# Recovering Probabilistic Information from Option Markets: Tests of Distributional Assumptions

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## INTRODUCTION

Option markets provide useful public information about the probability distributions of prices of the underlying assets [Gardner (1977)]. Because options' payoffs are uniquely determined functions of the possible prices of the underlying asset, interactions among option market participants result in collective expressions of expectations about future price distributions in current option prices. Hence, option pricing models can be used to recover probabilistic information about the prices of the underlying assets without eliciting opinions directly from market participants.

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Importantly, using options premiums for probability assessments enables the recovery of *ex ante* distributional parameters. The most common example is the use of Black's or Black and Scholes' option pricing models for obtaining estimates of the *ex ante* standard deviation of the underlying asset prices. A limitation of the recovery of implied volatilities in this manner is the dependence of the derived estimates on the pricing model imposed.

Despite its widespread application, the Black–Scholes model suffers from the imposition of a potentially restrictive price diffusion process and resulting end-of-period distribution that is frequently empirically rejected. Rubinstein (1994) also characterizes Black–Scholes type models as being increasingly unreliable and requiring increasingly sophisticated mechanisms for dealing with their known biases. He argues that financial economists should consider more flexible distributional specifications whose parameters could be simultaneously estimated from the complex of option strikes that trade on any given day. Rubinstein (1994) also questions the standard distributional assumptions made and notes that there have been only limited efforts to extract probabilistic information from option prices using more general distributional characterizations.

Under no-arbitrage restrictions, Cox and Ross (1976) derive a general asset pricing theory in which they show that the current price of any asset is equivalent to the stream of expected payoffs discounted to the present at the appropriate rate. Under fairly general conditions, expectations can be made with respect to an artificial distribution, called the risk-neutralized valuation measure. However, except for requiring the prices to be non-negative, economic theory provides little guidance regarding the distributional form of RNVM. In practice, the appropriate representation of price distributions is largely an empirical issue. The lognormal distribution is frequently used because of its relative simplicity and its correspondence to the Black and Black–Scholes option pricing models. However, because of its known problems, other distributions may be useful in characterizing expected prices.

Consistent with the spirit of Rubinstein's comments, this study employs a straightforward technique for simultaneously recovering all the parameters of an expected RNVM from options data alone, without the use of the underlying asset price. The procedure searches for probabilities which are most consistent with observed option premiums over multiple strike prices. The process is undertaken with two candidate parameterizations of the probability measure, the lognormal and the Burr III, using daily data on options on soybean futures for all contracts that traded over the period 1986–1991. Comparisons of the implied distributions are made between alternate characterizations and through time to assess the evolution of information and uncertainty about the underlying asset. Because the estimation makes no use of the underlying asset prices, important comparisons also can be made between the options and the underlying asset market regarding information contained in their respective prices.

This study is organized as follows. First, a brief discussion of option pricing theory under no-arbitrage conditions is provided. Next, data and methods are discussed including the rationale for the choice of Burr type III distribution as an alternative candidate distribution. Results are then presented including: comparisons of pricing errors under alternate parameterizations, comparisons of the location of the implied distributions to the futures prices, and comparisons of implied volatility to the resulting price variability. Summary remarks and suggestions for future research then follow.

## OPTION PRICING THEORY UNDER NO-ARBITRAGE

Cox and Ross (1976) show that the minimal assumption that no-arbitrage opportunities exist in an economy implies the existence of an artificial distribution termed the risk neutral valuation measure (RNVM), and that an equilibrium asset price is the properly discounted expected value of its payoffs. Within this framework, the current prices of European put and call options are described by the following:

$$V_p = b(T) \int_0^\infty \max(x_p - Y_T, 0)\, g(Y_T) dY_T \tag{1}$$

$$V_c = b(T) \int_0^\infty \max(Y_T - x_c, 0)\, g(Y_T) dY_T \tag{2}$$

where $V_p$ and $V_c$ are the prices of put and call options, respectively, expiring at time $T$; $x_p$ and $x_c$ are the strike prices for puts and calls, respectively; $Y_T$ is the random price of the underlying asset at expiration; $b(T)$ is the discount factor; and $g(.)$ is the probability density function of the asset price, $Y_T$, at maturity. This approach for option pricing does not require particular assumptions about the dynamics of price change, or other information about the time path of prices. The only required condition is that there are no arbitrage opportunities in the complex of options and underlying assets. In this context, "no-arbitrage" is simply the condition that any two portfolios with identical distributions of future payoffs have identical current prices. Under this approach, the assumption of a lognormal RNVM results in option pricing formulas for puts and calls on futures contracts which are equivalent to the familiar option pricing formulas derived by Black and Scholes and others [Fackler and King (1990); Garven (1986)].

Knowledge of the RNVM and the discount rate are sufficient to compute option premiums. Conversely, given observed option premiums and a discount rate, a RNVM can be recovered. The only requirements for the latter are an assumption regarding the distributional characterization for the underlying asset prices and an estimation criterion. The most common parameterization of the RNVM is lognormal. However, several authors have pointed out that lognormality is not an accurate description of some price distributions and have suggested the use of alternative candidate distributions, or mixtures of distributions [Hall, Brorsen, and Irwin (1989); So (1987)]. Stable Paretian, Burr XII, Burr III, gamma, Weibull, and exponential are among the several alternative candidate distributions suggested in the literature [Fackler (1990); McCulloch (1978); Sherrick, Irwin, and Forster (1992)]. Others have suggested modifications to the data-generating process to impose conditional dependence or otherwise admit nonindependent price changes [Myers and Hanson (1993)]. Nevertheless, the use of lognormality is appealing because of its consistency with the widely used equilibrium option pricing models and because of its relative simplicity. In this study, the performance of the candidate distribution which seems to be most representative of the data is tested against the lognormal which serves as a convenient benchmark.

## DATA AND METHODS

The data for the analysis consist of daily closing futures prices and option premia for options on 22 soybean futures contracts from 1988–1991 obtained from the Chicago Board of Trade. Trades listed with no corresponding volume, observations inconsistent with monotonic strike-premium patterns, and extraneous entries are eliminated. Characteristics of the resulting sample are presented in Table I. On average, there are 11.8 options per day with 61.7% of the options being calls. In addition to options data, a risk-free rate is needed. The daily three-month Treasury-bill yield obtained from the Federal Reserve Bank of Kansas City compounded over the relevant interval until expiration is used as the risk-free rate corresponding to $b(T)$.

The futures data are analyzed using moment-ratio diagrams of the observed data as suggested by Rodriguez (1977) or Tadikamalla (1980).

**TABLE I**

Descriptive Statistics of the Sample Data

| *Contract* | *# of Options Observations* | *% of Obs. Calls* | *Calendar Days Total* | *# of Trade Days* | *Avg # of Strikes Per Day* |
|---|---|---|---|---|---|
| Mar 88 | 437 | 59.95 | 46 | 34 | 12.85 |
| May 88 | 938 | 58.96 | 109 | 78 | 12.03 |
| Jul 88 | 1773 | 60.97 | 165 | 117 | 15.15 |
| Aug 88 | 1705 | 59.30 | 191 | 130 | 13.12 |
| Sep 88 | 1875 | 62.45 | 226 | 146 | 12.84 |
| Nov 88 | 4719 | 54.74 | 291 | 205 | 23.02 |
| Jan 89 | 2022 | 56.63 | 204 | 162 | 12.48 |
| Mar 89 | 2141 | 60.91 | 177 | 157 | 13.64 |
| May 89 | 2115 | 66.90 | 196 | 147 | 14.39 |
| Jul 89 | 3071 | 65.29 | 266 | 191 | 16.08 |
| Aug 89 | 1674 | 68.70 | 226 | 170 | 9.85 |
| Sep 89 | 1446 | 67.23 | 185 | 196 | 7.37 |
| Nov 89 | 3790 | 57.55 | 325 | 243 | 15.60 |
| Jan 90 | 1311 | 59.95 | 204 | 164 | 7.99 |
| Mar 90 | 1450 | 59.31 | 249 | 179 | 8.10 |
| May 90 | 1383 | 65.87 | 206 | 159 | 8.70 |
| Jul 90 | 2400 | 68.54 | 283 | 222 | 10.81 |
| Aug 90 | 1527 | 69.48 | 234 | 171 | 8.93 |
| Sep 90 | 1269 | 72.66 | 233 | 184 | 6.90 |
| Nov 90 | 2840 | 59.33 | 340 | 249 | 11.41 |
| Jan 91 | 1375 | 55.20 | 235 | 167 | 8.23 |
| Mar 91 | 1498 | 61.42 | 205 | 157 | 9.54 |
| Average | 1944 | 61.70 | 218 | 165 | 11.79 |

Moment-ratio diagrams are useful for preliminary analyses of data and can help identify suitable distributions to characterize sample data. Essentially, these diagrams, drawn in the skewness and kurtosis plane, indicate boundaries and regions of relative moments admitted under different distribution parameterizations. For example, the skewness and kurtosis of a normal distribution are 0 and 3, respectively. Hence, the normal distribution occupies a single point in moment-ratio diagrams. In contrast, Pearson distributions admit a wide array of relative moments, and hence, occupy a region rather than a point in the skewness–kurtosis plane. Plotting empirical moments highlights data characteristics and identifies distributions that are unable to generate similar data. Thus, although the procedures do not prescribe a single acceptable distribution, they do provide some evidence about distributions that are inconsistent with sample moments.

Table II provides results of the examinations of the daily soybean futures price data in both detrended levels and the log relative returns

**TABLE II**

Skewness, Kurtosis Statistics, and Normality Tests for the Sample Data

| | *Detrended Prices* | | | *Log Price Changes* | | |
|---|---|---|---|---|---|---|
| *Contract* | $\sqrt{b_1}$[a] | $b_2$[a] | *Jarque-Bera Statistic*[b] | $\sqrt{b_1}$[a] | $b_2$[a] | *Jarque-Bera Statistic*[b] |
| Mar 88 | 2.314 | 2.225 | 31.20 | 0.0012 | 1.922 | 1.597 |
| May 88 | 3.296 | 2.249 | 143.06 | 0.0014 | 2.324 | 1.467 |
| Jul 88 | 51.632 | 4.054 | 51989.28 | 0.0043 | 3.169 | 0.139 |
| Aug 88 | 30.989 | 3.646 | 16047.35 | 0.0081 | 2.964 | 0.007 |
| Sep 88 | 55.096 | 2.948 | 51098.92 | 0.0066 | 2.560 | 0.806 |
| Nov 88 | 79.680 | 4.303 | 21696.97 | 0.0069 | 3.723 | 4.448 |
| Jan 89 | 35.423 | 5.220 | 26168.17 | 0.0021 | 12.180 | 435.570 |
| Mar 89 | 11.079 | 2.697 | 2537.20 | 0.0062 | 3.072 | 0.028 |
| May 89 | 3.962 | 2.562 | 351.61 | 0.0008 | 3.559 | 1.732 |
| Jul 89 | 6.554 | 2.586 | 1290.01 | 0.0036 | 3.625 | 2.911 |
| Aug 89 | 2.589 | 2.381 | 142.85 | 0.0006 | 4.273 | 8.440 |
| Sep 89 | 12.854 | 3.392 | 3084.93 | 0.0035 | 3.258 | 0.308 |
| Nov 89 | 4.109 | 3.479 | 607.07 | 0.0039 | 3.981 | 8.593 |
| Jan 90 | 31.120 | 5.533 | 18754.60 | 0.0192 | 8.790 | 160.654 |
| Mar 90 | 20.057 | 402.27 | 10298.28 | 0.0042 | 13.624 | 681.967 |
| May 90 | 2.277 | 2.275 | 126.66 | 0.0016 | 3.005 | 0.0002 |
| Jul 90 | 12.072 | 3.934 | 9646.36 | 0.0014 | 3.375 | 1.113 |
| Aug 90 | 16.024 | 3.853 | 6295.61 | 0.0011 | 4.210 | 8.911 |
| Sep 90 | 10.219 | 3.353 | 5612.62 | 0.0004 | 3.144 | 0.110 |
| Nov 90 | 17.195 | 4.484 | 11009.41 | 0.0003 | 3.636 | 3.744 |
| Jan 91 | 5.715 | 2.210 | 678.23 | 0.0216 | 15.016 | 739.99 |
| Mar 91 | 9.881 | 3.221 | 2227.86 | 0.0013 | 3.079 | 0.036 |

[a] $\sqrt{b_1}$ and $b_2$ are scaled versions of the third and fourth moments.
[b] Under the null hypothesis of normality, the Jarque-Bera statistic has an asymptotic $\chi^2_{(2)}$ distribution. The critical value at 95% significance level is 5.99.

for each contract. The moment-ratio diagrams indicate that data in detrended levels fall in the regions with positive skewness and varying kurtosis, but often outside the range admitted by *any distribution* as a set of independent draws from a single distribution.[1] As a complementary test, normality is rejected using Jarque-Bera's statistic in all cases. Under returns (changes in log prices), normality is rejected for about a third of the 22 contracts, further indicating that the assumption of lognormality of prices may introduce inaccuracies. The implication of these results is that higher moment flexibility is a desirable property when modeling soybean futures price data.

[1] To remove a possible linear trend, the data are regressed on a time index and the effect of that estimated coefficient is removed from the data. Then, the results are conservative and do not suffer from the possibility of trending prices, although the absence of a time trend does not affect the conclusions as the time coefficient would simply be zero. Similar tests on levels that are not detrended, and in log levels produce similar results.

Previous studies, as well as the moment-ratio charts, suggest the use of a distribution which allows a wide range of skewness and kurtosis values. Because many of the moments fall in the space covered by Burr type III distribution, it is chosen as the alternate distribution to compare to the lognormal benchmark.

Although Burr distributions have received limited attention in the modeling of prices, they have been shown to be useful in describing other economic data with zero support, particularly for loss distributions in the insurance industry. Further, it is important to note that Burr III covers all the space regions in the skewness–kurtosis plane occupied by Pearson types IV, VI, and bell-shaped curves of Pearson type I, gamma, Weibull, normal, lognormal, exponential, and logistic distributions. Additional details about this distribution are provided in Tadikamalla (1980).

The Burr III cumulative distribution function (CDF) with parameters $a$, $\lambda$, and $\tau$ is:

$$F_{BR}(Y \mid a, \lambda, \tau) = \left(1 - 1 \Big/ \left(1 + \left(\frac{Y}{\tau}\right)^{a}\right)\right)^{\lambda}$$
$$\text{for } a, \lambda, \tau > 0; Y \geq 0, \tag{3}$$

and thus the density, (PDF) is,

$$f_{BR}(Y \mid a, \lambda, \tau) = \frac{a\lambda Y^{a\lambda-1}(\tau^{a} + Y^{a}) - \lambda a Y^{a(\lambda-1)}}{(\tau^{a} + Y^{a})^{(\lambda+1)}} \tag{4}$$

The cumulative distribution function for the lognormal with parameters $\mu$ and $\sigma$ is,

$$F_{LN}(Y \mid \mu, \sigma) = N((\ln Y - \mu)/\sigma) \tag{5}$$

where N $(\cdot)$ is the cumulative normal density function. The lognormal density is,

$$f_{LN}(Y \mid \mu, \sigma) = (2\pi)^{-1/2}(\sigma Y)^{-1} \exp[-(\ln Y - \mu)^{2}/(2\sigma^{2})] \tag{6}$$

Using the no-arbitrage pricing model described in (1) and (2), observed option premia are used to recover the parameters of the candidate price distributions by numerically searching for the values of parameters that result in implied option premia closest to the observed option premia. Intuitively, this procedure is similar to that of recovering an implied volatility except that the dimension of the choice variable vector corresponds to the number of parameters of the candidate distribution (two for lognormal distribution and three for the Burr III distribution).

The implied sets of parameters under each distributional assumption are obtained by minimizing squared error between observed and model option premia. Specifically, the following equation is solved for each day's data and for each contract to obtain the parameter vector, $\beta$, using the lognormal and Burr type III distributions for $g(Y_T)$,

$$\underset{\beta}{\text{Min}} \left[ \sum_{i=1}^{n} \left( \left( V_{c,i} - b(T) \int_{x_i}^{\infty} g(Y_T \mid \beta)(Y_T - x_i)\, dY_T{}^2 \right) \right. \right. \\ \left. \left. + \sum_{j=1}^{m} \left( \left( V_{p,i} - b(T) \int_{0}^{x_j} g(Y_T \mid \beta)(x_j - Y_T)\, dY_T \right)^2 \right) \right] \quad (7)$$

This procedure simply minimizes the sum of squared differences between the model premia, conditional upon the parameters of the distribution and the observed option premia. Daily samples of $m$ puts and $n$ calls at strike prices, $x_j$ and $x_{ij}$ and market premia, $V_{p,i}$ and $V_{c,i}$ are used, subject to the requirement that $(m + n)$ be greater than the number of parameters of each of the distributions. Equation (7) is solved using data from one day at a time using nonlinear least squares optimization methods.[2] To keep the problem computationally manageable, the process is restricted to no more than 150 trading days of each contract's life, or to the entire history for contracts that were active for less than 150 days. Thus, implied distributional parameters are obtained for each day under both Burr III and lognormal distributional assumptions for each of the 22 contracts. Figures 1 and 2 display a sample of the distributions as implied in option prices for the November, 1990 soybeans futures contract at several points in the contract life under the Burr III and lognormal parameterizations, respectively. The changing distributions over time reflect the resolution of uncertainty as the contracts near expiration. The evolution of parameters of the distributions over time likewise reflects the changing information contained in market premia.

## RESULTS

The relative performance of the two distributions is evaluated using a battery of tests. First, a comparison of average pricing errors between observed and estimated option prices implied by the alternative distributions is given. Next, comparisons of the differences between observed

[2]The minimization problems are solved using *Gauss-VMI* on an IBM compatible microcomputer. The numeric search routines are set to terminate when the relative gradient of the objective function to each parameter is less than 10e-6. The difference in dimension of the parameter vector between the two distributions may introduce differences in the performance of the numeric search.

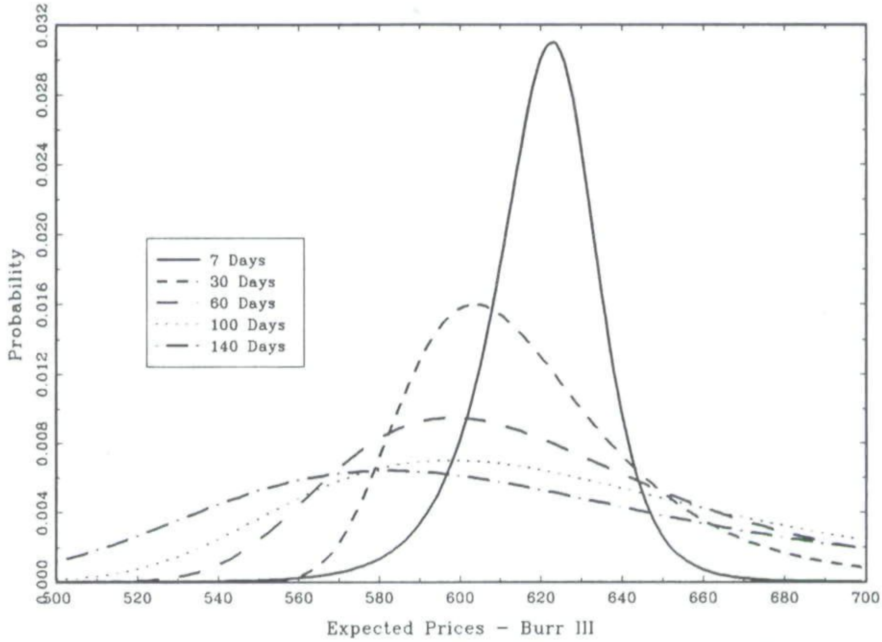


**FIGURE 1**
Option-based PDFs for November 1990 soybean futures.

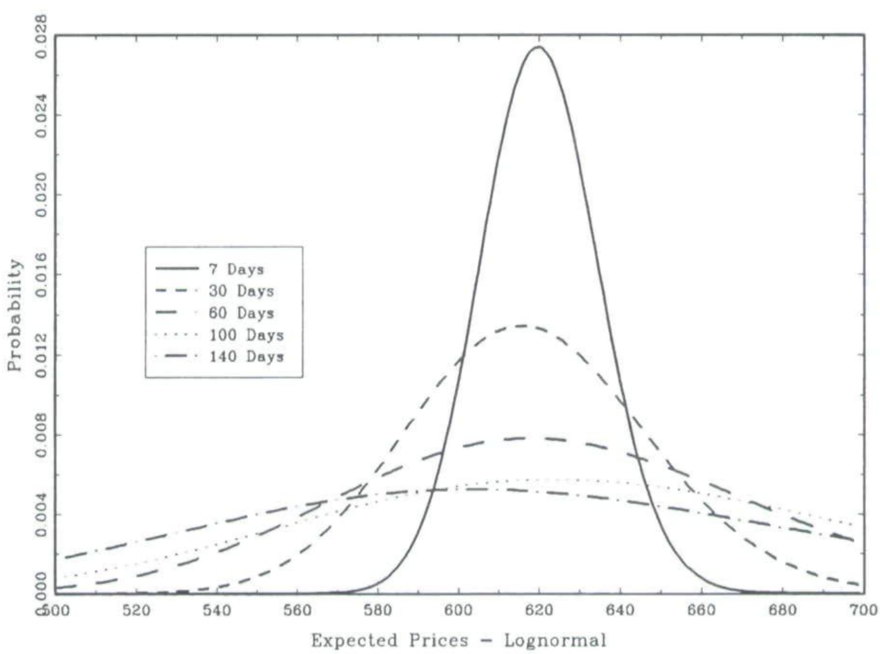


**FIGURE 2**
Option-based PDFs for November 1990 soybean futures.

**TABLE III**

Comparison of Option Pricing Errors

| | | *Burr III* | | *Lognormal* | |
|---|---|---|---|---|---|
| *Time to Maturity* | *Total # of Contracts* | *Contracts with Lowest Error*[a] | *Avg. Option Pricing Error*[b] *($)* | *Contracts with Lowest Error*[a] | *Avg. Option Pricing Error*[b] *($)* |
| 30 | 22 | 20 | 0.00045 | 2 | 0.00135 |
| 60 | 21 | 17 | 0.00066 | 4 | 0.00138 |
| 90 | 21 | 13 | 0.00117 | 8 | 0.00164 |
| 120 | 20 | 8 | 0.00199 | 12 | 0.00157 |
| 150 | 20 | 7 | 0.00289 | 13 | 0.00195 |

[a]Number of contracts with the lowest mean absolute average option price error over the time intervals.
[b]Computed as the unweighted average of mean absolute error between option premia and implied option premia across all contracts.

futures prices and the means of the implied distributions are provided. Finally, variances are compared between implied distributions and resulting data realizations. These and a few related issues are addressed in turn.

## Comparison of Average Pricing Errors

If prices are exact and continuous, and if the pricing model holds exactly for every single option, parameters can be recovered that result in zero error between the implied and observed prices. However, model and market imperfections introduce errors between model implied and observed prices.[3] Hence, the first test examines simple measures of error between the estimated option prices and observed option prices.

Summary statistics of the option pricing errors are reported in Table III. Although daily results are available, the findings are reported over roughly monthly intervals for brevity. The entries in Table III are calculated as follows. For each contract, the pricing errors between observed and implied option prices across all strikes are computed under both distributions. The average option pricing error is then computed as the unweighted average of the mean absolute difference between the option premia and the implied option premia over all contracts. Then, for each interval, the mean and variance of these average errors is calculated. The number of contracts for which each distribution registers lower average

[3]Market imperfections include trading errors, transactions costs, discrete tick reporting practices, and other frictions that prevent essentially continuous prices from being exactly represented.

pricing errors is reported in the table. For some cases, the total number of contracts is less than 22 where insufficient trading activity existed.

The magnitudes of the daily errors for both distributions are small, averaging only 0.14 and 0.16 cents under Burr III and lognormal assumptions, respectively. As expected, the magnitude of the pricing error decreases as time to maturity decreases. The Burr III has lower average pricing errors closer to maturity, the period of most heavy trading. The lognormal distribution performs slightly better for intervals of 120 to 150 days to maturity. While the Burr displays more cases with a lower average pricing errors, the economic significance of the improvement needs to be assessed to justify the complication of moving to a more flexible distribution. On the other hand, current computing technologies permit the movement to a more flexible distribution at a fairly low cost, and thus many situations may justify the efforts required to adopt the method with lower average pricing errors.

## Comparison of Realized Futures Prices and Implied Means

Because the futures prices are not used in the recovery of the implied distribution, meaningful comparisons can be made between the option implied price distributions and the contemporaneous futures price. If it is assumed that the current futures price is an unbiased estimate of the future price, the mean of the option-implied distribution can be compared to the contemporaneous futures price. Hence, simple comparisons are made under the two distributions to determine which corresponds more closely to the current futures price. Table IV summarizes these results, reporting findings for the same intervals presented in the previous section. The average futures price error is computed as the equally weighted average of the mean absolute difference between observed futures prices and futures prices implied by the models over all contracts. For each interval, the mean and variance of these average errors are calculated. The number of contracts with the lowest mean absolute average futures price error and lowest variance of the average futures price error under the alternate distributions are reported also. Again, the more flexible Burr marginally outperforms the lognormal in terms of the numbers of cases with lower errors. However, these results are tempered by the fact that neither distribution displays large differences between the implied means and the futures prices. The average absolute difference between the implied mean and the observed futures price over all contracts is very low, averaging only 0.351 and 0.391 cents per bushel ($0.00351

**TABLE IV**

Comparison of Implied Mean and Observed Futures Price

| | | Burr III | | | Lognormal | | |
|---|---|---|---|---|---|---|---|
| | | # of Contracts[a] | | | # of Contracts[a] | | |
| *Time to Maturity* | *Total # of Contracts* | *Mean* | *Var.* | *Average Futures Price Error*[b] | *Mean* | *Var.* | *Average Futures Price Error*[b] |
| 30 | 22 | 15 | 15 | 0.306 | 7 | 7 | 0.323 |
| 60 | 21 | 10 | 13 | 0.242 | 11 | 8 | 0.295 |
| 90 | 21 | 13 | 11 | 0.297 | 8 | 10 | 0.368 |
| 120 | 20 | 11 | 17 | 0.290 | 9 | 3 | 0.423 |
| 150 | 20 | 12 | 16 | 0.601 | 8 | 4 | 0.631 |

[a]Number of contracts with the lowest mean absolute average futures price error and lowest variance of the average futures price error over the time intervals.
[b]Computed as the unweighted average of mean absolute difference between the observed futures price and futures price implied by the model across all contracts.

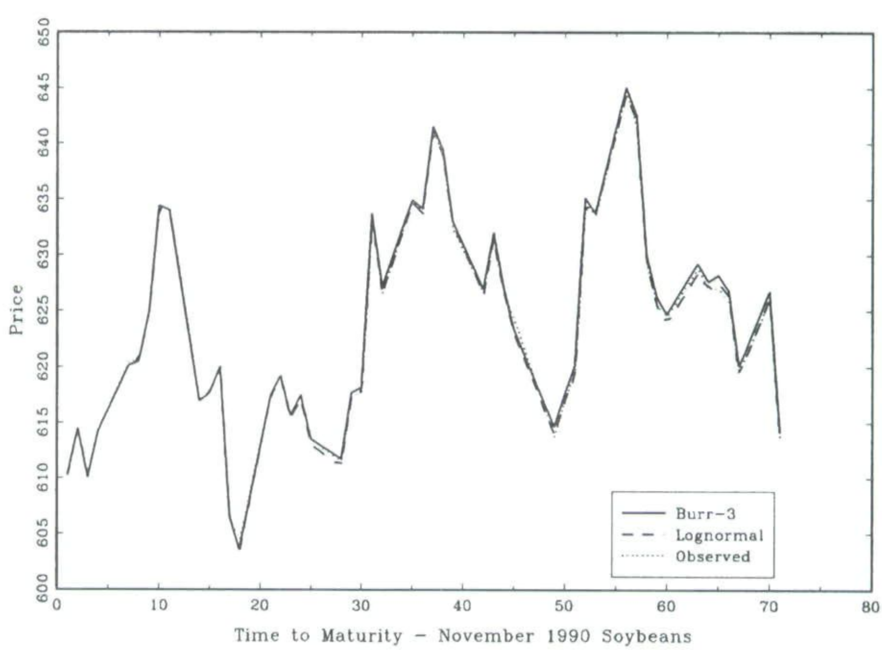


**FIGURE 3**
Option-implied means versus futures.

**TABLE V**

Comparison of Implied Variance by Distribution and Observed Variance

| *Time to Maturity* | *Total # of Contracts* | *Burr III (# of Contracts)*[a] | *Lognormal (# of Contracts)*[a] |
|---|---|---|---|
| 30 | 22 | 22 | 0 |
| 60 | 21 | 21 | 0 |
| 90 | 21 | 21 | 0 |
| 120 | 20 | 19 | 1 |
| 150 | 20 | 19 | 1 |

[a]Number of contracts with smallest difference between the implied variance and the observed variance.

and \$0.00391) under the Burr III and lognormal assumptions, respectively.

These results permit at least two interpretations. If the current futures price is thought to be an unbiased estimate of a future price, the option-implied means are unbiased as well; or, if the current futures price is somehow otherwise related to a future price, the options markets likewise contain similar information. To illustrate graphically, Figure 3 depicts the close relationship between the futures prices and the daily implied means for the November, 1990 contract.

## Comparison of Realized Price Variability and Implied Variance Estimates

If the implied distributions reflect market expectations of end-of-period prices, and if prices are from a data-generating process that is variance-additive-in-time, resulting price movements can be compared to forecasted variance from the options implied distributions. Even if prices are not from a data-generating process that is variance-additive-in-time, the implied distributions should reflect the probabilities of various outcomes.

For each day, the implied variance for the remaining interval until expiration is computed for both distributions and compared to the actual variance of resulting futures prices for the same interval. Absolute differences are calculated between the lognormal estimate and the realized variance and between the Burr III and the realized variance. The distribution with the smaller absolute difference is then recorded as the closer candidate for that interval. The results across all contracts are tabulated in Table V over the same intervals presented earlier. Burr III substantially outperforms the lognormal in tracking variance of the underlying price distribution across different time intervals. Figure 4 graphically depicts

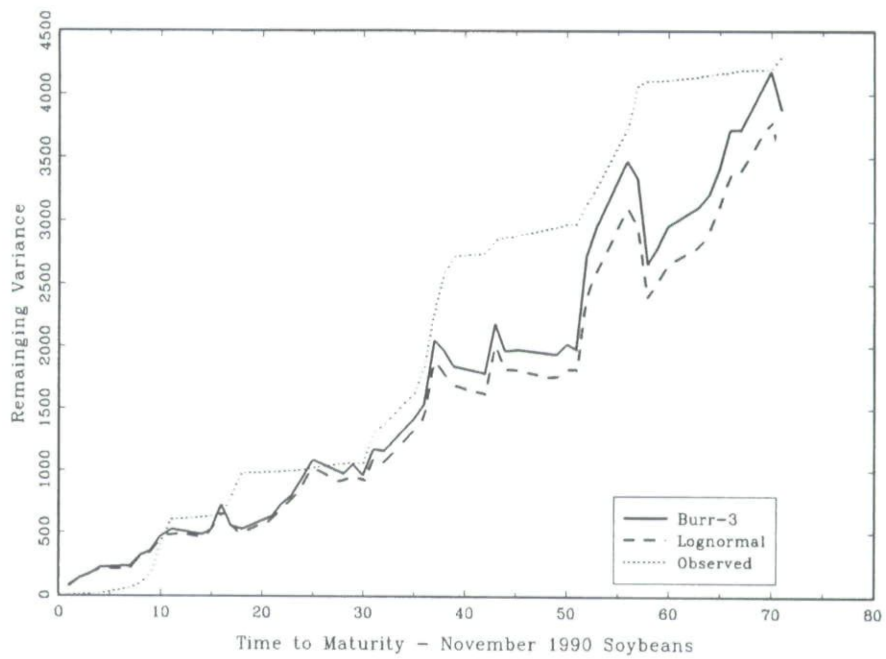


**FIGURE 4**
Variance comparisons.

these results demonstrating that the more flexible Burr III quite often outperforms the lognormal at depicting *ex ante* price variability. Note that although the Burr III is "better" than the lognormal in a vast majority of cases, the results do not imply that either are necessarily "good" indicators of resulting price behavior. Alternatively, both may accurately reflect the expectations of options market participants—who, for some reason, are overconfident about prices. Nevertheless, the relative improvement by the Burr III identifies the potential value in using more flexible distributions when higher ordered moments are of interest.

## SUMMARY AND CONCLUDING REMARKS

No-arbitrage models are used to recover market-based estimates of *ex ante* futures price distributions. Sample data properties imply that the Burr III distribution may provide a suitable alternative to compare to the lognormal. Using option price data, daily estimates under both lognormal and Burr III distributional assumptions are recovered and compared to each other as well as to the underlying asset prices. It should be reiterated that the underlying asset prices are not needed in the estimation process.

This feature alone is perhaps a sufficient improvement over implied volatility approaches to merit the use of the present model.

A series of tests are performed to compare the two functional forms including: evaluation of average pricing errors; comparisons of implied mean prices with futures prices; and assessment of the predictability of resulting futures price variability. In the first two cases, there is at least evidence of marginal improvement in moving to the more flexible specification. In depicting *ex ante* price variability, the Burr III substantially outperforms the lognormal. These benefits come at the cost of the loss of some convenience in estimation and the loss of familiarity with more traditional models.

Implications of the results depend on the context of the use of the information. For example, if an estimate of the implied futures prices are needed during a day with a futures price limit-move, but during which options continue to trade, either distribution will likely give very accurate results in recovering an implied futures price. However, for higher moment estimates, more flexible distributions are just that—more flexible and are less likely to have induced inaccuracies in estimating higher moments. Thus, on that same limit-move day, a more flexible distribution will probably provide a more accurate estimate of the remaining volatility for the life of the contract.

The merits of moving toward more flexible distributional specifications appear greatest under conditions that are likely to involve more complicated information. In the present context, the greatest likelihood of detecting additional economic value of more flexible specifications is under conditions with large amounts of information impacting the market-implied distributions. These conditions exist during heavily traded intervals close to maturity when much probabilistic information is summarized in option prices.

These procedures and the implied distributions provide a rich background for additional work. Future tests should consider the use of trading strategies based on differences in implied distributions and the data. For example, the data indicate that the options-based estimates of variance are often too low far from maturity. Hence, a strategy of buying option straddles far from maturity may prove profitable. Further research also should consider other distributional candidates and procedures for matching data characteristics to distributions for use in option pricing models.

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