int C^{BS}(\bar{V})h(\bar{V}|\sigma_t^2)d\bar{V} \tag{3}$$

where h is the conditional distribution of $\bar{V}$, and $\bar{V}$ is the mean variance over the time interval, $[0, T]$,

[3]The valuation of options when the interest rate is stochastic has been considered by Merton (1973) on stock options, by Grabbe (1983), Hilliard, Madura, and Tucker (1991) on currency options. However, Hull (1993) and Scott (1993) showed that interest rate volatility usually has little impact on short-term option prices since it is very much smaller compared with the volatility of the underlying asset.

$$\bar{V} = \frac{1}{T}\int_0^T \sigma^2(t)dt$$

Hull and White (1987) proposed a power series approximation technique based on expanding $C^{BS}(\bar{V})$ in a Taylor series about its expected value, $E(\bar{V})$,

$$C^{HW}(S_t, \sigma_t^2) \approx C^{BS}(E(\bar{V})) + \frac{1}{2}\frac{\partial^2 C^{BS}(E(\bar{V}))}{\partial \bar{V}^2}\mathrm{Var}(\bar{V}) + \frac{1}{6}\frac{\partial^3 C^{BS}(E(\bar{V}))}{\partial \bar{V}^3}\mathrm{Skew}(\bar{V}) \quad (4)$$

where Var (V) and Skew $(\bar{V})$ are the second and third central moments of $\bar{V}$ [Hull and White (1987)]. The second and third terms in eq. (4) are essential to capture the so-called "smile" effect for foreign currency options traded on PHLX documented by Bodurtha and Courtadon (1987). This equation is the basis for extracting the market expectation of future variance that appears below.

The quality of the approximation given by eq. (4) depends on the values of the parameters, μ and ξ. Hull and White (1987) suggest that for $\mu = 0$ and sufficiently small values of $\xi^2(T - t)$ (ξ with approximate range of 1–4), this approximation is very accurate. They argued that the choice of $\mu = 0$ is empirically justified on the grounds that, for any nonzero μ, options of different maturities would exhibit markedly different implied volatilities, but this was never observed empirically. Values of ξ are estimated from the GARCH(1,1) parameters estimates reported in Table I. Nelson (1991) showed that as the time interval goes to zero, the GARCH parameter, a_1, approaches $\xi\sqrt{dt/2}$. Using daily exchange rates data from 1986 to 1993, this suggests an estimate of ξ of 1.957 for the dollar/mark and 1.889 for dollar/yen.

American Style Option Pricing

Since the currency options used in this article are American-style options, it is important to adjust for the early exercise premium. This is done by using the Barone-Adesi and Whaley (1987) quadratic approximation method. As an example, American-style call option prices, $C(S, T)$ are calculated as

$$C(S, T) = c(S, T) + A_2(S/S^*)^{q_2},\ S < S^*$$
$$C(S, T) = S - X,\ S \geq S^* \quad (5)$$

TABLE I

In-Sample Regression and Tests of Information Content

Panel A: Dollar/Mark Rate From January 1986 to February 1993

Variance	λ_0	a_0	a_1	β_1	δ	Log-Likelihood	$\chi^2_a(1)$	$\chi^2_b(2)$
(G1)	0.00304	0.0038			1.2297	1394.43		
	(1.2168)	(3.8264)			(13.7051)			
(G2)	0.0048	0.0111	0.2041			1338.09		
	(1.8286)	(22.9121)	(5.8838)					
(G2′)	0.0038	0.0027	0.1021		1.0507	1401.96	127.74	15.06
	(1.5323)	(2.6159)	(3.3931)		(11.1072)			
(G3)	0.0043	0.00058	0.0871	0.8719		1386.81		
	(1.7356)	(3.6061)	(5.9211)	(40.0651)				
(G3′)	0.0038	0.0018	0.1022	0.2768	0.7232	1404.11	34.6	19.63
	(1.5417)	(2.0516)	(3.6051)	(1.6451)	(3.6152)			

Panel B: Dollar/Yen Rate From January 1986 to February 1993

Variance	λ_0	a_0	a_1	β_1	δ	Log-Likelihood	$\chi^2_a(1)$	$\chi^2_b(2)$
(G1)	0.0041	0.0021			0.8553	1554.73		
	(1.7589)	(2.403)			(9.1813)			
(G2)	0.0055	0.0099	0.1146			1505.10		
	(1.2414)	(24.5422)	(3.9981)					
(G2′)	0.0042	0.0021	0.0354		0.8111	1555.62	101.00	1.74
	(1.7923)	(2.4967)	(1.5123)		(8.3921)			
(G3)	0.0041	0.0008	0.0837	0.8453		1542.91		
	(1.7493)	(3.7068)	(5.4359)	(28.4491)				
(G3′)	0.0041	0.0008	0.0672	0.5125	0.3635	1558.13	30.4	6.74
	(1.7497)	(1.5621)	(3.0601)	(3.2321)	(2.5342)			

This table reports the ARCH(1) and GARCH(1, 1) parameter estimates and tests of the information content of implied variance in the GARCH specification of conditional variance. The implied variance, σ^2_I, is extracted from the model of Hull and White (1987). The return R_t is the change in the logarithm of the exchange rate. The foreign currencies used are the dollar/mark and dollar/yen rates. The sample period is from January, 1986 to February, 1993. The t-statistics in parentheses are computed using the robust inference procedure of Bollerslev and Wooldridge (1992). $\chi^2_a(1)$ statistics are for tests of the restricted model (G2) [or (G3)] against the alternative given by (G2′) [or (G3′)], $\chi^2_b(2)$ statistics are for tests of the restricted model (G1) against the alternative given by (G2′) [or (G3′)]. The various specifications of the conditional variance are

$$R_t = \lambda_0 + \varepsilon_t \qquad \varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, h^2_t)$$

$$\text{(G1)}\ h^2_t = a_0 + \delta\sigma^2_{I,t-1}$$

$$\text{(G2)}\ h^2_t = a_0 + a_1\varepsilon^2_{t-1}$$

$$\text{(G2}')\ h^2_t = a_0 + a_1\varepsilon^2_{t-1} + \delta\sigma^2_{I,t-1}$$

$$\text{(G3)}\ h^2_t = a_0 + a_1\varepsilon^2_{t-1} + \beta_1 h^2_{t-1}$$

$$\text{(G3}')\ h^2_t = a_0 + a_1\varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \delta\sigma^2_{I,t-1}$$

where $c(S, T)$ is the price for an European-style call option from the model of Hull and White (1987), S is the spot price, X is the strike price, and $A_2 = (S^*/q_2)\{1 - e^{(r_F - r_D)T} N[d_1(S^*)]\}$ [q_2 and S^* are defined in Barone–Adesi and Whaley (1987)].

Implied Variance Estimation Procedure

Suppose that the option market is efficient and that option traders incorporate their expectation of future volatility into the actual trading, then the market expectation of future volatility should be contained in the market prices. The *benchmark* implied volatilities are usually estimated by matching the model prices to the *at-the-money* options prices, and then inverting the Black–Scholes model. As discussed earlier, an advantage of using the stochastic volatility model is to capture the "smile" effect. The approximation of the Hull and White (1987) model illustrated in eq. (4) allows extraction of implied variance from currency options with a wider range of moneyness. Because it is an unbiased estimate of the variance that can be estimated from eq. (4), the term *implied variance* (instead of *implied volatility*) is used throughout the article. Consequently, the daily averaged implied variance is estimated by pooling together options with strike price/spot price ratio (moneyness) between 0.8 to 1.2.

Due to the computational costs and small differences typically generated by various weighting schemes, the equally weighted nonlinear least squares approach of Whaley (1982) is used to estimate the implied variance.[4] For each day, t, in the sample, the daily average implied variance, $\bar{V}_t = E(\bar{V}|\Omega_t)$, is estimated by nonlinear least squares. That is, $\bar{V}_t$ is chosen to minimize the distance between observed option prices on day, t, and the predicted option prices from the Hull and White (1987) model:

$$\min_{\bar{V}_t} SSE(\bar{V}_t) = \sum_i [CP_{t,i} - CP_{t,i}^{HW}]^2 \tag{6}$$

where i is an index over observations (calls or puts) in a given day; $CP_{t,i}$ is the observed market price; $CP_{t,i}^{HW}$ is the theoretical option price given the interest rate differential, $r_D - r_F$, strike price/spot price ratio, $(X/S)_{t,i}$, and the remaining life of an option, $\tau_{t,i}$.

Since the main objective is to test the predictive power of implied variance over future variance, it is necessary to estimate the implied variance from options with maturities that can match the fixed forecasting horizon. In addition, increasing the number of option prices used to extract the implied variance is also useful in further reducing the measure-

[4]In the literature, different weighting schemes are applied to pool average implied volatilities from different options to obtain an accurate estimate of implied volatility. Latane and Rendleman (1976) and Chiras and Manaster (1978) weighted each implied volatility according to the sensitivity of the option to its volatility, which is called Weighted Implied Standard Deviation (WISD). Beckers (1981) used only the option whose price is most sensitive to volatility. Day and Lewis (1992) introduced GLS with trading volumes as the weight.

ment error. Therefore, option contracts with maturities ranging from 30 to 90 calendar days (or 20 to 60 trading days) are pooled in the daily implied variance estimation.[5] The number of trades in a day varies significantly across the sample. The average number of trades is around 100 for the dollar/mark options and 50 for the dollar/yen options.

THE PREDICTIVE POWER OF IMPLIED VARIANCE

The purpose of this section is to examine whether the implied variance represents a sufficient statistic for forecasting the future variance of the exchange rate. The future variance of the exchange rate can be decomposed into two orthogonal parts

$$V_{t+1} = E[V_{t+1}|\Omega_t] + \varepsilon_{t+1} \tag{7}$$

where ε_{t+1} is random shock that is orthogonal to the conditional variance. If the implied variance is the optimal estimator of the conditional variance, it should be an *unbiased* forecast and its forecast error should be *orthogonal* to the information set, Ω_t. The tests of *unbiasedness* and *orthogonality* restrictions are based on the joint hypothesis that, (a) the foreign currency option markets are informationally efficient, and (b) the Hull and White (1987) option pricing model is valid.

Information Content and In-Sample Tests

The *information content* of implied variance, defined to be the one day ahead relationship between the implied variance and future variance, is tested by examining the significance of implied variance within the GARCH specification of conditional variance. This approach is in the same spirit as the work of Day and Lewis (1992) and Lamoureux and Lastrape (1993). However, the problem of the "forecast horizon mismatch" between realized (future) variance and implied variance confines this study to address only whether there is useful information contained in the implied variance, rather than a formal test of the *unbiasedness* and *orthogonality* restrictions.

[5]The downside of pooling implied variances over different maturities is that it may affect the results of implied variance as an unbiased predictor of future variance, if the slope of the term structure within this maturity range is very steep. However, Campa and Chang (1995) suggested that the term structure of the dollar/mark volatility quotes is relatively flat between one-month to three-month maturities (their Table 1). Furthermore, Campa and Chang (1995) were unable to reject the expectation hypothesis in testing the term structure of implied volatilities in the foreign currency option market.

The ARCH(1) model of Engle (1982), GARCH(1,1) model of Bollerslev (1986), and the GARCH-Implied Variance specification of conditional variance are formulated as:

$$
\begin{aligned}
R_t &= \lambda_0 + \varepsilon_t, \quad \varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, h_t^2) \\
\text{(G1) } h_t^2 &= a_0 + \delta\sigma_{I,t-1}^2 \\
\text{(G2) } h_t^2 &= a_0 + a_1\varepsilon_{t-1}^2 \\
\text{(G2}') \ h_t^2 &= a_0 + a_1\varepsilon_{t-1}^2 + \delta\sigma_{I,t-1}^2 \\
\text{(G3) } h_t^2 &= a_0 + a_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \\
\text{(G3}') \ h_t^2 &= a_0 + a_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + \delta\sigma_{I,t-1}^2
\end{aligned}
\tag{8}
$$

where $R_t = \sqrt{252}\ln(S_t/S_{t-1})$ is the change in the logarithm of the exchange rate from time, $t - 1$ to t; ε_t is assumed to be conditionally normal with mean zero and conditional variance, h_t^2. $\sigma_{I,t-1}^2$ represents the implied variance at time, $t - 1$. It is obvious that specification (G2) is ARCH(1), and (G3) is GARCH(1,1). Specification (G2′) and (G3′) include the implied variance as an exogenous variable in the GARCH specification for the conditional variance. Specification (G1) imposes the restriction that the GARCH parameters in (G3′) are zero. The parameter estimates in the variance equations are constrained to be positive, $a_0 > 0$, $a_1 > 0$, $\beta_1 > 0$, and $a_1 + \beta_1 < 1$. The GARCH models are estimated by Maximum Likelihood, and the t-statistics are computed using the robust inference procedure proposed by Bollerslev and Wooldridge (1992).

The parameter estimates of ARCH(1) and GARCH(1,1) models from dollar/mark and dollar/yen currencies are reported in Table I. For the dollar/yen rate, the estimated GARCH parameters of a_1 and β_1 are 0.0837 and 0.8453, with t-statistics of 5.44 and 28.4, respectively; for the dollar/mark rate, the estimated GARCH parameters of a_1 and β_1 are 0.087 and 0.872, with t-statistics of 5.92 and 40.07, respectively. These results are consistent with previous findings that the conditional heteroscedasticity in daily spot rates can be represented by a GARCH(1,1) specification [Hsieh (1988, 1989); Baillie and Bollerslev (1989)]. Daily exchange rate changes appear to have zero unconditional means, and almost no serial correlation.

The significant positive estimates of δ in (G2′) and (G3′) of Table I suggests that the implied variance contains information in addition to that captured by the GARCH models. This conclusion is further supported by the likelihood ratio tests of (G2) vs. (G2′) and (G3) vs. (G3′). The $\chi_a^2(1)$ statistics strongly reject the null hypothesis of no information content of implied variance at the 1% level.

In general, the in-sample tests suggest that implied variance contains incremental information in the GARCH specification of conditional variance. Although conditional variance estimates from GARCH models have statistically significant explanatory power in the dollar/mark regressions, the improvement is only marginal.

Measures of Variance and Descriptive Statistics

This section introduces methods to construct various variance measures: the MA(60) variance, the realized variance, and the average GARCH variance. The exchange rates are measured as dollar/mark and dollar/yen. Daily observations from January, 1986 to February, 1993 are used. The observation interval is taken as a trading day.

First, the MA(60) variance, $\sigma^2_{H,t}$, is estimated from a moving average window over the past $N = 60$ trading days, which is taken as a simple variance forecast of the underlying exchange rate

$$\sigma^2_{H,t} = \frac{1}{N-1}\sum_{i=1}^{N}(R_{t-i} - \bar{R})^2 \tag{9}$$

Second, the realized (future) variance, $\sigma^2_{F,t}$, is estimated as the sample variance of the next $T = 60$ trading days from time, t, which is constructed to be compatible with the forecasting horizon of implied variance

$$\sigma^2_{F,t} = \frac{1}{T-1}\sum_{i=1}^{T}(R_{t+i} - \bar{R})^2 \tag{10}$$

Third, the average GARCH variance forecast is constructed as follows. The GARCH(1,1) model of Bollerslev (1986) as specified as

$$R_t = \lambda_0 + \varepsilon_t, \quad \varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, h_t^2) \tag{11}$$

$$h_t^2 = a_0 + a_1\varepsilon^2_{t-1} + \beta_1 h^2_{t-1} \tag{12}$$

$$\begin{aligned} h^2_{t+k,t} &= a_0 + a_1 E[\varepsilon^2_{t+k-1}|\omega_t] + \beta_1 h^2_{t+k-1,t} \\ &= a_0 + (a_1 + \beta_1)h^2_{t+k-1,t}, \qquad k = 2, \ldots, N \end{aligned} \tag{13}$$

where $R_t = \sqrt{252}\ln(S_t/S_{t-1})$ is the change in the logarithm of the exchange rate from time, $t-1$ to t; ε_t is assumed to be conditionally normal with mean zero and conditional variance, h_t^2.

To ensure that the forecast horizon of the average GARCH variance forecast is identical to that of the implied variance, the T-day average

GARCH variance forecast is constructed by recursive substitution of the one period ahead GARCH(1,1) variance forecast, $h^2_{t+1,t}$, to obtain k period ahead prediction, $h^2_{t+k,t}$; and then the average is taken over the forecast period. Parameters estimated from the past 2 years are chosen as starting values for the GARCH specification to reestimate the sample variance at each time, t.[6] $h^2_{t+k,t}$ is the k period ahead prediction of conditional variance. By mathematical induction, one obtains

$$h^2_{t+k,t} + a_0\left[\frac{1-(a_1+\beta_1)^{k-1}}{1-(a_1+\beta_1)}\right] + (a_1+\beta_1)^{k-1}\,h^2_{t+1,t} \tag{14}$$

Therefore, the T period ahead average GARCH variance forecast is calculated as

$$\sigma^2_{G,t} = \frac{1}{T}\left(\sum_{k=1}^{T} h^2_{t+k,t}\right) \tag{15}$$

Next, the issue of the timing of observations and the matching of implied variance with future variance has to be addressed. The forecast performance is evaluated by comparing the predictions of implied variance, average GARCH variance, and MA(60) variance with the realized (future) variance over a fixed period of time. The empirical examination of weekend effects by Baillie and Bollerslev (1989) suggested that the exchange rate is generated by a process operating closer to trading time rather than calendar time. In contrast, the interest rate over the weekend follows a calendar time process. This difference can be reconciled by using a trading time variance and a calendar time interest rate [French (1984)]. However, in practice, French's (1984) adjustment makes little difference except for very short life options [Hull (1993)]. Thus, the implied variance is estimated with the remaining life of contract measured in calendar days.

Tables II and III provide description of means, standard deviations, and autocorrelations for different measures of variance: the realized (future) variance, the MA(60) variance, the average GARCH variance, and the implied variance.

The last entries in Tables II and III report the tests of the unit root hypothesis for all measures of variance. The reported Augmented Dickey–Fuller tests [Dickey and Fuller (1979); Said and Dickey (1984)] suggest the rejection of the unit root hypothesis and in favor of a highly persistent

[6]This may give the GARCH model an advantage of using *ex post* parameter estimates. However, even with the advantage of *ex post* parameter estimates, statistical time-series models are still outperformed by option-implied forecasts.

TABLE II

Summary Statistics for Variance Measures (Mark)

Variance	$\sigma^2_{F,t}$	$\sigma^2_{M,t}$	$\sigma^2_{G,t}$	σ^2_I
Mean	0.013776	0.013663	0.013938	0.01412
Std.	0.00701	0.00708	0.0026	0.00448
ρ_1	0.996	0.996	0.955	0.950
ρ_2	0.990	0.990	0.906	0.923
ρ_3	0.984	0.983	0.858	0.900
ρ_4	0.976	0.975	0.810	0.883
ρ_5	0.967	0.966	0.765	0.866
ρ_6	0.958	0.957	0.727	0.849
ρ_7	0.949	0.947	0.693	0.832
ρ_8	0.938	0.937	0.663	0.815
ρ_9	0.927	0.925	0.633	0.800
ρ_{10}	0.914	0.913	0.606	0.786
ρ_{11}	0.901	0.900	0.579	0.770
ρ_{12}	0.888	0.886	0.550	0.752
ρ_{16}	0.827	0.824	0.442	0.685
ρ_{20}	0.758	0.754	0.335	0.616
ρ_{24}	0.688	0.682	0.261	0.559
Unit root test stat.	−5.093	−4.849	−5.092	−3.983

This table reports the mean, standard deviation, and autocorrelation coefficients for several variance measures: $\sigma^2_{F,t}$ is the realized (future) variance over the next 60 trading days; $\sigma^2_{M,t}$ is the MA(60) variance; $\sigma^2_{G,t}$ is the average GARCH variance; and σ^2_I is the implied variances extracted from the model of Hull and White (1987). The sample period is from January, 1986 to February, 1993. Augmented Dickey–Fuller test statistics [Dickey and Fuller (1979); Said and Dickey (1984)] for unit root hypothesis are reported for each measure of variance.

but stationary process at the 5% level for all the variance measures. Statistics for the realized (future) and the MA(60) variances display high autocorrelations for both currencies.[7] The realized variance averages about 0.013776 per annum for the dollar/mark rate and 0.011058 per annum for the dollar/yen rate.

The average GARCH forecasts are stationary but highly persistent series, with average values of 0.013938 for the dollar/mark and 0.011268 for the dollar/yen. The values of $\alpha_1 + \beta_1$ are around 0.96 for the dollar/mark and 0.93 for the dollar/yen. These estimates imply that a shock to the variance has a half-life of 17 days [ln(0.5)/ln(0.96)] and 10 days for the dollar/mark and the dollar/yen rates, respectively.

The implied variances from the dollar/mark and the dollar/yen options display patterns which are typical with highly persistent series. The first-order autocorrelation is 0.950 for the dollar/mark, and 0.928 for the

[7]Large autocorrelation often occurs in calculating variance measures on overlapping observations. This may or may not be due to a unit root. Hejazi (1994) contains simulation results on the relationship of autocorrelation and overlapping observations.

TABLE III

Summary Statistics for Variance Measures (Yen)

Variance	$\sigma^2_{F,t}$	$\sigma^2_{M,t}$	$\sigma^2_{G,t}$	σ^2_{I}
Mean	0.01106	0.01101	0.01127	0.01058
Std.	0.00508	0.00511	0.00106	0.00425
ρ_1	0.994	0.994	0.918	0.928
ρ_2	0.986	0.986	0.845	0.893
ρ_3	0.978	0.978	0.780	0.872
ρ_4	0.970	0.969	0.725	0.841
ρ_5	0.960	0.959	0.670	0.818
ρ_6	0.950	0.948	0.615	0.793
ρ_7	0.939	0.937	0.566	0.775
ρ_8	0.928	0.926	0.528	0.747
ρ_9	0.916	0.914	0.484	0.729
ρ_{10}	0.903	0.900	0.442	0.708
ρ_{11}	0.890	0.887	0.401	0.678
ρ_{12}	0.876	0.873	0.366	0.660
ρ_{16}	0.818	0.814	0.247	0.594
ρ_{20}	0.760	0.755	0.175	0.530
ρ_{24}	0.697	0.691	0.113	0.469
Unit root test stat.	−4.725	−4.728	−4.657	−3.391

This table reports the mean, standard deviation, and autocorrelation coefficients for several variance measures: $\sigma^2_{F,t}$ is the realized (future) variance over the next 60 trading days; $\sigma^2_{M,t}$ is the MA(60) variance; $\sigma^2_{G,t}$ is the average GARCH variance; and σ^2_I is the implied variances extracted from the model of Hull and White (1987). The sample period is from January, 1986 to February, 1993. Augmented Dickey–Fuller test statistics [Dickey and Fuller (1979); Said and Dickey (1984)] for unit root hypothesis are reported for each measure of variance.

dollar/yen. The average values of the implied variances are 0.01412 for dollar/mark options, and 0.010588 for dollar/yen options. However, the hypothesis that implied variances have the same mean as realized future variances cannot be rejected.

Figures 1 to 4 display the time series plots of the realized variance, the MA(60) variance, the average GARCH variance, and the implied variances for the dollar/mark and dollar/yen currencies.

Estimation and Statistical Inference

Three statistical problems must be addressed to test the *unbiasedness* and *orthogonality* restrictions: error-in-variables, serial correlation in the error terms, and finite sample bias in the regression coefficients.

First, the regression is subject to the error-in-variables problem if the implied variances are estimated with error. To minimize this possibility, the section 2 "Implied Variance Estimation" improved the implied variance extraction procedure by reducing data measurement errors, adopting

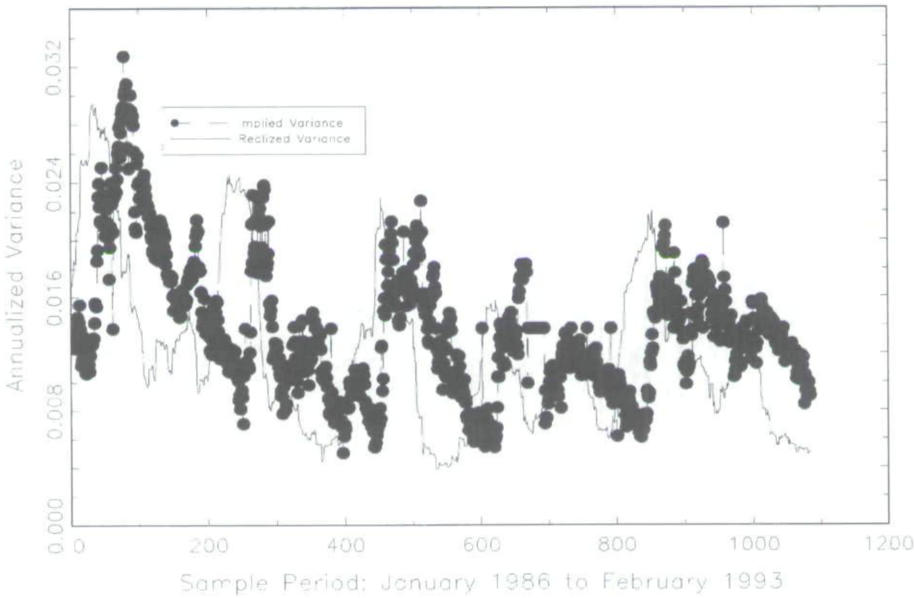

FIGURE 1
Historical variance versus realized variance (mark).

a logically consistent stochastic volatility model, and employing an efficient estimator.

Second, serial correlation arises by using overlapping observations rather than restricting the analysis to independent observations. Although the parameter estimates from ordinary least squares (OLS) are still unbiased, the OLS estimates of variance–covariance matrix are inconsistent.

Third, the estimation and statistical inference is further complicated by the finite sample bias in the regression coefficients that occurs in the presence of highly persistent series. For illustrative purposes, suppose the Data Generating Process (DGP) for an exogenous variable, x_t, and an endogenous variable, y_t, to be

$$y_t = \beta x_t + u_t, \qquad u_t \sim N(0, \sigma_u^2) \tag{16}$$

$$x_t = \phi x_{t-1} + v_t, \qquad v_t \sim N(0, \sigma_v^2) \tag{17}$$

where x_t can be made either $I(0)$ and $I(1)$ by choice of ϕ; $[u_t, v_t]'$ is weakly stationary with zero mean vector, and autocorrelated and intercorrelated.[8]

[8]Mankiw and Shapiro (1986) suggested that the asymptotic distribution of OLS test statistics can be extremely misleading when the time series examined is highly autoregressive. In practice, a practitioner relying on the asymptotic distribution will reject the true model too frequently. Stambaugh (1986) finds that the bias is negatively related with the sample size, and positively related to the autocorrelation in the exogenous variable and of the contemporaneous correlation between innovations in the two variables.

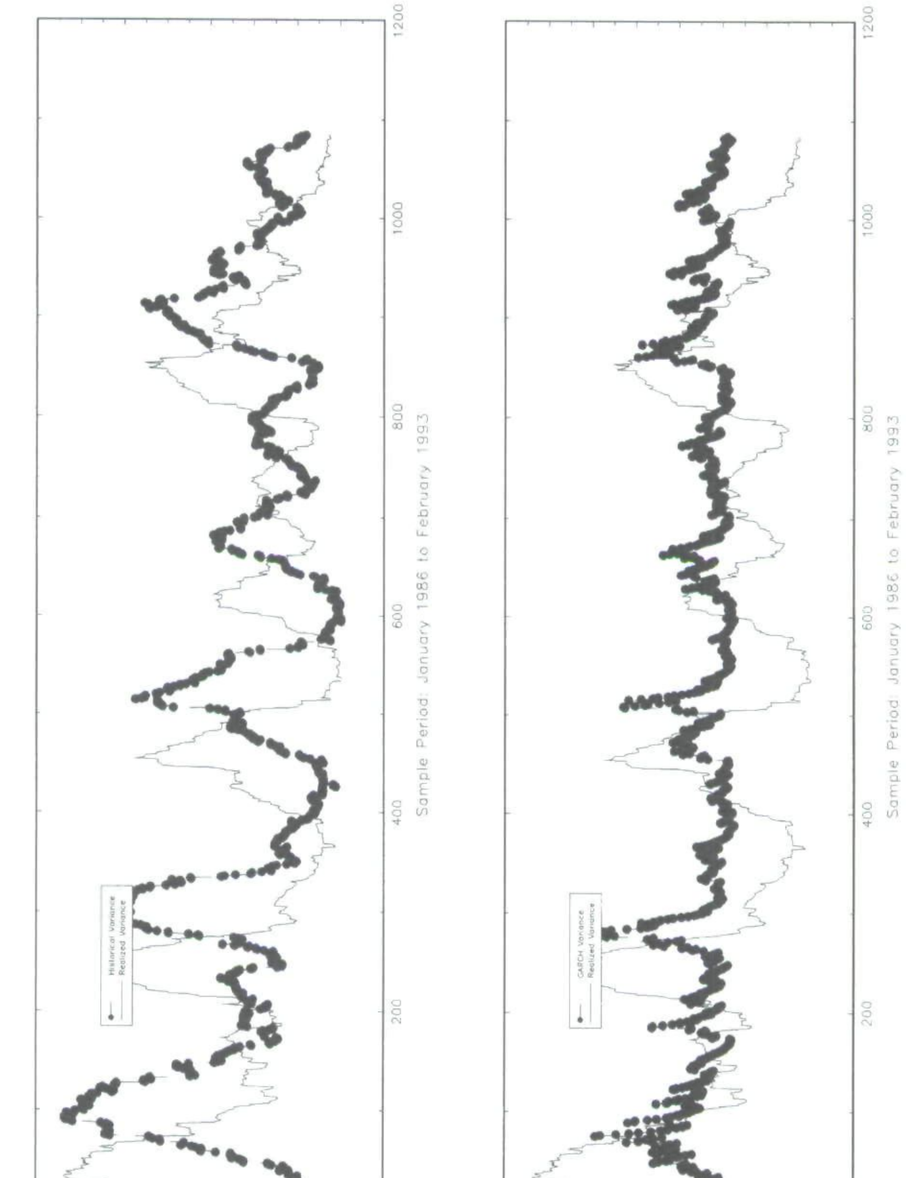

FIGURE 2
Implied variance (Hull–White) vs. realized variance (mark).

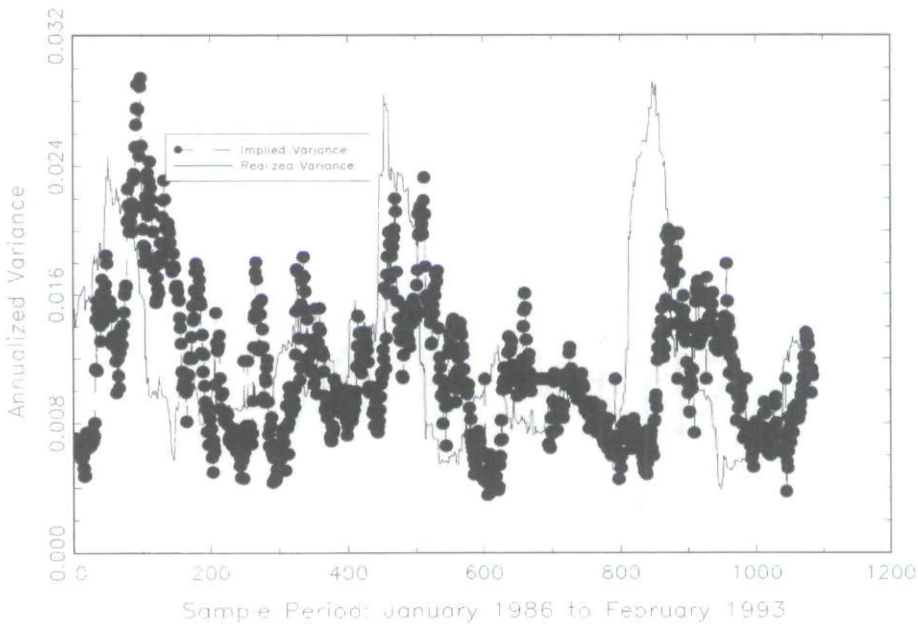

FIGURE 3
Historical variance vs. realized variance (yen).

For single-equation co-integrated regression, Banerjee, Dolado, Galbraith, and Hendry (1993) suggested that although the OLS parameter estimates are super consistent, their finite sample properties are not good; a large number of observations may be necessary before the bias becomes small. This study expects to encounter the same sort of finite sample bias as observed in static OLS parameter estimates of co-integrated economic variables. However, since all the variance measures are highly persistent but stationary, the sole issue here is finite sample bias of regression coefficients.

This study resolves the presence of serial correlation and the finite sample bias of regression coefficients by employing the Fully Modified least square estimator of Phillips and Hansen (1990), along with the Newey and West (1987) approach to estimate the variance–covariance matrix.[9] The reason for applying an estimator developed for nonstationary series to a near-integrated stationary model is purely a pragmatic one.[10]

[9]To correct for the finite sample bias in single-equation co-integrated regression, several asymptotically equivalent estimators are proposed: the FM estimator of Phillips and Hansen (1990), the "leads and lags" estimator of Saikkonen (1991) and Stock and Watson (1991), and the system procedure of full maximum likelihood estimation of Johansen (1988).

[10]Evans and Savin (1981) suggest that if the autoregressive parameter of an AR(1) model is close to one, the unit root asymptotic distribution actually provides a better finite sample approximation than

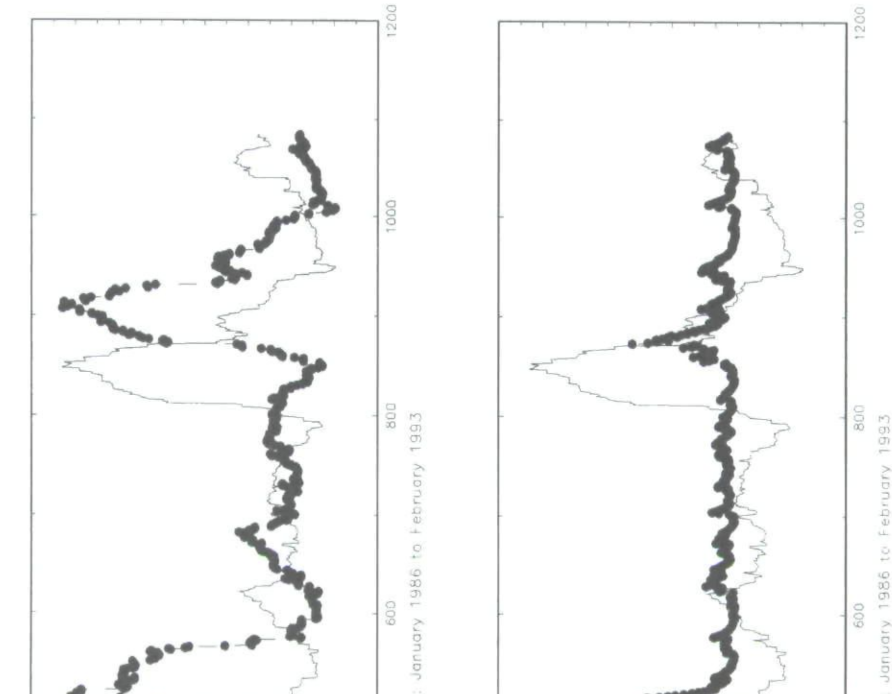

FIGURE 4
Implied variance (Hull–White) vs. realized variance (yen).

The Appendix describes the Phillips and Hansen's (1990) estimator, and demonstrates the statistical properties of the modified Wald statistics when the time series are highly persistent.

Unbiasedness and Orthogonality Restriction Tests

The *unbiasedness* of forecasts using a variance estimator is tested in the regression

$$\sigma^2_{F,t+1} = b_0 + b^*\sigma^2_t + \xi_{t+1} \tag{18}$$

where $\sigma^2_{F,t+1}$ is the realized variance, and σ^2_t is taken in turn to be implied variance, average GARCH variance, and MA(60) variance. If a forecast of future variance is unbiased, the estimates of b_0 and b^* should be insignificantly different from zero and one [Pagan and Schwert (1990)]. This joint hypothesis can be examined by using the modified Wald Statistics, χ^2_B, [Phillips and Hansen (1990)] with two degrees of freedom. Regressions (i), (ii), and (iii) in Tables IV and V summarize the results for testing the *unbiasedness* restriction for the three variance estimators.

Regression (i) in Tables IV and V examines the predictive power of implied variance. The Wald statistics, χ^2_A, indicate that the estimates of intercept and slope parameters are significantly different from zero; for example, the slope coefficient is 0.404 with t-statistic of 3.28 for dollar/yen. However, the *unbiasedness* restriction is rejected for both currencies; the χ^2_B statistics suggest the estimates of b_0 and b_1 are significantly different from zero and one jointly at the 1% level. Furthermore, the $\bar{R}^2$ statistics, with value of 0.0429 for the dollar/mark and 0.0923 for the dollar/yen, indicate that implied variance has significant explanatory power in predicting future variance. Finally, the positive intercept plus the slope coefficient less than one suggests that implied variance tends to underpredict in the low-variance state, but overpredict in the high-variance state.

Regression (ii) examines the predictive power of the GARCH variance forecasts. The estimates of b_0 and b_1 are significantly different from zero. The *unbiasedness* restriction is rejected for the dollar/mark but not for the dollar/yen. However, the GARCH variance forecasts have lower explanatory power than implied variances.

As for the MA(60) variance, the Wald statistics, χ^2_B, in regression (iii)

the asymptotically correct normal distribution for the limiting distribution of the least squares estimator. Campbell and Perron (1991) further suggest that it may be better to use integrated asymptotic theory for near-integrated stationary models, and stationary asymptotic theory for near-stationary integrated models.

TABLE IV

Out-of-Sample Regressions and Predictive Power Tests (Mark)

Regression	b_0	b_1	b_2	b_3	$\bar{R}^2$	χ^2_A	χ^2_B
Panel A: Unbiasedness Tests							
(i)	0.009	0.299			0.0429	221.83	32.49
	(4.18)	(2.32)				[0.000]	[0.000]
(ii)	0.0076		0.412		0.032	238.01	12.73
	(2.65)		(2.29)			[0.000]	[0.000]
(iii)	0.0096			0.349	0.001	123.78	7.29
	(2.7)			(1.35)		[0.000]	[0.000]
Panel B: Orthogonality Tests							
(iv)	0.0071	0.192	0.249		0.0449	225.89	35.19
	(2.49)	(2.19)	(1.136)			[0.000]	[0.000]
(v)	0.0091	0.486		−0.179	0.0641	213.13	37.81
	(4.16)	(2.46)		(−1.299)		[0.000]	[0.000]
(vi)	0.0076		0.4518	−0.0348	0.0373	249.75	
	(2.53)		(2.06)	(−0.34889)		[0.000]	
(vii)	0.0061	0.4196	0.3527	−0.229	0.0719	245.46	44.12
	(2.05)	(2.115)	(1.61)	(−1.625)		[0.000]	[0.000]

This table reports the out-of-sample tests for *unbiasedness* and *orthogonality* restrictions for the implied variance, the average GARCH variance, and the MA(60) variance of dollar/mark. The sample period is from January, 1986 to February, 1993. Implied variance $\sigma^2_{I,t}$ is estimated from the Hull and White (1987) model. The Fully Modified Least Square Estimator of Phillips and Hansen (1990) along with the Newey and West's (1987) variance–covariance estimator is employed. *t*-statistics are reported in parentheses. χ^2_A, χ^2_B-statistics (with [p-value]) are reported for the following null hypotheses, respectively, H^A_0: $b_i = 0$, $i = 0, 1, 2, 3$, H^B_0: $b_0 = 0$, $b^* = 1$ for (i), (ii), (iii) and H^B_0: $b_1 = 1$, $b_i = 0$, $i = 0, 2, 3$ for (iv), (v), (vii).

$$\sigma^2_{F,t+1} = b_0 + b_1\sigma^2_{I,t} + b_2\sigma^2_{G,t} + b_3\sigma^2_{M,t} + \eta_{t+1}$$

strongly reject the *unbiasedness* restriction. Indeed, the estimates of b_3 are not significantly different from zero for both currencies.

The comparisons of alternative variance forecasts suggest that the implied variance exhibits higher explanatory power than the MA(60) and GARCH models. The MA(60) variance forecast has the least explanatory power. In Tables IV and V the encompassing tests investigate whether the MA(60) and average GARCH variance forecasts contain additional information that is not contained in the forecasts of implied variances.

The relative predictive power of implied variance, average GARCH variance, and MA(60) variance can be examined using encompassing principle of Hendry and Richard (1982), and Fair and Shiller (1990). The encompassing principle is concerned with the ability of a model to explain the behavior embodied in the relevant characteristics of rival models. If a variance forecast contains no useful information regarding the dependent variable, one would expect the estimate of the coefficient for the

TABLE V

Out-of-Sample Regressions and Predictive Power Tests (Yen)

Regression	b_0	b_1	b_2	b_3	$\bar{R}^2$	χ^2_A	χ^2_B
			Panel A: Unbiasedness Tests				
(i)	0.0066	0.4039			0.0923	265.89	24.49
	(4.75)	(3.28)				[0.000]	[0.000]
(ii)	0.000187		0.947		0.0415	244.28	1.37
	(0.049)		(2.82)			[0.000]	[0.000]
(iii)	0.00648			0.472	0.0104	182.25	6.61
	(2.44)			(1.84)		[0.000]	[0.000]
			Panel B: Orthogonality Tests				
(iv)	0.0059	0.393	0.0588		0.0925	247.82	25.33
	(1.48)	(2.21)	(0.1295)			[0.000]	[0.000]
(v)	0.0073	0.549		−0.181	0.1079	263.51	28.48
	(4.98)	(3.75)		(−1.621)		[0.000]	[0.000]
(vi)	0.00257		1.008	−0.0523	0.0398	249.818	
	(0.0668)		(2.85)	(−0.502)		[0.000]	
(vii)	0.00527	0.492	0.242	−0.195	0.1094	280.97	28.33
	(1.37)	(2.74)	(0.558)	(−1.71)		[0.000]	[0.000]

This table reports the out-of-sample tests for *unbiasedness* and *orthogonality* restrictions for the implied variance, the average GARCH variance, and the MA(60) variance of dollar/yen. The sample period is from January, 1986 to February, 1993. Implied variance $\sigma^2_{I,t}$ is estimated from the Hull and White (1987) model. The Fully Modified Least Square Estimator of Phillips and Hansen (1990) along with the Newey and West's (1987) variance–covariance estimator is employed. t-statistics are reported in parentheses. χ^2_A, χ^2_B-statistics (with [p-value]) are reported for the following null hypotheses, respectively, H_0^A: $b_i = 0$, $i = 0, 1, 2, 3$, H_0^B: $b_0 = 0$, $b^* = 1$ for (i), (ii), (iii) and H_0^B: $b_1 = 1$, $b_i = 0$, $i = 0, 2, 3$ for (iv), (v), (vii).

$$\sigma^2_{F,t+1} = b_0 + b_1\sigma^2_{I,t} + b_2\sigma^2_{G,t} + b_3\sigma^2_{M,t} + \eta_{t+1}$$

particular forecast to be statistically insignificant. A general model for the encompassing tests is formulated as

$$\sigma^2_{F,t+1} = b_0 + b_1\sigma^2_{I,t} + b_2\sigma^2_{G,t} + b_3\sigma^2_{M,t} + \xi_{t+1} \tag{19}$$

where $\sigma^2_{F,t+1}$ is the future variance; $\sigma^2_{I,t}$ is the implied variance; $\sigma^2_{G,t}$ is the average GARCH variance; and $\sigma^2_{M,t}$ is the MA(60) variance. The *orthogonality* restriction implies that the MA(60) variance and average GARCH variance should contain no information in addition to the information contained in the implied variance for predicting future variance.

As in the *unbiasedness* tests, the slope coefficients of implied variance in regressions (iv) and (v) are significantly different from zero but not equal to one. The slope coefficients of the GARCH and MA(60) variance forecasts are smaller and insignificant with the presence of implied variance in regressions (iv) and (v). In addition, the $\bar{R}^2$s of the regressions for GARCH and MA(60) forecasts improve substantially.

Regression (vi) compares the predictive power of the two time-series models, the GARCH and MA(60) variance forecasts. For both currencies, the slope coefficients of the MA(60) forecasts become insignificant with the presence of the GARCH forecasts, whereas the $\bar{R}^2$s improve significantly. These results indicate that the GARCH variance dominates the MA(60) variance forecast in the class of time-series models.

Regression (vii) reports the encompassing tests for the implied variance, the average GARCH variance, and the MA(60) variance. The $\bar{R}^2$s indicate that neither the average GARCH variance nor the MA(60) variance contributes significantly to the explanatory power with the presence of the implied variance. Indeed, the parameter estimates of the GARCH and MA(60) variances are not significantly different from zero when implied variance is present as an explanatory variable.

It appears that implied variances extracted from the foreign currency options perform better in predicting the future variance of the underlying asset than those from the stock index and stock options. After adopting a stochastic volatility model, reducing data measurement errors, and improving the techniques of estimation and statistical inference, this study finds that the predictive power of implied variance cannot be rejected. These results are consistent with Fleming's (1993) and Jorion's (1995) findings that implied volatility represents a dominant, but biased estimator in terms of *ex ante* forecasting power.

CONCLUSION

This article investigates the predictive power of implied variances extracted from the dollar/mark and dollar/yen currency options. Foreign currency option contracts traded on the Philadelphia Stock Exchange (PHLX) are examined from January, 1986 to February, 1993. The simultaneous recording of exchange rates and option prices from the PHLX database alleviates dramatically the "nonsynchronous trading problem" that exists in the studies of stock index options. The existing literature is improved by (a) reducing the measurement errors, (b) adopting a stochastic volatility model, and (c) improving techniques of statistical inference.

In contrast with recent studies on stock index options, this article finds that implied variance is an efficient, but biased forecast of future variance, and that the variance forecasts from the GARCH and the MA(60) models do not contain significant incremental information in predicting future variance with the presence of implied variance.

There are at least two potential explanations for the rejection of *unbiasedness* restriction: there could be errors in the extraction of implied variance, and the option market may not be efficient. It is possible that the observed bias could be due to model misspecification, in particular the assumption of zero skewness or zero risk premium of variance risk in the model of Hull and White (1987). Because the daily dollar/mark and dollar/yen exchange rates display virtually no skewness [Knoch (1992); Hsieh (1988,1989)], the documented biases are unlikely to be caused by the assumption of a zero value of ρ. However, the assumption of a zero risk premium for variance risk could be a potential source of bias. Melino and Turnbull (1990) found that a stochastic volatility model with nonpositive price of volatility risk explains observed option prices better than the constant volatility models. The existence of a nonzero risk premium of variance risk is further confirmed by Guo (1995), who found that the risk premium parameter implicit in currency option prices is nonzero and time varying. An alternative explanation of the bias could be that the option market is inefficient, and option traders overstate or understate future variance during a specific period. In this case, it would be interesting to study whether the biases of implied variance are economically significant under a set of dynamic trading rules.

Finally, the forecasting performance of implied variance extracted from the model of Hull and White (1987) demonstrates the usefulness of stochastic volatility model in the study of implied variance. The evidence suggests that the practice of using of implied variance as a sufficient statistic of market's *ex ante* forecast of future variance is justifiable. After adjustment for the linear bias, the implied variance extracted from a stochastic volatility model is a high quality approximation of the market expectation of future variance in the foreign exchange market. The implied variance is also useful in pricing and hedging options, and in constructing the term structure of volatilities, etc.

APPENDIX

The semiparametric method of Phillips and Hansen (1990) and Hansen (1992) is a two-step estimator in which the first step estimates the covariance parameters, and the second step estimates the regression parameters.

$$y_t = \beta x_t + u_{1t}$$

$$x_t = x_{t-1} + u_{2t}$$

$u_t = [u_{1t},$ and $u_{2t}]'$ is weakly stationary with mean vector, $[0,0]'$, and long-run variance–covariance matrix

$$\Omega = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{n} E(u_j u_t')$$

$$\Lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{t} E(u_j u_t')$$

In practice, to ensure that the estimator for long run variance–covariance matrix, Ω, is positive semidefinite, kernel estimators are applied

$$\hat{\Omega} = \hat{\Omega}_0 + \sum_{j=1}^{m} w(j, m)\,(\hat{\Omega}_j + \hat{\Omega}_j')$$

$$\hat{\Lambda} = \hat{\Lambda}_0 + \sum_{j=1}^{m} w(j, m)\,(\hat{\Lambda}_j + \hat{\Lambda}_j')$$

where $w(\cdot)$ is a weight function (kernel) and m is a bandwidth parameter. The Bartlett kernel in Newey and West (1987) has the form, $w(j, m) = 1 - j/m+1$. A plug-in bandwidth estimator for the Bartlett kernel recommended by Andrews and Monahan (1992) is

$$\hat{m} = 1.1147(\hat{\alpha}(1)n)^{1/3}$$

where

$$\hat{\alpha}(1) = \sum_{a=1}^{p} \frac{4\hat{\rho}_a^2\hat{\sigma}_a^2}{(1-\hat{\rho}_a)^6\,(1+\hat{\rho}_a)^2} \Bigg/ \sum_{a=1}^{p} \frac{\hat{\sigma}_a^2}{(1-\hat{\rho}_a)^4}$$

and $(\hat{\rho}_a, \hat{\sigma}_{\mathrm{a}}^2)$ denote the autoregressive and innovation variance estimates for the ath element of residual, $\hat{e}_t$.

The second step is to estimate the parameters. Partition $\hat{\Omega}$ and $\hat{\Lambda}$, and set $\hat{\Omega}_{1\cdot 2} = \hat{\Omega}_{11} - \hat{\Omega}_{12}\hat{\Omega}_{22}^{-1}\hat{\Omega}_{21}$ and $\hat{\Lambda}_{21}^{+} = \hat{\Lambda}_{21} - \hat{\Lambda}_{22}\hat{\Omega}_{22}^{-1}\hat{\Omega}_{21}$ and the transformed dependent variable, $y_t^+ = y_t - \hat{\Omega}_{12}\hat{\Omega}_{22}^{-1}\hat{u}_{2t}$. The FM estimator of parameter, β, is

$$\hat{\beta}^{+} = \left(\sum_{t=1}^{n} (y_t^+ x_t' - (0\ \hat{\Lambda}_{21}^{+\prime}))\right)\left(\sum_{t=1}^{n} x_t x_t'\right)^{-1}$$

The Modified Wald Statistics of Phillips and Hansen (1990) is

$$\text{Wald} = (\beta^{+} - \beta_0)\left[\Omega_{11.2} * \left(\sum_{t=1}^{n} x_t x_t'\right)\right]^{-1} \sim \chi_k^2$$

Percentage Acceptance of Wald Statistics at Various Levels

The Data Generating Processes for y_t and x_t are

$$y_t = \beta x_t + u_t$$

$$x_t = \phi x_{t-1} + v_t$$

$u_t \sim N(0, 1)$ and $v_t \sim N(0, 1)$ are independent, and x_t can be made to be either $I(0)$ or $I(1)$ by the choice of ϕ. y_t is then regressed on x_t. This sequence is repeated 10,000 times for each value of ϕ. Entries denote acceptances of the null, H_0: $\beta = 1$. The sample sizes are $T = 300, 500$. The number of lags in the Bartlett kernel of Newey and West's (1987) variance–covariance estimator is chosen to be NLAG=2,10.

TABLE A.I

ϕ	*90 %*	*95 %*	*97.5 %*	*99 %*	*Wald*	*90 %*	*95 %*	*97.5 %*	*99 %*	*Wald*
$T = 300$			NLAG = 2					NLAG = 10		
1.0	89.00	94.40	97.05	98.66	1.056	86.69	92.80	95.67	97.97	1.194
0.99	88.80	94.11	96.96	98.75	1.068	86.29	92.46	95.36	97.79	1.228
0.98	89.06	94.31	96.94	98.66	1.055	86.42	92.32	95.66	97.76	1.222
0.97	89.15	94.24	96.82	98.59	1.065	86.60	92.03	95.29	97.71	1.232
0.96	88.88	94.02	96.70	98.66	1.075	85.96	91.82	95.31	97.73	1.257
0.95	88.31	93.83	96.79	98.64	1.098	85.48	91.77	95.41	97.61	1.285
0.94	88.76	94.04	96.60	98.31	1.087	85.88	91.84	95.11	97.45	1.271
0.93	88.32	93.54	96.46	98.44	1.105	85.46	91.70	94.87	97.46	1.291
0.92	88.29	93.80	96.81	98.69	1.108	85.65	91.64	95.34	97.54	1.299
0.91	88.11	93.85	96.71	98.50	1.106	85.43	91.44	95.18	97.72	1.274
0.90	88.22	93.73	96.53	98.38	1.120	85.58	91.85	95.14	97.51	1.297
$T = 500$			NLAG = 2					NLAG = 10		
1.0	88.96	94.31	97.09	98.84	1.053	87.39	93.26	96.33	98.24	1.145
0.99	89.16	94.11	96.89	98.78	1.046	87.54	93.18	96.00	97.99	1.157
0.98	88.79	93.98	96.86	98.60	1.079	86.47	92.25	95.68	97.90	1.213
0.97	89.02	94.07	97.02	98.83	1.047	87.06	92.79	95.91	98.09	1.172
0.96	88.82	93.97	96.83	98.59	1.063	86.84	92.40	95.73	97.90	1.200
0.95	88.46	93.76	96.51	98.55	1.093	86.15	92.01	95.41	97.63	1.244
0.94	88.77	94.08	96.86	98.75	1.076	86.61	92.17	95.53	97.90	1.232
0.93	88.60	93.90	96.58	98.51	1.091	86.23	92.12	95.28	97.81	1.242
0.92	88.25	93.64	96.72	98.48	1.104	86.19	92.08	95.46	97.75	1.247
0.91	88.23	93.90	96.96	98.65	1.115	85.75	92.10	95.83	97.80	1.271
0.90	87.98	93.51	96.55	98.51	1.119	85.95	91.74	95.48	97.74	1.259

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