Storage Profitability and Hedge Ratio Estimation*

SERGIO H. LENCE
MARVIN L. HAYENGA
MICHAEL D. PATTERSON

Farmers and other agents involved in the production, marketing, and processing of storable commodities often store them to profit from the expected increase in their cash prices. In such situations, marketing experts and economists often advise hedging against the price risk associated with commodity storage by using futures markets. Since commodity hedging is typically a direct hedge,[1] producers often take a short position in the futures market which is equal to their long position in the physical commodity. This is known as a "one-to-one" hedge. In more advanced hedging approaches, regression techniques are used to calculate hedge ratios which tell producers what percentage of the physical commodity stock to hedge in the futures market.

The majority of the empirical hedging literature published in recent years has dealt with the estimation of minimum-variance hedge ratios. For a given cash position, the minimum-variance hedge is the hedge that minimizes the variance of returns to the combined cash/futures position.

[1]A direct hedge consists of hedging in the futures contract for the same commodity as the one in the cash market.

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- *Sergio H. Lence is an Assistant Professor of Economics, Iowa State University.*
- *Marvin L. Hayenga is a Professor of Economics, Iowa State University.*
- *Michael D. Patterson is a Commodity Analyst for the Louis Dreyfus Corporation.*

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The proper specification of the minimum-variance hedge model has been debated extensively. Interestingly, however, the question of profitability has received very little attention. This omission is commonly justified by the fact that empirical studies have usually found futures prices to be unbiased. If this is true, the expected profitability of the futures position is zero. But unbiased futures markets do not ensure profitability in the cash position. For example, it is quite possible for the basis[2] to move in a relatively predictable fashion throughout the year, and to also have storage costs exceeding the expected increase in cash prices.

Economic theory suggests that cash and futures prices of storable commodities behave in a predictable fashion when supplies are abundant in the cash market. In this situation, exploitation of arbitrage opportunities should prevent cash prices from being below futures prices by more than the cost of carrying inventories into future months. If this were not the case, speculative profits could be made by buying a commodity in the spot market, selling futures contracts, and delivering the commodity at expiration to cancel the futures obligation. In the case of a cash market shortage, however, cash prices can shoot theoretically well above futures prices because there is no mechanism analogous to storage arbitrage that can put an upper limit to the difference between cash and futures prices.

Because of the aforementioned asymmetry in the behavior of cash and futures prices in periods of cash market surplus versus periods of shortage, the covariance between cash and futures prices is likely to be different under these two scenarios. But if such covariance is different under surpluses as opposed to shortages, minimum-variance hedge ratios calculated without differentiating between the two market situations will generally be inappropriate for use in storage hedge models. This is true because minimum-variance hedge ratios are obtained as the ratio of the covariance between futures and cash prices to the variance of futures prices. Only hedge ratios corresponding to the surplus scenario are to be used if, as it seems reasonable to assume, profitable storage and hedging occurs only when the commodity is abundant.

This study presents and discusses an alternative approach to the standard methods of hedge ratio estimation for the special case of storable commodities. The innovative feature of the model presented is that it allows one to test whether the expected profitability of storage affects the relationship between cash and futures prices, thereby affecting hedge ratios as well. The model is illustrated with applications to corn and soybeans. For these particular examples, it is found that the size of the hedge

[2]Basis is defined as cash prices minus futures prices.

ratio estimate is negatively influenced by the expected profitability of storage. Such impact implies that corn and soybean minimum-variance hedge ratios decrease in size as storage is expected to be more profitable.

RELATED WORK ON HEDGE RATIO ESTIMATION

The traditional minimum-variance hedge ratio is obtained from a simple regression of cash prices on futures prices or of cash price changes on futures price changes [Ederington (1979)]. The slope estimates in these simple regressions are interpreted as the minimum-variance hedge ratios. Benninga, Eldor, and Zilcha (1984) showed that minimum-variance hedges are also optimal—in the sense of maximizing expected utility—when futures markets are unbiased. They also pointed out that minimum-variance hedge estimates from regression models are empirically useful because their data requirements are minimal, futures are typically found to be unbiased [Kolb (1992)], and no assumptions about utility functions are necessary.

Much of the recent debate in the hedging literature centers around the proper specification of the minimum-variance hedge model. For example, researchers have argued about the suitability of hedge ratio estimation techniques which involve the regression of cash price changes on futures price changes versus those which use a regression of cash price levels on futures price levels. Hill and Schneeweis challenged the validity of price level models on both theoretical and statistical grounds using foreign currency futures markets as a case study. They found that the hedge ratios and hedging effectiveness measures obtained from price change models differed significantly from those calculated from price level models. According to Hill and Schneeweis, the main statistical problem with price level models is that they typically exhibit autocorrelated errors. When error terms are correlated, standard OLS regressions yield coefficient estimates that are unbiased and consistent, but inefficient and with conventionally estimated standard errors smaller than their true standard errors. This underestimation causes coefficient estimates to appear more accurate than they actually are. Hill and Schneeweis also cite theoretical problems with price level models. They argue that the purpose of hedging with currency futures is to reduce exposure to exchange rate changes. Thus, they claim that "an empirical analysis that examines relationships between levels leads to misspecified hedge ratios and measures of hedging effectiveness" [Hill and Schneeweis (1981, p. 659)].

Witt, Schroeder, and Hayenga (1987) examined optimal hedge ratios through price level regressions, price change regressions, and percentage price change regressions, and found that no one of the techniques was statistically superior to the others. They instead concluded that, from a practical standpoint, the appropriate hedge ratio estimation model should depend on the hedger's objective function. Price level models would be more appropriate when dealing with nonstorable commodities, whereas price change models would be more appropriate for storage models.

More recently, Lence (1995) showed theoretically that hedge ratio estimates from regression models are suboptimal (i.e., do not maximize expected utility) in general, unless futures are unbiased and prices are specified in levels or in level changes. Lence's findings rule out regressions in price logarithms or in price ratios to obtain hedge estimates consistent with expected utility maximization.

From a statistical standpoint, however, the choice between price level and price difference regression models can be settled straightforwardly by using the concept of integration [Engle and Granger (1987); Hendry (1986)]. If prices are stationary (i.e., if prices are integrated of order zero), then the coefficient corresponding to futures in a regression of cash prices on futures prices gives the hedge ratio. But if prices have a unit root (i.e., prices are integrated of order one), such coefficient represents the *long-term* relationship between those price series and typically will not provide the appropriate hedge ratio for decision makers who want to reduce *short-term* price risk. The appropriate hedge ratio will then be obtained as the coefficient on futures price changes in a cash price change error correcting model. Therefore, testing for stationarity is essential for a proper specification of the hedge regression model.

A related line of research was undertaken by Myers and Thompson (1989) who argued that the hedge ratio estimates traditionally used in the literature are inappropriate except under special circumstances.[3] Their assertion is true because the traditional literature calculates the slope coefficient as the ratio of the unconditional covariance between cash and futures prices (or cash and futures price changes) to the unconditional variance of futures prices (or futures price changes). The authors suggested a conditional information model that takes into account the information which is available at the time the hedge is placed. Viswanath (1993) developed a modified version of the Myers and Thompson model where changes in cash prices are regressed on changes in futures prices and the basis at the time the hedge is placed. In several

[3]As an aside, they also argued correctly that the approach suggested by Witt, Schroeder, and Hayenga (1987) regarding the choice between price level and price difference regression models was inappropriate. Myers and Thompson (1989, p. 859) stated that "... different decision problems will lead to different hedging rules, but the type of problem does not prescribe how the parameters ... of those hedging rules should be estimated."

instances, he found that his basis-corrected conditional model yielded smaller hedged portfolio return variances than the traditional unconditional model.

THE DECISION TO STORE AND THE STORAGE HEDGE

The decision as to whether to store a commodity or not is often based on historical price patterns for the storage period in question. Many producers store without the price risk protection of a futures market position. These producers rely on the hope that cash prices will improve enough to cover their storage costs. However, producers can use the futures market to reduce the price risk involved with storage. While the hedge ratios discussed in the preceding section are optimal in the sense that they are regression coefficients which minimize the variance around cash price levels or cash price changes, they typically neglect the issue of whether the cash position being hedged is expected to be profitable or not.

There are periods within the marketing year during which storing may be a rational decision. For example, soybeans are usually in abundant supply at harvest time, from the middle of September through the middle of October. Soybean cash prices are generally low at this time in comparison with other times of the year. Thus, storing soybeans from October through February, for example, may make sense if March futures prices plus the expected basis are sufficiently high to cover the cost of purchasing and carrying soybeans over the storage horizon. Conversely, soybean cash prices tend to fall during the period from June through harvest. A producer or commercial agent who is holding soybeans in June would generally not want to store over a horizon where returns to storage tend to be negative.

Because there are situations where expected returns to storage are negative, there is essentially a two-step procedure implicit in the decision to store and hedge. In the first step, the decision maker decides whether to store or not based on the expected profitability of storage (which is determined by the expected change in the basis if futures prices are unbiased). If he or she decides to store, the second step consists of calculating the optimal hedge for that physical position. In the price change regression method of hedge ratio estimation, hedge ratios are calculated by minimizing the variance around the change in cash price over the storage period. But this technique makes little sense in situations where storing is not rational in the first place (i.e., in situations where the expected profits from storage are negative). It seems obvious that the traditional price change models should be modified to take into account alternative market conditions: periods with expected profits for storage as opposed to periods with expected losses for storage. Analogous considerations apply to price level hedging regression models as well.

The article's central contribution can be summarized as follows: If storage is sometimes rational and sometimes not, and prices behave differently under the two scenarios, then one should only care about the price behavior when storage is expected to be profitable. In contrast, standard hedge-ratio estimation models do not differentiate between the two alternative scenarios. Therefore, traditional models yield hedge ratios that are some sort of "average" of the relevant (i.e., profitable scenario) and "irrelevant" (i.e., unprofitable scenario) hedge ratios.

The preceding intuitive concepts can be formalized using the standard model of hedging under expected utility maximization. Assume a decision maker with a utility function of terminal wealth ($U(W_T)$, $U' > 0$, $U'' < 0$), and terminal wealth defined as

$$W_T \equiv W_0 + P_T Q - P_0 Q - C(Q) + (F_0 - F_T)X \tag{1}$$

Here, W_0 is initial wealth, Q ($Q \geq 0$) is the quantity of commodity stored between the initial and terminal dates, P_T (P_0) is the commodity cash price on the terminal (initial) date, $C(Q)$ ($C(0) = 0$, $C' \geq 0$, $C'' \geq 0$) is the cost of carry, and X is the amount of futures contracts sold at the futures price, F_0, on the initial date and repurchased at the futures price, F_T, on the terminal date.

As of the decision date (0), it is assumed that W_0, P_0, and F_0 are fixed variables, P_T and F_T are random variables, and Q and X are decision variables. The agent is hypothesized to choose Q and X so as to maximize expected utility, i.e., the objective function is

$$\begin{aligned}\max_{Q,X} \pounds = \max_{Q,X} E\{U[W_0 + P_T Q \\ - P_0 Q - C(Q) + (F_0 - F_T)X]\}\end{aligned} \tag{2}$$

where $E(\cdot)$ is the expectations operator. The first order conditions corresponding to (2) are

$$\frac{\partial \pounds^*}{\partial Q} = E\{[P_T - P_0 - C'(Q^*)]U'(W_T^*)\} \leq 0, Q^* \geq 0, Q^* \frac{\partial \pounds^*}{\partial Q} = 0 \tag{3}$$

$$\frac{\partial \pounds^*}{\partial X} = E[(F_0 - F_T)U'(W_T^*)] = 0 \tag{4}$$

where $W_T^* \equiv W_0 + P_T Q^* - P_0 Q^* - C(Q^*) + (F_0 - F_T)X^*$, and asterisks denote optimal quantities.

Under the standard assumption of unbiased futures prices [i.e., $E(F_T) = F_0$], optimal storage is strictly positive if and only if the expected basis change plus the marginal cost of carry evaluated at $Q = 0$ is strictly positive. That is,

$$Q^* > (=)\ 0 \Leftrightarrow E(B_T) - B_0 + C'(0) > (\leq)\ 0, \tag{5}$$

where $B_T\ (\equiv P_T - F_T)$ and $B_0\ (\equiv P_0 - F_0)$ are the bases at terminal and initial dates, respectively. The proof of (5) is provided in the Appendix. Lence has also shown that, in the presence of unbiased futures prices, the optimal hedge, X^*, is a fixed proportion, β_0, of *any* physical position, Q, regardless of the utility function, $U(\cdot)$, *if and only if*

$$P_T = \beta_0 F_T + g_0(e_T) \tag{6}$$

where β_0 is an arbitrary constant, $g_0(\cdot)$ is an arbitrary function, and e_T is a random variable such that F_T is conditionally independent of e_T.

The results in the previous paragraph imply that the decision process can be divided conceptually into two stages when futures prices are unbiased. First, evaluate whether storage is profitable or not by looking at the expected basis and the marginal cost of carry evaluated at $Q = 0$. Storage should be undertaken only if the expected basis change plus the marginal cost of carry at $Q = 0$ is positive. Second, if storage is expected to be profitable, calculate the optimal hedge as $X^* = \beta_0 Q^*$, where β_0 is the minimum-variance hedge ratio.

The subscripts in (6) make it clear that both β_0 and $g_0(\cdot)$ are fixed as of the initial date. However, β_0 and $g_0(\cdot)$ may be different under alternative initial market conditions. The minimum-variance hedge ratio, β_0, is typically obtained by means of regression techniques. Traditional unconditional regression models have assumed simplistically that both β_0 and $g_0(\cdot)$ are constant, regardless of the market situation at the initial date. Myers and Thompson (1989) recognized this flaw in the unconditional models, and postulated a regression model allowing $g_0(\cdot)$ (but not β_0) to depend on the initial market conditions. Although in Myers and Thompson's (1989) conditional model, β_0 is assumed to remain unchanged under alternative initial market scenarios, allowing $g_0(\cdot)$ to depend on the latter yields conditional β_0 *estimates* that are generally different from unconditional β_0 *estimates*. Viswanath further contributed to the conditional model by hypothesizing that the initial basis should explain most of the dependence of $g_0(\cdot)$ on the initial market situation.

Here, it is argued that (i) β_0 [in addition to $g_0(\cdot)$] should be allowed to depend on the market situation at the decision date, and (ii) the expected change in the basis should capture most of such dependence. The first hypothesis is based on the fact that economic theory suggests that the relationship between cash and futures prices is likely to be different in the presence of surpluses as opposed to shortages. The second hypothesis follows from the combined observations that storage arbitrage is a fundamental factor affecting the relationship between cash and futures prices, and that storage makes sense only when it is expected to be profitable. Although (5) implies that storage profitability depends not only upon the expected basis change but also upon the marginal cost of carry evaluated at $Q = 0$, in the empirical section, attention is restricted to the term $[E(B_T) - B_0]$ because of lack of data about $C'(0)$.

The remainder of this article discusses modifications of the traditional minimum-variance hedge models aimed at capturing the effects of alternative market conditions on storage hedge ratios, and at testing whether such effects are significant. The model advanced is then applied to the estimation of hedge ratios for two typical storable agricultural commodities, namely, corn and soybeans.

TRADITIONAL MODEL SPECIFICATIONS AND AN ALTERNATIVE APPROACH

The simplest hedge ratio estimation method consists of regressing cash price levels on futures price levels. The hedge ratio is calculated as the ratio of the covariance between futures and cash price levels to the variance of futures price levels. The hedge ratio thus obtained is interpreted as the proportion of the physical (storage) position to hedge in the futures market. Traditionally, this ratio has been calculated in an unconditional framework, with futures price level as the only explanatory variable in the regression equation. The unconditional price level model can be expressed as

$$P_t = \alpha^{ul} + \beta^{ul} F_t + e_t^{ul} \tag{7}$$

where P_t denotes the cash price at the end of the storage period, t, F_t is the futures price at the end of the storage period; α^{ul} and β^{ul} are constants; and e_t^{ul} is an error term. In (7), β^{ul} is interpreted as the hedge ratio.

Another simple storage hedge model specification is a regression of cash price changes against futures price changes. The unconditional price change model can be written as follows:

$$\Delta P_t = \alpha^{u\Delta} + \beta^{u\Delta} \Delta F_t + e_t^{u\Delta} \tag{8}$$

where $\Delta P_t \equiv P_t - P_{t-j}$, $\Delta F_t \equiv F_t - F_{t-j}$, and j is the storage period. The symbols, $\alpha^{u\Delta}$ and $\beta^{u\Delta}$, denote constants, and $e_t^{u\Delta}$ represents an error term. The hedge ratio in the price change model, $\beta^{u\Delta}$, is interpreted in the same manner as in the price level model, (β^{ul}), but the estimation procedure differs slightly. In the price change model, (8), the regression equation is estimated by minimizing the variance around a cash price difference. In the price level model, (7), the regression equation is estimated by minimizing the variance around the price level at the time the hedge is lifted.

As discussed earlier, on statistical grounds the choice between the price level regression model and the price difference regression model depends on whether the price series are stationary or not. If prices are stationary, then the appropriate model is the one with prices specified in levels. Otherwise, the price difference model is preferable.

There are two specific omissions in the traditional unconditional models, (7) and (8), which should be addressed by an improved storage hedge model specification. First, as discussed by Myers and Thompson (1989), regression models should be conditional on the information available at the time of placing the hedge. Viswanath (1993) advocated including basis at the time the hedge is placed as an explanatory variable in the regression model. Typically, hedgers are concerned with the *current* basis (that is, basis at the time the hedge is placed) and the *expected* basis at the time when the hedge will be lifted. The difference between the current basis and the expected basis is a critical determinant of the expected profitability of the joint cash/futures position. The current basis is readily available to the hedger. The expected basis can be inferred using historical cash-futures price patterns at or near contract maturity. Therefore, at a minimum, a more complete hedging model should incorporate the current basis and historical bases at the time of placing the hedge.

Second, as argued earlier, traditional regression models have ignored the question of profitability. It clearly does not seem rational for a farmer or elevator operator to store when the expected basis change is either negative or positive but not enough to cover storage costs. Despite this fact, hedge ratios are generally calculated without differentiating between the scenario with expected profits to storage and the situation where storage is expected to yield a loss. Hence, hedge ratios calculated in the traditional manner may be misleading if the relationship between cash and futures prices is affected by the alternative market scenarios. Ideally, storage hedge models should attempt to uncover the effect of the alternative market scenarios, and help decision makers arrive at the appropriate hedge ratio for the situation they face at the time of placing the hedge.

Given the previous discussion, the conditional regression models, (9) and (10), below are specified to calculate hedge ratios for storable commodities:

$$P_t = \alpha^{cl} + \beta^{cl}F_t + \gamma^{cl}[E_{t-j}(B_t) - B_{t-j}]F_t + \delta^{cl}B_{t-j} + \phi^{cl}B_{t-1\text{year}} + \eta^{cl}B_{t-j-1\text{year}} + e_t^{cl} \quad (9)$$

$$\Delta P_t = \alpha^{c\Delta} + \beta^{c\Delta}\Delta F_t + \gamma^{c\Delta}[E_{t-j}(B_t) - B_{t-j}]\Delta F_t + \delta^{c\Delta}B_{t-j} + \phi^{c\Delta}B_{t-1\text{year}} + \eta^{c\Delta}B_{t-j-1\text{year}} + e_t^{c\Delta} \quad (10)$$

where $E_{t-j}(\cdot)$ denotes the expectation operator conditional on information available at the time of placing the hedge, $(t - j)$; $B_t(\equiv P_t - F_t)$ is the basis at the time of lifting the hedge; B_{t-j} is the basis at the time the hedge is placed; $B_{t-j-1\text{year}}$ $(B_{t-1\text{year}})$ is the previous year's basis for the time at which the hedge is placed (planned to be lifted); and α^{cl}, β^{cl}, γ^{cl}, δ^{cl}, ϕ^{cl}, η^{cl}, $\alpha^{c\Delta}$, $\beta^{c\Delta}$, $\gamma^{c\Delta}$, $\delta^{c\Delta}$, $\phi^{c\Delta}$, and $\eta^{c\Delta}$ are constants. The variables, e_t^{cl} and $e_t^{c\Delta}$, are error terms.

Inclusion of the terms, $E_{t-j}(B_t)$, B_{t-j}, $B_{t-1\text{year}}$, and $B_{t-j-1\text{year}}$, in regressions, (9) and (10), takes care of the conditionality problem, by rendering the corresponding hedge ratio estimates conditional on the information available at the time of placing the hedge. Furthermore, inclusion of the cross-product terms, $\{[E_{t-j}(B_t) - B_{t-j}]F_t\}$ and $\{[E_{t-j}(B_t) - B_{t-j}]\Delta F_t\}$, as explanatory variables in (9) and (10), respectively, allows one to account for the possible influence of the expected profitability of storage on the relationship between cash and futures prices.

The cross-product terms in (9) and (10) are aimed at capturing the differential effect of expected profitability on the relationship between cash and futures prices. If cash and futures behave the same in times of shortages relative to times of surpluses, then the expected profitability of storage should not affect the relationship between cash and futures prices, which implies that $\gamma^{cl} = 0$ or $\gamma^{c\Delta} = 0$. As discussed earlier, however, economic theory suggests that price movements for storable commodities should not be symmetric under opposite market conditions. Therefore, the cash/futures relationship when storage is expected to be most profitable (i.e., highest $[E_{t-j}(B_t) - B_{t-j}]$) is likely to be different from the relationship when storage is expected to be least profitable (i.e., lowest $[E_{t-j}(B_t) - B_{t-j}]$), in which case $\gamma^{cl} \neq 0$ or $\gamma^{c\Delta} \neq 0$.

If γ^{cl} or $\gamma^{c\Delta}$ are found to be significantly different from zero, the cross-product terms should be useful to "adjust" the hedge ratio at the time the hedger is making his/her storage decision. The adjusted hedge ratio can then be calculated as

$$HR^{cl} = \beta^{cl} + \gamma^{cl}[E_{t-j}(B_t) - B_{t-j}] \tag{11}$$

and

$$HR^{c\Delta} = \beta^{c\Delta} + \gamma^{c\Delta}[E_{t-j}(B_t) - B_{t-j}] \tag{12}$$

for the conditional price level and price change models, respectively. The adjusted hedge ratio (HR) indicates the proportion of the farmer or commercial agent's cash position that should be hedged in the futures market to minimize price risk.

Inclusion of the cross-product terms raises the issue of how best to estimate the expected basis change $[E_{t-j}(B_t) - B_{t-j}]$. In two seminal studies, Working (1953) and Heifner (1966) used a simple regression of basis change on initial basis to forecast gross returns to storage. Here it is hypothesized that the basis change is described reasonably well by the slightly more elaborate model

$$\Delta B_t = a^b + \delta^b B_{t-j} + \phi^b B_{t-1\text{year}} + \eta^b B_{t-j-1\text{year}} + e_t^b \tag{13}$$

where $\Delta B_t \equiv B_t - B_{t-j}$, e_t^b is an error term, and a^b, δ^b, ϕ^b, and η^b denote constants. Hence, the expected change in the basis is given by

$$E_{t-j}(B_t) - B_{t-j} = a^b + \delta^b B_{t-j} + \phi^b B_{t-1\text{year}} + \eta^b B_{t-j-1\text{year}} \tag{14}$$

More sophisticated (e.g., structural) models of basis behavior should be used if they exhibit better fit or if a more complete data set is available. Here, however, (13) is chosen intentionally to keep the presentation simple to focus on the major contributions of the analysis.

DATA

The advocated conditional hedging regression models and the basis model are estimated using corn and soybean prices for the crop years, 1974–1975 through 1993–1994. The extended time series purposely is chosen to allow for a wide range of market situations regarding the expected profitability from storage. Storage and hedging decisions are assumed to be made in either October, February, or June for the next four months. That is, the storage periods considered are October–February, February–June, and June–October.

Cash prices are the spot prices in North Central Iowa on the Thursday closest to the 15th of the months beginning or ending each storage period (i.e., October, February, and June). Cash prices are from the Iowa State University Market News. Futures prices are the corresponding clos-

TABLE I

Augmented Dickey–Fuller Tests for Unit Roots, 1974–1975 Through 1993–1994

Models:

$$\Delta \mathbf{x}_t \equiv \mathbf{x}_t - \mathbf{x}_{t-1} = \mathbf{a}_0 + \mathbf{a}_1 \mathbf{x}_{t-1} + \mathbf{a}_2\, time + a_{2+1}\, \Delta x_{t-1} + \cdots + a_{2+q}\, \Delta x_{t-q} + e_t$$
$$\Delta x_t \equiv x_t - x_{t-1} = b_0 + b_1 x_{t-1} + b_{1+1}\, \Delta x_{t-1} + \cdots + b_{1+q}\, \Delta x_{t-q} + e_t$$

Series ($\mathbf{x}_t$)	Lags (q)	Null Hypothesis	Evaluation Statistics	Critical Value 5%	Critical Value 1%
Corn cash prices	3	$a_1 = 0$	$\hat{\tau}_\tau = -3.62$	−3.49	−4.13
	3	$a_1 = a_2 = 0$	$\Phi_3 = 10.06$	6.70	9.26
	3	$a_1 = 0$	$z = -3.62$	−1.65	−2.33
Corn futures prices	0	$a_1 = 0$	$\hat{\tau}_\tau = -3.30$	−3.49	−4.13
	0	$a_1 = a_2 = 0$	$\Phi_3 = 5.44$	6.70	9.26
	0	$b_1 = 0$	$\hat{\tau}_\mu = -3.18$	−2.93	−3.57
Soybeans cash prices	0	$a_1 = 0$	$\hat{\tau}_\tau = -4.36$	−3.49	−4.13
Soybeans futures prices	0	$a_1 = 0$	$\hat{\tau}_\tau = -4.24$	−3.49	−4.13

Note: $\hat{\tau}_\tau$, $\hat{\tau}_\mu$, and Φ_3 are Dickey–Fuller tests (Dickey and Fuller). z is the standard normal test.

ing prices on various Chicago Board of Trade futures contracts as reported in the *Wall Street Journal*. For corn (soybeans), the futures contracts corresponding to the storage periods October–February, February–June, and June–October are March, July, and December (March, July, and November), respectively. Therefore, the data set contains 3 observations per year over 20 years, for a total of 60 observations.[4]

ESTIMATION RESULTS AND DISCUSSION

Unit root tests on the price series are conducted first because the choice between regression models (9) and (10) depends on whether prices are stationary or not. Table I summarizes the results of applying the methodology advocated by Dolado, Jenkinson, and Sosvilla-Rivero (1990) to test for unit roots.[5] On this basis, it is concluded that the price level

[4]Note that the construction of the basis for a particular month depends on whether that month is the beginning or the end of the storage period under consideration. For example, for the period October–February, the relevant futures contract is March. Hence, the corresponding basis in February (i.e., the month at which the hedge is planned to be *lifted*) is obtained as the difference between the North-Central Iowa spot price and the *March* closing futures price on the Thursday closest to the 15th of February. In contrast, for the February–June period, the basis in February (i.e., the month at which the hedge is being *placed*) is calculated as the difference between the North-Central Iowa spot price and the *July* closing futures price on the Thursday closest to the 15th of February. The reason for doing so is that the futures contract for the February–June period is July.

[5]Dolado, Jenkinson, and Sosvilla-Rivero (1990) proposed performing first an augmented Dickey–

TABLE II

Ordinary Least Squares Regressions Corresponding to Basis Model, 1974–1975 Through 1993–1994

Model:

$$\mathbf{B}_t - \mathbf{B}_{t-j} = \boldsymbol{\alpha}^b + \boldsymbol{\delta}^b \mathbf{B}_{t-j} + \boldsymbol{\phi}^b \mathbf{B}_{t-1\text{year}} + \boldsymbol{\eta}^b \mathbf{B}_{t-j-1\text{year}} + \mathbf{e}_t^b$$

Commodity		*Corn*	*Soybeans*
Coefficients:	α^b	−0.116[b]	−0.196[b]
		(0.037)[c]	(0.048)
	δ^b	−0.856[b]	−0.859[b]
		(0.076)	(0.051)
	ϕ^b	0.587[b]	0.507[b]
		(0.111)	(0.113)
	η^b	−0.100	−0.132[a]
		(0.070)	(0.051)
R^2		0.804	0.874
Number of observations		57	57
Test for autocorrelation of e_t^b		$Q'(1)$[d] = 1.82 [0.18][e]	$Q'(1)$ = 1.06 [0.30]
		$Q'(3)$ = 7.03 [0.07]	$Q'(3)$ = 3.87 [0.28]
Test for ARCH of e_t^b		LMA(1)[f] = 0.68 [0.41]	LMA(1) = 0.02 [0.89]
		LMA(3) = 1.72 [0.63]	LMA(3) = 4.03 [0.26]
Test for normality of e_t^b		LMN[g] = 14.65 [0.001]	LMN = 0.45 [0.80]

[a,b]Significantly different from zero at the 5 (1) % level of significance based on the two-tailed t-statistic.
[c]Numbers between parentheses denote standard deviations of the corresponding coefficient estimates.
[d]$Q'(i)$ is the Ljung–Box portmanteau test or modified-Q statistic for i-order autocorrelation [Ljung and Box (1978)].
[e]Numbers between brackets denote p values for the corresponding test statistics.
[f]LMA(i) is the Lagrange multiplier test for i-order autoregressive conditional heteroscedasticity [Engle (1982)].
[g]LMN is the Lagrange multiplier test for normality [Jarque and Bera (1980)].

model, (9), is the appropriate one to use in the regressions for both corn and soybeans.

Results for the basis model are presented and discussed before the results for the price level hedge regression model because the latter can be estimated only after estimating the former. Ordinary least squares regression estimates and goodness-of-fit measures for the basis model are reported in Table II. Parameter estimates for corn are very similar to those

Fuller test on a regression which contains time as an explanatory variable and testing whether the coefficient on the lagged level (a_1) is significantly negative by means of the $\hat{t}_\tau$ test statistics. If $\hat{t}_\tau$ rejects the null hypothesis that $a_1 = 0$, then it can be concluded that the series is stationary. On this basis, both soybean price series were found stationary. When $\hat{t}_\tau$ does not reject the null hypothesis that $a_1 = 0$, as was the case for corn prices, the null hypothesis that $a_1 = a_2 = 0$ must be tested by means of the Φ_3 test statistics. If Φ_3 rejects the null, as for corn cash prices, then the null hypothesis that $a_1 = 0$ must be tested using the standard normal z test statistics. z rejected the null for corn cash prices, which ruled out the presence of a unit root. If Φ_3 does not reject the null, as for corn futures prices, then the null hypothesis that $b_1 = 0$ must be tested using the $\hat{t}_\mu$ test statistics on a regression which does not include time as an explanatory variable. If $\hat{t}_\mu$ rejects the null that $b_1 = 0$, as was the case for corn futures prices, one can conclude that the series is stationary.

for soybeans, lending support to the model. The three exogenous variables considered explain over 80% of the basis change variability. The greater the initial basis, the greater the unfavorable basis change (i.e., the greater the decline of cash prices relative to futures prices).[6] In contrast, the greater the closing basis the previous year, the greater the favorable basis change.

The null hypothesis that the regression coefficients are the same across the four-month storage periods selected is tested against the alternative hypothesis that the null is false. Using the likelihood ratio test with the generalized method of moments (Davidson and Mackinnon [1993, Chapter 17]), the test statistics are 9.22 for corn and 10.14 for soybeans, which are distributed as chi-squared with 12 degrees of freedom. The calculated test statistics have $p = 0.68$ for corn and $p = 0.60$ for soybeans, which imply that there is not enough evidence to reject the hypothesis that the regression coefficients are the same across the four-month storage periods selected. The Ljung–Box test indicates that the residuals from the basis change regressions exhibit no autocorrelation.[7] Similarly, the Lagrange multiplier test proposed by Engle reveals no problems of autoregressive conditional heteroscedasticity in the residuals. Finally, the null hypothesis of normally distributed residuals cannot be rejected at standard levels of significance for soybeans, but is soundly rejected for corn. Therefore, strictly speaking, it is inappropriate to use the standard t-statistics to test the significance of the estimated coefficients in the corn basis regression.

Table III provides information regarding the predictive power of the basis model. Predictions from the basis model are correct 82.5% of the time for either commodity when predicting whether basis changes would be above or below the corresponding four-month storage period's mean. Based on this evidence and on the other properties discussed earlier, it seems fair to conclude that the basis model behaves reasonably well.

Regressions for the price level hedging model, (9), along with goodness-of-fit statistics are shown in Table IV.[8,9] The coefficients for corn again are strikingly similar in magnitude (with the exceptions of α^{cl} and δ^{cl}) to those for soybeans. The model has high explanatory power, as the

[6]This result is consistent with the findings by Working (1953) and by Heifner (1966).

[7]Recall that lag 3 corresponds to the same storage period in the previous year.

[8]Note that the predicted basis changes necessary to estimate in the hedging regressions are obtained from the corresponding basis regressions reported in Table II.

[9]In this application, the price change regression model, (10), is not justified because the null hypothesis of stationary price series can not be rejected. Fitting (10) with the soybeans data, however, yields virtually identical results as (9). In contrast, when model (10) is fitted with the corn data, it yields errors that are clearly non-normally distributed (LMN is 27.64, which has $p < 10^{-6}$).

TABLE III

Comparison of Predicted Versus Actual Basis Changes, 1974–1975 Through 1993–1994

	October–February		*February–June*		*June–October*	
	Predicted Above Mean (Observations)	*Predicted Below Mean (Observations)*	*Predicted Above Mean (Observations)*	*Predicted Below Mean (Observations)*	*Predicted Above Mean (Observations)*	*Predicted Below Mean (Observations)*
A *Corn*						
Actual above mean	7	2	6	2	11	2
Actual below mean	1	9	2	9	1	5
B *Soybeans*						
Actual above mean	4	3	6	3	13	0
Actual below mean	2	10	2	8	0	6

Note: "Predicted above (below) mean" means number of predicted basis changes that are above (below) the average for the corresponding four-month period. Similarly, "Actual above (below) mean" means number of actual basis changes that are above (below) the average for the corresponding four-month period.

exogenous variables included account for 95.6% of the variability in corn cash prices, and for 98.6% of the variability in soybean cash prices. Residuals are nicely behaved, without evidence of problems with autocorrelation or with autoregressive conditional heteroscedasticity. Furthermore, the null hypothesis of normally distributed residuals cannot be rejected, implying that standard t-statistics can be used to conduct tests of significance regarding coefficient estimates.

As it is typically found in hedging regression models for storable commodities, the coefficient corresponding to futures prices is positive and close to unity. Of particular interest for this analysis is the magnitude and significance of the estimated coefficients corresponding to the cross product term, $\{[E_{t-j}(B_t) - B_{t-j}]F_t\}$, i.e., γ^{cl}. The estimate of γ^{cl} is negative and significantly different from zero at the 5% level for corn, and negative and significantly different from zero at the 1% level for soybeans. The magnitude of the γ^{cl} estimates is very similar for both commodities (-0.206 for corn and -0.188 for soybeans). Therefore, the data support

TABLE IV

Ordinary Least Squares Regressions Corresponding to Conditional Price Level Hedging Model, 1974–1975 Through 1993–1994

Model

$$P_t = \alpha^{cl} + \beta^{cl} F_t + \gamma^{cl} [E_{t-j}(B_t) - B_{t-j}] F_t + \delta^{cl} B_{t-j} + \phi^{cl} B_{t-1\text{year}} + \eta^{cl} B_{t-j-1\text{year}} + e_t^{cl}$$

Commodity		*Corn*	*Soybeans*
Coefficients:	α^{cl}	−0.101	−0.456[a]
		(0.089)[c]	(0.182)
	β^{cl}	0.960[b]	1.000[b]
		(0.031)	(0.020)
	γ^{cl}	−0.206[a]	−0.188[b]
		(0.102)	(0.070)
	δ^{cl}	−0.304	−0.822[a]
		(0.236)	(0.352)
	ϕ^{cl}	0.836[b]	0.966[b]
		(0.175)	(0.217)
	η^{cl}	−0.190[a]	−0.324[b]
		(0.076)	(0.089)
R^2		0.956	0.986
Number of observations		57	57
Test for autocorrelation of e_t^{cl}		$Q'(1)^d = 2.14$ [0.14][e]	$Q'(1) = 2.16$ [0.14]
		$Q'(3) = 5.89$ [0.12]	$Q'(3) = 3.59$ [0.31]
Test for ARCH of e_t^{cl}		LMA(1)[f] = 1.34 [0.25]	LMA(1) = 0.58 [0.45]
		LMA(3) = 5.54 [0.14]	LMA(3) = 4.65 [0.20]
Test for normality of e_t^{cl}		LMN[g] = 2.89 [0.24]	LMN = 0.40 [0.82]

[a,b]Significantly different from zero at the 5 (1) % level of significance based on the two-tailed *t*-statistic.
[c]Numbers between parentheses denote standard deviations of the corresponding coefficient estimates.
[d]$Q'(i)$ is the Ljung–Box portmanteau test or modified-Q statistic for i-order autocorrelation [Ljung and Box (1978)].
[e]Numbers between brackets denote p values for the corresponding test statistics.
[f]LMA(i) is the Lagrange multiplier test for i-order autoregressive conditional heteroscedasticity [Engle (1982)].
[g]LMN is the Lagrange multiplier test for normality [Jarque and Bera (1980)].

the hypothesis that market conditions affect the relationship between cash and futures prices for these storable commodities. More specifically, there is evidence that the ratio of the conditional covariance between cash and futures prices to the conditional variance of futures prices is negatively affected by the conditional expected change in the basis.

To assess the impact of expected profitability on hedge ratio estimates, panel A of Table V provides information about the actual basis changes that occur in the period analyzed, and about the expected basis changes estimated using the basis models reported in Table II. The actual basis change fluctuates within a wide range, from a low of −0.85 (−1.31) dollars per bushel to a high of 0.45 (0.76) dollars per bushel for corn (soybeans). Actual basis changes are positive, on average, which

TABLE V

Basis Changes and Hedge Ratio Estimates, 1974–1975 Through 1993–1994

Commodity	*Corn*	*Soybeans*
A. Actual basis changes and expected basis change estimates (in dollars per bushel)		
Actual basis changes		
Minimum	−0.85	−1.31
Average	0.06	0.12
Maximum	0.45	0.76
Expected basis change estimates:		
Minimum	−0.53	−1.10
Average	0.06	0.12
Maximum	0.39	0.78
B. Hedge ratio estimates		
Hedge ratio estimates assuming that expected basis change equals		
Minimum historical value	1.134	1.246
Minimum historical estimate	1.069	1.206
Zero	0.960	1.000
Maximum historical value	0.868	0.858
Maximum historical estimate	0.880	0.853

indicates that, ignoring costs of carry, storage and one-to-one hedging is profitable, on average.

Panel B of Table V shows how hedge ratios should be adjusted under alternative market scenarios. For example, the recommended hedge ratio, assuming no expected change in the basis, is 0.960 for corn and 1.000 for soybeans. Assuming that the expected basis change equals the maximum historical *increase,* the adjusted hedge ratios are 0.868 for corn and 0.858 for soybeans. In the opposite situation of expecting a basis change matching the maximum historical *decrease,* the adjusted hedge ratios are 1.134 for corn and 1.246 for soybeans. The former (latter) figures represent hedge *reductions* (*increases*) of 9.6 (18.1)% and 14.2 (24.6)% for corn and soybeans, respectively, relative to the scenario with no expected basis change. Therefore, adjustments in the hedge ratios can be sizable and ignoring them may lead to hedging a substantially different proportion of the physical position in futures markets.

Finally, Table VI provides a comparison of the ex-post profits for the recommended hedging strategy versus zero hedging and one-to-one hedging. The mean ex-post profits are similar for the three strategies, albeit greatest for one-to-one hedging and smallest for zero-hedging. In terms of risk, however, the zero-hedging strategy is clearly inferior to the other two hedging strategies. Zero-hedging yields the greatest difference be-

TABLE VI

Ex-Post Profitability of Alternative Hedging Strategies for Four-Month Storage Periods in Which Storage Was Expected to be Profitable, 1974–1975 through 1993–1994

Commodity	*Hedging Strategy*	*Minimum Ex-Post Profit (Cents/Bushel)*	*Mean Ex-Post Profit (Cents/Bushel)*	*Maximum Ex-Post Profit (Cents/Bushel)*	*Standard Deviaton of Ex-Post Profit (Cents/Bushel)*	*Periods with Negative Ex-Post Profits (%)*
Corn	Recommended	−9.3	14.6	39.6	11.3	9.8
Corn	Zero hedging	−71.2	12.8	80.6	27.8	26.8
Corn	One-to-one hedging	−13.3	14.9	44.8	12.2	9.8
Soybeans	Recommended	−7.9	22.8	59.3	16.2	6.8
Soybeans	Zero hedging	−142.5	21.0	289.8	89.8	36.4
Soybeans	One-to-one hedging	−7.7	24.0	75.6	19.7	9.1

Note: Ex-post profits are defined as $\pi_T \equiv (P_T - P_0) - (F_T - F_0)\,h$, where h = HR for the recommended hedging strategy, $h = 0$ for the zero-hedging strategy, and $h = 1$ for the one-to-one hedging strategy. All strategies are evaluated only on four-month periods in which storage is expected to be profitable at the decison time ($t = 0$). Storage is considered to be profitable at decision time whenever the basis models reported in Table II predict a positive basis change. There are 41 (44) out of 57 four-month periods in which storage is expected to be profitable for corn (soybeans). Of those 41 (44) four-month periods for corn (soybeans), 19 (19) correspond to the four-month period October–February, 17 (16) to the four-month period February–June, and 5 (9) to the four-month period June–October.

tween minimum and maximum ex-post profits, has the greatest standard deviations of ex-post profits, and has the greatest number of periods with negative ex-post profits. Furthermore, on the same three counts, the recommended hedging strategy fares better than one-to-one hedging. In summary, although Table VI does not provide conclusive evidence in favor of the recommended hedging strategy, the reported figures are consistent with the risk-minimization objective upon which the recommended hedging strategy is being advocated.

SUMMARY AND CONCLUSIONS

This study presents and discusses a method to estimate minimum-variance hedge ratios for storable commodities. The model incorporates recent innovations regarding hedge ratio estimation, namely, that hedge regressions should be conditional on the information available at the time of placing the hedge, and that the basis should be included in that information set. The model departs from the existing literature, however, in that it allows one to test whether the size of the hedge ratio estimate is influenced by the expected profitability of storage. Economic theory suggests that such an hypothesis is appealing because of the intrinsic asymmetry that should characterize the price behavior of storable commodities in periods of shortage relative to periods of excess supply. The rationale for this assertion is that storage implies that exploitation of arbitrage opportunities should prevent cash prices from being far below futures prices, but there is no mechanism analogous to storage arbitrage that can prevent cash prices from being well above futures prices.

The advocated method of hedge ratio estimation is applied to corn and soybeans. The empirical results strongly support the hypothesis that hedge ratio estimates are influenced by the expected profitability of storage for the commodities analyzed. Furthermore, such impact implies that corn and soybean minimum-variance hedge ratios decrease in size as storage is expected to be more profitable. An implicit suggestion for future research is to test whether this approach actually reduces hedging risk using out-of-sample data.[10]

[10]Here, no attempt is made to use out-of-sample data for three reasons. First, there is not enough data available to conduct a reasonable out-of-sample test of performance. Second and more importantly, the proposed model must be measured in terms of its ability to reduce the variance of the combined cash-futures position when storage is expected to be profitable. Because the proper measure of performance is the conditional reduction in variance, the corresponding performance test is exceedingly difficult. Third, any such measure of variance reduction is conditional on expected profitability, which makes it almost impossible to disentangle the individual performance of the hedging model from that of the basis model.

In concluding, it is also important to highlight that the model proposed here requires almost no additional effort in terms of data or estimation techniques. Hence, there are no obvious reasons to preclude the adoption of the proposed method to estimate hedge ratios for storable commodities. The proposed method does require the estimation of a model to forecast basis changes, but estimation of such a model should precede that of the hedging model in virtually all situations involving rational decision making. This assertion is true because, under unbiased futures prices, the storage (and therefore hedging) decision should be driven by the expected basis change.

APPENDIX: DERIVATION OF EXPRESSION (5)

First-order conditions (3) and (4) can be rewritten as (A1) and (A2), respectively:

$$\frac{\partial \pounds^*}{\partial Q} = E[P_T - P_0 - C'(Q^*)]\, E[U'(W_T^*)] + \operatorname{cov}[U'(W_T^*), P_T] \leq 0,\ Q^* \geq 0,\ Q^* \frac{\partial \pounds^*}{\partial Q} = 0 \qquad \text{(A1)}$$

$$\frac{\partial \pounds^*}{\partial X} = E(F_0 - F_T)\, E[U'(W_T^*)] + \operatorname{cov}[U'(W_T^*), F_T] = \operatorname{cov}[U'(W_T^*), F_T] = 0 \qquad \text{(A2)}$$

where cov(·) denotes the covariance operator.[11] The product term in first order condition (A2) equals zero because of the unbiased futures assumption.

Expression (5) is proven by showing sequentially the following statements:

$$Q^* > 0 \Rightarrow E[P_T - F_T - P_0 + F_0 - C'(0)] > 0 \qquad \text{(A3)}$$

$$E[P_T - F_T - P_0 + F_0 - C'(0)] > 0 \Rightarrow Q^* > 0 \qquad \text{(A4)}$$

$$Q^* = 0 \Rightarrow E[P_T - F_T - P_0 + F_0 - C'(0)] \leq 0 \qquad \text{(A5)}$$

$$E[P_T - F_T - P_0 + F_0 - C'(0)] \leq 0 \Rightarrow Q^* = 0 \qquad \text{(A6)}$$

Proof of (A3). Assume $Q^* > 0$. Then first order condition (A1) implies

[11] Expressions (A1) and (A2) follow from (3) and (4), respectively, because $\operatorname{cov}(y, z) = E(y\,z) - E(y)\,E(z)$, for any pair of random variables, y and z.

that $\partial £^*/\partial Q = 0$. Therefore, $\partial £^*/\partial Q$ and $\partial £^*/\partial F$ can be added term-by-term and manipulated slightly to yield:

$$E[P_T - F_T - P_0 + F_0 - C'(Q^*)]\, E[U'(W_T^*)] + \frac{1}{Q^*} \operatorname{cov}[U'(W_T^*), W_T^*] = 0 \tag{A7}$$

But (i) $E[U'(W_T^*)] > 0$ because $U' > 0$ by assumption; (ii) $Q^* > 0$ by assumption; and (iii) $\operatorname{cov}[U'(W_T^*), W_T^*] < 0$ because $\partial U'(W_T^*)/\partial W_T^* = U''(W_T^*) < 0$ by assumption and $\partial W_T^*/\partial W_T^* = 1 > 0$. Hence, (A7) can only be satisfied if $E[P_T - F_T - P_0 + F_0 - C'(Q^*)] > 0$. Expression (A3) follows immediately because $E[P_T - F_T - P_0 + F_0 - C'(0)] \geq E[P_T - F_T - P_0 + F_0 - C'(Q^*)]$ by the assumption that $C'' \geq 0$.

Proof of (A4). Assume $E[P_T - F_T - P_0 + F_0 - C'(0)] > 0$. Suppose further that $Q^* = 0$. Then first-order condition (A2) can be expressed as

$$\operatorname{cov}[U'(W_T^*), F_T] = \operatorname{cov}\{U'[W_0 - C(0) + (F_0 - F_T)X^*], F_T\} = 0 \tag{A8}$$

It is clear from (A8) that $\operatorname{cov}[U'(W_T^*), F_T] = 0$ can hold only if $X^* = 0$. But if $Q^* = X^* = 0$, then $\operatorname{cov}[U'(W_T^*), P_T] = \operatorname{cov}\{U'[W_0 - C(0)], P_T\} = 0$, and the first term in first-order condition (A1) simplifies to

$$\frac{\partial £^*}{\partial Q} = E[P_T - P_0 - C'(0)], U'[W_0 - C(0)] \leq 0. \tag{A9}$$

For (A9) to hold, it is necessary that $E[P_T - P_0 - C'(0)] \leq 0$ because $U'[W_0 - C(0)] > 0$ by assumption. But this contradicts the initial assumption that $E[P_T - F_T - P_0 + F_0 - C'(0)] > 0$ because $E[P_T - P_0 - C'(0)] = E[P_T - F_T - P_0 + F_0 - C'(0)]$ by the unbiased futures condition. Hence, it must be true that $Q^* > 0$.

Proof of (A5). Assume $Q^* = 0$. Suppose further that $E[P_T - F_T - P_0 + F_0 - C'(0)] > 0$. But the latter contradicts (A4). Hence, it must be true that $E[P_T - F_T - P_0 + F_0 - C'(0)] \leq 0$.

Proof of (A6). Assume $E[P_T - F_T - P_0 + F_0 - C'(0)] \leq 0$. Suppose also that $Q^* > 0$. But the later contradicts (A3). Therefore, $Q^* = 0$ must hold.

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